Design and Analysis of IMC based PID Controller for Unstable Systems for Enhanced Closed Loop Performance

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Abstract: In this paper, design of PID controller is analyzed for unstable second order processes with time delay based on IMC method and H_2 minimization. A new desired closed loop transfer function is obtained based on which the PID controller is designed. Maclaurin series is used to approximate the controller expression as a PID controller. Improved closed loop performances are achieved with the proposed method when compared to the recently reported methods in the literature. Comparative analysis has also been carried out with modified Smith predictor schemes and showed that the proposed method is superior. Further, an analysis is carried out based on maximum sensitivity for arriving at systematic guidelines for selection of the closed loop tuning parameter which is essential for unstable systems. The bounds for this tuning parameter are analyzed using the maximum sensitivity.

Keywords: Unstable process, steady state, sensitivity analysis, robustness, PID controller.

1. INTRODUCTION

Open loop unstable systems exist in many chemical and biological processes and these systems are fundamentally difficult to control than that of stable systems. The difficulty increases if the system has time delay. The difficulty further increases when the system contains a positive zero. In unstable systems, controlling a first order system is comparatively easy than that of a second order one (system with two unstable poles). These exists many PID design methods (Nasution *et al*., 2011; Shamsuzzoha and Lee, 2008a; Sree *et al*. 2004; Lee *et al*., 2000) for controlling unstable first order systems. However, very few methods (Shamsuzzoha and Lee, 2008b; Rao and Chidambaram, 2006) are available for unstable second order systems. Existence of second order unstable systems is well described by Sree and Chidambaram (2006). The PID controller cannot provide stabilized responses when the time delay to time constant ratio is greater than 1.2 for unstable systems. Modified Smith predictor based control schemes (Liu *et al*., 2005) and modified IMC based control schemes (Tan *et al*., 2003) are also developed for unstable first order and second order systems. However, these modified schemes also are not applicable when the time delay to time constant ratio exceeds 1.2. Also, more control effort is required for these schemes. Hence, keeping the simplicity into account, properly designed PID controller is better than these modified schemes. However, the designed PID controller should provide good nominal and robust closed loop responses and smooth manipulated variable responses. To achieve these objectives, in this work, IMC is used to design PID controller for second order unstable processes based on H_2 minimization. Here, the design is addressed for second order unstable systems because there already exist many design methods for first order unstable systems and also controlling a second order unstable systems is difficult than that of first order one. Once the controller is designed, an analysis is carried out for proper selection of the IMC tuning parameter. The present design method is based on the work described in Nasution *et al*. (2011). For clear illustration, the design is addressed in the next section followed by simulation results and conclusions.

2. PROPOSED DESIGN METHOD

The block diagram of the IMC scheme is shown in Fig. 1 where G_{imc} is the IMC controller, G_p is the process and G_m is the process model.

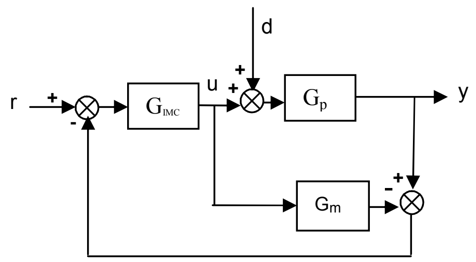

Fig. 1. IMC control scheme

The second order unstable process considered here is

$$G_p(s) = ke^{-\theta s}/(\tau_1 s-1)(\tau_2 s-1) \quad (1)$$

The equivalent controller in a conventional feedback form can be written as $G_c = G_{imc}/(1-G_{imc}G_m)$. According to H_2 optimal controller design (Morari and Zafiriou, 1989), the

