IFAC Conference on Advances in PID Control
PID'12
Brescia (Italy), March 28-30, 2012 FrPS.9

Automatic design of robust PID controllers based on QFT specifications

R. Comasòlivas*, T. Escobet*
J. Quevedo*

* *Advanced Control Systems (SAC), Technical University of Catalonia (UPC)*
Rambla Sant Nebridi, 10, 08222 Terrassa (Spain)
(Email: [ramon.comasolivas, teresa.escobet, joseba.quevedo]@upc.edu)

Abstract. This paper proposes an optimization algorithm for the automatic design of robust PID controllers using Quantitative Feedback Theory (QFT) specifications. The proposed algorithm is based on a criterion to minimize the energy of the control effort. To illustrate the methodology, the pitch angle of a laboratory helicopter is used as a model application with structured uncertainty. The results show that the design of robust controller can be formulated using an objective function and a number of restrictions that are developed as specifications.

Keywords: Robust Control, PID controller, Quantitative Feedback Theory, Automatic Loop Shaping, Helicopter

1. INTRODUCTION

This article proposes a method for the automatic design of a robust proportional integral derivative (PID) controller that combines the Quantitative Feedback Theory (QFT) – QFT automatic loop shaping – method with optimization techniques to automatically design PID controllers minimizing the control effort. The design method applies to the case study which consists of the pitch angle control of a laboratory helicopter with structured uncertainty (Garcia-Sanz *et al*., 2006).

The QFT technique can be used to design controllers with no predetermined structure for systems with parametric uncertainties that are modeled by intervalar linear models. The classic design of controllers to ensure that specifications are met involves the frequency response, where the controller gain is attenuated by appropriately manipulating the poles and zeros of the transfer function of the controller. This design can be implemented quite efficiently using support software, such as the Matlab QFT toolbox (Borghesani *et al*., 1994). If a controller with a fixed low order structure is supposed, the designer has to manipulate the parameters of the controller to ensure that all specifications of the closed loop are met.

An open problem in QFT is the automatic design of controllers, which is known as *automatic loop shaping* (ALS). The idea of the automatic design of controllers QFT was introduced by Gera and Horowitz (1992). In the literature, there are various approaches to the automatic design of loop controllers for a plant with uncertainty, so that specifications lead to robustness (stability, etc.) and minimize the controller gain. Techniques that have been used include genetic algorithms (Chen *et al*., 1998), linear programming (Nataraj and Nandkishor, 2007) and optimization algorithms (Chait *et al.*, 1997), (Sachin, 2005).

Some authors (Nandakumar *et al*., 2002) used a hybrid technique involving optimization and propagation of uncertainty, combining interval global optimization and local nonlinear optimization. It is usual to propose ALS methods fixing the controller structure, for example in Yaniv and Nagurka (1995). Nataraj and Deshpande (2008) propose the synthesis of a controller as a constraint satisfaction problem with interval variables. Zolotas and Halikias (1999) propose an ALS method for PID controllers based on searching over a dense set of controllers. Molins and Garcia-Sanz (2009) describe a QFT method for designing controllers that is based on a combination of techniques, evolutionary algorithms and genetic algorithms. The same authors, Garcia-Sanz and Molins (2010) proposed an ALS method, by grouping previously all usually QFT specifications into two expressions, one in terms of sensitivity function, and other in terms of the complementary sensitivity function.

In this paper, a loop-shaping design for PID controllers that minimize the energy of the control effort by means of optimization tools is proposed.

As the problem to solve is non linear and non convex, the commercial global optimization package *Tomlab* (Pintér *et al*., 2005), which is based on branch and bound algorithms, is used.

In Section 2 we formulate an automatic controller design problem, to minimize the energy of the control effort. Section 3 shows the results of the design applied to a laboratory helicopter model system, and Section 4 presents the main conclusions.

2. DESIGN METHOD FOR ROBUST CONTROLLERS

2.1 *Formalizing the problem*

The usual control structure in the QFT technique is shown in Figure 1, where P, G, F and H represent the transfer functions of the plant, the controller, the pre-filter and the sensor, respectively. R, Y, U, E and Q represent the set point, the output, the action control, the control error and the sensor output signals. Finally, W, V, D and N indicate added input disturbances at the input of the controller, the input of the plant and the output of the plant as well as noise disturbances, respectively.

