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Robustness Considerations on PID Tuning for Regulatory Control of Inverse Response Processes

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Abstract: In general, to control industrial processes that show an inverse response output is difficult. This difficulty arises because achievable performance and robustness levels are competing factors that the designer has to deal with. In this paper we focus on proportional integral derivative (PID) control of inverse response processes, in particular with the Direct Synthesis design for disturbance rejection *DS-d* tuning method of Chen and Seborg. The paper presents equations that link the control system design performance parameter (closed-loop control system speed) with its minimum robustness level, measured with the maximum sensitivity M_S, for the mentioned tuning method. In this way, the proposed result allows to obtain robust PID control systems with $M_S \leq 2$ for inverse response processes. The examples show the effectiveness of the proposed design parameter lower limit estimation for the *DS-d* tuning method.

Keywords: Inverse Response Processes, PID Control, Robustness.

1. INTRODUCTION

Inverse response processes originated by two parallel competing dynamics are present in several industrial and chemical systems, such as boilers and chemical reactors (Marlin, 2000). These processes present serious challenges for control design due to these competing dynamics and impose fundamental limitations to the performance and robustness attainable levels (Skogestad and Postlethwaite, 1996).

Proportional integral derivative (PID) control of inverse response processes have received attention regarding control structures such as the internal model control (IMC) (Morari and Zafirou, 1990) and PID parameters tuning (Luyben, 2003). Analytical IMC-based tuning of PI/PID controllers for inverse response controlled processes are presented in Chen et al. (2006); Chien et al. (2003); Scali and Rachid (1998). All these methods included one design parameter, normally denoted as λ (the IMC-controller filter time constant), that is selected using only performance considerations, although it also affects the control systems relative stability (its robustness to changes in the controlled process characteristics).

Robustness is an important attribute for control systems, because the design procedures are usually based on the use of low-order linear models identified at the control system operation point. Due to the non-linearities of most of the industrial processes, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system. Therefore, the design of the closed-loop control system must take into account the system *performance* and its *robustness* considering the well-known *trade-off* between them.

In particular we consider the *Direct Synthesis design for disturbance rejection* DS-d tuning method of Chen and Seborg (2002). This method uses a regulatory control third-order target closed-loop transfer function with only one tuning parameter, τ_c, defined as the ratio of the closed-loop system time constant T_c to the controlled process main time constant T, ($\tau_c \doteq T_c/T$). For the cited *DS-d* tuning method, there is no criteria for the selection of the performance design parameter τ_c that provides a suitable robustness level.

In this paper we propose an estimation of the minimum τ_c value (maximum speed) that limits the control system maximum sensitivity to $M_S \leq 2$. This estimation is given as a function of the plant model dimensionless parameters a and b, the ratio of the two time constants and the relative position of the right half plane zero, respectively. In this way, the designer can determine what is the maximum performance level τ_c attainable with the particular inverse

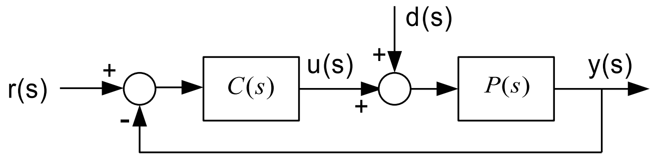

Figure 1. Feedback Control System.

response process at hand with a robustness limit $M_S \leq 2$. This is a mandatory step to do before attempting any PID tuning. Otherwise, non-robust tunings can be selected.

The paper is organized as follows. After the problem formulation in Section 2, the paper main result is established in Section 3, in which the equations that relate the performance level τ_c with the robustness margin M_S are derived as functions of just two parameters a and b. In Section 4 it is shown the result applicability on the PID tuning for a continuous stirred tank reactor (CSTR). The paper finishes with some conclusions.

2. PROBLEM FORMULATION

Consider the closed-loop control block diagram in Fig. 1, where $P(s)$ is the inverse response controlled process model transfer function and $C(s)$ is the controller transfer function.

