Actuator Fault Tolerant PID Controllers

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Abstract: This paper deals with issues of actuator fault tolerant PID controllers. It is studied, how residual generation based on a linear model is affected by the robust properties of a PID used in feedback. To get the sensitivity of the residual with respect to faults and to keep the robustness of the controller, a scheme is proposed with a non linear residual generator and a family of PIDs interconnected in such a way that the stability is held even with actuator faults. The conditions reported by Bhattacharyya for a family of stabilizing PID controllers are used to select the fault tolerant PID parameters considering actuator faults. Using the benchmark of the three tank system with two pumps, simulations and experimental results show the advantages of the fault tolerant PID scheme.

Keywords: fault tolerant PID, non linear model based residual, actuator faults, stabilizing PID.

1. INTRODUCTION

Automated processes are vulnerable to faults and the consequences of this fact may be a complete failure, or a disaster. Actuator faults, erroneous sensor readings, and faulty components affect the performance of the automatic system. For such reasons Fault Tolerant Control (FTC), as it has been described by Blanke et al. (2006) is a crucial developing area in automatic control where several disciplines and system theoretic issues are combined to obtain a unique functionality.

Active FTC can be considered as a feedback system in which the diagnostic system and the controller are coupled with antagonistic properties; the first one designed to be as sensitive as possible and the latest is designed to be as insensitive as possible. Since, the integral action with a PID is active, as long as the steady state error is not zero, it hides the most types of actuator faults. Then, the fault detectability could be changed depending on the exogenous signals. A standard PID does not distinguish between disturbances and faults and any deviation in the operation point changes the properties of the residual if the diagnostic system is designed on the base of a linear analytical model (Ding, 2008) or a statistical method (Qin, 2003). The robustness properties of a PID makes difficult the diagnosis issue. As consequence, as it is known, it is difficult to decide if large deviations in closed loops are caused by uncertainties or by faults (Isermann, 2006). Then, the model based residual must be designed on the base of non linear behavior.

On the other hand the PID's have recently been reemerging and the stabilizing family for a given plant can be obtained if its relative degree and the number of poles and zeros in the right half plane are known (Bhattacharyya et al., 2000).

The above facts motivated this work in which it is suggested a scheme with two PIDs to manage the actuator fault issue. Each PID is designed in such a way that if both are working or one of them is turned off, the feedback system is stable and moreover holds zero steady state error. The deactivation function of the PID's is managed by non linear residual designed using two observers.

The paper is organized as follows. Section 2 discusses the behavior of two sets of residuals for actuator faults; one designed on the base on the linear model and the other with the non linear version for a three tank system. The discussion considers the open and closed loop with a PID controller. Section 3 introduces the fault tolerant control scheme integrated by the fault diagnostic system and two PIDs. Section 4 shows the results by simulation and experimental data. Finally, in Section 5 some remarks and conclusions are given.

2. RESIDUAL GENERATOR

The capacity to detect faults and the isolation issue of each possible fault in a dynamic system when uncertainties and disturbances exist is a main topic in the automatic operation of complex process. The feasibility of a solution for a specific set of faults is a system structural property which leads to the search for system invariant under some given transformations. Diverse tools have been proposed to study the detectability and isolability of faults: the Geometric Theory (Massoumnia et al., 1989) and the Structural Analysis (Blanke et al., 2006). To design the residual generator, the functional observers with unknown inputs are used with the linear and non linear version. To make robust an observer in the presence of a perturbation, it can be taken advantage of the structure of the model and transform it with an output vector injection, so that the state space is divided into two subspaces, one sensitive and another insensitive to the unknown inputs. This last one is the base to estimate the state or part of it. In the diagnostic case, the objective is to determine the subspaces that are insensitive to perturbations and sensitive to faults with the property of observability using measurements, this formulation makes that the perturbations or uninterested faults do not take effect in the residual.

To design the residual most of the methods assume a plant in open loop. However, this assumption together with the linearity can modify the pattern of the symptoms, as shown in the following subsection. This fact is not frequently pointed out.

