A Class of PID Controllers Tuned in Fractional Representation

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Abstract: The contribution is focused on design and tuning of simple controllers for continuous-time SISO systems without and with time delays by algebraic methods. The control synthesis is based on general solutions of linear Diophantine equations in the ring of proper and Hurwitz stable rational functions (R_{PS}). Both, feedback (1DOF) a feedback and feed forward (2DOF) structures of the control system are considered. A scalar positive parameter is introduced for tuning and influencing control responses. In the paper, this parameter is used for aperiodic tuning of PI controllers. The methodology is utilized for autotuning principles and for robust applications. Interval perturbations in controlled systems and robustness of proposed algorithms are outlined through the value set concept, zero exclusion condition and Kharitonov theorem. For both applications user-friendly program packages were developed in the Matlab+Simulink environment with the support of Polynomial Toolbox.

Keywords: Algebraic approaches, control system design, PID controllers, autotuners, uncertainty.

1. INTRODUCTION

Continuous-time controllers of PID type have been widely used in many industrial applications for decades. There are several features of their success, e.g. structure simplicity, reliability, robustness in performance, etc., see e.g. (Åström and Hägglund, 1995; Bennet, 2000; O´Dwyer, 2003). However, the choice of the individual weighting of the three actions, i.e. proportional, integral and derivative has been a vexing problem in many applications, see Yu (1999). Moreover, the number of tuning parameters in more sophisticated PID modifications proposed in Åström, *et al.*, (1992) is even higher. The classical tuning algorithms were derived from Ziegler and Nichols method (Åström and Hägglund, 1995). The method is based on the ultimate cycle technique and it is not suitable for unstable and time delay systems. Then a period of transfer functions and polynomial utilization started and algebraic notions as rings, ideals, linear (Diophantine, Bezout) equations seemed to be a powerful tools (Grimble and Kučera, 1996; O´Dwyer, 2003; Åström and Hägglund, 1995,). The necessity of robust control was naturally developed by the situation when the nominal plant (used in control design) differs from the real (perturbed and controlled) one. Hence, a polynomial description of transfer functions had to be replaced by another one. A convenient description adopted from (Vidyasagar, 1985; Kučera 1993; Doyle, *et al.*, 1992) is a factorization approach where transfer functions are expressed as a ratio of two Hurwitz stable and proper rational functions. A suitable tool for parameter uncertainty is the infinity norm. Then, all stabilizing controllers are obtained and parameterized via linear equations in an appropriate ring. Robustness also influenced design and tuning of PID controllers (Morari and Zafiriou, 1989; Prokop and Corriou, 1997). The similar philosophy and tools can be employed for delay systems through the ring of meromorphic functions, see (Pekař and Prokop, 2011).

2. SYSTEM DESCRIPTION AND CONTROL DESIGN OVER RINGS

2.1 System description

The traditional engineering design approach of PID like controllers was performed either in the frequency domain or in polynomial representation, see e. g. Åström and Hägglund, (1995). Nevertheless, the fractional approach developed in (Vidyasagar, 1985; Kučera, 1993) and analyzed in (Prokop and Corriou, 1997; Prokop *et al.*, 2005, 2010) enables a deeper insight into control tuning and a more elegant expression of all suitable controllers.

It is well known that a set of polynomials is a ring. However, there are other rings which can be used for the system description. The fractional approach supposes that transfer functions of continuous-time linear causal systems in R_{PS} can be expressed as a ratio of two elements:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)} \qquad (1)$$

2.2 Control design

A typical control problem for a given system can be formulated as follows: Find a controller (or family of controllers) such that the feedback system is stable and some additional properties (tracking, disturbance rejection, optimality, strict properness,…) are fulfilled.

Consider a two degree-of-freedom (2DOF) control system depicted in Fig. 1. Note that the traditional one degree-of-freedom (1DOF, FB) system is obtained simply by $R=Q$.