In this system, the variables of interest can be described as follows:

- $d(s)$ is the load-disturbance input,
- $r(s)$ is the set-point for the process output,
- $u(s)$ is the controller output signal,
- $y(s)$ is the process output (controlled variable).

Also, the inverse response controlled process is represented by the second-order plus a right-half-plane-zero model given by

$$P(s) = \frac{K(-bTs+1)}{(Ts+1)(aTs+1)}, \tag{1}$$

were K, T, a and b are the process gain, the main time constant, the ratio of the two time constants and the relative position of the right half plane zero, respectively.

We consider a standard proportional integral derivative (PID) controller (Åström and Hägglund, 1995) given by

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1}\right), \tag{2}$$

where K_p is the proportional gain, T_i is the integral time constant, and T_d is the derivative time constant. The derivative filter constant N usually takes values within the range from 5 to 20 (Åström and Hägglund, 2006; Visioli, 2006). Without the loss of generality, $N = 10$ is used (Chen and Seborg, 2002).

The PID controller is tuned using the *DS-d* method (Chen and Seborg, 2002). This is a direct synthesis design for disturbance rejection, regulatory control, with a third-order target closed-loop transfer function given by

$$M_{yd}(s) = \frac{T_i/K_p s(-bTs+1)}{(\tau_c Ts+1)^3}, \tag{3}$$

where the design parameter τ_c is the ratio of the closed-loop system time constant T_c ($T_c = \tau_c T$) to the controlled process main time constant T. Normalized equations of the *DS-d* tuning are shown in the Appendix A.

3. ROBUSTNESS ANALYSIS

The design parameter τ_c selection has a direct influence over the control system performance (the closed-loop system speed) and its robustness (the control system sensitivity to changes in the controlled process characteristics). Then its selection must take into account the existing *performance/robustness trade-off.*

The closed-loop control system robustness (relative stability) is measured using the maximum sensitivity M_S, which is defined as follows:

$$M_S \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \left| \frac{1}{1 + C(j\omega)P(j\omega)} \right|, \tag{4}$$

Recommended values for M_S are typically within the range from 1.4 to 2.0 (Åström and Hägglund, 2006).

The use of the maximum sensitivity as a robustness measure, has the advantage that lower bounds to the gain margin, A_m, and phase margin, ϕ_m, can be assured according to (Åström and Hägglund, 2006)

$$A_m > \frac{M_S}{M_S - 1}, \tag{5}$$

$$\phi_m > 2\sin^{-1}\left(\frac{1}{2M_S}\right). \tag{6}$$

Therefore, to assure $M_S = 2.0$ provides what is commonly considered the minimum robustness requirement (that translates to $A_m > 2$ and $\phi_m > 29^o$).

In this sense, (4) is evaluated in order to determine the design parameter, τ_c, lower limit to obtain the standard minimum robustness level of $M_S = 2.0$.

For the analysis, the controlled process model time constants ratio a is selected within the range from 0.2 to 1.0 to include controlled processes with nearly first-order, over damped second-order, and dual-pole dynamics. The relative position of the right half plane zero b is taken within the range from 0.25 to 4.0 due to the limitations to obtain a robust control system as will be showed bellow.

Fig. 2 shows the lower limits for the design parameter (for five a ratios), to achieve a system robustness higher than the usual minimum ($M_S \leq 2.0$) with PID controllers tuned with the *DS-d* method.

As it can be seen, model parameters a and b, affect the control system performance/robustness relation. For any model time constants ratio a, if the right half plane zero moves towards the origin (i.e. b increases), it is necessary to decrease the closed-loop speed (increasing the design parameter τ_c). This will maintain the system robustness above the usual minimum level ($M_S \leq 2.0$).