2.1 Linear Residual Generator

Assuming a linear model with additive faults given by

$$\dot{x}(t) = Ax(t) + Bu(t) + E_1\overline{f}(t) + F_1 f(t) \quad (1)$$
$$y(t) = Cx(t) + Du(t) + E_2\overline{f}(t) + F_2 f(t) \quad (2)$$

with E_i the vector associated to uninterested faults and F_i corresponds to the vector associated to the faults of interest, the starting point to design the residual insensitive to $\overline{f}$ is the structure of the observer

$$\dot{z}(t) = Fz(t) + TBu(t) + Ly(t) \quad (3)$$
$$\hat{x}(t) = z(t) + Hy(t) \quad (4)$$

The necessary and sufficient conditions of the existence of this observer are reduced to

(1) The rank$(CE_1) = rank(E_1)$;
(2) The pair (C, A_1) is detectable with

$$A_1 = A - E_1((CE_1)'(CE_1))^{-1}(CE_1)'CA$$

or equivalently, the transmission zeros of the input $\overline{f}$ to the measurement vector $y(t)$ must be stable (Chen, 1984)

For the observer design (3,4) there are some degrees of freedom in the matrix (F, T, L, H) which are adjusted with the purpose to have a stable dynamic system and a residual

$$R(t) = (I - CH)y(t) - Cz(t) \quad (5)$$

which deviates from zero if the fault vector $f(t) \neq 0$.

For actuator faults the condition

$$TF_1 \neq 0 \quad (6)$$

must be satisfied. This condition could be more specific, if simultaneous faults are not allowed.

To analyze the behavior of the residual generator given by (3) and (4) a benchmark is used. The non linear model of a three tank system with two pumps is considered (Appendix A).

Linearizing the non linear model of the hydraulic system (Appendix A), around an operation point (x^*, u^*) the set of matrices (A, B, C, D) are taken to design two residuals using linear observers. One has to be sensitive to the fault in the pump 1 and insensitive to pump 2. On the contrary the other residual is insensitive to fault in pump 1 and sensitive to 2.

Following the steps reported in (Hou and Mueller, 1994) for the set

$$(A, B, C, D, E_1, F_1), \quad E_1 = F_2$$

one gets the subsystem

$$\dot{x}_1 = -0.0118x_1 + 64.97u_1 + 0.0118y_3 \quad (7)$$
$$y_1 = x_1 \quad (8)$$

with output injection y_3 which is free of faults in actuator u_2. This subsystem allows to design an observer sensitive to faults in actuator 1 with the output estimation error

$$R_1(t) = y_1(t) - \hat{y}_1(t) \quad (9)$$

as residual. To detect faults in actuator 2 the residual has to be insensitive to faults of actuator 1. Thus, similarly to the above case, one obtains the subsystem

$$\dot{x}_2 = -0.0239x_2 + 64.97u_2 + 0.0123y_3 \quad (10)$$
$$y_2 = x_2 \quad (11)$$

with output injection y_3 which is not affected by the signal u_1. Then, one observer can be designed and the output estimation

$$R_2(t) = y_2(t) - \hat{y}_2(t) \quad (12)$$

plays the role of a residual for pump 2.

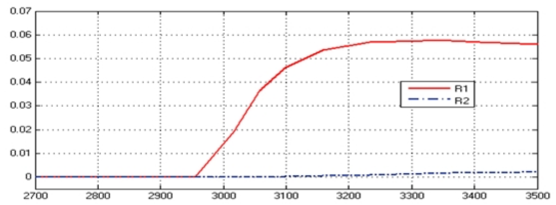
Experiments. a) *Open Loop.* To validate the above residuals, the system in open loop with faults in pump 1 and 2 is simulated. Fig. 1 (a) shows the evolution of the residual R_1 when a bias is emulated in actuator 1. One can see that R_1 alerts a fault in actuator 1 and R_2 stays in zero. The results with a fault in the pump 2 are given in Fig. 1 (b); the evolution of $R_2(t)$ detects the fault in actuator 2 while residual $R_1(t)$ stays in zero.

An advantage of the linear model to obtain the residual is the simplicity to generate the submodels and the observer design even for high order systems. However, since linearity is assumed, the residual behavior does not hold for any operation point. Large deviations of it modify the residual.

