# Multivariable PID control by inverted decoupling: Application to the Benchmark PID 2012

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**Abstract:** This paper deals with the boiler control problem proposed as a benchmark for the IFAC Conference on Advances in PID Controllers (PID'12). This boiling process is a multivariable nonlinear system that shows interactions and is subjected to input constraints. As proposal, in this work, a PID control by inverted decoupling with feedforward compensation is developed. The design simplicity and easiness of implementation are highlighted. Experiment simulations considered in the benchmark show that the proposed design achieves better performance indexes than those of the reference cases.

*Keywords:* centralized control, decoupling, PID control, multivariable control, boilers.

## 1. INTRODUCTION

In most power plants, steam generation systems and, subsequently, boiler control problem are critical tasks to cope with the frequent load changes and sudden load disturbances. These boiler systems are multivariable processes showing great interactions and nonlinear dynamics under a wide range of operating conditions (Åström and Bell, 2000). In order to obtain a good performance, multivariable control strategies are usually required.

In recent years, many researchers have paid attention to the control of boiler systems using different approaches, such as robust control, genetic algorithm based control, gain-scheduled, predictive control, nonlinear control and so on (Tan, Marquez, Chen and Liu, 2005). The authors of this paper have already dealt with the boiler control problem (Garrido, Morilla and Vázquez, 2009) working with methodologies based on decoupling control.

The pure centralized strategies under the paradigm of "decoupling control", propose to find a controller K(s), such that the closed loop transfer matrix $G(s)\cdot K(s)\cdot[I+G(s)\cdot K(s)]^{-1}$ is decoupled over some desired bandwidth. This goal is ensured if the open loop transfer matrix G(s)·K(s) is diagonal. For this reason, the techniques used in decoupling control are quite similar to those used to design decouplers.

Most of these methodologies use the conventional scheme of centralized control depicted in Fig. 1, which has received considerable attention for several years (Wang, Zhang and Chiu, 2003; Morilla, Vázquez and Garrido, 2008). Nevertheless, the proposed controller uses another centralized control scheme, which is shown in Fig. 2 and was exposed in (Garrido, Vázquez and Morilla, 2010). It is based on the structure of inverted decoupling, which is rarely mentioned in the literature (Wade, 1997; Garrido, Vázquez and Morilla, 2011a), although it has important advantages from a practical point of view (Garrido, Vázquez and Morilla, 2011b).

Using the scheme of Fig. 2, it is possible to achieve the desired requirements with very simple $k_{ij}(s)$ elements in the controllers. In addition, the elements of the open loop process G(s)·K(s) are much less complicated than those using the conventional centralized decoupling control.

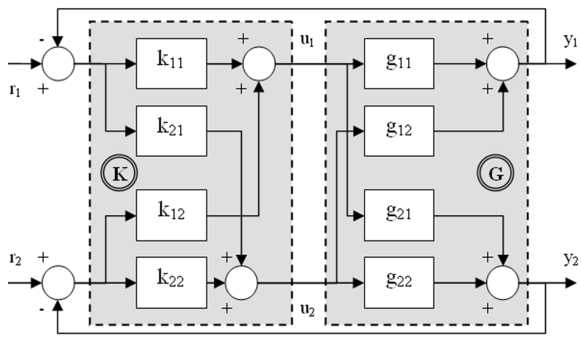


Fig. 1. 2x2 conventional centralized control with four controllers.

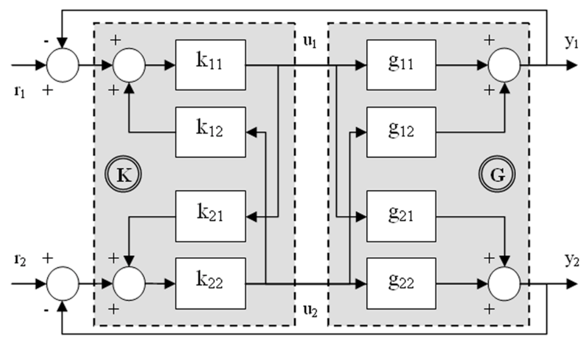


Fig. 2. 2x2 inverted centralized control with four controllers.

This paper illustrates the application of a multivariable PID control by inverted decoupling with feedforward compensation to the multivariable boiler considered in the benchmark problem for the IFAC Conference on Advances in PID Controllers (PID'12). In section 2, some aspects of the boiler system are commented, and a linearized model is presented in order to carry out the control design. The methodology of centralized PID control by inverted decoupling is discussed in section 3. In section 4, the design is apply to the benchmark and the results are evaluated. Finally, section 5 summarizes the conclusions.

