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Event-triggered PI control subject to actuator saturation

D. Lehmann * **K. H. Johansson** *

* *Automatic Control Laboratory, KTH Royal Institute of Technology, Stockholm, 100 44 Sweden (e-mail: dlehmann,kallej@kth.se).*

Abstract: Event-triggered control aims at reducing the communication load over the feedback link in networked control systems by adapting the information exchange to the current needs. This paper firstly extends a common approach to event-triggered control by incorporating a PI controller and by showing the setpoint-tracking properties of the extended scheme. Second, it investigates the consequences of actuator saturation on the behavior of the event-triggered PI-control loop. Simulations show that the effect of actuator saturation depends on the selection of the event threshold which might even destabilize the closed-loop system due to integrator windup. Finally, anti-windup techniques are discussed to overcome this problem.

Keywords: Event-triggered control, PI control, Stability, Setpoint tracking, Actuator saturation, Anti windup.

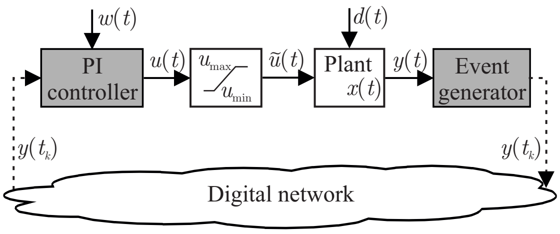

Fig. 1. Event-triggered control loop

1. INTRODUCTION

1.1 Event-triggered control

The communication network used in networked control systems (NCS) has a considerable influence on the closed-loop performance since its load affects the quality of service by inducing delays or packet losses which degrade the system performance or even cause the instability of the control loop (Nair et al. (2007)).

In this context, the analysis of event-triggered control has gained attention by considering event-triggered control as a means to reduce the communication load of the network (Årzén (1999); Åström and Bernhardsson (2002); Bemporad et al. (2010); Tipsuwan and Chow (2003)). The main aim to be reached by this feedback structure is the adaptation of the communication among the components of the feedback loop to the current needs. In fact, by reducing the information exchange to the minimum communication that is necessary to ensure the required system performance, an overload of the digital communication network can be avoided.

The event-triggered control loop as considered in this paper is depicted in Fig. 1. It consists of

- the plant with state $x(t)$, input $\tilde{u}(t)$ subject to actuator limitations, output $y(t)$ and disturbance $d(t)$,
- a smart sensor node which incorporates the event generator based on which a communication is invoked,
- and a PI controller which gets the reference input $w(t)$ and produces the control input $u(t)$.

The controller and the sensor node are connected by means of a digital network. Only at event times t_k $(k = 0, 1, 2, ...)$ determined by the event generator, the output $y(t_k)$ is sent from the sensor node towards the controller which is indicated by the dashed lines. The solid lines indicate continuous-time signals.

1.2 Literature

Almost all approaches to event-triggered control, e.g. by Anta and Tabuada (2010); Cervin and Henningsson (2008); Heemels et al. (2008); Lunze and Lehmann (2010); Wang and Lemmon (2009), consider an event-triggered control using a proportional controller. Here, the analysis is concentrated on showing the stability as well as the communication properties of the event-triggered control loop.

However, in practical applications further requirements become important especially setpoint tracking for constant reference and disturbance signals. In this context, first papers by Årzén (1999); Otanez et al. (2002); Vasyutynskyy and Kabitzsch (2006) showed by simulations that event-triggered PID control is able to significantly reduce the computational and communication effort while only slightly degrading the control performance.

Lehmann and Lunze (2011) presented a model-based approach to event-triggered PI control. It provides a theoretical framework to analyze the stationary behavior of the event-triggered PI-control loop for constant and time-varying reference and disturbance signals. However, the scheme proposed is computationally demanding because

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the sensor node has to run the model used for producing the control input $u(t)$ which also requires a continuous access to the reference input $w(t)$.

Donkers and Heemels (2010) considered a very general dynamical event-triggered controller. For this purpose, they proposed an extended event-generating mechanism which simultaneously monitors the evolution of the measured state $x(t)$ and the control input $u(t)$ produced by the controller. Using linear matrix inequalities, the behavior of the event-triggered control loop was analyzed in terms of its stability and $\mathcal{L}_\infty$ properties.

However, event-triggered PI control brings about further questions which are important in practice. One of these questions concerns the consequences of actuator saturation (see Åström and Hägglund (1995); Zaccarian and Teel (2011)) on the event-triggered PI-control loop which, up to now, has not been treated in literature.

1.3 Contribution of this paper

The contributions of this paper are the following:

(1) It proposes an event-triggered PI control which uses the event condition introduced by Otanez et al. (2002) and provides new theoretical results with respect to the steady-state properties and the minimum inter-event time of the closed-loop system (Theorem 2).
(2) It investigates the consequences of actuator saturation (Proposition 1) illustrated by simulations which show that the closed-loop performance depends on the selection of the event threshold.
(3) To improve the behavior of the event-triggered PI-control loop in terms of overcoming integrator windup, anti-windup methods are evaluated by simulations showing promising results.

