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A look into *robustness/performance* and *servo/regulation* issues in PI tuning ⋆

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Abstract: This paper addresses the model-based tuning of Proportional-Integral (PI) controllers focusing on the *robustness/performance* and *servo/regulation* trade-offs. First, a tuning rule is derived analytically by solving a simple $\mathcal{H}_\infty$ weighted sensitivity problem, where the weight is chosen so that two design parameters permit to adjust the considered conflicting objectives. This way, the resulting tuning expressions show clearly how the controller's parameters should be changed to shift each trade-off, giving insight into the tuning task. Then, we proceed to study the two trade-offs at hand and the interplay between them.

Keywords: Servo/Regulator control, Robustness, Performance, PI controllers, H-infinity control

1. INTRODUCTION

The design of feedback compensators is challenging because the different conflictive objectives demand a *good balance* which may be difficult to achieve. For example, assuming the classical unity feedback scheme, we know that, in general, the higher the *performance*, the lower the *robustness* (Boulet and Duan, 2007). On the other hand, the feedback controller faces both *servo* (set-point tracking) and *regulatory* (input or "load" disturbance rejection) performance specifications (Skogestad and Postlethwaite, 2005), which are usually conflictive too. Therefore, for design purposes, the system designer should take into account both *robustness/performance* and *servo/regulation* issues. This is precisely the theme of this paper, where we look at these two *inherent* trade-offs from an $\mathcal{H}_\infty$ control perspective. Note that, because disturbances entering at the output of the plant can be regarded as unmeasured set-point changes, the servo/regulation trade-off should also be considered in the context of Two-Degree-of-Freedom control schemes (Alcántara et al., 2011a).

To make the discussion as clear and insightful as possible, an specific control setup is adopted from Alcántara et al. (2011b) which leads to a controller of Proportional-Integral (PI) type. As a result, it is possible to establish a link between the considered $\mathcal{H}_\infty$ (weighted sensitivity) problem and the PI parameters, somehow bridging the gap between *practical* (e.g., PID) control and more *academic* methods. Not surprisingly, there have been other attempts to establish such a link. For example, a loop shaping approach is followed in Tan et al. (1998); Panagopoulos and Åström (2000); Boulet and Duan (2007), whereas Vilanova (2008) presents an analytical model matching strategy, to cite just a few. Essentially, these approaches deal solely with the robustness/performance trade-off. In this regard, a distinguishing feature of the suggested procedure is that servo and regulatory performance aspects are also dealt with explicitly within the problem formulation.

Note that, although tuning rules aimed at optimizing set-point tracking or disturbance rejection responses (with or without imposing robustness constraints) are very common: Zhuang and Atherton (1993); Visioli (2001); Astrom and Hagglund (2004); Vilanova (2008); Sanchís et al. (2010), it is much less common to find tuning rules obtained under the spirit of *balanced* servo/regulation operation. Two main alternative routes have been followed in the literature:

- *Analytical approach*, for example, based on Internal Model Control (IMC) (Morari and Zafiriou, 1989). A representative sample of this category is given by Lee and Shi (2002); Skogestad (2003); J.Shi and W.S.Lee (2004). The basic idea here is to increase the integral gain of the controller in IMC-based (*servo-type*) settings to yield improved (load) disturbance response. In particular, Skogestad (2003) proposes an analytical expression for the integral gain, which eventually results into remarkably simple and effective *balanced* tuning rules.
- *(Numerical) Optimization-based approach*. In this group, we have, for example, the works by Arrieta et al. (2010); Arrieta and Vilanova (2011). In Arrieta et al. (2010), *extreme* tuning rules aimed at servo and regulation operation are considered in order to find an *intermediate tuning*. As for Arrieta and Vilanova (2011), a more conventional optimization approach is adopted, including a robustness constraint through the peak of the sensitivity function.