IMC controller is designed as $G_{imc}(s)=\tilde{G}_{imc}(s)F(s)$ where F is the filter to make G_{imc} as a realizable controller and also to maintain robustness. $\tilde{G}_{imc}(s)$ is designed to achieve H_2 optimal performance for a specific input type, $\nu(s)$. The process model and the input are divided into minimum phase part and non-minimum phase part as $G_m(s)=G_{m-}(s)G_{m+}(s)$ and $\nu(s)=\nu_-(s)\nu_+(s)$ where " – " refers to the minimum phase part and " + " refers to non-minimum phase part. Further, the Blachke product of RHP poles of $G_m(s)$ and $\nu(s)$ are introduced as

$$b_m(s)=\prod_{i=1}^{k}\left(-s+p_{mi}\right)/(s+\bar{p}_{mi})$$

$$b_v(s)=\prod_{i=1}^{k}\left(-s+p_{vi}\right)/(s+\bar{p}_{vi})$$

Where p and $\bar{p}$ are the RHP pole and its conjugate. Then the H_2 optimal controller is derived using

$$\tilde{G}_{imc}(s)=b_p\left(\tilde{G}_{m-}b_v\nu_-\right)^{-1}\left\{\left(b_p\tilde{G}_{m+}\right)^{-1}b_v\nu_-\right\}_* \tag{2}$$

Where $\{\ldots\}_*$ is defined as the operator obtained after omitting all terms involving the poles of $\left(\tilde{G}_{m+}\right)^{-1}$ after taking partial fraction expansion. In the present work, for the second order unstable process (1), the required quantities for the operator are obtained as

$$\tilde{G}_{m-}(s)=\frac{k}{\tau_1\tau_2\left(-s+\frac{1}{\tau_1}\right)\left(-s+\frac{1}{\tau_2}\right)};\tilde{G}_{m+}(s)=e^{-\theta s}$$

$$\nu_-(s)=\frac{k}{\tau_1\tau_2\left(-s+\frac{1}{\tau_1}\right)\left(-s+\frac{1}{\tau_2}\right)s};\nu_+(s)=1$$

$$b_p(s)=\left(-s+\frac{1}{\tau_1}\right)\left(-s+\frac{1}{\tau_2}\right)\Big/\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)$$

$$b_v(s)=\left(-s+\frac{1}{\tau_1}\right)\left(-s+\frac{1}{\tau_2}\right)\Big/\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)$$

Substituting in (2) IMC controller is obtained as

$$\tilde{G}_{imc}=\frac{(\tau_1 s-1)(\tau_2 s-1)}{k}\left[\frac{\tau_1\tau_2(\tau_1-\tau_2-\tau_1e^{\theta/\tau_1}+\tau_2e^{\theta/\tau_2})s^2+(\tau_1{}^2e^{\theta/\tau_1}-\tau_2{}^2e^{\theta/\tau_2}+\tau_2{}^2-\tau_1{}^2)s+(\tau_1-\tau_2)}{(\tau_1-\tau_2)}\right]$$

Considering the filter as $F(s)=(\alpha_2s^2+\alpha_1s+1)/(\lambda s+1)^4$, the IMC controller is obtained as $G_{imc}(s)=\tilde{G}_{imc}(s)F(s)$, where λ is the tuning parameter which should be selected carefully so that good nominal and robust closed loop performances are achieved. With that the desired closed loop transfer function is obtained as

$$H(s)=\left(y/r\right)_d$$
$$\left[\frac{\tau_1\tau_2(\tau_1-\tau_2-\tau_1e^{\theta/\tau_1}+\tau_2e^{\theta/\tau_2})s^2+(\tau_1{}^2e^{\theta/\tau_1}-\tau_2{}^2e^{\theta/\tau_2}+\tau_2{}^2-\tau_1{}^2)s+(\tau_1-\tau_2)}{(\tau_1-\tau_2)}\right]Fe^{-\theta s}$$

Then the equivalent controller in a conventional feedback form is obtained from IMC structure as $G_c=G_{imc}/(1-G_{imc}G_m)$. After substituting G_{imc} and G_m, G_c will be obtained as a higher order numerator and denominator expression. To simplify this expression to a PID controller form, Maclaurin series is used here. To do that, let us define $J(s) = sG_c(s)$. Expand $J(s)$ using Maclaurin series expansion to obtain the controller G_c as

$$G_c(s)=\frac{1}{s}\left(J(0)+J^{'}(0)s+\frac{J^{''}}{2!}s^2+.......\right)$$

By considering this as a PID controller in the form

$$G_c(s)=k_c\left(1+\frac{1}{\tau_i}s+\tau_ds\right)$$

the PID controller parameters are obtained as $k_c=J^{'}(0)$, $\tau_i=J^{'}(0)/J(0)$ and $\tau_d=J^{''}(0)/2J^{'}(0)$