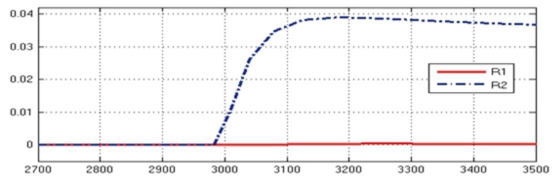
The following subsection analyzes the behavior of the residual given by eqs (9) and (12) for the three tank system with a PID controller.

b) *Closed Loop.* To control the water level of tank 3 the variable $y_3 = x_3$ is considered as the output of a feedback scheme with a PID controller. The input flow u_1 is selected as an action variable. The laboratory prototype allows to emulate bias faults in the actuators. A fault in actuator 2 is considered and the behavior of both residuals is given in Fig. 1 (c). Since both residuals deviate from zero, one concludes that both actuators are faulty; this fact is false. The deviation of the residual R_1 is produced by a change in an operating point of the plant. This is justified because the PID increases the control action to get an asymptotic zero error.

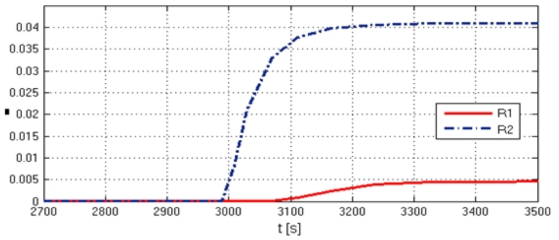
From the above results, it is concluded that the residual generators based on a linearized model work only when the



(a) Residual with a fault in actuator 1 for the system in open loop



(b) Residual of a fault in actuator 2 for the system in open loop



(c) Residual of a fault in actuator 2 for the system in closed loop

Fig. 1. Residual evolution with faults in actuators

system is operating around an specific operating point. Having the system working in closed loop, the presence of a fault in any of the actuators makes that the control law changes the operating point to another point; this deteriorates the residual response. This fact leads us to implement a residual generator based on a nonlinear model.

2.2 Non linear Residual Generator

To design the non linear residual generators, the redundant relations are obtained using structural analysis tools. Following the proposition to get redundant relations from bipartite graphs (Verde and Sánchez-Parra, 2010), one obtains the graph shown in Fig. 2 from the set of constraints (t1-t9) for the three tank system (Appendix A). From the structure, two redundant graphs for the actuator faults $\{f_{u1}, f_{u2}\}$ can be obtained. The first one given by

$$\mathcal{GR}_1(t_1, t_4, t_7, t_9, u_1, y_1, y_3) \tag{13}$$

assumes $u(1)$ and $y(3)$ as input and y_1 as output and involves the set of constraints (t_1, t_4, t_7, t_9). One can see that $\mathcal{GR}_1$ is sensitive to the presence of a fault in actuator 1. It means that one can get a subsystem that only requires u_1 and y_3 as inputs and obtains y_1 as an output. Any non linear observer for this subsystem allows to generate the residual 1. The second graph by symmetry is given by

$$\mathcal{GR}_2(t_2, t_5, t_8, t_9, u_2, y_2, y_3) \tag{14}$$

with u_2 and y_3 as input and y_2 as output. From the variables and constraints involved in $\mathcal{GR}_2$, one can see that exists a subsystem considering u_2 and y_3 as inputs and y_2 as an output. An observer for this subsystem allows the generation of the residual 2. The derivation of all graphs for all faults in the system are reported in (Mina, 2008).

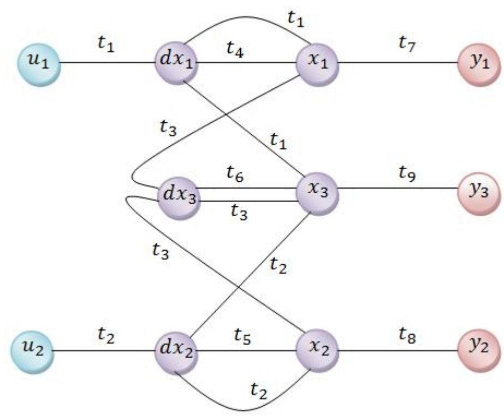


Fig. 2. bipartite graph for the three tank system

Then, the signature matrix is reduced to Table 1.