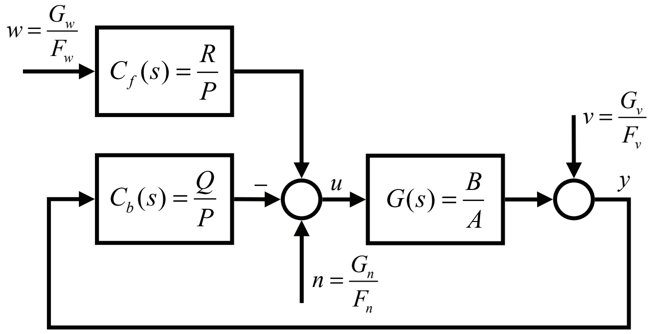

Fig. 1. Two-degree-of-freedom closed loop system

External signals $w=\frac{G_w}{F_w}$, $n=\frac{G_n}{F_n}$ and $v=\frac{G_v}{F_v}$ represent the reference, load disturbance and disturbance signal, respectively. The general control law (2DOF structure) is governed by:

$$P(s)u(t)=R(s)w(t)-Q(s)y(t) \tag{2}$$

Basic relations following from Fig. 1 are

$$y=\frac{B}{A}u+v \qquad u=\frac{R}{P}w-\frac{Q}{P}y \tag{3}$$

where w, v are independent inputs into the closed loop system (reference and disturbance). Further, the following equations hold:

$$y=\frac{AP}{AP+BQ}\frac{G_v}{F_v}+\frac{BR}{AP+BQ}\frac{G_w}{F_w} \tag{4}$$

$$e=w-y=\frac{AP}{AP+BQ}\frac{G_v}{F_v}+\left(1-\frac{BR}{AP+BQ}\right)\frac{G_w}{F_w} \tag{5}$$

For the 1DOF structure the last relation gives the form:

$$e=\frac{AP}{AP+BQ}\frac{G_v}{F_v}+\frac{AP}{AP+BQ}\frac{G_w}{F_w} \tag{6}$$

The basic task is now to ensure internal stability of the system in Fig. 1. All stabilizing feedback controllers are given by all solutions of the linear Diophantine equation – see (Vidyasagar, 1985; Kučera, 1993):

$$AP+BQ=1 \tag{7}$$

with a general solution $P=P_0+BT$, $Q=Q_0-AT$; where T is free in R_{PS} and P_0, Q_0 is a pair of particular solutions (Youla – Kučera parameterization of all stabilizing controllers). Details and proofs can be found e.g. in (Vidyasagar, 1985; Kučera, 1993; Prokop and Corriou, 1997). Then (5) takes the form:

$$e=AP\frac{G_v}{F_v}+(1-BR)\frac{G_w}{F_w} \tag{8}$$

For asymptotic tracking then follows:

a) F_w divides P for 1DOF

b) F_w divides *(1-BR)* for 2DOF

The last condition and the 2DOF structure give the second Diophantine equation in the form:

$$F_wS+BR=1 \tag{9}$$

Another control problem of practical importance is disturbance rejection and disturbance attenuation. In both cases, the effect of disturbances v and n should be asymptotically eliminated from the plant output. Since the both disturbances are external inputs into the feedback part of the system, the effect must be processed by a feedback controller. In other words, a multiple F_v, F_n must divide P. The details are discussed in Kučera (1993), Prokop and Corriou (1997), Matušů and Prokop, (2010).

2.4 PI and PID controllers

Simplest cases for SISO systems (1) is a first order controlled system (plus time delay FOPDT)

$$G(s)=\frac{K}{Ts+1}\cdot e^{-\Theta s} \tag{10}$$

and a second order equivalent transfer function SOPDT has the form:

$$G(s)=\frac{K}{(T^2s^2+2T\xi s+1)^2}\cdot e^{-\Theta s} \tag{11}$$

Diophantine equation (7) for the first order systems (1) without a time delay term can be easily transformed into polynomial equation:

$$(Ts+1)p_0+Kq_0=s+m \tag{12}$$

with general solution:

$$\begin{aligned}P&=\frac{1}{T}+\frac{K}{s+m}\cdot Z\\ Q&=\frac{Tm-1}{TK}-\frac{Ts+1}{s+m}\cdot Z\end{aligned} \tag{13}$$

where Z is free in the ring R_{PS}. Asymptotic tracking is achieved by the choice $Z=-\frac{m}{TK}$ and the resulting PI controller is in the form:

$$C(s)=\frac{Q}{P}=\frac{q_1s+q_0}{s} \tag{14}$$

where parameters q_1 a q_0 are given by:

$$q_1=\frac{2Tm-1}{K} \qquad q_0=\frac{Tm^2}{K} \tag{15}$$

For the SOPDT the design equation takes the form:

$$(T^2s^2+2T\xi+1)^2\cdot(p_1s+p_0)+K\cdot(q_1s+q_0)=(s+m)^3 \tag{16}$$

and after similar manipulations the resulting PID controller gives the transfer function:

$$C(s)=\frac{Q}{P}=\frac{q_2s^2+q_1s+q_0}{s(s+p_0)} \tag{17}$$

For both systems FOPDT and SOPDT the scalar parameter *m>0* seems to be a suitable „tuning knob" influencing control behavior as well as robustness properties of the closed loop system. The derivation for SOPDT can be found in Prokop et al. (2011). Naturally, both derived controllers correspond to classical PI and PID ones. It is clear that (14) represents the PI controller and (17) represents a PID in the standard four-parameter form, see e.g. Åström and Hägglung (1995).