Where

$$J(0)=1/p_m(0)D(0)$$

$$J^{'}(0)=-[p_m^{'}(0)D(0)+p_m(0)D^{'}(0)]/[p_m(0)D(0)]^2$$

$$J^{''}(0)=$$
$$J^{'}(0)\left[\frac{p_m^{''}(0)D(0)+2p_m^{'}(0)D^{'}(0)+p_m(0)D^{''}(0)}{p_m^{'}(0)D(0)+p_m(0)D^{'}(0)}+\frac{2J^{'}(0)}{J(0)}\right]$$

$$D(0)=4\lambda-p_A^{'}(0);D^{'}(0)=[12\lambda^2-p_A^{''}(0)]/2$$

$$D^{''}(0)=[24\lambda^3-p_A^{'''}(0)]/3$$

$$p_A^{'}(0)=\frac{(\tau_1-\tau_2)(-\theta+\alpha_1)+\tau_1b-\tau_2a}{\tau_1-\tau_2}$$

$$p_A^{''}(0)=\frac{(\tau_1-\tau_2)(\theta^2-2\alpha_1\theta+2\alpha_2)+2(\tau_1b-\tau_2a)(-\theta+\alpha_1)+2\tau_1\tau_2(a-b)}{\tau_1-\tau_2}$$

$$p_A^{'''}(0)=\frac{(\tau_1-\tau_2)(-\theta^3+3\alpha_1\theta^2-6\alpha_2\theta)+(\tau_1b-\tau_2a)(3\theta^2-6\alpha_1\theta+6\alpha_2)+2\tau_1\tau_2(a-b)(-3\theta+3\alpha_1)}{\tau_1-\tau_2}$$

$$p_m(0)=k;p_m^{'}(0)=\frac{-k\left((\tau_1-\tau_2)(-\tau_2-\tau_1+\alpha_1)+\tau_1b-\tau_2a\right)}{(\tau_1-\tau_2)}$$

$$p_m^{''}(0)=\frac{\begin{array}{l}-k[(\tau_1-\tau_2)2(\tau_1-\tau_2)(\tau_1\tau_2-\alpha_1\tau_2-\alpha_1\tau_1+\alpha_2)+\\2(\tau_1b-\tau_2a)(-\tau_2-\tau_1+\alpha_1)+2\tau_1\tau_2(a-b)-\\2\left((\tau_1-\tau_2)(-\tau_2-\tau_1+\alpha_1)+\tau_1b-\tau_2a\right)^2]\end{array}}{(\tau_1-\tau_2)^2}$$

In which α_1 and α_2 values are obtained from the requirements to satisfy internal stability in IMC based control schemes. The condition for internal stability is

$$(1-G_{imc}G_m)|_{s=\frac{1}{\tau_1},\frac{1}{\tau_2}}=0$$

Thus $\alpha_1=\left(\frac{\lambda}{\tau_1}+1\right)^4\frac{\tau_1^2e^{\theta/\tau_1}}{x}-\left(\frac{\lambda}{\tau_2}+1\right)^4\frac{\tau_2^2e^{\theta/\tau_1}}{y}-\tau_1-\tau_2$

$$\alpha_2=\left(\frac{\lambda}{\tau_1}+1\right)^4\frac{(\tau_1-\tau_2)}{x}\tau_1^2e^{\theta/\tau_1}-\tau_1^2-\alpha_1\tau_1$$

$$x=\frac{\tau_2}{\tau_1}(a-b)+\frac{(\tau_1b-\tau_2a)}{\tau_1}+\tau_1-\tau_2$$

$$y=\frac{\tau_1}{\tau_2}(a-b)+\frac{(\tau_1b-\tau_2a)}{\tau_2}+\tau_1-\tau_2$$

$$a=\tau_2(e^{(\theta/\tau_2)}-1);b=\tau_1(e^{(\theta/\tau_1)}-1)$$

3. SIMULATION STUDIES

To analyze the performance of the proposed design method, three examples are considered.