Table 1. Signature matrix

	f_{u1}	f_{u2}
$\mathcal{GR}_1$	•	
$\mathcal{GR}_2$		•

Because the redundant relations obtained from the *GRs* does not assume linearity, non linear residuals are directly obtained. Moreover, because dynamic systems (13) and (14) have the generic form

$$\begin{pmatrix} \dot{\xi} \\ \dot{u} \end{pmatrix} = f \begin{pmatrix} \xi \\ u \end{pmatrix}, y(t) = h(\xi, u) \tag{15}$$

their respective residuals can be generated, estimating the state with an approximated observer of the form

$$\dot{z}(t) = f(z(t), u(t)) + H[y(t) - \hat{y}(t)] \tag{16}$$

$$\hat{y}(t) = h(z(t)) \tag{17}$$

where H has to be designed such that has an asymptotically stable error in normal operation. To analyze the behavior of residuals in closed loop, the same scheme and PID used in section 2.1 is considered. The results are given in Fig.3 and Fig. 4.

The residue constructed with this observer is sufficiently sensible to faults as it is required. Both residues are generated using the output estimation, exactly as it was done in (9) and (12). It has to be stated that when a fault occurs, the observer output can not reach the behavior of the real plant. This fact allows us to know when the fault takes place.

Fig. 3 shows the residual evolution R_2 and one can see the correct presence of a fault in the actuator 2 while the residual R_1 remains in zero. In Fig. 4 the response of the residual to a fault in the actuator 1 is shown, here the residual detects the fault of the pump 1 while the residual 2 remains in zero.

The above analysis confirms that the PID controller and the residual generator has to be designed as a whole system

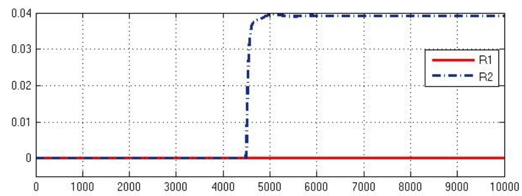


Fig. 3. Nonlinear residual evolution with a fault in actuator 2 and PID controller for level x_3

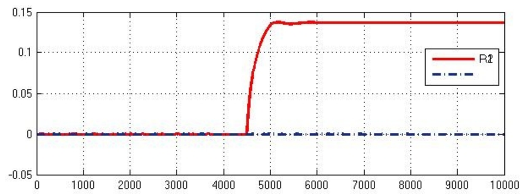


Fig. 4. Nonlinear residual evolution with a fault in actuator 1 and PID controller for level x_3

such that the robustness of the PID controller does not diminish the effect of the faults in the residuals. This is an open problem in the field of fault tolerant control.

3. FAULT TOLERANT PID CONTROLLER

Diverse schemes have been proposed to tackle the fault tolerant controller issues. Adaptive and learning techniques are proposed by Zhang and Jiang (2008) including the auto-tuning PID controller reported in (Ding-Li et al., 2005). Other option is the switching scheme with a set of faulty models for specific PID controllers (Sánchez-Parra et al., 2010).

If more than one actuator is available, an idea in the framework of switching scheme is the design of a control law in such a way that in normal conditions, all the control signals u_i work together to achieve a good performance and in fault conditions the control signal associated to the fault is deactivated. This requires the stability of subsets of feedback loops which can be tested off line in a straightforward manner using the software application reported in (Mora, 2011) for PID controllers. The algorithm is designed on the base of the stabilizing family of PIDs for a given plant, if the relative degree of the plant and the number of poles and zeros in the right half plane are known (Bhattacharyya et al., 2000).

The novel switching scheme for the three tanks with the output variable, the level x_3, is shown in Fig. 5. Thus, one PID controller generates the action u_1 and the second control signal is applied to actuator 2. The deactivation function corresponds to the evaluation of each residual.

To adjust the PIDs such that all configurations have good performance, the nominal transfer matrix

$$G(s) = [\,0\ 1\ 0\,]\,(sI - A)^{-1}B = [\,G_1(s)\ \ G_2(s)\,]$$

is considered with the matrices given in Appendix A and the structure of each controller

$$C_m(s) = \frac{K_{Dm}s^2 + K_{Pm} + K_{Im}}{s} \quad \text{for} \quad m = 1, 2$$

This scheme is equivalent to the block diagram given in Fig. 6 with the constraint $Ref_1 = Ref_2$. Two loops are identified: the blue one with direct trajectory $C_2(s)G_2(s)$; and the green one with direct trajectory $C_1(s)G_1(s)$. This diagram is also equivalent to Fig. 7.