Here, we revisit the analytical tuning rules derived in Alcántara et al. (2011b). A salient feature of them is that both servo/regulation and robustness/performance considerations are taken into account by means of two design param-

⋆ This work is a sequel of the previously published article by Alcántara et al. (2011b).

eters. Based on their clear engineering meaning, a numerical optimization study (along the lines of Arrieta and Vilanova (2011) or Grimholt (2010)) is conducted afterwards with a two-fold purpose: first, to provide tuning guidelines; second, to understand both the robustness/performance and servo/regulator trade-offs more deeply, as well as the interplay between them. These points were left open in Alcántara et al. (2011b) and constitute the main contribution here, where different performance indices for balanced servo/regulation operation are examined.

Finally, it should be emphasized that, even if Proportional-Integral-Derivative (PID) controllers have been around for more than seven decades, they still constitute the most extended form of feedback in the process industry today (Kano and Ogawa, 2010), with a dominance of PI's over PID's. Also, it is noteworthy that the simplicity of such controllers — there are only two tuning parameters in the PI case; ideally *three* if derivative action is added — means anything but that it is easy to find good settings for them (Skogestad, 2003), as confirmed by the larger and larger number of tuning rules appearing in the literature (Astrom and Hagglund, 2005). This paper partly aims at giving some new insight into the problem.

2. $\mathcal{H}_\infty$ SETUP FOR PI TUNING

Consider the basic unity feedback configuration depicted in Fig. 1: P and K are the plant and the controller, respectively, and

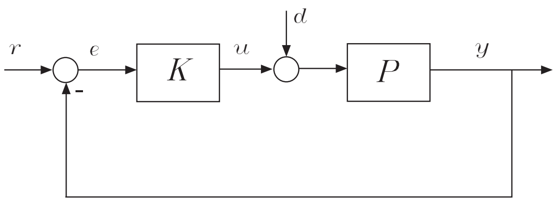

Fig. 1. Basic feedback configuration.

r, y, u, e, d denote (in the same order) the reference, output, control, error and input (or load) disturbance signals. A basic problem in $\mathcal{H}_\infty$ control is the weighted sensitivity problem (Zames and Francis, 1983; Skogestad and Postlethwaite, 2005):

$$\min_{K\in\mathcal{C}} \|\mathcal{N}\|_\infty = \min_{K\in\mathcal{C}} \|WS\|_\infty \tag{1}$$

where

- $\mathcal{C}$ denotes the set of internally stabilizing controllers.
- $\|\mathcal{N}\|_\infty \doteq \sup_\omega |\mathcal{N}(j\omega)|$ (the peak of the magnitude frequency response).
- S is the sensitivity function: $S = \frac{1}{1+PK}$.
- W is a weight responsible for the shaping of S (*the design key point*).

The control setup is completed by choosing

- An stable First Order Plus Time Delay (FOPTD) model for the plant (Skogestad, 2003; Astrom and Hagglund, 2005), i.e.:
$$P = K_g \frac{e^{-sh}}{\tau s + 1} \tag{2}$$
where K_g, τ, h represent the gain, time constant and (effective) delay of the process.
- A weight of the form
$$W = \frac{(\lambda s + 1)(\gamma s + 1)}{s(\tau s + 1)} \tag{3}$$

where $\lambda > 0, \gamma \in [\lambda, \tau]$. The rationale behind this choice is explained next. We start by considering $\lambda \approx 0$, then

- If $\gamma = \tau$, $|W| = |1/s|$, and the optimization problem (1), taking P as in (2), is equivalent to $\min_{K\in\mathcal{C}}\|S\|_\infty$ subject to integral action. As $e = Sr$, this choice of γ corresponds to a *servo* specification (Kristiansson and Lennartson, 2006).
- If $\gamma = \lambda$, $|W| = \frac{1}{|K_g|}|P/s|$. Since the constant $\frac{1}{|K_g|}$ plays no role, the optimization problem (1) is now equivalent to $\min_{K\in\mathcal{C}}\|PS\|_\infty$ subject to integral action. As $e = -PSd$, this choice of γ corresponds to a *regulation* objective (Kristiansson and Lennartson, 2006).
- Intermediate values of γ correspond to a balance between the purely servo and regulation situations.