3.1 Example-1: An unstable second order process $G_p(s)=2e^{-0.3s}/(3s-1)(s-1)$ is considered here (Tan *et al.*, 2003). To select the tuning parameter (λ), an analysis is carried out here based on maximum sensitivity. Fig. 2 shows the variation of Ms with respect to λ. There exist a large value for Ms corresponding to λ = 0.62 after which the Ms decreases up to λ = 1.45 (Ms = 3.1) and then Ms increases. Hence λ should be selected in the range of 0.72 – 2.7. Within this range of λ, the maximum value for Ms will be 10. If λ is selected outside this range the closed loop performance is not good or is not stable. Note that the minimum value of Ms achievable in this range of λ is 3.1. It can be observed from the figure that one should not select λ without proper analysis. For the purpose of comparison, method proposed by Panda (2009) is considered. The PID settlings obtained by Panda (2009) are k_c = 0.881, τ_i = 5.11, τ_d = 3.42 for a tuning parameter value 0.948 which corresponds to Ms of 4.82. To have fair comparison, for the proposed method also, λ is selected as 0.9 which gives Ms value of 4.82. Note that this λ value is after the peak in the graph in Fig. 1. The PID controller parameters are obtained as k_c = 0.9718, τ_i = 2.4591, τ_d = 4.066. With these controller settings, the methods are simulated by giving a unit step change in set point and a unit negative input in the load at t = 50 sec respectively. The closed loop performance for perfect parameters is shown in Fig. 3. Perturbations of +10% in time delay and -10% in process gain are given and the corresponding closed loop responses are shown in Fig. 4. It can be observed that the proposed design performs better and also the control action responses are smooth for the proposed method that that of Panda (2008). The same improvement is also shown in terms of the performance index integral absolute error (IAE). To further analyze the robustness, Ms values are plotted for different values of the time delay and λ and is shown in Fig. 5. It can be observed that as θ increases, λ need to be increased so that the system robustness is maintained as per the requirements and within the possible limits.

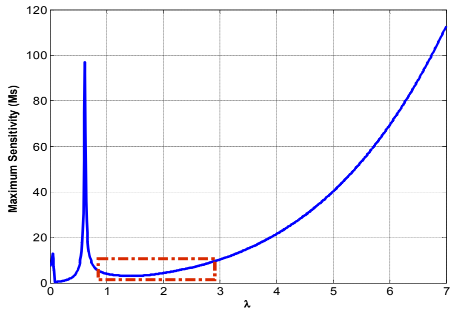

Fig. 2. Ms versus λ for example-1

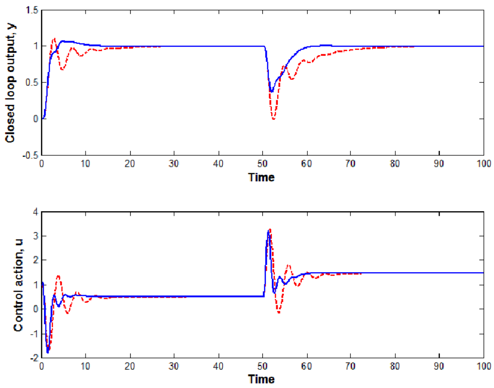

Fig. 3. Closed loop responses for perfect conditions for example-1, solid-proposed method (IAE = 4.437), dash-Panda (IAE = 8.831).

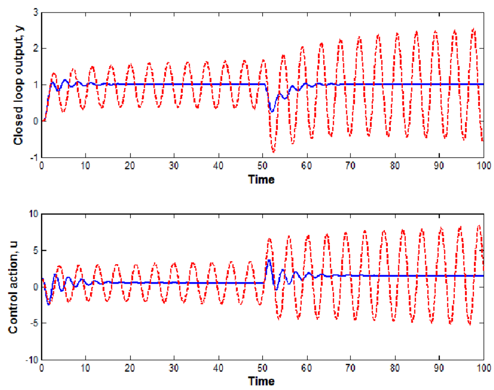

Fig. 4. Closed loop response for example-1 for perturbations of +10% in time delay and -10% in gain, solid-proposed method (IAE = 4.702), dash-Panda (IAE = 64.74).