According to Bhattacharyya *et al.* (1995), an uncertain plant can be described by an interval model:

$$P(\theta,s)=\frac{B(\theta,s)}{A(\theta,s)} \tag{1}$$

where B and A are polynomials in the s domain, and θ is the vector of uncertain parameters of dimension p with their values bounded by a compact set $\theta\in\Theta$ of box type, i.e. $\Theta=\left\{\theta\in\Re^{p}\left|\underline{\theta}\le\theta\le\overline{\theta}\right.\right\}$

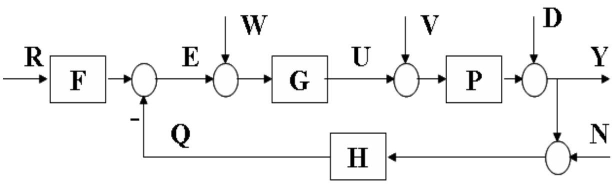

Fig. 1. QFT generic control structure

A classical problem in QFT (Horowitz, 1993; Houpis, and Rasmussen, 1999; Yaniv, 1999) is to synthesize the feedback controller $G(s)$ and the prefilter $F(s)$ as strictly proper, rational and stable transfer functions. Thus, that some specifications are satisfied, while the bandwidth of the controller is kept as low as possible, despite the presence of uncertainty in $P(s)$. In general, feedback control is defined as

$$G(k_c,s)=\frac{B_c(k_c,s)}{A_c(k_c,s)} \tag{2}$$

where B_c and A_c are polynomials in the s domain; and k_c is the vector controller parameter of dimension g.

Controller design using QFT is carried out by formulating several of the system's frequency specifications, according to the requirements. In this study, three types of specifications are used: robust stability, robust disturbance attenuation, and minimization of the control effort.

When the nominal open loop is given as $L(\theta,j\omega)=P(\theta,j\omega)G(j\omega)$ and $H(s)$=1 and $F(s)$=1, a robust stability specification is expressed as:

$$\left|T_R(\theta,j\omega)\right|=\left|\frac{L(\theta,j\omega)}{1+L(\theta,j\omega)}\right|\le\delta_1(\omega) \tag{3}$$

where $\delta_1(\omega)$ is the maximum magnitude of the closed-loop tracking transfer function $\dfrac{Y(j\omega)}{R(j\omega)}$.

Secondly, the requirement for disturbance rejection at the plant output, or sensitivity, is expressed as:

$$\left|S(\theta,j\omega)\right|=\left|\frac{1}{1+L(\theta,j\omega)}\right|\le\delta_s(\omega) \tag{4}$$

where $\delta_s(\omega)$ is the magnitude of the disturbance rejection, that is the upper bound of the magnitude of the $\dfrac{Y(j\omega)}{D(j\omega)}$ transfer function.

Finally, the control effort specification is expressed as:

$$\left|T_U(\theta,j\omega)\right|=\left|G(j\omega)S(\theta,j\omega)\right|\le\delta_c(\omega) \tag{5}$$

where $S(\theta,j\omega)=\dfrac{1}{1+L(\theta,j\omega)}$ is the sensitivity, and $\delta_c(\omega)$ is the magnitude of the control effort, that is the upper bound of the frequency magnitude of the transfer function $\dfrac{U(j\omega)}{W(j\omega)}$.