3. CONTROLLER TUNING

3.1 Aperiodic PI controllers

Control engineers have produced an abundant number of controller tuning rules over the past 60 years after the introduction of Ziegler-Nichols rule in 1942. Hundreds of them are collected and listed in table form in O´Dwyer, (2003), many of them for PI controllers. A frequent requirement from control applications is aperiodic behavior for controlled outputs. A simple choice for the tuning parameter $m>0$ can be easily derived analytically in the case of FOPDT and PI controller in the R_{PS} representation. The proposed tuning rule is then used for an autotuning application.

For the FOPDT with $\Theta=0$ and PI controller (14)-(15), the closed-loop transfer function K_{wy} is:

$$K_{wy} = \frac{BQ}{AP+BQ} = BQ = \frac{(2Tm-1)s+Tm^2}{(s+m)^2} \tag{18}$$

The step response of (18) can be expressed by Laplace transform:

$$h(t) = L^{-1}\left\{\frac{K_{wy}}{s}\right\} = L^{-1}\left\{\frac{k_1 s + k_0}{s(s+m)^2}\right\} =$$

$$= L^{-1}\left\{\frac{A}{s} + \frac{B}{(s+m)} + \frac{C}{s+m)^2}\right\}, \tag{19}$$

where A,B,C are calculated by comparing appropriate fractions in (22) and $k_1=2mT-1$, $k_0=Tm^2$. The response $h(t)$ in time domain is then

$$h(t) = A + Be^{-mt} + Cte^{-mt} \tag{20}$$

The overshoot or undershoot of this response is characterized by the first derivative condition $h'(t)=0$. The time of the extreme of response $h(t)$ is then easily calculated as:

$$t_e = \frac{C-mB}{mC} = \frac{1}{m} - \frac{B}{C} \tag{21}$$

Since the aperiodic response means that the extreme does not exist for positive t_e, it implies $t_e < 0$ and after all substitutions of A,B,C, k_1, k_0 relation (21) takes the simple form

$$1 < m\frac{B}{C} = \frac{1}{\frac{1}{Tm} - 1} \tag{22}$$

The denominator of (22) must be positive and less than 1 and $m>0$ which implies the inequality:

$$\frac{1}{2T} < m < \frac{1}{T} \tag{23}$$

Any parameter $m>0$ from (22) ensures the aperiodic response. It is a question for further investigation and simulation what choice from interval (22) is the best one. For autotuning philosophy time constant T is always estimation and then the middle value of (22) would be reasonable, it means

$$m = \frac{3}{4 \cdot T} \tag{24}$$

Also other tuning principles for aperiodic tuning certainly exist. For the mentioned algebraic synthesis, the equalization method developed by Gorez and Klán (2000) can be adopted. The idea goes out from PI controller in the form (16). The tuning rule is very simple and it leads in relations:

$$K_P = \frac{1}{2K} \quad T_I = 0.4 \cdot T_u \tag{25}$$

where K is a process gain and T_u is the ultimate period obtained from the Ziegler-Nichols experiment. However, the fulfillment of (23) by unique value of $m>0$ is impossible, see Prokop, et al. (2010). The exact fulfillment of both relations in (23) could be obtained in the case of distinct poles at the right side of (16).

3.2 Autotuning

The estimation of the process or ultimate parameters is a crucial point in all autotuning principles. The relay feedback test can utilize various types of relay for the parameter estimation procedure. The classical relay feedback test (Åström and Hägglund, 1984) was proposed for stable processes by symmetrical relay without hysteresis. Following sustained oscillation is then used for determining the critical (ultimate) values and the standard tuning rule is then utilized.