## 2. THE BOILER MODEL

This work is focused on the boiler control problem associated to the multivariable proposition in the benchmark PID 2012. In this case, the boiling process can be approached as a multivariable system with two variables (steam pressure and water level) that can be controlled by two manipulated variables (fuel flow and water flow). Additionally, there is a measurable disturbance variable (load level), and an indirect controlled variable (oxygen level) used as quality performance variable. All of these variables are expressed in percentage. The input variables are subjected to the range of [0-100] %, and the fuel flow has a slew-rate limit of ±1 %. More information about the boiler model can be found in the website: www.dia.uned.es/~fmorilla/benchmarkPID2012/.

In order to carry out the proposed design in this work, it is necessary to start from a linear model of the plant. Using the Matlab identification toolbox, a linearized model of the boiling system has been obtained around the normal operation point: fuel flow ≅ 35.21 %, water flow ≅ 57.57 %, load level ≅ 46.36 %, steam pressure ≅ 60 %, oxygen level ≅ 50 %, and water level ≅ 50 %. The obtained continuous model is given by (1), where G(s) is the transfer matrix relating the controlled variables to manipulated variables, and where $G_d(s)$ relates the controlled variables to the measurable disturbance variable (load level). The oxygen level is not shown because it will not be taken account in the design.

$$G(s)=\begin{pmatrix} \dfrac{0.308}{28.96s+1} & \dfrac{-0.159}{183.7s+1} \\ \dfrac{-0.0055872\cdot(-166.9s+1)}{s(26.38s+1)} & \dfrac{0.010645}{s} \end{pmatrix}$$
$$G_d(s)=\begin{pmatrix} \dfrac{-0.72384}{(195.5s+1)(40.5s+1)} \\ \dfrac{-0.0013778\cdot(-76.32s+1)}{s(7.882s+1)} \end{pmatrix} \quad (1)$$

The open loop dynamic behaviours of this process are the following. The first output (steam pressure) response is stable for the three input signals (both flows and load level). There is a non-minimum phase behaviour in the second output (water level) associated to the first input (fuel level) and the load level. Moreover, the water level shows an integrating response for all of input signals.

## 3. PID CONTROL BY INVERTED DECOUPLING

Considering the unity output feedback 2x2 control system in Fig. 2, and assuming that the open loop transfer matrix L(s) should be diagonal, the elements of the centralized inverted decoupling are given by

$$k_{11}=\frac{l_1}{g_{11}} \quad k_{12}=\frac{-g_{12}}{l_1} \quad k_{21}=\frac{-g_{21}}{l_2} \quad k_{22}=\frac{l_2}{g_{22}}, \quad (2)$$

where the Laplace operator s has been omitted, and where $l_1(s)$ and $l_2(s)$ are the desired open loop transfer functions. The proof can be found in (Garrido, Vázquez and Morilla, 2010). The main advantage of (2) is the simplicity of the $k_{ij}$ elements in comparison with that of the elements in (3), obtained with the conventional centralized control of Fig. 1.

$$K=\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}=\begin{pmatrix} g_{22}l_1 & -g_{12}l_2 \\ -g_{21}l_1 & g_{11}l_2 \end{pmatrix}/(g_{11}g_{22}-g_{12}g_{21}) \quad (3)$$

The controller elements in (1) do not contain sum of transfer functions, whereas those in (2) may result very complicated even if the elements of G(s) have simple dynamics. Additionally, the open loop transfer functions $l_i(s)$ may keep very simple in such a way that the performance requirements can be specify easily.

Nevertheless, the structure of centralized inverted decoupling control presents an important disadvantage: because of stability problems it cannot be applied to processes with multivariable right half plane (RHP) zeros, that is, RHP zeros in the determinant of G(s). Fortunately, the linear model in (1) does not have multivariable RHP zeros, so this method can be applied.

In order to obtain the four $k_{ij}(s)$, it is only necessary to specify the two transfer functions $l_i(s)$. They can be selected freely as long as the controller elements are realizable.