2.2 *Proposed design methodology*

In this paper, we propose an algorithm for *automatic loop shaping*. It is based on minimizing the control effort impulse response energy, given by (5), for a plant with a particular value of θ and controller k_c:

$$E(\theta,k_c)=\int_0^{\infty}\left|T_U(\theta,\omega,k_c)\right|^2 d\omega \tag{6}$$

while the energy for the whole family of plants will be given by

$$\int_{\underline{\theta}}^{\overline{\theta}}E(\theta,k_c)d\theta=\int_{\underline{\theta}}^{\overline{\theta}}\int_0^{\infty}\left|T_U(\theta,\omega,k_c)\right|^2 d\omega d\theta \tag{7}$$

A discrete approximation of (7), given a discrete grid of frequencies $\omega_1,\omega_2,\ldots$ and a grid of uncertain parameters of the plant $\theta_1,\theta_2,\ldots$, could be:

$$\int_{\underline{\theta}}^{\overline{\theta}}E(\theta,k_c)d\theta\cong\sum_{i=1}^{np}\sum_{j=1}^{nw}\left|T_U(\theta_i,\omega_j,k_c)\right|^2\Delta\omega\Delta\theta \tag{8}$$

where *np* and *nw* are the number of plants and frequencies considered, respectively.

The *Algorithm* shows the procedure to compute the optimal solution of the controller parameters $k_c\in KC$, where KC is the initial bounded set of controller parameters, $KC=\left\{k_c\in\Re^{g}\left|\underline{k_c}\le k_c\le\overline{k_c}\right.\right\}$, which minimize the control effort (8) taking into account the restrictions gave by (3), (4) and (5), in case of a generic controller structure.

Moreover, although it is not a predefined requirement, we chose a structure of control that is similar to the one used in the referenced articles, i.e. a PID with a filter in the derivative action:

$$G(k_c,s)=K_p(1+\frac{1}{T_i s}+\frac{T_d s}{T_f s+1}) \tag{9}$$

where K_p is the proportional gain, T_i is the integral time, T_d is the derivative time and T_f is the high frequency filter parameter. Usually filter is not been regarded as a part of the design but added afterwards to prevent that the high frequency gain of the controller growing up. In the proposed approach the four parameters has been considering, being the controller parameter vector $k_c=\left[K_p,T_i,T_d,T_f\right]$. In this case, the set of a minimum and maximum parameter bounds are $\underline{k_c}=\left[\underline{K_p},\underline{T_i},\underline{T_d},\underline{T_f}\right],\overline{k_c}=\left[\overline{K_p},\overline{T_i},\overline{T_d},\overline{T_f}\right]$.

Algorithm. Automatic loop shaping

Step 1. Choose the grid of frequencies $\omega_1,\omega_2,\ldots$ and parameters of the plant $\Theta=\{\theta_i|\theta_1,\theta_2,\ldots\}$.

Step 2. Solve the following optimization problem. For each frequency $\omega_1,\omega_2,\ldots$, and for the set of plant parameters Θ:

$$\min_{k_c}\sum_{i=1}^{np}\sum_{j=1}^{nw}\left|T_U(\theta_i,\omega_j,k_c)\right|^2\Delta\omega\Delta\theta$$

Subject to:

For i=1,…np

For *j=1,…nw*

$$\left|T_R\left(\theta_i,\omega_j,k_c\right)\right|\le\delta_1(\omega_j)$$

$$\left|S\left(\theta_i,\omega_j,k_c\right)\right|\le\delta_s\left(\omega_j\right)$$

$$\left|T_U\left(\theta_i,\omega_j,k_c\right)\right|\le\delta_c\left(\omega_j\right)$$

End

End

$$\underline{k_c}\le k_c\le\overline{k_c}$$

Step 3. If the solution has not been found, the controller parameters bounds should be increased or/and the specifications should be relaxed and go to **Step 1**.

Fig. 2. Algorithm for the automatic design of the controller

Wallén *et al.* (2002) presents a study of the advantages to constrain the ratio T_i/T_d to a fixed value $n\geq4$ when loop shaping is applied to design PID controllers. Also a low pass high frequency filter, with a priori selected time constant T_f, is used in the derivative term in order to limit the susceptibility of the controller to measurement noise. T_f has been selected as one tenth of the derivative time following the suggestions of Astrom *et al.* (1995).

According this, we fixed some restrictions to obtain consistent parameters, and to reduce the searching space of PID parameters; some restrictions have been fixed between them:

$$T_d=T_i/4;T_f=0.1*T_d \tag{10}$$

These two conditions are added in the algorithm as new constraints. In summary, the PID automatic loop shaping is based to solve the optimization problem (Algorithm, Fig. 2), adding the 2 new mentioned constraints (10), for all $\omega_i\in\omega$ and $\theta_i\in\Theta$.