As we increase λ, the minimization of $|S|$ at higher frequencies is emphasized, preventing large peaks on S at the expense of closed-loop bandwidth. Thus, once γ is fixed, λ can be used to select a compromise between robustness and performance.

For the simple considered setup, the analytical solution can be easily found[1]. In particular, if the approximation $e^{-sh} \approx -sh + 1$ is used for the time delay, a PI controller is obtained:

$$K = K_c\left(1 + \frac{1}{T_i s}\right) \tag{4}$$

The expressions for the controller parameters K_c, T_i have been collected in Table 1.

Table 1. (WS)PI tuning rule.

K_c	T_i	
$\frac{1}{K_g}\frac{T_i}{\lambda+\gamma+h-T_i}$	$\frac{\tau(h+\lambda+\gamma)-\lambda\gamma}{\tau+h}$	$\lambda > 0, \gamma \in [\lambda, \tau]$

3. ROBUSTNESS/PERFORMANCE AND SERVO/REGULATION ISSUES

In this section, we examine how to select the λ and γ parameters to yield a *well-balanced* closed-loop. Let us start by illustrating how these parameters affect the final design. In view of Fig. 2, the roles of λ and γ are intuitively clear: λ has a direct connection with the closed-loop bandwidth, and it can be mainly used to adjust the compromise between robustness and performance.

Remark 1. In this paper, the closed-loop bandwidth, w_B, is taken as the frequency where $|S(j\omega)|$ first crosses $1/\sqrt{2} = 0.707$ (≈-3dB) from below (Skogestad and Postlethwaite, 2005).

On the other hand, the primary concern of γ is that of balancing the servo/regulation properties. Having said that, one can easily imagine that there exists some interaction between λ and γ. To clarify this point, we first recall that the peak of the sensitivity function

$$M_S \doteq \|S(j\omega)\|_\infty \doteq \sup_\omega \left|\frac{1}{1+L(j\omega)}\right| \tag{5}$$

is usually used as a robustness indicator. This comes from the fact that M_S equals the inverse of the (shortest) distance from the Nyquist curve to the critical point $-1+0j$. Thus, the lower the value of M_S, the higher the robustness level (in practice, $M_S \in [1.4, 2]$ for acceptable designs). Now, from Fig. 2 one

[1] Consult Alcántara et al. (2011b) for the details

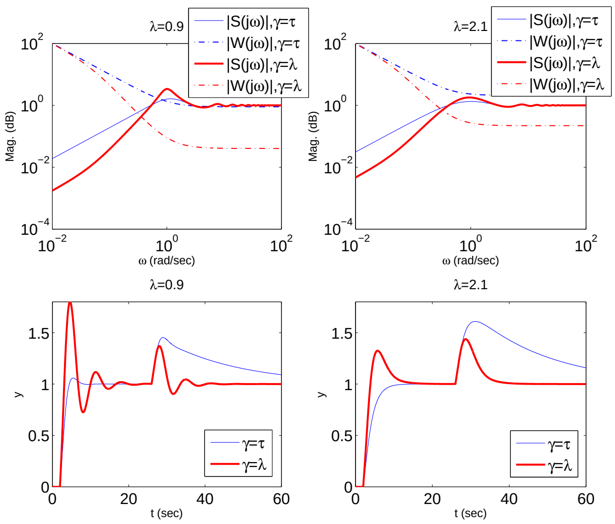

Fig. 2. Influence of λ and γ parameters on closed-loop properties for $P = \frac{5e^{-s}}{20s+1}$: frequency domain (top) and time domain responses for a unity set-point change and load disturbance at $t = 1$ and $t = 25$, respectively (bottom).

can see that decreasing γ to improve the regulatory performance brings as a consequence an increment in M_S and (albeit less noticeable) in the closed-loop bandwidth w_B. In summary, improving the regulatory performance via γ has a deteriorating effect on (*midfrequency*) robustness.