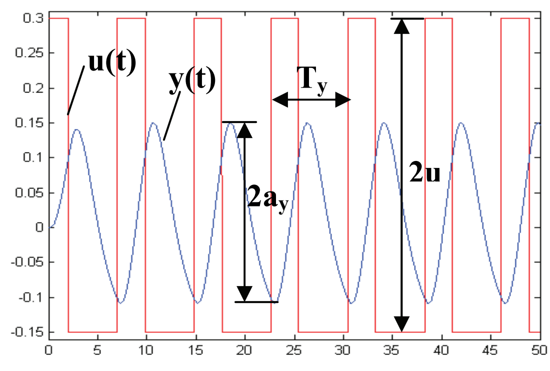

Fig. 2. Asymmetrical relay oscillation of stable process

Asymmetrical relays with or without hysteresis bring further progress (Yu, 1999; Kaya and Atherton, 2001). After the relay feedback test, the estimation of process parameters can be performed. A typical data response of such relay experiment is depicted in Figure 2. The relay asymmetry is required for the process gain estimation in (10), (11) while a symmetrical relay would cause the zero division in the appropriate formula.

In this paper, an asymmetrical relay with hysteresis was used. This relay enables to estimate transfer function parameters as well as a time delay term. For the purpose of the aperiodic tuning the time delay was not exploited. For both, FODPT and SOPDT the term K is estimated by

$$K = \frac{\int_0^{iT_y} y(t)dt}{\int_0^{iT_y} u(t)dt}; \quad i = 1,2,3,... \tag{26}$$

The time delay terms for FODPT and SODPT are given by (Vítečková and Víteček, 2005):

$$\Theta = \frac{T_y}{2\pi} \cdot \left[\pi - arctg \frac{2\pi T}{T_y} - arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \tag{27}$$

where a_y and T_y are depicted in Fig.2 and ε is hysteresis of the relay. Then the time constants for FOPDT and SOPDT term can be estimated according to (Vítečková and Víteček, 2005) by the relations:

$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{16 \cdot K^2 \cdot u_0^2}{\pi^2 \cdot a_y^2} - 1}$$
$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{4 \cdot K \cdot u_0}{\pi \cdot a_y} - 1} \tag{28}$$

The second equality is valid for SOPDT with $\xi=0$.

3.3 Uncertainty and robustness

Uncertainty, inaccuracy and approximations are notions accompanying control theory for decades. There are two principal uncertainty types and consequently two sort of uncertain models. Parametric uncertainty model is often used when precise knowledge of the actual parameters is not known, while nonparametric model of uncertainty is more suitable at disregarding of fast dynamics, nonlinearities, etc.

Systems with parametric uncertainty can be described by several basic types of uncertain models with the following inclusion: interval ⊂ affine linear ⊂ multilinear ⊂ polynomic. In this paper, only the case of controlled plant with interval uncertainty, which leads to the closed-loop polynomial with linear affine uncertainty structure, is considered in e.g. Barmish, (1994), Tan and Atherton (2000), Matušů and Prokop, (2005).

Robust stability analysis of the closed loop system connecting a plant with parametric uncertainty and a fixed controller can be studied through a closed-loop characteristic polynomial. If this polynomial is stable not only for nominal values of uncertain parameters or particular set of them but for all values $q_i \in \left[q_i^-; q_i^+\right]$, then it is called robustly stable.

The important and effective tool used for investigation of robust stability is the value set concept. Suppose an uncertain polynomial $p(s,q)$ and an uncertainty bounding set Q, then, at a fixed frequency $\omega \in R$, the value set is the subset of the complex plane consisting of all values which can be assumed by $p(j\omega,q)$ as q ranges over Q. Said another way, $p(j\omega,Q)$ is the range of $p(j\omega,\cdot)$. The value set concept is applied in following fundamental theorem which is known as the zero exclusion condition. Assume that a family of polynomials $P=\{p(s,q): q \in Q\}$ has invariant degree with associated uncertainty bounding set Q which is pathwise connected, continuous coefficient functions $q_i(q)$ for i = 0,1,2,…,n and at least one stable member $p(s,q^0)$. Then P is robustly stable if and only if the origin, z = 0, is excluded from the value set $p(j\omega,Q)$ for all frequencies $\omega \geq 0$; i.e. P is robustly stable if and only if $0 \notin p(j\omega,Q)$ for all frequencies $\omega \geq 0$. The great advantage of the value set concept and combination with zero exclusion theorem is that its usage is general-purpose. In fact, there is no need to know any analytical results for given uncertainty structure and only value sets have to be computed. Further details of the zero exclusion condition and subsequent results can be find e.g. in (Barmish, 1994).