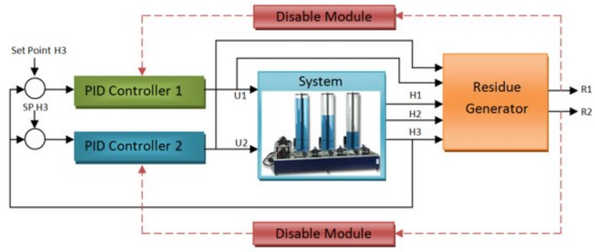


Fig. 5. Two controllers scheme with deactivation function produced by the residual evaluation

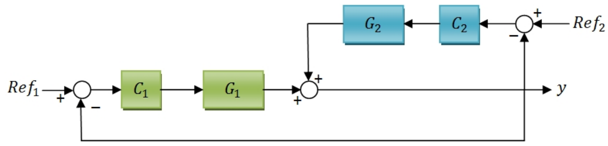


Fig. 6. Control scheme for the three tanks system

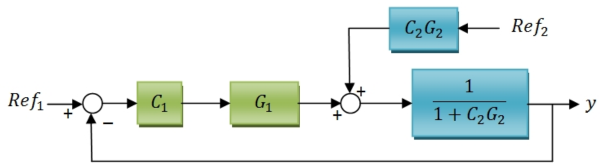


Fig. 7. Reduced control block diagram for the three tanks system

As first step, a controller $C_2(s)$ is selected which stabilizes the transfer function

$$\frac{C_2(s)G_2(s)}{1 + C_2(s)G_2(s)}$$

This controller ensures the stability of the system if actuator 1 is faulty and $C_1(s)$ is disconnected. By the symmetry of the channels, similarly one can select a stabilizing controller $C_1(s)$. This ensures the stability of the transfer function

$$\frac{C_1(s)G_1(s)}{1 + C_1(s)G_1(s)}$$

Thus, the occurrence of a fault in actuator 2 and the disconnection of $C_2(s)$ does not affect the stability of the closed loop.

Finally, assuming known the two controllers $C_1(s)$ and $C_2(s)$, one can verify the stability of the whole transfer function

$$\frac{C_1(s)G_1(s) + C_2(s)G_2(s)}{1 + C_1(s)G_1(s) + C_2(s)G_2(s)}$$

which is associated with the nominal closed loop system without faults.

In this study the controllers $C_1(s)$ and $C_2(s)$ are designed using a software that implements the stability conditions shown in (Bhattacharyya et al., 2000) which establishes that for a given plant of order n it is possible to reduce the problem of determining all PID stabilizing controllers to the problem of solving a set of linear inequalities in terms of the constants K_P, K_I and K_D of the controller. The set of inequalities is solved using linear programming (Mora, 2011)

The stabilizing regions for C_1, forming a volume, are shown in Fig. 8 where the stabilizing values are the points of the shaded region. Similar regions K_P, K_I and K_D are

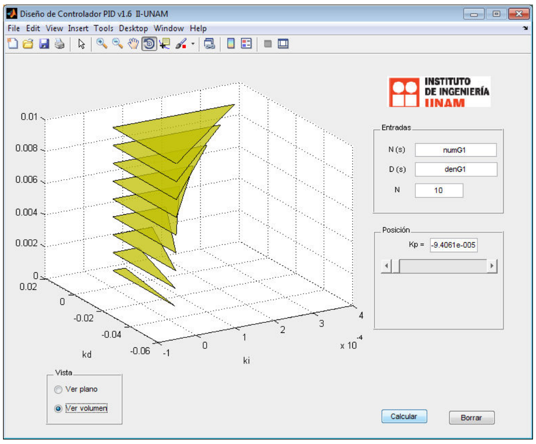


Fig. 8. Stabilizing regions of controller C_1

obtained for C_2. According to the considered plant and using the SISOTOOL of Matlab, the selected constants for C_1 are

$$K_{P1} = 1.5 \times 10^{-4}, \quad K_{I1} = 2.5 \times 10^{-6}, \quad K_{D1} = 5 \times 10^{-4}$$

and for C_2

$$K_{P2} = 9 \times 10^{-5}, \quad K_{I2} = 1.1 \times 10^{-6}, \quad K_{D2} = 5 \times 10^{-4}$$

Fig. 9 shows that the transient response of the standard feedback with one PID and with the novel scheme; the response of the system considering two actions has a better performance that the system with only one PID. In fault conditions, the evolution of the output is shown in Figs 13 and 14; as it is expected in fault conditions the effect of the faults are canceled out by the PID which is actived during the fault conditions.