For equal b values, the maximum allowed speed to achieve a robust control system, is lower for systems with similar time constants ($a \approx 1.0$), than for systems with very different time constants (low a). It is also noted that for some combinations of system parameters (a, b), it is not possible to achieve a robust control system. For the

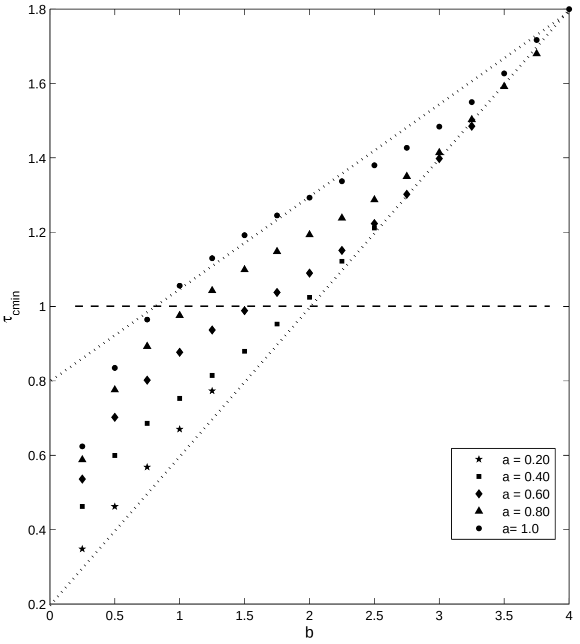

Figure 2. Design Parameter τ_c Lower Limits for $M_S \leq 2.0$.

Table 1. Constants a_i for (7)

a	b_{max}	a_0	a_1	a_2	a_3
0.20	1.25	0.2212	0.5386	-0.1377	0.04800
0.40	2.50	0.3263	0.6437	-0.2792	0.06578
0.60	3.25	0.3947	0.7064	-0.2804	0.05164
0.80	3.75	0.4388	0.7648	-0.2795	0.04405
1.0	4.0	0.4659	0.8250	-0.2850	0.40860

considered a values in the analysis, the proximity of the zero position to the origin is limited by the b_{max} value, as stated in Table 1.

Therefore, using the information in Fig. 2, it is possible to include a minimum system robustness level into the design stage, estimating a recommended maximum speed (τ_{cmin}). Then, for the model time constants ratios a in the analysis, by using the MATLAB®Curve Fitting Toolbox a set of equations of the form

$$\tau_{cmin} = a_0 + a_1 b + a_2 b^2 + a_3 b^3, \qquad (7)$$

is obtained, where the a_i constants are listed in Table 1.

Equations (7) allow to select the performance design parameter such that $\tau_c \geq \tau_{cmin}$ to obtain control systems with a robustness $M_S \leq 2.0$. In case that the model time constants ratio a does not coincide with the values in Table 1 an interpolation is necessary using the equations for the two nearest a values.

It can also be seen from Fig. 2, that the recommended lower limit for the design parameter should be roughly bounded between the two dotted right lines given by

$$\tau_c^l = 0.2 + 0.40b, \qquad (8)$$

$$\tau_c^h = 0.8 + 0.25b, \qquad (9)$$

where τ_c^l and τ_c^h state for the *low* and *high* limits of τ_{cmin}, respectively.

Table 2. Example 1 - Process Model Parameters

α	K	T	a	b
0.5	1.0	1.368	0.515	0.371
1.0	1.0	1.345	0.557	0.759
2.0	1.0	1.309	0.565	1.564
3.0	1.0	1.285	0.557	2.387

Table 3. Example 1 - *DS-d* Tuning PID Parameters and Robustness

α	τ_{cmin}	τ_c	K_p	T_i	T_d	M_S^r
0.5	0.576	0.60	1.879	1.938	0.448	1.969
1.0	0.749	0.80	0.929	2.047	0.465	1.929
2.0	1.051	1.10	0.404	1.833	0.374	1.739
3.0	1.352	1.40	0.209	1.464	0.136	1.809

Then, a general approximated estimation of τ_{cmin} for any a and b values within the studied range, can be obtained assuming that for each b value the τ_{cmin} corresponding to $a \in \{0.20, 0.40, 0.60, 0.80, 1.0\}$ are uniformly distributed between these two lines, and given by

$$\tau_{cmin} = 0.05 + 0.75a + 0.475b - 0.1875ab, \qquad (10)$$

where the parameters must satisfy the following relation

$$b \leq -0.2 + 8.2a - 2a^2, \qquad (11)$$

imposed by the right half plane zero position limit b_{max} stated in Table 1.