Analysis for interval polynomials can be studied with help of the Kharitonov theorem (Kharitonov, 1979). The Kharitonov theorem states that the stability of an interval polynomial can be determined by testing the stability of just four polynomials which can be easily obtained using upper and lower values of the uncertain parameters. Thus, the value sets are called Kharitonov rectangles.

Typical robust stability instruments for the case of uncertain polynomial with affine linear structure are the edge theorem, the thirty-two edge theorem and the sixteen plant theorem (Barmish, *et al.*, 1992). An alternative approach for robust stability investigating is to "overbound" original more general and complicated structure by the simple interval polynomial. Thus, the dependence of parameters is ignored. Then, by testing the stability of the "overbounding" interval polynomial, the sufficient condition (with some conservatism) for the stability of the original uncertainty structure is obtained.

Practical robust stability checking via the value sets and zero exclusion condition can be easily and comfortable done with assistance of Polynomial Toolbox for Matlab – some basic routines description can be found in (Šebek, *et al.*, 2000).

4. EXAMPLES

Example 1: The first example demonstrates the utilization of proposed methodology in parts 2.4 and 3.1 in an autotuning application. A fourth order system is supposed with the transfer function

$$G(s) = \frac{2}{(0.5s+1)^4} \tag{29}$$

Then the asymmetrical relay tests gave according to (26)-(28) a FODPT and SODPT estimated transfer functions:

$$\tilde{G}(s) = \frac{2}{1.54s+1} \cdot e^{-0.98s}, \quad \tilde{\tilde{G}}(s) = \frac{2}{0.66s^2 + 1.62s + 1} \cdot e^{-0.58s} \tag{30}$$

Step responses for comparison of approximation are shown in Fig. 3. Then both transfer functions (33) without time delay term Θ were taken for control synthesis according to the methodology mentioned in 2.4 and a PI and PID controllers were designed. The control responses of (33) in the 1DOF structure for both controllers are depicted in Fig. 4.

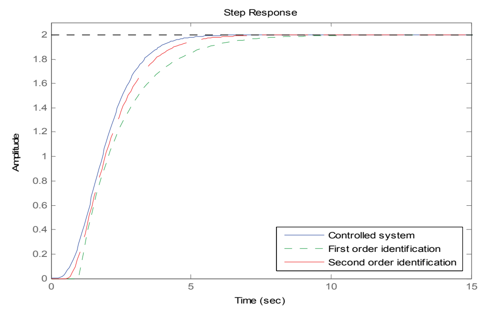

Fig. 3. Step responses of systems (32) – (33)

The control responses confirm two facts. The first one is that the aperiodic response is achieved also in the case where the control system is of higher order and the approximated system is of the first order with aperiodic PI tuning. The second one declares that a second order approximation does not have to achieve a better response than a first one.

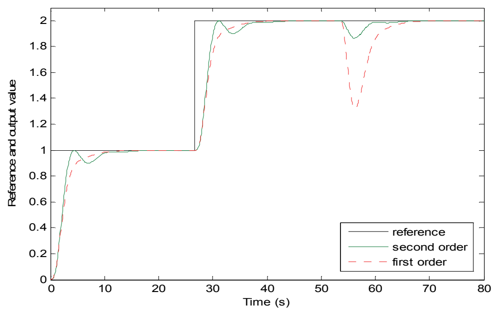

Fig. 4. Control responses

<u>Example 2:</u> The example illustrates the robust methodology mentioned in part 3.3. A controlled system is assumed to be given as a second order interval transfer function:

$$G(s, b_j, a_i) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{31}$$

where $b_1, b_0, a_1, a_0 \in [0.5;\ 1.5]$. The system (31) with parameters $b_1 = b_0 = a_1 = a_0 = 1$ is supposed as a nominal one. The 2DOF asymptotic tracking controller was designed and tuned with m=1. The coefficients can be computed according to relations in Prokop, *et al.* (2002). Thus the controller parts take the form:

$$C_b(s) = \frac{Q(s)}{P(s)} = \frac{2s^2 + 2s + 1}{s^2 + s} \tag{32}$$

$$C_f(s) = \frac{R(s)}{P(s)} = \frac{s^2 + 2s + 1}{s^2 + s} \tag{33}$$

The closed-loop characteristic polynomial of the control system (Fig. 1) with feedback part of the controller (32) and interval plant (31) has the affine linear uncertainty structure. Robust stability of this polynomial has been investigated via two principal ways. The first method consists in construction of the "overbounding" interval polynomial using minimal and maximal possible values of polynomial coefficients and consequent robust stability test of the resulting interval polynomial. The main advantage of this approach is simplicity of applied tools. On the other hand, the investigation of robust stability for "overbounding" interval polynomial can be superfluously conservative as will be shown later. The other utilized technique then directly verifies the robust stability of the original polynomial with affine linear uncertainty structure.