The remainder of this paper is organized as follows. Section 2 introduces some basic notations and the continuous-time PI-control loop. In Sec. 3, the continuous-time control is used as a reference system to evaluate the event-triggered PI control without actuator saturation. The effect of actuator saturation on the event-triggered PI-control loop is studied in Sec. 4. Finally, this section also evaluates anti-windup techniques applied to event-triggered control by simulations.

2. PRELIMINARIES

Throughout this paper the plant is given by the scalar state-space representation

$$\dot{x}(t) = ax(t) + b\tilde{u}(t) + ed(t), \quad x(0) = x_0 \tag{1}$$
$$y(t) = x(t), \tag{2}$$

where $x \in \mathbb{R}$ denotes the state of the plant with initial state x_0, $\tilde{u} \in \mathbb{R}$ is the input given by $\tilde{u}(t) = \mathrm{sat}(u(t))$ with

$$\mathrm{sat}(u(t)) = \begin{cases} u_{\max}, & \text{for } u(t) > u_{\max}; \\ u(t), & \text{for } u_{\min} \le u(t) \le u_{\max}; \\ u_{\min}, & \text{for } u(t) < u_{\min}, \end{cases} \tag{3}$$

and $d \in \mathbb{R}$ is an unknown disturbance which is assumed to be bounded according to

$$|d(t)| \le d_{\max}, \quad \forall t \tag{4}$$

with $|\cdot|$ the absolute value. Moreover, the output $y(t)$ is assumed to be given by the plant state $x(t)$.

Consider first, that

$$\tilde{u}(t) = u(t), \quad \forall u(t) \tag{5}$$

holds (no actuator saturation). Then, it is a well known fact that the PI controller

$$\dot{x}_{\mathrm{CTI}}(t) = y(t) - w(t), \quad x_{\mathrm{CTI}}(0) = 0 \tag{6}$$
$$u(t) = k_{\mathrm{I}} x_{\mathrm{CTI}}(t) + k_{\mathrm{P}}(y(t) - w(t)) \tag{7}$$

guarantees setpoint tracking

$$\lim_{t\to\infty} |y(t) - w(t)| = 0 \tag{8}$$

for constant exogenous signals

$$w(t) = \bar{w}, \;\; |\bar{w}| < w_{\max} \; \forall t \qquad d(t) = \bar{d}. \tag{9}$$

In a relaxed form, this aim can be formulated as

$$\lim_{t\to\infty} \mathrm{dist}(y(t), \Omega_{\mathrm{y}}(\bar{w})) = 0 \tag{10}$$

where $\Omega_{\mathrm{y}}(\bar{w})$ is the set of acceptable output values around the desired setpoint $\bar{w}$ and the term $\mathrm{dist}(y(t), \Omega_{\mathrm{y}}(\bar{w}))$ denotes the distance between the output $y(t)$ and the set $\Omega_{\mathrm{y}}(\bar{w})$ according to

$$\mathrm{dist}(y(t), \Omega_{\mathrm{y}}(\bar{w})) := \min_{\hat{y}\,\in\,\Omega_{\mathrm{y}}(\bar{w})} |y(t) - \hat{y}|.$$

With plant (1), (2), controller (6), (7) and by considering condition (5), the resulting closed-loop system is described by

$$\begin{pmatrix} \dot{x}_{\mathrm{CT}}(t) \\ \dot{x}_{\mathrm{CTI}}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} a + bk_{\mathrm{P}} & bk_{\mathrm{I}} \\ 1 & 0 \end{pmatrix}}_{\bar{\boldsymbol{A}}_{\mathrm{I}}} \underbrace{\begin{pmatrix} x_{\mathrm{CT}}(t) \\ x_{\mathrm{CTI}}(t) \end{pmatrix}}_{\boldsymbol{x}_{\mathrm{CTI}}(t)} + \underbrace{\begin{pmatrix} -bk_{\mathrm{P}} \\ -1 \end{pmatrix}}_{\boldsymbol{f}_{\mathrm{I}}} w(t)$$
$$+ \underbrace{\begin{pmatrix} e \\ 0 \end{pmatrix}}_{\boldsymbol{e}_{\mathrm{I}}} d(t), \quad \begin{pmatrix} x_{\mathrm{CT}}(0) \\ x_{\mathrm{CTI}}(0) \end{pmatrix} = \underbrace{\begin{pmatrix} x_0 \\ 0 \end{pmatrix}}_{\boldsymbol{x}_{\mathrm{I0}}} \tag{11}$$

$$y(t) = \underbrace{\begin{pmatrix} 1 & 0 \end{pmatrix}}_{\boldsymbol{c}_{\mathrm{I}}^{\mathrm{T}}} \begin{pmatrix} x(t) \\ x_{\mathrm{I}}(t) \end{pmatrix}, \tag{12}$$

where x_{CT} is used in the following to denote the plant state of the continuous-time PI-control loop without actuator saturation and $(\cdot)^{\mathrm{T}}$ denotes the transpose of a vector.