### *3.1 Controller realizability*

The realizability requirement for the controller is that its elements should be proper, causal and stable. For processes with time delays or RHP zeros, direct calculations can lead to elements with prediction or unstable poles. Apart from the scheme of Fig.2 with the elements in (2), there is an alternative scheme for centralized inverted decoupling, in which the elements in the direct path are alternated (Garrido, Vázquez and Morilla, 2010). Its controller elements are given by

$$k_{11}=\frac{-g_{11}}{l_1} \quad k_{12}=\frac{l_2}{g_{21}} \quad k_{21}=\frac{l_1}{g_{12}} \quad k_{22}=\frac{-g_{22}}{l_2}, \quad (4)$$

Next, the conditions that a specified configuration, (2) or (4), needs to satisfy in order to be realizable are commented. Additionally, the constraints on the open loop transfer functions $l_i(s)$ are stated. There are three aspects to take into account and to be inspected by row:

1- Non causal time delays $\tau_{ij}$ must be avoided in controller elements. If $g_{ik}(s)$ is the transfer function of the row *i* with the smallest time delay $\tau_{ik}$, the element $k_{ki}(s)$ of K(s) should be selected to be in the direct path between the process and the reference error. In addition, the time delay ($\tau_i$) of the $l_i(s)$ transfer function must fulfil

$$\min(\tau_{ij}) \le \tau_i \le \max(\tau_{ij}) \quad j = 1,2 \; ; \tag{5}$$

where $\tau_{ij}$ represents the time delay of $g_{ij}(s)$, *min* represents the minimum function, and *max*, the maximum function.

2 - Decoupler elements must be proper, that is, the relative degree $r_{ij}$ must be greater or equal than zero. Similarly to the previous case, the element $k_{ki}(s)$ should be in the direct path if the transfer function $g_{ik}(s)$ has the smallest relative degree $r_{ik}$ of the row *i*. In addition, the relative degree ($r_i$) of the $l_i(s)$ transfer function must fulfil

$$\min(r_{ij}) \le r_i \le \max(r_{ij}) \quad j = 1,2 \; . \tag{6}$$

3 - When some transfer function $g_{im}(s)$ has a RHP zero, the element $k_{mi}(s)$ of K(s) should not be selected in the direct path, in order to avoid this zero becomes a RHP pole in some controller element. When the zero appears in all elements of the same row, it is necessary to check its multiplicity $\eta_{ij}$ in each element. Again, if $g_{ik}(s)$ is the transfer function of the row *i* with the smallest RHP zero multiplicity $\eta_{ik}$, the element $k_{ki}(s)$ should be selected to be in the direct path. This RHP zero must appear in the $l_i$ open-loop transfer function with a multiplicity ($\eta_i$) that fulfils

$$\min(\eta_{ij}) \le \eta_i \le \max(\eta_{ij}) \quad j = 1,2 \; . \tag{7}$$

From (5), (6) and (7), note that when the value (time delay, relative degree or RHP zero multiplicity) is shared by both transfer functions of the row, there are more possibilities to choose the configuration, but the flexibility (time delay or relative degree) of the open-loop process $l_i(s)$ is limited to the common value of row.

When two elements of K(s) have to be selected necessarily in the same column to satisfy the previous conditions in both rows, there is no realizable configuration. Then, it is necessary to insert an additional block N(s) between the system G(s) and the inverted controller K(s) in order to modify the process and to force the non-realizable elements into realizability. Then, centralized inverted control would be applied to the new process $G_N(s)=G(s)\cdot N(s)$. This problem is well discussed in (Garrido, Vázquez and Morilla, 2011a).

For the boiler process (1), the inverted decoupling scheme in Fig. 2 is realizable without adding any extra dynamics N(s); therefore, expressions in (2) must be used.

### *3.2 How to specify the $l_i(s)$*

Every open loop transfer function $l_i(s)$ used in (2) must take into account the dynamic of the two processes $g_{i1}(s)$ and $g_{i2}(s)$ to obtain realizability, and the achievable performance specifications of the corresponding closed loop system. Since the closed loop must be stable and without steady state errors due to set point or load changes, the open loop transfer function $l_i(s)$ must contain an integrator. Then, the following general expression for $l_i(s)$ is proposed:

$$l_i(s) = k_i \bar{l}_i(s)\frac{1}{s} \, . \tag{8}$$

Parameter $k_i$ becomes a tuning parameter in order to meet design specifications and the $\bar{l}_i(s)$ must be a rational transfer function taking into account the not cancellable dynamic of $g_{i1}(s)$ and $g_{i2}(s)$, and the conditions (6) and (7).