3. APPLICATION TO THE MODEL OF A LABORATORY HELICOPTER

3.1 Physical model of the helicopter

To test the Algorithm proposed in the previous section, we used a scale laboratory helicopter with two parallel rotors. This application was designed by *Quanser* and described in detail in García-Sanz *et al.* (2006). Fig. 3 shows its general appearance.

Fig. 3. General appearance of the *Quanser* Helicopter 3 DOF

García-Sanz *et al.* (2006) presents a detailed description of the helicopter characteristics, and the physics equations that can be used to obtain a linear time invariant model (LTI) of the pitch angle, by preprocessing appropriate linearizations and simplifications. The final model provided a second order LTI model, with parametric or structured uncertainty obtained from experimental data. A more complete model is described in Egaña *et al.* (2001) and García-Sanz *et al.* (2002), in which the angles of roll, pitch and yaw are included as three degrees of freedom (3 DOF).

The pitch angle model is used to show the results of the Algorithm described in Fig. 2. From the estimates of physical parameters (length, mass, friction) in Egaña *et al.* (2001), a second order transfer function is defined in which the system's signal input is voltage, the output is the pitch angle and the parameter are unknown but bounded, as it is indicated in (11).

$$P(s)=\frac{k\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\begin{cases}k\in[0.07, \quad 0.12]\\ \xi\in[0.1, \quad 0.16]\\ \omega_n\in[0.55, \quad 0.60]\end{cases} \qquad (11)$$

3.2 Design specifications

The aims of the robust controller design are similar to those proposed in Egaña *et al.* (2001), particularly robust stability, sensitivity and control effort.

Regarding this previous design, the following limits of performances have been selected: $\delta_1(\omega)=2$, $\delta_s(\omega)=\frac{2(j\omega+0.01)(j\omega+0.3)}{((j\omega)^2+4j\omega+4)}$ and $\delta_c(\omega)=5000$. First and third, constrain the ratio between output-input and control effort-input, respectively, at all range of frequencies. Second, sensitivity bound is a frequency dependent specification.

In the frequency spectrum, the same range was taken for all the specifications and only for low frequencies: $\omega\in[0.1, \quad 100]$ rad/sec. We chose this frequency range because it includes the frequencies that are apparently more problematic in terms of meeting the specifications.

The domain of the control parameters has been selected taking into account the previous work of Egaña (2001) and initial results of our methodology with a large domain of control parameters. Now, we restrict this domain to the following bounded set (time parameters in seconds):

$$\begin{Bmatrix}K_p\in[10,300]\\ T_i\in[1,20]\\ T_d\in[0.25,5]\\ T_f\in[0.025,0.5]\end{Bmatrix} \qquad (12)$$

3.3 Results

In order to solve the optimization of the Algorithm that corresponds to a non convex and non-smooth problem, a global optimization commercial toolbox has been used to solve this problem. In particular, the optimization environment Matlab/*Tomlab* toolbox (Pintér *et al.*, 2005) using the Efficient Global Optimization algorithm, *ego,* (Jones *et al.*, 1998) has been applied. To obtain the optimal controller solution, objective function (8) and constraints (3), (4), (5) and (10), has been properly manipulated to obtain the equivalent polynomial expressions.

The Algorithm, proposed in Section 2.2, was applied by taking a finite number of 8 plants, obtained by all the combinations of lower and upper parameter bounds, and choosing the following grid of frequencies: $\omega=[0.1 \quad 1 \quad 2.5 \quad 5 \quad 10 \quad 50 \quad 100]\,rad/\sec$. We selected this limited number of plants and frequencies because we considered that it provided enough information to illustrate the method. When we increased the number of plants and the frequency grid, we found that the method was still valid, but the tradeoff was a higher computational cost.