Equation (10) gives a less precise estimation of the design parameter lower limit τ_{cmin} than (7) but does not require any addition work for the cases when the model time constants ratio a is not one of the values listed in Table 1.

Therefore, for an inverse response controlled process (1), it is possible to design a robust regulatory control system with a PID controller (2) using the *DS-d* normalized tuning method (18) to (20) and following the proposed recommendation for τ_{cmin} (7) or (10), selecting the design parameter as $\tau_c \geq \tau_{cmin}$, in order to achieve a robust control system with $M_S \leq 2.0$.

4. EXAMPLES

4.1 Example 1

Consider the following second order inverse response controlled process

$$P_1(s) = \frac{1 - \alpha s}{(s+1)^2}, \qquad (12)$$

with $\alpha \in \{0.5, 1.0, 2.0, 3.0\}$ that is a variation of the inverse response benchmark plant proposed in Åström and Hägglund (2000). Using the inverse response model identification procedure proposed in Balaguer et al. (2011) and summarized in Appendix B, the corresponding models were obtained whose parameters are show in Table 2.

Models information of Table 2 were used to obtain the PID parameters using (18) to (20) where the corresponding design parameter was selected with (10).

The *DS-d* tuning controller parameters for the selected design parameter τ_c are shown in Table 3 along with the achieved robustness (M_S^r).

As it can be seen in Table 3, following the proposed recommendation to obtain a robust control system (10),

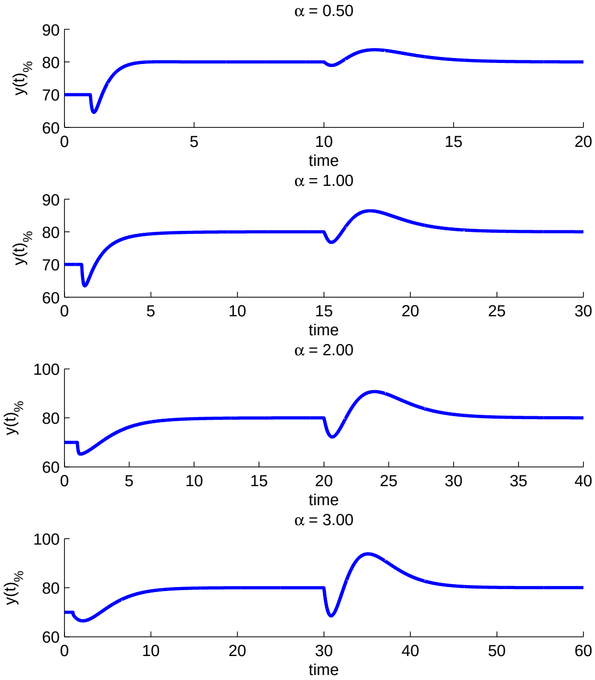

Figure 3. Example 1 - Control System Responses for P_1

selecting the design parameter $\tau_c \geq \tau_{cmin}$, it is possible to get good values for M_S. Therefore, achieving a robust control system ($M_S \leq 2.0$).

The control system response to a 10% set-point step change followed by a 10% load-disturbance step of such systems are shown in Fig. 3, for each case of α. It was supposed that at its normal operation point the system output y is at 70% of their normal operation range.