Substituting (8) into (2) the general expressions of the controller elements are obtained as follows

$$k_{ii}(s) = k_i \frac{\bar{l}_i(s)}{s \cdot g_{ii}(s)} \text{ and } k_{ij}(s) = -\frac{s \cdot g_{ij}(s)}{k_i \cdot \bar{l}(s)} \, . \tag{9}$$

In the boiler process under review (1), $\bar{l}_1(s)$ =1 is chosen for $l_1(s)$, because the processes associated to this row are stable and minimum phase systems. In this case, the closed loop transfer function has the typical shape of a first order system:

$$h_1(s) = \frac{k_1/s}{1+k_1/s} = \frac{1}{T_1 s+1} \, , \tag{10}$$

with time constant $T_1=1/k_1$. Therefore, after specifying a desired time constant of the closed loop system $T_1$=20 s, it is obtained that $k_1$=0.05.

On the other hand, $\bar{l}_2(s)$ =(s+z)/s is chosen for $l_2(s)$ because the processes of the second row are stable, except in s=0, and minimum phase systems. The corresponding closed loop transfer function is given by (11), a second order system with a zero at s=-z.

$$h_2(s) = \frac{k_2(s+z)/s^2}{1+k_2(s+z)/s^2} = \frac{k_2(s+z)}{s^2+k_2 s+k_2 z)} \tag{11}$$

Its poles are characterized by the natural frequency and the damping factor

$$\omega_n = \sqrt{k_2 z} \quad \xi = \sqrt{k_2/4z} \; . \tag{12}$$

In the controller design, a critical damping and $\omega_n$=0.0628 are selected. From (12), $k_2$=0.1257 and z=0.0314 are obtained.

Consequently, after selecting the two transfer functions $l_i(s)$, the diagonal equivalent open loop process L(s) is

$$L(s) = \begin{pmatrix} \dfrac{0.05}{s} & 0 \\ 0 & \dfrac{0.1257\cdot(s+0.0314)}{s^2} \end{pmatrix} . \tag{13}$$

After defining L(s), and from (9), the following controller elements are achieved:

$$k_{11}(s) = \frac{4.7011(s+0.03453)}{s} \qquad k_{22}(s) = \frac{11.805(s+0.03142)}{s} \tag{14}$$

$$k_{12}(s) = \frac{3.18s}{183.7s+1} \qquad k_{21}(s) = \frac{-0.28129s(s-0.005991)}{(s+0.0379)\cdot(s+0.03142)} \tag{15}$$

*3.3 Using PID structure*

The two resulting controllers in (14) have directly PI structure. The other two controllers in (15) are compensators with derivative action. Note that the derivative action should be filtered to avoid amplification of high frequency noise and to be implementable. In this work, it is proposed to reduce the controller elements in (15) to the structure of filtered derivative action like (16), where $K_{Dij}$ is the derivative gain and $N_{ij}$ is the derivative filter constant. Therefore, only $k_{21}(s)$ needs to be approximated.

$$k_{ij}^{D}(s) = s\left(\frac{K_{Dij}}{N_{ij}s+1}\right) \quad (16)$$

The model reduction technique used in this work is based on balanced residualization (Skogestad and Postlethwaite, 2005). The approximated element $k_{21}(s)$ obtained in this way is given by

$$k_{21}^{ap}(s) = \frac{-2.4689s}{6.9156s+1} \ . \quad (17)$$

*3.4 Feedforward compensation*

In order to compensate the disturbances generated by the load level and identified by $G_d(s)$ in (1), a feedforward compensator is developed. This is designed according to the scheme of Fig. 3, where each compensator $c_{FFi}(s)$ sees a monovariable process $l_i(s)$, thanks to the decoupling carried out previously. In this way, the feedforward design is considerably simplified. If the feedforward action is added directly to the control signal $u_i$, it would be necessary to invert G(s) and to use four feedforward blocks to maintain the system decoupled. The expression for $c_{FFi}(s)$ is given by

$$c_{FFi}(s) = \frac{-g_{di}(s)}{l_i(s)} \ . \quad (18)$$

By using (18), the feedforward compensators in (19) are obtained. Since they are compensators with derivative action, they are approximated to the same structure of filtered derivative action in (16), using balanced residualization.

$$\begin{aligned} c_{FF1}(s) &= \frac{14.476s}{7918s^2+236s+1} \approx \frac{8.1599s}{133.0285s+1} \\ c_{FF2}(s) &= \frac{-26.634s^2+0.349s}{250.8982s^2+39.7132s+1} \approx \frac{-0.4443s}{3.4702s+1} \end{aligned} \quad (19)$$

*3.5 Practical considerations*

*3.5.1 Filtering measured signals*

Due to the noise at process outputs, and in order to reduce the possible subsequently noise at the control signals, the controlled variables are filtered by a second order filter with relative damping factor ξ=1/√2. The expression of the filter is given by

$$G_f(s) = \frac{1}{1+T_f s+(T_f s)^2/2} \ . \quad (20)$$

The filter-time constant $T_f$ is chosen as $T_i/N$ for the PI controllers in (14), with N=20, as it is recommended in (Aström and Hägglund, 2006). $T_{f1}$=1.448 and $T_{f2}$=1.5915 are obtained.