By mentioned conditions, the optimal PID controller computed is:

$$G(s)=\frac{93.76(1.5434s^2+2.4294s+1)}{s(0.05921s+1)} \qquad (13)$$

corresponding to a controller (9) with the following parameter values (time parameters in seconds):

$$K_p=222.2;\ T_i=2.37;\ T_d=0.592;\ T_f=0.0592; \qquad (14)$$

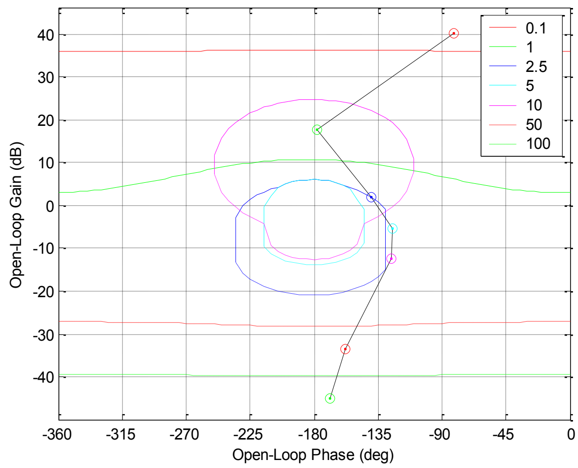

Fig. 3. Nichols chart with the designed controller. That includes the three specifications

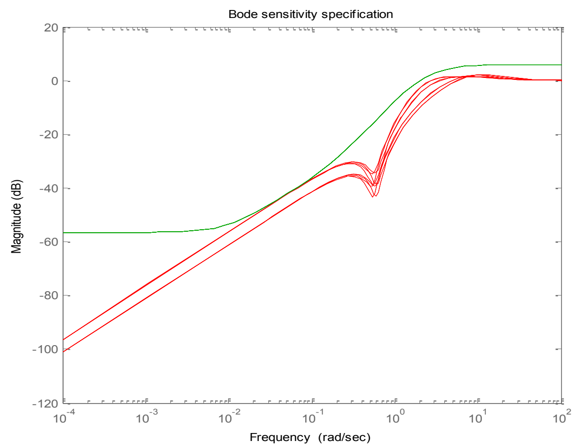

Fig. 4. Sensitivity specification and Bode diagram module of a set of 8 controlled plants.

The *QFT toolbox* (Borghesani *et al.*, 1994) allows validate the PID computed. Fig. 3 shows, in the Nichols chart, the QFT bounds for each ω_i. Also, in a solid black line, shows the nominal open loop plant with the synthesized controller, $L(j\omega_i)$ (the circle 'o' denote the response at each ω_i frequency), where nominal plant corresponds with all the parameters at their lower values. Notice that $L(j\omega_i)$ satisfies the bound at some frequencies at the limit, concluding that the optimal controller were obtained, for the set of frequencies chosen.

As redundant information, there is shown in Figure 4 the module on the Bode diagram of the sensitivity specifaction. The red lines show the sensitivity module for the eight plants selected and the green line shows the maximum magnitude of the disturbance rejection, $\delta_s(\omega)$, to note that it is the upper bound of the magnitude of the $\dfrac{Y(j\omega)}{D(j\omega)}$ transfer function.

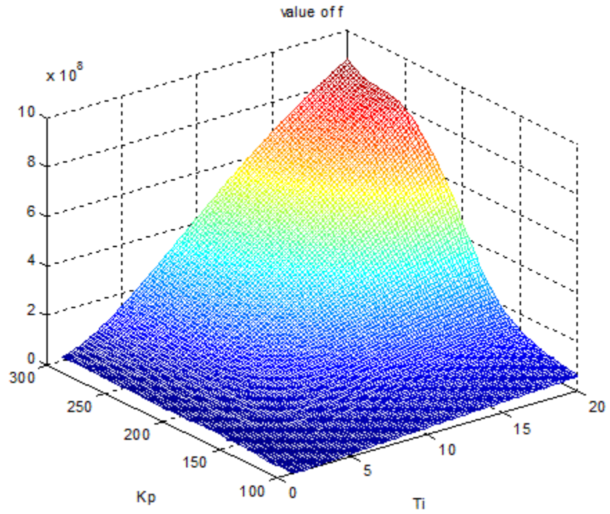

Fig. 5. Representation of the function to minimize, without considering constraints and depending on the parameters K_p and T_i

An additional information is available in Fig. 5. The objective function is computed using equation (8) for different values of controller parameters, $K_p \in [100, 300]$ and $T_i \in [1, 20]$, where T_d and T_f have been restricted by (10).