Considering the two adjusted PID controllers, the residual responses are evaluated. Fig. 12 shows the behavior when a fault is emulated in the pump 2 of the pilot plant and the residual detects the fault which is presented in u_2 while the residual generator reports that actuator 1 is working correctly. Fig. 13 shows the residual when a fault is emulated in pump 1 and it allows to locate the bias, in this case the residual R_2 stays nearby zero. In a real process the residues do not exactly stay in zero because of modeling errors or uncertainties in the parameters of the system. However a proposed solution is to use Adaptive thresholds (Isermann, 2006) which, correctly designed, make the fault tolerant system robust to any of this troubles. The adaptive thresholds use a low pass filter and a proportional enlargement added by a constant. In Fig. 11 is shown a nonlinear residue, which construction was based on a model with uncertainties. Under this condition the deactivation function stops the failing pump when the residue is bigger than the threshold. It only happens when a fault in actuator is presented.

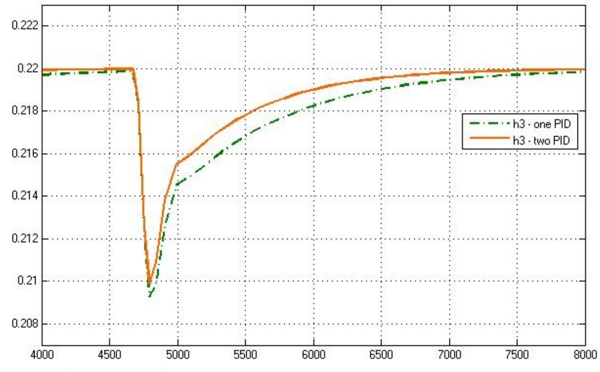


Fig. 9. Evolution of the output y_3 working with one PID controller and with two PID controllers

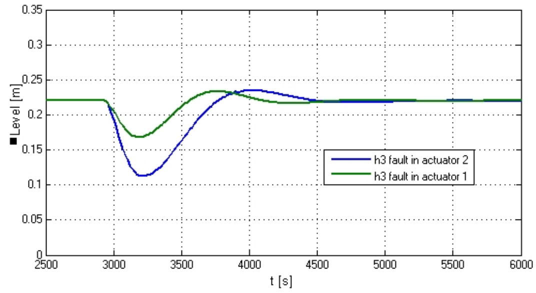


Fig. 10. Behavior of the output y_3 with a fault in actuator 2 and 1, working with one PID

4. CONCLUSION

This paper discusses the limitation of a linear residual generator in the framework of a fault tolerant control. It is clearly stated that when a fault changes the operation point of the model, the residue generator induces false alarms even when the error of the feedback with a PID controller is zero. The study is made considering the linear and non linear model of the three tank benchmark. Moreover, one suggests to include for the control task, two PIDs interconnected in such a way that the occurrence of a fault does not change the robust properties of a PID. The PIDs are adjusted off line using a software based on the stabilizing PID controller. It is seen that the matching of a PID controller and the non linear residual generators achieve a good performance. Future research is dedicated to design a robust residue generator that does not need the control inputs of the plant. This characteristic would be so useful in the case when they are not measurable.