Definition 1. (Khalil (2002)) The solution $x_{\mathrm{CT}}(t)$ of the continuous-time PI-control loop (11), (12) is globally uniformly ultimately bounded (GUUB) if, for every $x_{\mathrm{CT}}(0) \in \mathbb{R}$, there exists a positive constant p and a time $\bar{t}$ such that holds:

$$x_{\mathrm{CT}}(t) \in \Omega_{\mathrm{x}} = \{x_{\mathrm{CT}} : |x_{\mathrm{CT}}| \le p\}, \;\; \forall t \ge \bar{t}.$$

Theorem 1. (Khalil (2002)) The plant state $x_{\mathrm{CT}}(t)$ of the continuous-time PI-control loop (11), (12) is GUUB if the matrix $\bar{\boldsymbol{A}}_{\mathrm{I}}$ is Hurwitz and the signals $w(t)$ and $d(t)$ are bounded.

Henceforth, it is assumed that controller (6), (7) is designed such that the state $x_{\mathrm{CT}}(t)$ of the continuous-time PI-control loop (11), (12) is GUUB ($\bar{\boldsymbol{A}}_{\mathrm{I}}$ is Hurwitz) and possesses the setpoint-tracking property (8) for constant exogenous signals (9), and that $a + bk_{\mathrm{P}} < 0$ holds.

3. EVENT-TRIGGERED PI CONTROL WITHOUT ACTUATOR SATURATION

3.1 Setpoint-tracking property

This section firstly investigates the event-triggered PI-control loop without actuator saturation according to condition (5) in order to show the setpoint-tracking properties of the unconstrained scheme. The main difference to the continuous-time PI-control loop is given by the fact, that the controller gets new information about the plant output $y(t)$ only at event times t_k $(k = 0, 1, 2, ...)$ invoked by the event generator (Fig. 1). That is, the controller can be described in the time interval $[t_k, t_{k+1})$ by

$$\dot{x}_{\mathrm{I}}(t) = y(t_k) - w(t), \quad x_{\mathrm{I}}(t_k) = x_{\mathrm{I}k} \tag{13}$$
$$u(t) = k_{\mathrm{I}} x_{\mathrm{I}}(t) + k_{\mathrm{P}}(y(t_k) - w(t)), \quad t \in [t_k, t_{k+1}).$$

By introducing the output error

$$e(t) = y(t) - y(t_k), \tag{14}$$

this controller can be rewritten according to

$$\dot{x}_{\mathrm{I}}(t) = y(t) - e(t) - w(t), \quad x_{\mathrm{I}}(0) = 0 \tag{15}$$
$$u(t) = k_{\mathrm{I}} x_{\mathrm{I}}(t) + k_{\mathrm{P}}(y(t) - e(t) - w(t)) \tag{16}$$

which holds for all times $t \geq 0$.

With this controller representation and plant (1), (2) and by introducing the augmented state vector

$$\boldsymbol{x}_{\mathrm{I}}(t) = \left(\, x(t) \;\; x_{\mathrm{I}}(t) \, \right)^{\mathrm{T}},$$

the state-space model

$$\dot{\boldsymbol{x}}_{\mathrm{I}}(t) = \bar{\boldsymbol{A}}_{\mathrm{I}} \boldsymbol{x}_{\mathrm{I}}(t) + \boldsymbol{f}_{\mathrm{I}} w(t) + \boldsymbol{g}_{\mathrm{I}} e(t) + \boldsymbol{e}_{\mathrm{I}} d(t) \tag{17}$$
$$\boldsymbol{x}_{\mathrm{I}}(0) = \boldsymbol{x}_{\mathrm{I}0}$$
$$y(t) = \boldsymbol{c}_{\mathrm{I}}^{\mathrm{T}} \boldsymbol{x}_{\mathrm{I}}(t) \tag{18}$$

results with $\bar{\boldsymbol{A}}_{\mathrm{I}}$, $\boldsymbol{f}_{\mathrm{I}}$, $\boldsymbol{e}_{\mathrm{I}}$, $\boldsymbol{c}_{\mathrm{I}}^{\mathrm{T}}$ defined above and $\boldsymbol{g}_{\mathrm{I}} = \left(-bk_{\mathrm{P}} \;\; -1 \right)^{\mathrm{T}}$. Note that this description holds independent of the event condition used and, hence, also in the sampled-data case.

To prove the stability and the setpoint-tracking property of the event-triggered control loop, the output error $e(t)$ has to be bounded (cf. Theorem 1). An obvious choice for limiting this signal is given by performing deadband sampling according to Otanez et al. (2002). The proposed event condition invokes a communication whenever the output error $e(t)$ reaches a predefined threshold $\bar{e}$, i.e.

$$|e(t)| = |y(t) - y(t_k)| = \bar{e}. \tag{19}$$

Theorem 2. The event-triggered PI-control loop (17), (18) using event condition (19) has the following properties:

- Its plant state $x(t)$ is GUUB.
- It possesses the relaxed setpoint-tracking property (10) with $\Omega_{\mathrm{y}}(\bar{w})$ given by
$$\Omega_{\mathrm{y}}(\bar{w}) = \{\hat{y} \; : \; |\bar{w} - \hat{y}| \leq \bar{e}\}. \tag{20}$$
- The minimum inter-event time $T_{\min} = \min_k\{t_{k+1} - t_k\}$ $(k = 0, 1, 2, ...)$ is lower bounded by
$$\bar{T} = \arg\min_t \{z_1 x_{\max} + z_2(|bk_{\mathrm{I}}| x_{\mathrm{I}\max} + |bk_{\mathrm{P}}|(\bar{w} + \bar{e}) + |e| d_{\max}) = \bar{e}\} \tag{21}$$
with
$$z_1 = \left| \mathrm{e}^{\bar{a}t} - 1 \right| \tag{22}$$
$$z_2 = \int_0^t \mathrm{e}^{\bar{a}t} \, \mathrm{d}s, \tag{23}$$
$x_{\max}$, $x_{\mathrm{I}\max}$ given by Eqs. (32), (33) and $\bar{a} = (a + bk_{\mathrm{p}})$.

Proof. The proof can be found in the appendix. □

The theorem shows that the steady-state error of event-triggered PI control solely depends on the event threshold $\bar{e}$ (Eq. (20)). Hence, by decreasing $\bar{e}$, the error can be made arbitrarily small which, however, increases the possible information exchange (Eq. (21)).

3.2 Simulation results

Given is the first-order plant

$$\dot{x}(t) = 0.1x(t) + \tilde{u}(t) + 0.1d(t), \quad x(0) = 0 \tag{24}$$
$$y(t) = x(t). \tag{25}$$

With the PI controller

$$\dot{x}_{\mathrm{I}}(t) = y(t) - w(t), \quad x_{\mathrm{I}}(0) = 0$$
$$u(t) = -x_{\mathrm{I}}(t) - 1.6(y(t) - w(t)), \tag{26}$$

the exogenous signals

$$w(t) = \bar{w} = 1.5$$
$$d(t) = \bar{d} = 0.1,$$

and under the assumption that condition (5) holds, the continuous-time closed-loop system is given by

$$\begin{pmatrix} \dot{x}_{\mathrm{CT}}(t) \\ \dot{x}_{\mathrm{CTI}}(t) \end{pmatrix} = \begin{pmatrix} -1.5 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_{\mathrm{CT}}(t) \\ x_{\mathrm{CTI}}(t) \end{pmatrix} + \begin{pmatrix} 1.6 \\ -1 \end{pmatrix} \bar{w} + \begin{pmatrix} 0.1 \\ 0 \end{pmatrix} \bar{d}, \quad \begin{pmatrix} x_{\mathrm{CT}}(0) \\ x_{\mathrm{CTI}}(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

with $y_{\mathrm{CT}}(t) = x_{\mathrm{CT}}(t)$. Figure 2 shows the behavior of the continuous-time PI-control loop (grey dotted curves) which stationarily reaches the setpoint $\bar{w}$.

The black curves in Fig. 2 depict the behavior of the corresponding event-triggered PI-control loop using event condition (19) with

$$\bar{e} = 0.2,$$

where the black dashed curve indicates the output information $y(t_k)$ used by the controller between two consecutive event times. The middle plot shows the control input $u(t)$ produced by the controller, and the event times t_k $(k = 0, 1, 2, ...)$ are indicated in the lower plot. With 18 events occurring in the time interval considered the event-triggered PI control is able to drive the output $y(t)$ into the set

$$\Omega_{\mathrm{y}}(\bar{w}) = \{y \; : \; |1.5 - y| \leq 0.2\}$$

and to stationarily keep it there despite the exogenous disturbance $d(t)$ (see Theorem 2) while only slightly degrading the performance compared to the continuous-time PI control.

However, the stationary limit cycle leads to undesired communication events. To avoid this stationary behavior either the PI controller has to be adapted, e.g. by using

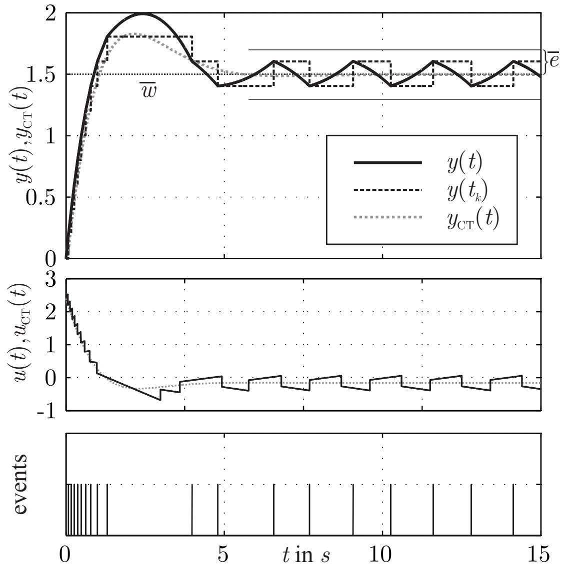

Fig. 2. Behavior of the continuous-time and the event-triggered PI-control loop

PIDPLUS as proposed by Araújo (2011); Song et al. (2006) or an alternative event condition has to be used, see Lehmann and Lunze (2011).