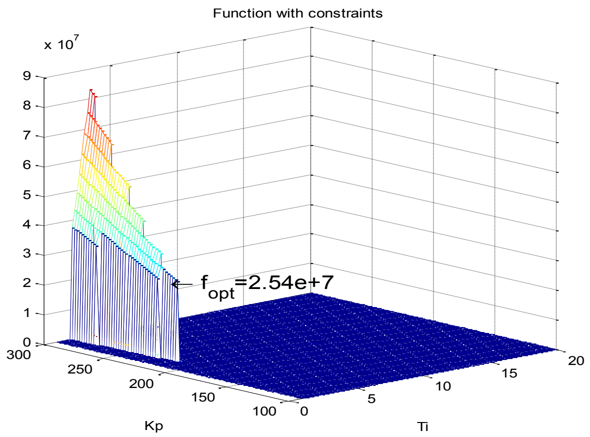

Fig. 6. Representation of the function to minimize, considering constraints, depending on parameters K_p and T_i

When constraints are added, only one of them, specifically the sensitivity specification (4), is not fulfilled for the whole range of the parameters K_p and T_i . This is shown in Fig. 6 by the abrupt jump in the graph that appears to be modified. The optimal controller corresponds to the lowest point of the boundary line, which coincides with the values of K_p and T_i found by the optimizer given in (14). The value of the objective function for the optimum point (minimum) is f_{opt}=2.54e+7. All the constraints are satisfied for this optimal point, corresponding to the controller found.

3.4 Time domain analysis

The described benchmark (García-Sanz *et al*., 2006) proposes the design of a controller that can follow a given temporal reference signal, as shown in Fig. 7, where are also shown the time responses of eight plants chosen at random.

Fig. 7 shows that the response of the closed loop system is fast, with a peak at around 20% when the input signal is a step. This peak could be reduced by lowering the threshold value of robust stability. The speed of the response entails a significant increase in control effort. This oscillation can be reduced by decreasing the value of the control effort. In this case, a large value $\delta_c(\omega)$ is considered as a constraint.

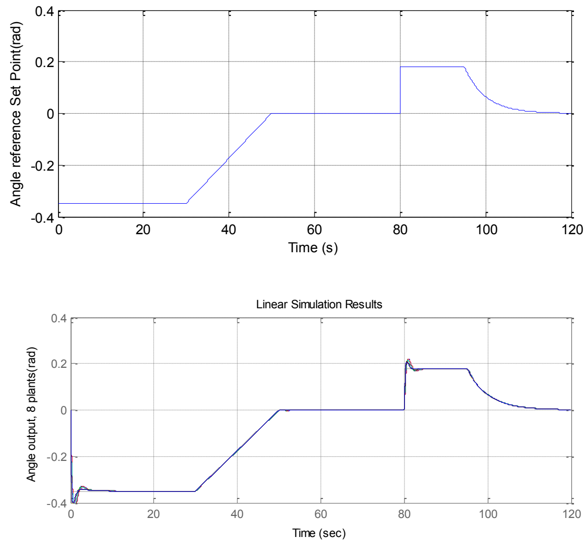

Fig. 7. Time response of a set of 8 plants in response to a reference set point

4. CONCLUSIONS

In this paper, we proposed an algorithm for the automatic design of controllers. It is based on optimization of an index associated with control effort. Some studies in the literature aim to obtain a similar objective of automatic loop shaping based on optimization. The algorithm has been applied to the specific case of controller design for a model of the pitch angle of a laboratory helicopter, with structured uncertainty. It was supported by a commercial global optimization tool, *Tomlab*, with satisfactory results, as validated by simulations.

ACKNOWLEDGEMENTS

This work has been supported by the Spanish Ministry of Economy and Competitiveness under grant SHERECS (CICYT DPI2011-26243) and the EU project ISENSE (FP7-IST-270428).

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