3.1 Servo/Regulation trade-off (γ tuning)

It has been pointed out that, although it is natural to use γ for servo/regulation adjustments, some caution is necessary to keep the robustness level intact. If we select M_S as the robustness indicator, some retuning for λ will be necessary after modifying the value of γ. The idea is that, if we reduce γ for regulatory improvement, λ must be increased to compensate for the robustness loss, see Fig. 3. Therefore, for each robustness level, we can define

$$\Lambda\Gamma_k \doteq \{(\lambda, \gamma); M_S = k, \lambda > 0, \gamma \in [\lambda, \tau]\} \qquad (6)$$

The problem we study now is how to select γ (restricted to $\Lambda\Gamma_k$) to yield a good balance between servo and regulatory performance. Before embarking on the tuning of γ, we note that the length of the interval $[\lambda, \tau] \ni \gamma$ can be taken as an indication of how stringent is the trade-off between servo and regulation operation. In this sense, as we increase λ to make the system more robust, the trade-off becomes less stringent. This can also be appreciated from Fig. 3. Also, the lower the value of τ, the less stringent the trade-off. This is explained next:

- If we go to the extreme $\tau \to 0$ (a pure dead-time process, $P \approx K_g e^{-sh}$), one has the influence on the error of the reference and the disturbance are (except for a time delay) exactly the same. Therefore, for $h/\tau \gg 1$, there is no trade-off between set-point and disturbance response (only *smooth/tight* control aspects (Skogestad, 2006)).
- The trade-off gets more pronounced as we go to the other extreme: $\tau \to \infty$, corresponding to an integrating process ($P \approx K'_g \frac{e^{-sh}}{s}, K'_g = K_g/\tau$). In this case, it is well-known that a proportional controller can yield excellent reference response; however, in such a case, the disturbance response would exhibit steady-state error.

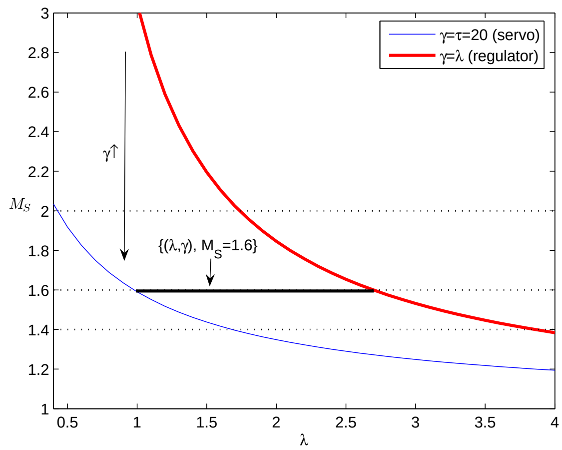

Fig. 3. M_S over λ for extreme values of the γ parameter and $P = \frac{e^{-s}}{20s+1}$.

Based on the second point above, we will set the focus on the range $h/\tau < 1$ (for $h/\tau \geq 1$, it is meaningless to bother about servo/regulation issues, see Remark 2).

Remark 2. For the FOPTD model (2), the main performance limitation comes from the time delay h. In this sense, an starting rule of thumb for the λ parameter could be choosing $\lambda \approx h$. Then, the interval for γ, $[\lambda, \tau]$, reduces to a point for balanced lead/lag processes (i.e., $h/\tau = 1$), because $\gamma \in [\lambda, \tau] = [h, h]$. This indicates that there is no need to worry about servo/regulation aspects in this case.

Remark 3. Note that, for $\gamma = \tau$, the tuning rule in Table 1 simplifies to

$$K_c = \frac{1}{K_g} \frac{\tau}{\lambda + h} \qquad T_i = \tau \qquad (7)$$

which coincides with the IMC-based PI settings (Skogestad, 2003). This observation, together with Remark 2, shows that, for $h/\tau \geq 1$, the conventional IMC approach will give good results in terms of servo/regulation considerations.