4.2 Example 2

In order to add completeness to the analysis, a case-study example is provided. We consider the isothermal continuous stirred tank reactor (CSTR), as the one in Fig. 4, where the isothermal series/parallel Van de Vusse reaction (Kravaris and Daoutidis, 1990; Van de Vusse, 1964) is taking place. The reaction can be described by the following scheme

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C, \qquad (13)$$
$$2A \xrightarrow{k_3} D.$$

Doing a mass balance, the system can be described by the following model

$$\frac{dC_A(t)}{dt} = \frac{F_r(t)}{V}\left[C_{Ai} - C_A(t)\right] - k_1 C_A(t) - k_3 C_A^2(t),$$
$$\frac{dC_B(t)}{dt} = -\frac{F_r(t)}{V} C_B(t) + k_1 C_A(t) - k_2 C_B(t), \qquad (14)$$

where F_r is the feed flow rate of product A, V is the reactor volume which is kept constant during the operation, C_A and C_B are the reactant concentrations in the reactor, and

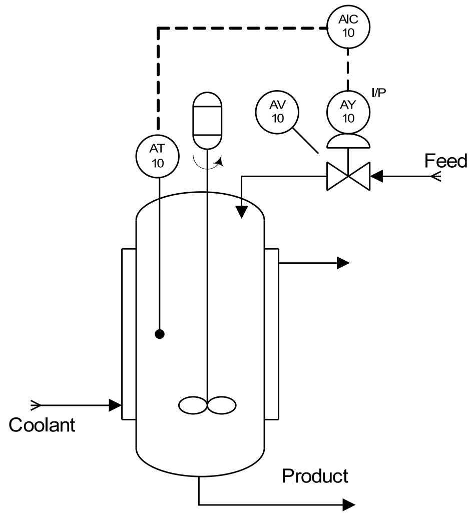

Figure 4. Example 2 - CSTR System

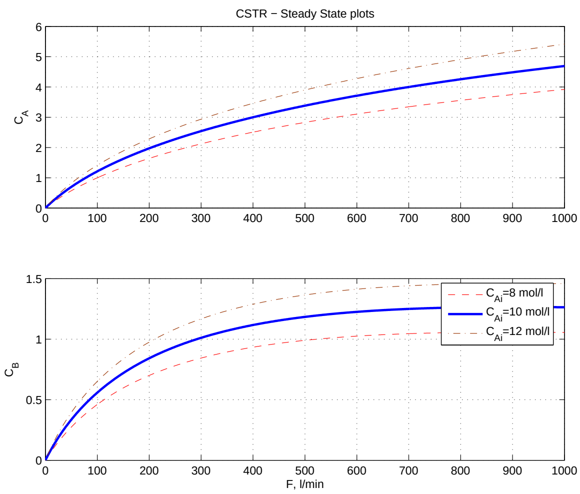

Figure 5. Example 2 - Steady-State Characterization for the Reactor

k_i $(i = 1, 2, 3)$ are the reaction rate constants for the three reactions.

In this case, the variables of interest are: the concentration of B in the reactor (C_B as the controlled variable), the flow through the reactor (F_r as the manipulated variable), and the concentration C_{Ai} of A in the feed flow (whose variation can be considered as the disturbance). The kinetic parameters are chosen to be $k_1 = 5/6\ min^{-1}$, $k_2 = 5/3\ min^{-1}$, and $k_3 = 1/6\ l \cdot mol^{-1} \cdot min^{-1}$. Also, is assumed that the nominal concentration of A in the feed (C_{Ai}) is $10\ mol \cdot l^{-1}$ and the volume $V = 700\ l$.

Using (14) and the parameters values, the characterization of the steady-state for the process can be obtained as it is shown in Fig. 5, for three concentrations of C_{Ai}, where is easy to see the non-linearity of the system.

Initially, the system is at steady-state (therefore at the operational point) with $C_{Ao} = 2.9175\ mol \cdot l^{-1}$ and $C_{Bo} = 1.10\ mol \cdot l^{-1}$. From this, it can be selected the measurement range for C_B from 0 to $1.5714\ mol \cdot l^{-1}$ and