4. EVENT-TRIGGERED PI CONTROL INCLUDING ACTUATOR SATURATION

4.1 Problem formulation

In the previous analysis, it is assumed that the PI controller produces a control input $u(t)$ which can always be applied by the actuators of the plant (cf. Eq. (5)). However, in practical applications the actuators are almost entirely subject to physical limitations, so that condition (5) does not hold.

For continuous-time PI control, it is known that this nonlinear effect can deteriorate the performance of the closed-loop system and might even cause its instability (Åström and Hägglund (1995)). In the following, the influence of actuator saturation on the event-triggered PI control is investigated.

By introducing the control input error

$$e_{\mathrm{u}}(t) = \tilde{u}(t) - u(t), \tag{27}$$

the plant (1), (2) and the event-triggered PI controller (15), (16) yield the state-space model

$$\dot{\boldsymbol{x}}_{\mathrm{I}}(t) = \bar{\boldsymbol{A}}_{\mathrm{I}}\boldsymbol{x}_{\mathrm{I}}(t) + \boldsymbol{f}_{\mathrm{I}}w(t) + \boldsymbol{g}_{\mathrm{I}}e(t) + \boldsymbol{h}_{\mathrm{I}}e_{\mathrm{u}}(t) + \boldsymbol{e}_{\mathrm{I}}d(t) \tag{28}$$

$$\boldsymbol{x}_{\mathrm{I}}(0) = \boldsymbol{x}_{\mathrm{I}0}$$

$$y(t) = \boldsymbol{c}_{\mathrm{I}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{I}}(t) \tag{29}$$

with $\boldsymbol{h}_{\mathrm{I}} = (\, b \;\, 0 \,)^{\mathrm{T}}$.

Proposition 1. The state $x(t)$ of the event-triggered PI-control loop (28), (29) using event condition (19) and subject to actuator saturation (3) is GUUB provided that the control input error $e_{\mathrm{u}}(t)$ given by Eq. (27) is bounded.

Proof. The proof follows directly from Theorem 1 and Theorem 2. □

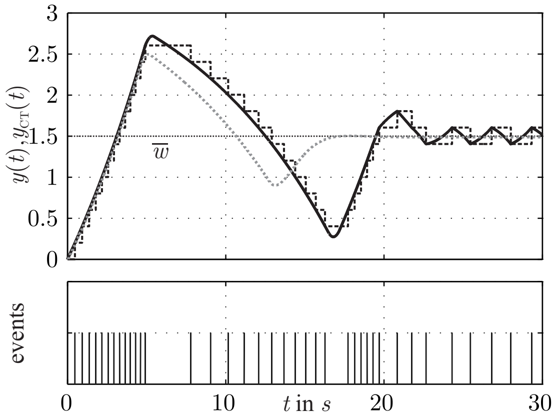

Fig. 3. Behavior of the continuous-time and the event-triggered PI-control loop subject to actuator saturation

This result brings about the following problems:

(1) Find a condition which guarantees that the control input error $e_{\mathrm{u}}(t)$ is bounded.
(2) Extend the event-triggered PI-control loop so that the effect of actuator saturation can be compensated.

Note that for the continuous-time control loop there exist several techniques to avoid an undesired increase of the control input error $e_{\mathrm{u}}(t)$ one of which is applied in Sec. 4.3 to event-triggered control. The next section investigates the consequences of actuator saturation on the event-triggered PI-control loop by simulations.

4.2 Simulation results

The plant (24), (25) is now considered to be affected by the actuator saturation

$$\tilde{u}(t) = \begin{cases} 0.4, & \text{for } u(t) > u_{\max}; \\ u(t), & \text{for } u_{\min} \leq u(t) \leq u_{\max}; \\ -0.4, & \text{for } u(t) < u_{\min}, \end{cases} \tag{30}$$

and $u(t)$ according to Eq. (26).

Compared to the unconstrained case (Fig. 2), Fig. 3 shows that the transient behavior of the continuous-time PI-control loop (grey dotted curve) and the event-triggered PI-control loop (black solid curve) with event threshold $\bar{e} = 0.2$ deteriorates due to a large control input error $e_{\mathrm{u}}(t) = \tilde{u}(t) - u(t)$ (see upper plot of Fig. 4). This is caused by integrator windup depicted in the lower plot of Fig. 4 which is increased in the event-triggered case since the controller uses old information $y(t_k)$ about the plant output between two consecutive event times. However, a stable behavior and the respective setpoint-tracking properties can be retained.

The consequences of increasing the event threshold $\bar{e}$ are highlighted in Fig. 5. For $\bar{e} = 0.45$, the integrator windup forces the control input $\tilde{u}(t)$ to be almost all times t at its saturation bounds (Fig. 4) which leads to an unstable behavior of the closed-loop system.

In summary, the simulation results show that integrator windup due to actuator saturation cannot be neglected when dealing with event-triggered PI control and that it strongly depends on the selection of the event threshold $\bar{e}$.