Choosing a performance index So far, we have gained some qualitative insight into the servo/regulation trade-off. The next step is to study how to set γ more systematically. For this purpose, we propose the following performance index to be minimized for balanced operation:

$$J_{sr} \doteq \max(\Delta_s, \Delta_r) \qquad (8)$$

where

$$\Delta_s = \frac{IAE_s}{IAE_s^o}, \Delta_r = \frac{IAE_r}{IAE_r^o} \qquad (9)$$

and

$$IAE \doteq \int_0^\infty |r(t) - y(t)|dt = \int_0^\infty |e(t)|dt \qquad (10)$$

In (9), the subindex $_{s(r)}$ stands for servo(regulator), and indicates that the IAE is calculated with respect to set-point(load disturbance)[2]. Finally, the superindex ^{o} is used to indicate the best attainable IAE along with $\Lambda\Gamma_k$. Note that, ideally, we want $J_{sr} = 1$ (this is the case where both optimal set-point and disturbance responses are achieved). In practice, however, $J_{sr} > 1$ because of the inherent servo/regulation trade-off.

[2] Throughout this study, both the reference r and the disturbance d are assumed to be unity step signals unless otherwise stated.

Tuning of γ based on J_{sr} Once the performance index J_{sr} has been defined, we can proceed to the systematic tuning of γ as illustrated in Fig. 4. To impose a *medium-high* robustness level, we took $k = 1.6$ (i.e., $M_S = 1.6$). In this example,

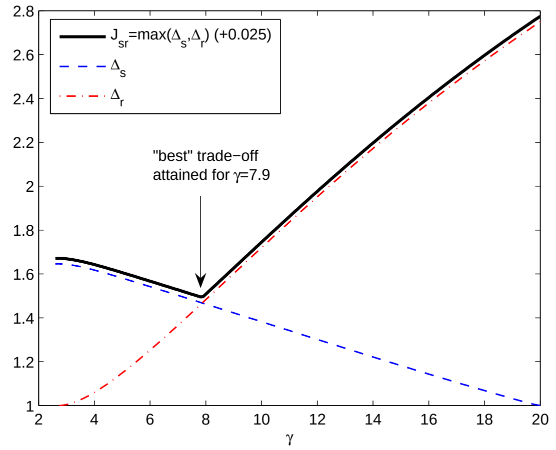

Fig. 4. Tuning of γ based on J_{sr} for $P = \frac{e^{-s}}{20s+1}$ $(h/\tau = 1/20 = 0.05)$. For every γ, λ is such that $(\lambda, \gamma) \in \Lambda\Gamma_k$.

the *best* trade-off is obtained for $\gamma = 7.9$. As we are moving through $\Lambda\Gamma_k$, the value for $\lambda(= 1.27)$ is also obtained from the conducted experiment.

3.2 Robustness/Performance trade-off (λ tuning)

Given a robustness level ($M_S = k$), the index J_{sr} was defined for balanced servo/regulation performance. In order to obtain a well-balanced tuning for the controller, we have also to ask for a good trade-off between robustness and performance. In previous subsections, a robustness level $M_S = 1.6$ was considered for all the simulations. We can now wonder whether $M_S = 1.6$ gives a good compromise between robustness and (servo/regulator) performance. In Fig. 5, we depict the relation between robustness (in terms of $M_S = k$, $k = 1.4..2$) and performance. In the latter case, we show both the servo and regulatory IAE's associated with the pair $(\lambda, \gamma) \in \Lambda\Gamma_k$ that minimizes J_{sr}. It is shown that, for the particular plant at hand,

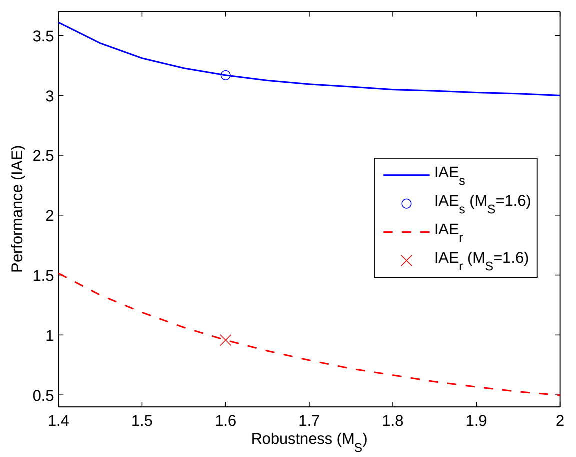

Fig. 5. Robustness vs Performance ($P = \frac{e^{-s}}{20s+1}$).