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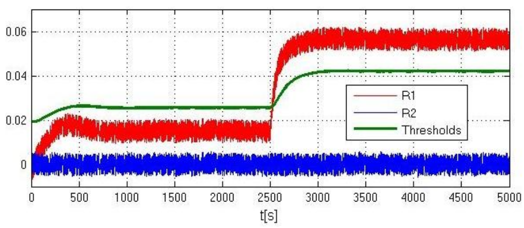

Fig. 11. Nonlinear Residue evolution with a fault in actuator 1 and its adaptive threshold

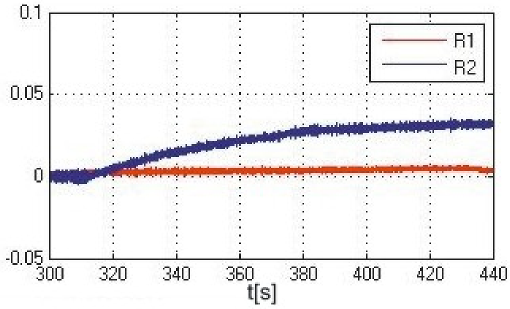

Fig. 12. Nonlinear Residue evolution with a fault in actuator 2 working with one PID

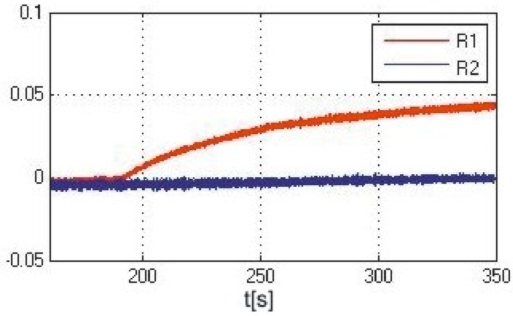

Fig. 13. Nonlinear Residue evolution with a fault in actuator 1 working with one PID

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Appendix A. THREE TANK SYSTEM

The model of the three tank system is taken from (GmbH, 1994)

$$A\dot{x}_1 = u_1 + R_{13}\rho(x_1, x_3) + \Delta Q_1 \quad (A.1)$$
$$A\dot{x}_2 = u_2 + R_2\rho(x_2, 0) + R_{32}\rho(x_3, x_2) + \Delta Q_2 \quad (A.2)$$
$$A\dot{x}_3 = R_{13}\rho(x_1, x_3) + R_{32}\rho(x_3, x_2) \quad (A.3)$$

where $x_i = h_i$ are the water levels, $u_i = Q_i$ the in-flows, the function $\rho(x_i, x_j) = sgn(x_i - x_j)sqrt(2g(x_i - x_j))$ and the coefficients R_{13}, R_2 and R_{32} are function of the valves between tanks V_1, V_2 and V_3 respectively. The measurable states correspond to the three levels (x_1, x_2, x_3). The faults considered are constant deviations in the pumps flows and are defined in the model as ΔQ_1 and ΔQ_2.

The structural model of system (A.1) can be written by

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_3, u_1, R_{13}, \Delta Q_1) && (t1)\\
\dot{x}_2 &= f_2(x_3, x_2, u_2, R_{20}, R_{32}, \Delta Q_2) && (t2)\\
\dot{x}_3 &= f_3(x_1, x_2, x_3, R_{13}, R_{32}) && (t3)\\
x_4 &= dx_1/dt && (t4)\\
x_5 &= dx_2/dt && (t5)\\
x_6 &= dx_3/dt && (t6)\\
y_1 &= x_1 && (t7)\\
y_2 &= x_2 && (t8)\\
y_3 &= x_3 && (t9)
\end{aligned}$$

where the parameters R_{ij} are constant parameters which depends on the pipe between tanks i and j. Considering the operating point

$$\begin{aligned}
&u_1^* = 2.764 \times 10^{-5}[m^3/s] \quad u_2^* = 2.487 \times 10^{-5}[m^3/s]\\
&x_1^* = 0.2964[m] \qquad\qquad x_2^* = 0.1471[m]\\
&x_3^* = 0.22[m]
\end{aligned}$$

and the values $R_{13} = 2.2576 \times 10^{-5}$, $R_2 = 3.091 \times 10^{-5}$ and $R_{32} = 2.3111 \times 10^{-5}$. Without faults, the matrices of the linearized model are given by

$$A = \begin{bmatrix} -0.0118 & 0 & 0.0118 \\ 0 & -0.0239 & 0.0123 \\ 0.0118 & 0.0123 & -0.0241 \end{bmatrix}, \quad B = \begin{bmatrix} 64.97 & 0 \\ 0 & 64.97 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the vectors associated with the actuator faults 1 and 2 are respectively

$$F_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$