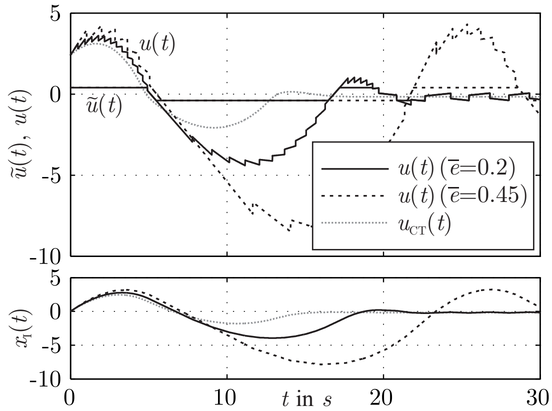

Fig. 4. Control inputs $\tilde{u}(t)$, $u(t)$ and integrator state $x_{\mathrm{I}}(t)$ for the continuous-time control loop (grey dotted curves), the event-triggered control loop with $\bar{e} = 0.2$ (black solid curves) and the event-triggered control loop with $\bar{e} = 0.45$ (black dashed curves)

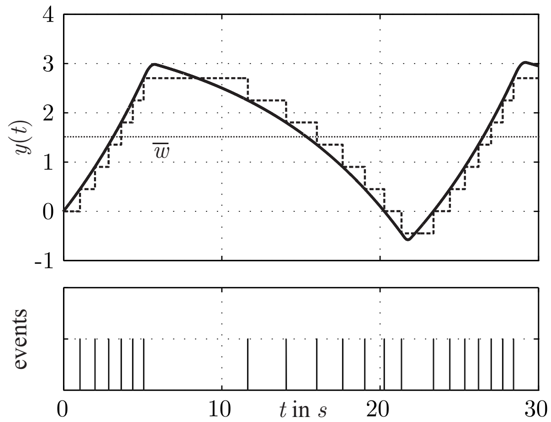

Fig. 5. Behavior of the event-triggered PI-control loop with $\bar{e} = 0.45$ and subject to actuator saturation

4.3 Anti-windup methods for event-triggered control

In continuous-time control, the performance degradation due to integrator windup is faced by applying anti-windup techniques. These methods generally use a static or dynamic feedback of the control input error $e_{\mathrm{u}}(t)$ to affect the evolution of the integrator state $x_{\mathrm{I}}(t)$ and to reduce the integrator windup (see e.g. Åström and Hägglund (1995); Zaccarian and Teel (2011)).

In the following, two methods are applied in order to overcome the problems due to integrator windup in event-triggered control:

(1) A continuous-time anti-windup scheme (CTAW), where a smart actuator includes the controller function and uses a dynamic feedback of the control input error $e_{\mathrm{u}}(t)$ (Fig. 6).
(2) An event-triggered anti-windup scheme (ETAW), where a separate node (EG) uses the information about the input saturation and the evolution of the input signal $u(t)$ in order to invoke additional sampling events and reset the integrator state $x_{\mathrm{I}}(t)$ (Fig. 7) whenever

$$|u(t) - u(t_k)| = \bar{e}_{\mathrm{c}} \quad \text{if} \quad e_{\mathrm{u}}(t) \neq 0$$

holds, where $\bar{e}_{\mathrm{c}}$ is a new threshold to be designed.

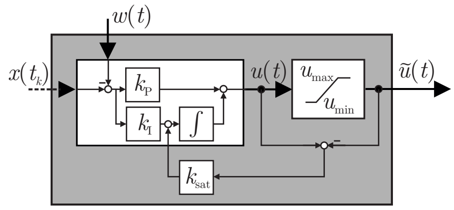

Fig. 6. Smart actuator including the controller and a dynamic anti-windup scheme (CTAW)

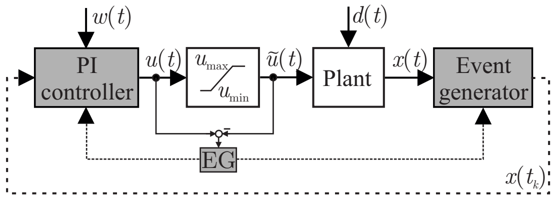

Fig. 7. Event-triggered anti-windup scheme (ETAW)

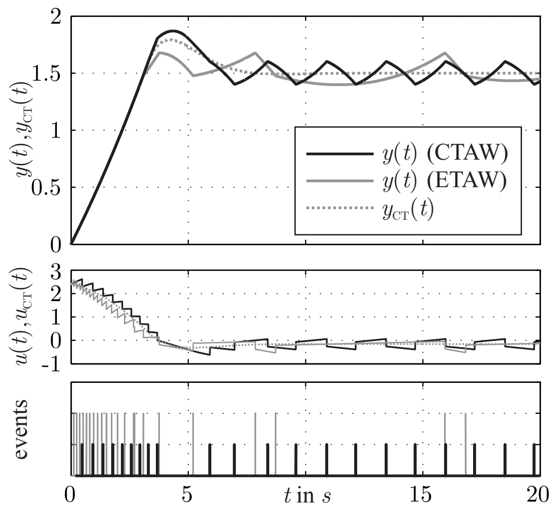

Fig. 8. Behavior of the continuous-time control loop and the event-triggered control loop using anti-windup techniques

The simulation results are depicted in Fig. 8. It is shown that by using the continuous-time anti windup, the transient behavior of both the continuous-time control loop (grey dotted curve) and the event-triggered control loop (black solid curve) can be significantly improved (cf. Fig. 3). The control inputs are shown in the middle plot of Fig. 8.