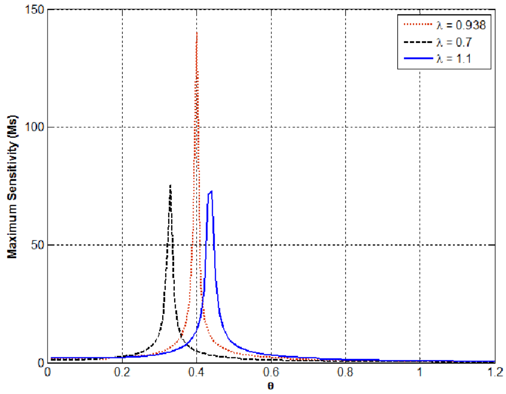

Fig. 5. Ms versus θ for different λ values for example-1

3.2 Example-2: A second order process with one integrator is considered here (Liu *et al.*, 2006; Rao and Chidambaram, 2006). The process transfer function is described as $G_p(s) = e^{-0.2s} / s(s-1)$. To use the proposed method, this process is converted for convenience as $G_m(s) = e^{-0.2s} /(s-0.01)(s-1)$ which in turn can be written as $G_m(s) = 100e^{-0.2s} /(100s-1)(s-1)$. For this process, the proposed method is applied. To understand the selection of λ, Ms values are plotted against λ and the graph is shown in Fig. 6. It can be observed from Fig. 6 that the selection of λ should be carried out as specified within the bracketed section so that the system robustness is maintained. Based on this understanding λ is selected as 0.65 (after the peak in the Fig. 6). With that the controller settings are obtained as k_c = 2.0662, τ_i = 2.6451, τ_d = 1.5062.

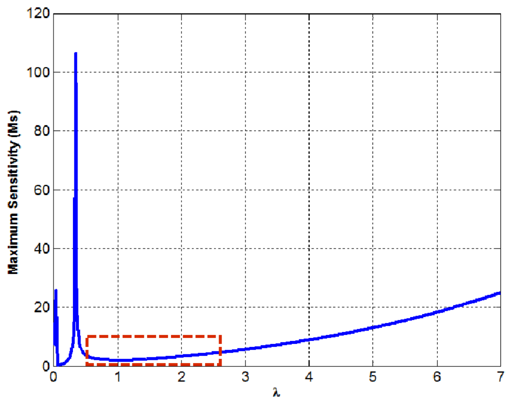

Fig. 6. *Ms versus λ for example-2*

For the purpose of comparison, method of Liu *et al*. (2005) is considered here which is a two degrees of freedom control scheme based on modified form of Smith predictor with three controllers. As this method uses a set point controller, for fair comparison, in the proposed method also, set point weighting (0.4) is considered. These two methods are simulated by giving a unit step change in the set point and a negative step change in the load at t = 50 sec respectively. Fig. 7 shows the closed loop responses for perfect model. The proposed method performs better. With set point weighting, the set point response is smooth and performs further better. Fig. 8 shows the responses for perturbation of +30% in time delay. It can be observed that the proposed method performs better. Note that the set point response is smoothened by using the set point weighting in the proposed method.

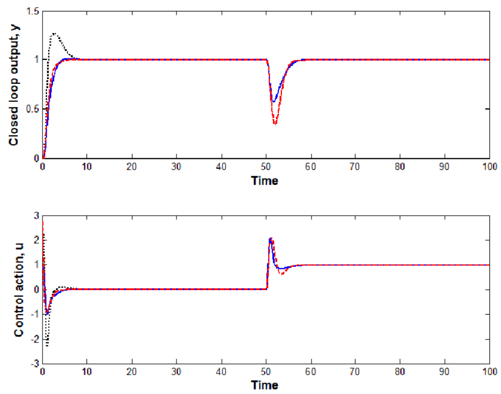

Fig.7. Closed loop responses for perfect conditions for example-2, solid-proposed method with set point weighting (IAE = 2.947), dot- proposed method without set point weighting (IAE = 3.037), dash-Liu *et al*. (IAE = 3.164).