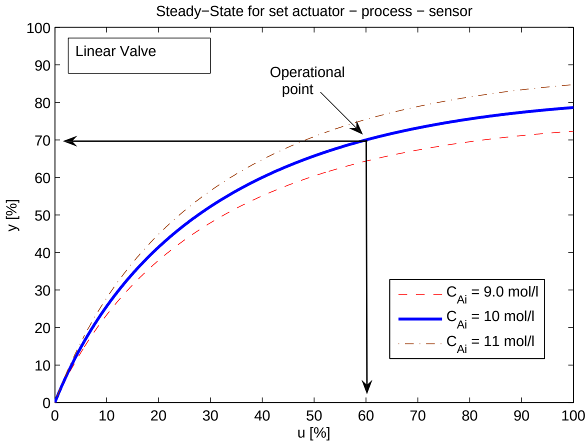

Figure 6. Example 2 - Steady-State Characterization for the Actuator-Process-Sensor Set

the capacity for the control valve with a maximum flow of 634.1719 $l \cdot min^{-1}$ (variation range of the flow from 0 to 634.1719 $l \cdot min^{-1}$) (Arrieta et al., 2008). The signals (y, u, r) will be in percentage (0 to 100%).

The sensor-transmitter element takes the form

$$y(t)_{\%} = \left(\frac{100}{1.5714}\right) C_B(t), \qquad (15)$$

and the control valve with a linear flow characteristic,

$$F_r(t) = \left(\frac{634.1719}{100}\right) u(t)_{\%}. \qquad (16)$$

Fig. 6 shows the steady-state characterization, taking into account the instruments represented by (15) and (16). This is the called *actuator-process-sensor set* and from this is clearly that for the selected steady-state, $r_o = 70\%$ and $u_o = 60\%$.

It is assumed that changes in the set-point would be not bigger than 10% and the possible disturbance in C_{Ai}, can variate around $\pm 10\%$.

Using the identification methodology in Appendix B (Balaguer et al., 2011), a linear model was obtained and given by

$$P_2(s) \approx \bar{P}_2(s) = \frac{0.3199(-0.3520s + 1)}{(0.5619s + 1)(0.3086s + 1)}. \qquad (17)$$

Figure 7 shows the process and the identified model (17) outputs for a step change in the process input ($u(t)$). It is possible to see that the identification procedure has a very good accuracy.

From (17) parameters of the inverse response model (1) are: $K = 0.3199$, $T = 0.5619$, $a = 0.5492$ and $b = 0.6264$.

Since model parameters, a and b, satisfy (11), using (10) the design parameter for tuning the PID controller can be selected as $\tau_c = 0.75 \geq \tau_{cmin} = 0.7$.

Controller parameters obtained with (18) to (20) for $\tau_c = 0.75$ are: $K_p = 3.467$, $T_i = 0.850$ and $T_d = 0.194$; and the robustness of the resulting control system is $M_S^r = 1.928$.

The control system responses to step-changes in the set-point and the disturbance are shown in Fig. 8