Moreover, the behavior of the event-triggered control loop subject to the event-triggered anti-windup scheme is drawn by the grey solid curve in Fig. 8. It shows that the behavior of the closed-loop system can be further improved by invoking more sampling events whenever the input is saturated (lower plot of Fig. 8). Additionally, this may even lead to a reduced stationary communication.

The simulations bring about, that continuous-time anti-windup techniques can be successfully applied to event-triggered control. However, alternative methods should be also taken into consideration as they might even better meet the requirements of event-triggered control including situations in which a continuous-time feedback of the control input error $e_{\mathrm{u}}(t)$ is not feasible.

5. CONCLUSION

This paper investigated an event-triggered PI-control scheme with respect to its setpoint-tracking properties and the consequences of actuator saturation. It has been shown that the influence of actuator saturation strongly depends on the selection of the event threshold but can be avoided by applying anti-windup techniques.

Future work will include the extension of the stability analysis for plants subject to actuator saturation, a theoretical investigation and a possible adaptation of common anti-windup techniques applied to event-triggered control, and the design of new event-triggered anti-windup methods.

APPENDIX

Proof of Theorem 2. The first property follows directly from Theorem 1 and the fact that the reference input $w(t)$ and the disturbance $d(t)$ are assumed to be bounded (Eqs. (4), (9)), and that the output error $e(t)$ is bounded due to event condition (19).

The second property can be obtained by comparing the continuous-time control loop (11), (12) with the event-triggered control loop (17), (18). Introducing the difference state

$$\boldsymbol{e}_{\mathrm{CTI}}(t) = \boldsymbol{x}_{\mathrm{I}}(t) - \boldsymbol{x}_{\mathrm{CTI}}(t)$$

the difference behavior is described by

$$\begin{aligned}\dot{\boldsymbol{e}}_{\mathrm{CTI}}(t) &= \dot{\boldsymbol{x}}_{\mathrm{I}}(t) - \dot{\boldsymbol{x}}_{\mathrm{CTI}}(t)\\ &= \bar{\boldsymbol{A}}_{\mathrm{I}}\boldsymbol{e}_{\mathrm{CTI}}(t) + \boldsymbol{g}_{\mathrm{I}}e(t), \quad \boldsymbol{e}_{\mathrm{I}}(0) = \boldsymbol{0}. \qquad (31)\end{aligned}$$

By overapproximating $e(t)$ according to $e(t) = \bar{e}$ (Eq. (19)) and due to the fact that $\boldsymbol{g}_{\mathrm{I}} = \boldsymbol{f}_{\mathrm{I}}$ holds and the continuous-time control loop (11), (12) provides setpoint tracking for constant references signals (9), the stationary difference $\lim_{t\to\infty} |e_{\mathrm{CT}}(t)| = \lim_{t\to\infty} |x(t) - x_{\mathrm{CT}}(t)|$ is upper bounded by

$$\lim_{t\to\infty} |e_{\mathrm{CT}}(t)| = \bar{e}.$$

With

$$\lim_{t\to\infty} |y_{\mathrm{CT}}(t)| = \bar{w},$$

$y(t) = x(t)$ and $y_{\mathrm{CT}}(t) = x_{\mathrm{CT}}(t)$, it follows

$$\Omega_{\mathrm{y}}(\bar{w}) = \{\hat{y} \, : \, |\bar{w} - \hat{y}| \le \bar{e}\}.$$

In order to prove the third property, Eq. (31) can be used to overapproximate the augmented state $\boldsymbol{x}_{\mathrm{I}}(t)$ according to

$$\|\boldsymbol{e}_{\mathrm{CTI}}(t)\| \le \bar{e} \cdot \int_0^\infty \left\| \mathrm{e}^{\bar{\boldsymbol{A}}_{\mathrm{I}} t} \boldsymbol{g}_{\mathrm{I}} \right\| = e_{\max}, \quad \forall t$$

where $e_{\max}$ is bounded since $\bar{\boldsymbol{A}}_{\mathrm{I}}$ is Hurwitz. This yields

$$\max_t |x(t)| = \max_t |x_{\mathrm{CT}}(t)| + e_{\max} = x_{\max} \qquad (32)$$

$$\max_t |x_{\mathrm{I}}(t)| = \max_t |x_{\mathrm{CTI}}(t)| + e_{\max} = x_{\mathrm{Imax}}, \qquad (33)$$

where $\max_t |x_{\mathrm{CT}}(t)|$ and $\max_t |x_{\mathrm{CTI}}(t)|$ can be determined by considering the continuous-time control loop (11), (12) for a fixed initial state $\boldsymbol{x}_{\mathrm{I0}}$ and a fixed reference signal $\bar{w}$. Consider the evolution of $x(t)$ for $t \ge t_k$ described by the differential equation

$$\begin{aligned}\dot{x}(t) &= ax(t) + bk_{\mathrm{I}}x_{\mathrm{I}}(t) + bk_{\mathrm{p}}(y(t) - w(t) - e(t)) + ed(t)\\ &= \bar{a}x(t) + bk_{\mathrm{I}}x_{\mathrm{I}}(t) - bk_{\mathrm{p}}(w(t) + e(t)) + ed(t)\end{aligned}$$