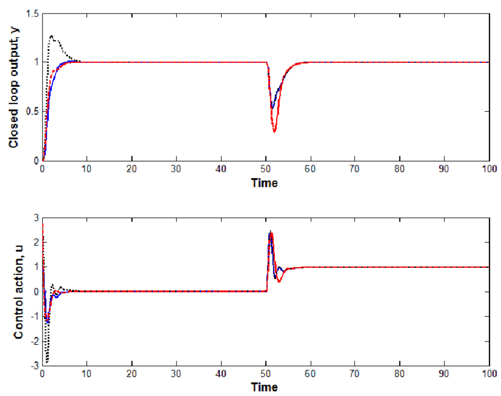

Fig. 8. Closed loop responses for perturbations of +10% in gain and time delay and -10% in time constant for example-2, solid-proposed method with set point weighting (IAE = 2.914), dot- proposed method without set point weighting (IAE = 2.997), dash-Liu *et al*. (IAE = 3.156).

To analyze the robustness further, sensitivity and complementary sensitivity functions are plotted for different values of λ and shown in Fig. 9. It can be observed that as λ increases, the peak values of the sensitivity functions decreases indicating more robust behaviour.

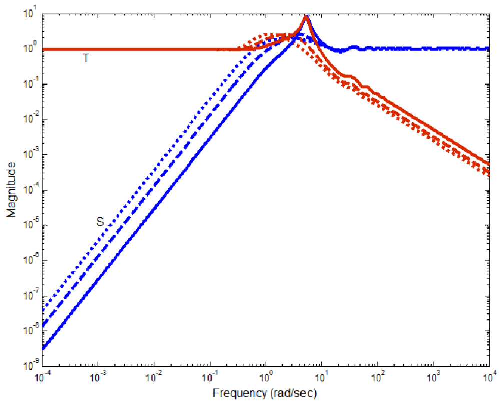

Fig. 9. Sensitivity and complementary sensitivity functions for different values of λ, solid - λ = 0.4; dash - λ = 0.65, dot - λ = 0.9.

3.3 Example-3: An unstable process with one stable pole is considered here (Lee *et al.*, 2000, Tan *et al.*, 2003). The process is described as $G_p(s) = e^{-0.939s}/(5s-1)(2.07s+1)$. To apply the proposed method, this process is converted as $G_m(s) = -e^{-0.939s}/(5s-1)(-2.07s-1)$ with k = -1, θ = 0.939, τ_1 = 5 and τ_2 = -2.07. To analyze the selection of the tuning parameter, Ms values are plotted for various values of λ and corresponding graph is shown in Fig. 10.

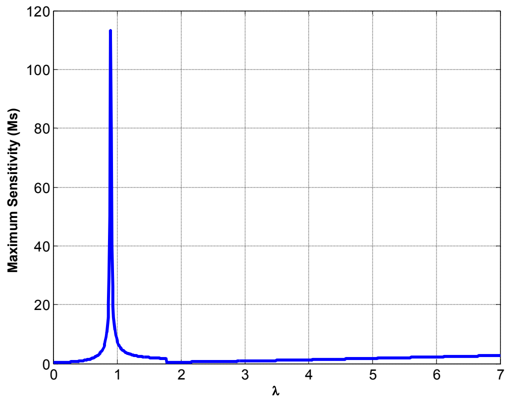

Fig. 10. Ms versus λ for example-3.

As explained earlier, based on this Fig. 10, λ is selected as 1.9. With this the PID controller parameters are obtained as k_c = 4.4432, τ_i = 10.25, τ_d = 1.932. Set point weighting (0.4) is used for the proposed. For the purpose of comparison, method of Tan *et al.* (2003) is considered which is based on the modified IMC scheme with three controllers. Fig. 11 shows the closed loop responses for perfect model for a unit step change in the set point and a negative step input at t = 75 sec respectively. Fig. 12 shows the closed loop responses for perturbations. The proposed method performance is marginally lower for perfect conditions but the proposed method performs better for perturbations. Note that the control action responses are not better for Tan *et al.* in both cases. This is evident from Fig. 11 that for Tan *et al.* method, to reach a step change from 0 to 1, the control action needs an effort to change from 25 to -1. This is not recommended.