$M_S = 1.6$ (for which $\lambda = 1.27$) offers indeed a very good trade-off. To end this subsection, Fig. 6 matches M_S with the λ parameter and the closed-loop bandwidth w_B. As expected, increasing λ has a monotonic incluence on M_S and w_B; the larger the value of λ, the more robust and slower the closed-loop.

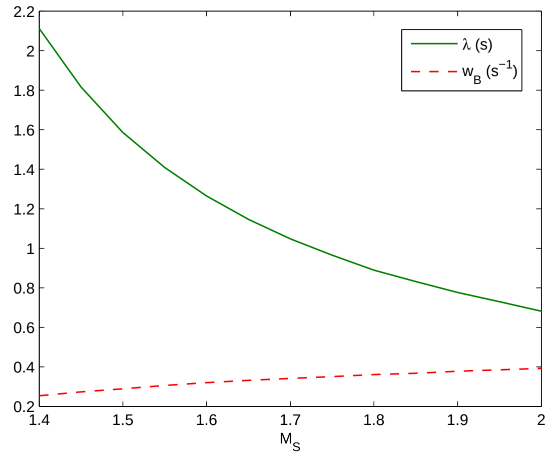

Fig. 6. Correlation between M_S, λ and w_B ($P = \frac{e^{-s}}{20s+1}$).

4. COMPARISONS AND EXAMPLES

In this subsection, we apply the procedure described in Section 3.1 for two different values of h/τ:

- $P_1 = \frac{e^{-s}}{5s+1}$, a slightly lag-dominant process ($h/\tau = 0.2$)
- $P_2 = \frac{e^{-s}}{30s+1}$, a lag-dominant process ($h/\tau = 0.033$)

For both P_1 and P_2, we will only consider $M_S = k = 1.6$ (however, it turns out that similar conclusions follow for other robustness levels). In addition, apart from J_{sr}, two alternative performance measures will be used for comparison purposes:

- $J_1 = \sqrt{(IAE_s - IAE_s^o)^2 + (IAE_r - IAE_r^o)^2}$
- $J_2 = \frac{1}{2}\left(\frac{IAE_s}{IAE_s^o} + \frac{IAE_r}{IAE_r^o}\right)$

The performance index J_1 is suggested in Arrieta and Vilanova (2011) and it is used there to derive automatic tuning rules for balanced servo/regulation operation. A first observation about J_1 is that it is gain-dependent. Regarding J_2, it has been recently reported that the SIMC PI tuning rule is close to (Pareto-)optimal with respect to it (Grimholt, 2010; Skogestad and Grimholt, 2012).

The results of applying the procedure in Section 3.1 to P_1 and P_2 are summarized in Figs 7–12. For P_1 (the slightly lag-dominant process), we see that the trade-off between servo and regulatory specifications is not very severe. Anyway, J_{sr} provides a compromise between them, and the same applies to J_1. On the other hand, J_2 coincides with the regulator mode tuning, and hence puts the emphasis on the regulation objective. For P_2 (the lag-dominated process), the servo/regulation trade-off becomes more stringent. Again, J_{sr} yields a reasonable compromise between set-point and disturbance response. Now, however, J_1 fails in the attempt, and leads to a very sluggish disturbance attenuation. As for J_2, it again results into a regulator-type tuning ($\gamma = \lambda$).