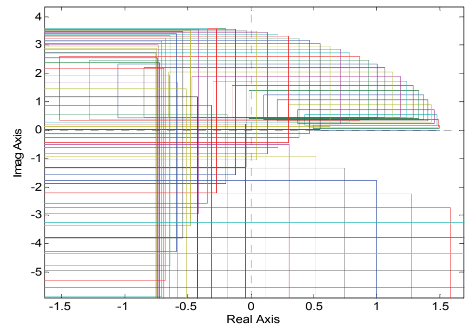

Fig. 5. Kharitonov rectangles

Kharitonov rectangles of "overbounding" interval polynomial are shown in Fig. 5. Fig. 6 then presents corresponding results of robust stability analysis represented by Kharitonov polynomials, coefficients of "overbounding" interval polynomial and comment of the outcomes. Both figures declare that the feedback controller (32) is not robustly stabilizing for $b_1, b_0, a_1, a_0 \in [0.5;\ 1.5]$.

Overbounding interval polynomial:

Interval polynomial (minus) = 0.5000 + 2s + 3s^2 + 2.5000s^3 + 1.0000s^4
Interval polynomial (plus) = 1.5000 + 6s + 9s^2 + 5.5000s^3 + 1.0000s^4

Kharitonov polynomials:

K1 = 0.5000 + 2s + 9s^2 + 5.5000s^3 + 1.0000s^4
K2 = 1.5000 + 6s + 3s^2 + 2.5000s^3 + 1.0000s^4
K3 = 1.5000 + 2s + 3s^2 + 5.5000s^3 + 1.0000s^4
K4 = 0.5000 + 6s + 9s^2 + 2.5000s^3 + 1.0000s^4

The overbounding closed-loop characteristic interval polynomial IS NOT STABLE!

The original (linear affine) closed-loop characteristic polynomial IS STABLE!

Fig. 6. Results of robust stability analysis

However, this conclusion is not true and it proves the superfluous conservatism of the overbounding method.

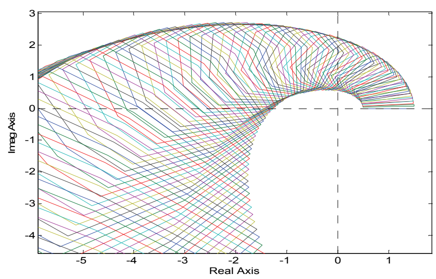

Fig. 7. Value sets for original affine linear structure

The original value sets of the closed-loop polynomial with linear affine uncertainty structure is shown in Fig. 7 and the robust stability is achieved (zero exclusion theorem). Control behavior of the selected amount of systems from the family given by controlled interval system can be seen in Fig. 8.

Control responses prove the robust stability of proposed 2DOF controller (32), (33).

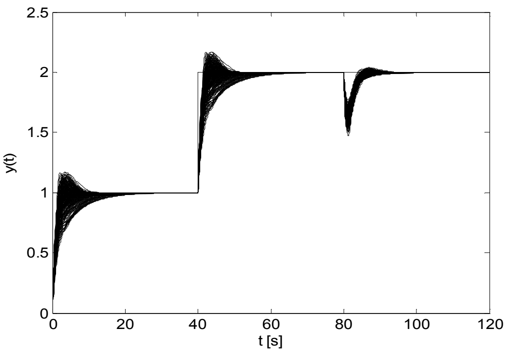

Fig. 8. Closed-loop control behavior

6. CONCLUSIONS

The paper shows the design and tuning of simple controllers for continuous-time SISO systems by algebraic tools. The control synthesis is based on general solutions of linear equations in the ring of proper and Hurwitz stable rational functions (R_{PS}). Two structures (1DOF, 2DOF) of the controlled system are considered. The proposed methodology enables to tune and influence the robustness and control behavior by a single scalar parameter $m>0$. The contribution shows how it is possible to utilize the tuning parameter for aperiodic tuning and for robust design. Robust stability is studied through the value set concept, zero exclusion condition and Kharitonov theorem. For both aims, the program systems designed in Matlab, Simulink and Polynomial Toolbox 2.5 environment were developed. Two illustrative examples demonstrate their capabilities.

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