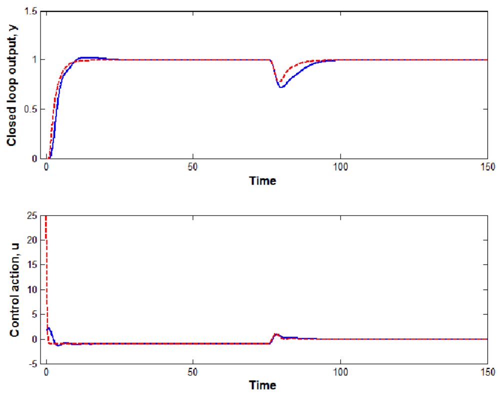

Fig. 11. Closed loop responses for perfect model for example-3, solid-proposed method (IAE = 6.68), dash-Tan *et al.* (IAE = 4.47).

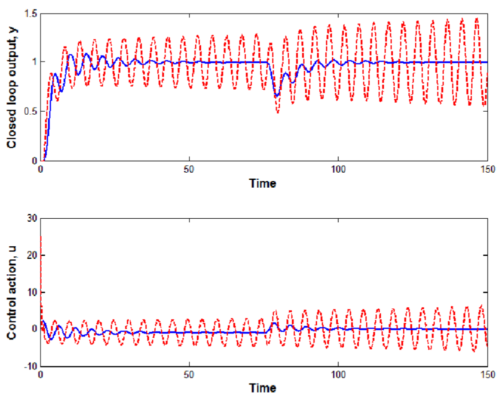

Fig. 12. Closed loop responses for perturbations of +15% in gain and time delay & -15% in τ_1 for example-3, solid-proposed method (IAE = 7.92), dash-Tan *et al.* (IAE = 33.19).

To analyze the effect of time delay on Ms, a graph is plotted for three different values of θ and shown in Fig. 13. It can be observed that as θ increases, λ need to be increased to maintain the system robustness.

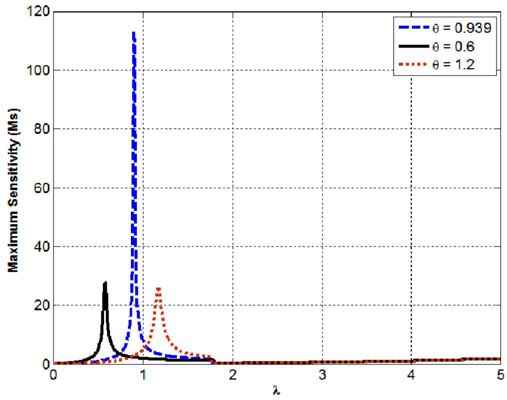

Fig. 13. Ms versus λ for different θ values for example-3

4. CONCLUSIONS

In this study, simple PID controller design using IMC method and H_2 minimization is presented. The following are the observations from this study.

1. For unstable systems, the selection of the tuning parameter is crucial for obtaining stable responses. Ms versus λ plot gives good understanding about this. For the same Ms value, there exist two λ values (one before the peak and another after the peak). If λ value is selected before the peak, then the closed loop response will be poor and also the resulting controller will not be robust (may lead to unstable behaviour). In the literature, for performance comparison of various methods, same Ms value is normally chosen as the proper comparative measure. However, if there exists two λ values for the same Ms value, the λ value after the peak in the Ms versus λ plot should only be considered.

2. With the present PID controller design, improved closed loop performance is achieved when compared to recently reported PID design methods in the literature.

3. When the performance is compared with complex control schemes (modified Smith predictor or modified IMC) which consist of more than two controllers, the proposed method still provides improved performances. Note that the improvement in the manipulated variable responses is significant. Even if there is a marginal improvement with the complex control schemes, PID will be preferred due to simplicity for practical usage.

4. The proposed method is also applied to second order unstable processes with RHP zeros. If RHP zero process is described as $G_p(s) = k(1-ps)e^{-\theta s}/(\tau_1 s-1)(\tau_2 s-1)$ then this process is converted as $G_p(s) = ke^{-(\theta+p)s}/(\tau_1 s-1)(\tau_2 s-1)$ and for this modified process the proposed design is applied and obtained good closed loop performances (graphs not shown here because of space limitation) when the p value is small (p < θ). A straight forward design is required to take into account large values of p.

5. Here, the analysis is carried out for second order unstable processes only. In our future research, a generalized procedure will be developed for all classes of unstable and integrating systems with and without RHP zeros for effective and analytical formulae for selection of the tuning parameter.

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