(Eqs. (1), (2), (16)) with $\bar{a} = (a + bk_{\mathrm{p}})$, and $x(t_k) = x_k$, $x_{\mathrm{I}}(t_k) = x_{\mathrm{I}k}$ the plant state and the integrator state at time t_k, respectively. Then, for $t \ge t_k$, the output error $e(t) = y(t) - y(t_k)$ is given by

$$\begin{aligned}e(t) = \left(\mathrm{e}^{\bar{a}(t-t_k)} - 1\right) x_k + \int_{t_k}^{t} \mathrm{e}^{\bar{a}(t-s)} (bk_{\mathrm{I}}x_{\mathrm{I}}(s)\\ -bk_{\mathrm{p}}(w(s) + e(s)) + ed(s))\mathrm{d}s.\end{aligned}$$

As an event is generated whenever $|e(t)| = \bar{e}$ holds, this relation can now be used to derive a lower bound $\bar{T} \le T_{\min}$ on the minimum inter-event time which is given by

$$\begin{aligned}\bar{T} = \arg\min_t \{ z_1 x_{\max} + z_2(|bk_{\mathrm{I}}|x_{\mathrm{Imax}}\\ +|bk_{\mathrm{P}}|(\bar{w} + \bar{e}) + |e|d_{\max}) = \bar{e}\}\end{aligned}$$

with z_1 and z_2 given by Eqs. (22), (23).

REFERENCES

Anta, A. and Tabuada, P. (2010). To sample or not to sample: Self-triggered control for nonlinear systems. *IEEE Transactions on Automatic Control*, 55(9), 2030–2042.

Araújo, J. (2011). *Design and Implementation of Resource-Aware Wireless Networked Control Systems.* Licentiate thesis, KTH Royal Institute of Technology.

Årzén, K.E. (1999). A simple event-based PID controller. In *Proceedings of IFAC World Congress*, 423–428. Beijing.

Åström, K.J. and Bernhardsson, B. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of IEEE Conference on Decision and Control , Vol. 2*, 2011–2016. Las Vegas.

Åström, K.J. and Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning.* ISA.

Bemporad, A., Heemels, W.P.M.H., and Johansson, M. (2010). *Networked Control Systems.* Lecture Notes in Control and Information Sciences. Springer Verlag.

Cervin, A. and Henningsson, T. (2008). Scheduling of event-triggered controllers on a shared network. In *Proceedings of IEEE Conference on Decision and Control*, 3601–3606. Cancun.

Donkers, M.C.F. and Heemels, W.P.M.H. (2010). Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved event-triggering. In *IEEE Conference on Decision and Control.* Atlanta.

Heemels, W.P.M.H., Sandee, J., and van den Bosch, P. (2008). Analysis of event-driven controllers for linear systems. *Intern. J. of Control*, 81(4), 571–590.

Khalil, H.K. (2002). *Nonlinear Systems.* Prentice Hall, New Jersey.

Lehmann, D. and Lunze, J. (2011). Extension and experimental evaluation of an event-based state-feedback approach. *Control Engineering Practice*, 19(2), 101–112.

Lunze, J. and Lehmann, D. (2010). A state-feedback approach to event-based control. *Automatica*, 46(1), 211–215.

Nair, G.N., Fagnani, F., Zampieri, S., and Evans, R.J. (2007). Feedback control under data rate constraints: an overview. *Proceeding of the IEEE*, 95(1), 108–137.

Otanez, P.G., Moyne, J.G., and Tilbury, D.M. (2002). Using deadbands to reduce communication in networked control systems. In *Proceedings of American Control Conference*, 3015–3020. Anchorage.

Song, J., Mok, A.K., Chen, D., Nixon, M., Blevis, T., and Wojsznis, W. (2006). Improving PID control with unreliable communication. In *Proceedings of ISA EXPO Technical Conference.*

Tipsuwan, Y. and Chow, M.Y. (2003). Control methodologies in networked control systems. *Control Engineering Practice*, 11, 1099–1111.

Vasyutynskyy, V. and Kabitzsch, K. (2006). Implementation of PID controller with send-on-delta sampling. In *Proceedings of International Control Conference.* Glasgow.

Wang, X. and Lemmon, M.D. (2009). Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability. *IEEE Transactions on Automatic Control*, 54(3), 452–467.

Zaccarian, L. and Teel, A.R. (2011). *Modern Anti-windup Synthesis: Control Augmentation for Actuator Saturation.* Princeton.