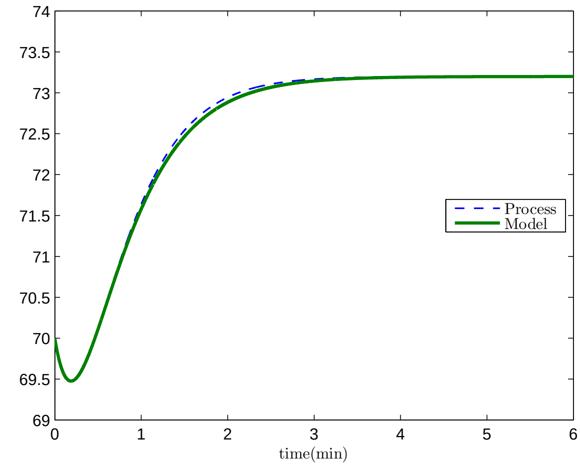

Figure 7. Example 2 - Reaction Curve for Process and Model

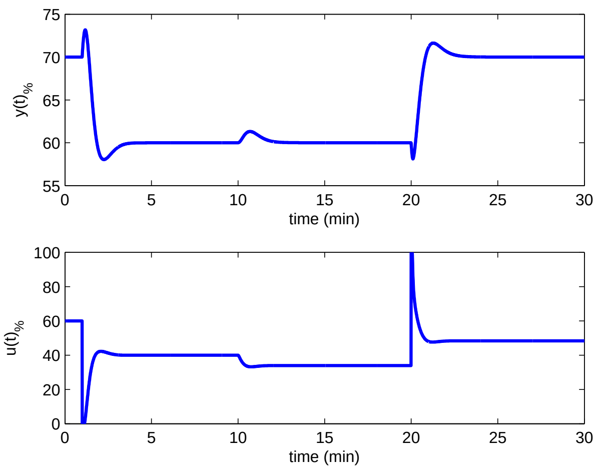

Figure 8. Example 2 - Control System Responses to Step-Changes in the Set-Point and the Disturbance

5. CONCLUSIONS

In this paper the relation of the closed-loop PID control system performance τ_c with the maximum sensitivity M_S was analyzed for the *DS-d* tuning method of Chen and Seborg (2002). The result shows in a quantitative way that regarding the inverse response process characteristics, defined by the model dimensionless numbers a and b, there are some performance levels that are not robustly attainable.

Its usefulness in control practice has been shown by means of the PID tuning of some examples, including a continuous stirred tank reactor (CSTR).

The proposed selection criteria for the design parameter τ_c allows the designer to obtain robust PID control systems with $M_S \leq 2.0$ for inverse response processes using the *DS-d* tuning rule.

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APPENDICES

A. DS-d tuning rule

The tuning equations from Chen and Seborg (2002) in a normalized form for controller (2) with a model (1) are:

$$\kappa_p \doteq K_p K = \frac{-3\tau_c^2 b + [a + (1+a)b](3\tau_c + b) - \tau_c^3}{(\tau_c + b)^3} \tag{18}$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{-3\tau_c^2 b + [a + (1+a)b](3\tau_c + b) - \tau_c^3}{a + (1 + a + b)b} \tag{19}$$

$$\tau_d \doteq \frac{T_d}{T} = \frac{(-b - 1 - a)\tau_c^3 + 3\tau_c^2 a + ab(3\tau_c + b)}{-3\tau_c^2 b + [a + (1+a)b](3\tau_c + b) - \tau_c^3} \tag{20}$$

with τ_c as a design parameter.

B. Identification procedure

Simple algorithms for identification of inverse response models from step response are difficult to obtain because analytically it is required the solution of a system of coupled nonlinear equations. In this Appendix we summarize the simple identification procedure for second order inverse response processes based on the plant step response presented in Balaguer et al. (2011).

In order to identify the parameters of model (1) following information take from Fig. 9 is needed: the input step change Δu, the output total change Δy, the inverse response minimum point (y_p in %, t_p), and the times take by the output to reach the 47% (t_{47}) and the 90% (t_{90}) of its total change.

Model identification equations are (Balaguer et al., 2011):

$$\begin{aligned} K &= \frac{\Delta y}{\Delta u}, \\ T &= 0.60(t_{90} - t_{47}), \\ b &= 1 - \frac{1 - y_p/\Delta y}{e^{-t_p/T}}, \\ a &= \frac{t_{47} - 0.7538 + 0.6262b + 0.0696b^2}{0.9275 - 0.1794b - 0.0161b^2}. \end{aligned} \tag{21}$$

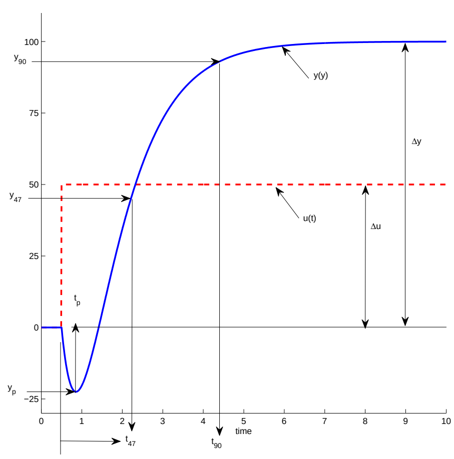

Figure 9. Identification from the step response