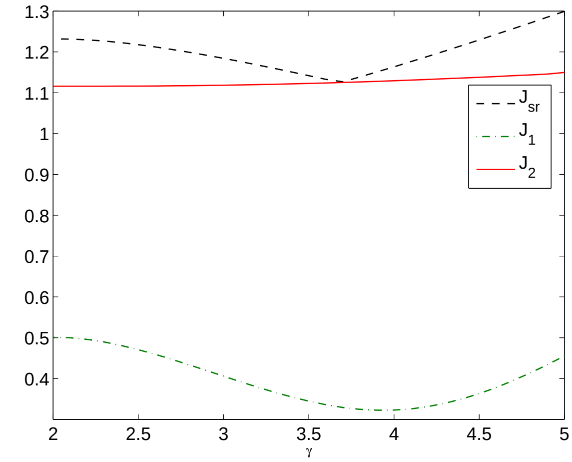

Fig. 7. Different performance measures for P_1.

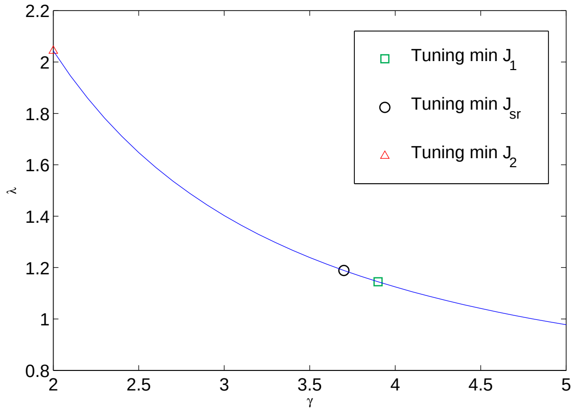

Fig. 8. $\Lambda\Gamma_k$ $(k = 1.6)$ for P_1.

Generally, J_{sr} yields good balanced operation (this has been confirmed for other h/τ ratios ranging from slightly lag-dominant to integrating plants). On the other hand, when using J_{sr}, there is always the danger of concentrating only on the worst case (recall the definition of J_{sr} given in (8)) and fail to get the *almost for free* benefits on the part not considered. In this regard, even if J_2 has resulted into regulatory-type tunings, this should not be interpreted as indicating a *poor* balance between servo and regulation objectives. The fact that J_2 tends to put the emphasis on regulatory performance just means that we get more benefit in terms of the IAE by stressing the regulatory performance. Besides, the balanced criterion J_2 has the advantage of giving a simple rule for the tuning of γ, that is, choosing $\gamma = \lambda$. Finally, although J_1 yielded a good trade-off for the slightly lag-dominated process, it failed in the more lag-dominant case. This, in conjunction with the fact that J_1 is gain-dependent, makes J_2 and J_{sr} preferable measures.

5. CONCLUSIONS

Based on $\mathcal{H}_\infty$ weighted sensitivity, this paper has addressed the balanced tuning of PI controllers in terms of the robustness/performance and servo/regulation trade-offs. First, an analytical tuning rule dependent on two intuitive parameters (λ, γ) has been revisited. The main reason for using the parameters λ, γ, instead of the controller's original ones K_c, T_i, is that they

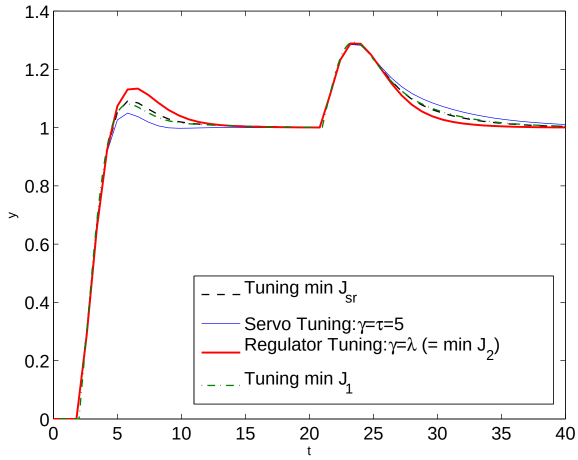

Fig. 9. Time domain responses for P_1: unity set-point change applied at $t = 1$; load-disturbance of magnitude 1 entering at $t = 20$.

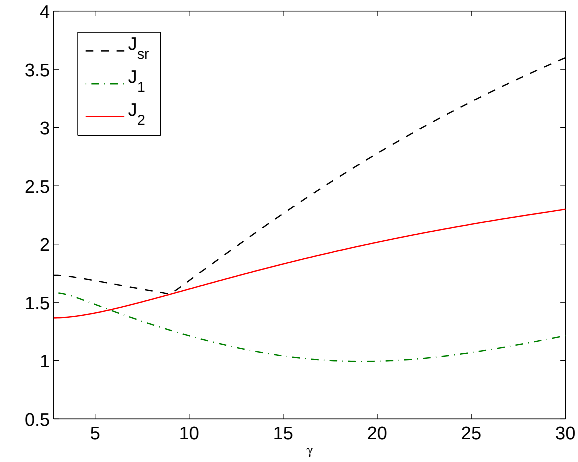

Fig. 10. Different performance measures for P_2.

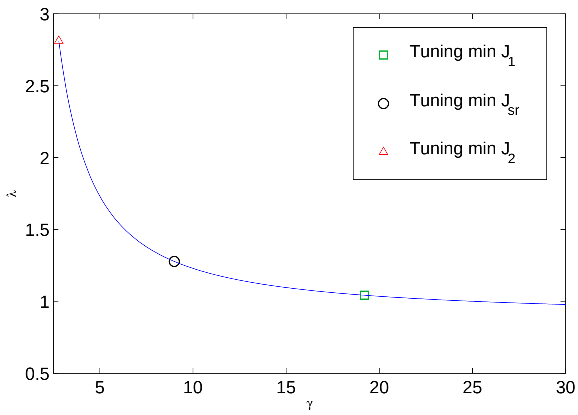

Fig. 11. $\Lambda\Gamma_k$ $(k = 1.6)$ for P_2.

are clearly associated with the considered trade-offs: roughly speaking, λ governs the compromise between robustness and performance, whereas γ is more related to servo/regulation considerations.

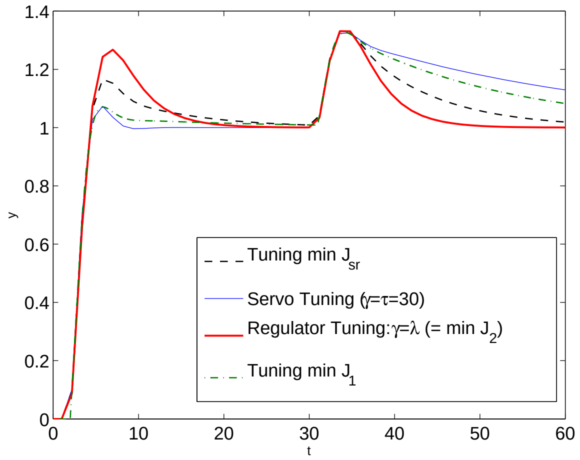

Fig. 12. Time domain responses for P_2: unity set-point change applied at $t = 1$; load-disturbance of magnitude 5 entering at $t = 30$.

Despite the intuitive nature of the λ, γ parameters, selecting good values for them is not so easy, in part because there is some interaction between them. In order to elucidate the interplay between servo/regulation and robustness issues, an optimization-based procedure has been presented to search for the *best* values of the tuning parameters. Here, *better* is understood in terms of different performance indices that capture well the asymmetry of the servo/regulation performance degradation. The overall conclusion is that both J_{sr} and J_2 are reasonable *balanced* performance indices. If the user chooses J_2, the tuning guideline is to go for the regulatory-type controller, hence providing a simple tuning guideline towards the best *balanced* choice of γ (i.e., $\gamma = \lambda$). Nevertheless, if the user wants to put more emphasis on the servo performance (to improve the reference tracking or the rejection of output disturbances), then the newly introduced J_{sr} index provides a sensible alternative.

To continue this work, the insight gained within the weighted sensitivity framework will be fully exploited, taking advantage of it to obtain well-balanced (automatic) tuning rules for both PI and PID controllers. In addition, unstable processes are planned to be included in the disussion too.

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