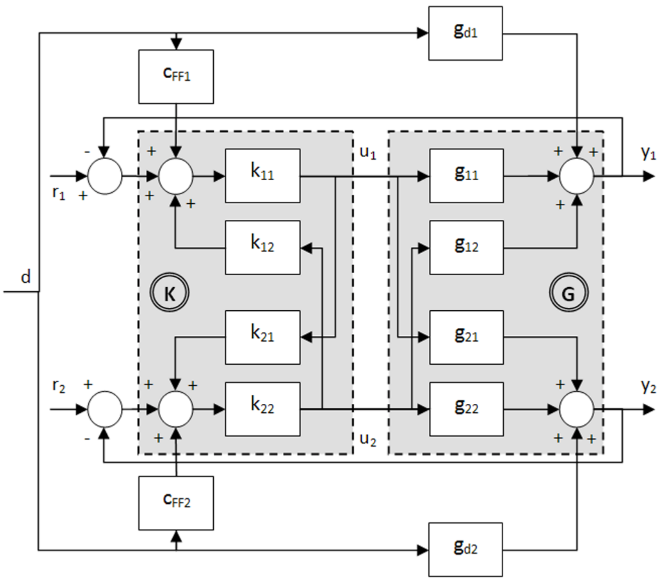


Fig. 3. 2x2 inverted centralized control with four controllers and two feedforward compensators.

*3.5.2 Anti-windup scheme*

In order to cope with the input constraints of the nonlinear boiler avoiding the windup in the PI controllers, the simple anti-windup scheme in Fig. 4 is implemented in $k_{11}(s)$ and $k_{22}(s)$. This scheme, which is used for monovariable PID controllers, is based on back-calculation (Åström and Hägglund, 2006). It uses an input constraint model inside the controller, where input saturations and slew-rate limits are considered. When the saturated input is different from the PI output, the controller works in tracking mode following the saturated signal. In this multivariable case, it is possible to use this simple monovariable scheme due to the structure of the inverted decoupling control. In the conventional scheme of Fig.1, it is more difficult to implement an anti-windup strategy.

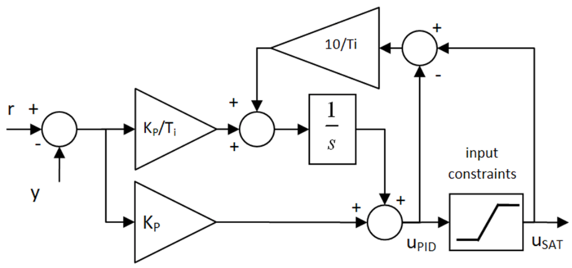


Fig. 4. PI controller with anti-windup.

## 4. SIMULATION RESULTS

In this section, the proposed control is tested for the three types of experiments in the benchmark, and results are compared with the two reference cases presented in the benchmark. The same performance indexes of the benchmark are used for comparison. The considered reference control 2 is the evaluated control in the original benchmark. Performance indexes for each experiment are listed in Table1.

**Table 1. Performance indexes for the different tests**

| | $RIAE_1$ | $RIAE_2$ | $RIAE_3$ | $RITAE_1$ | $RITAE_3$ | $RIAVU_1$ | $RIAVU_1$ | $J_M(0.25)$ |
|---|---|---|---|---|---|---|---|---|
| | **Standard test** | | | | | | | |
| Reference control 2 | 0.2682 | 0.9993 | 0.4954 | - | - | 1.6138 | 2.6508 | 0.8083 |
| Proposed control | 0.1169 | 1.0026 | 0.1541 | - | - | 1.1753 | 2.8690 | **0.6528** |
| | **Test type 1** | | | | | | | |
| Reference control 2 | 0.2645 | 0.9996 | 0.3142 | - | - | 1.5218 | 1.6868 | 0.6801 |
| Proposed control | 0.0892 | 1.0067 | 0.1738 | - | - | 1.1319 | 1.6594 | **0.5621** |
| | **Test type 2** | | | | | | | |
| Reference control 2 | 0.5210 | 1.1540 | 1.1298 | 0.3696 | - | 2.6260 | 4.4489 | 1.0985 |
| Proposed control | 0.4541 | 0.9358 | 0.0753 | 0.2679 | - | 1.0747 | 1.6722 | **0.5378** |

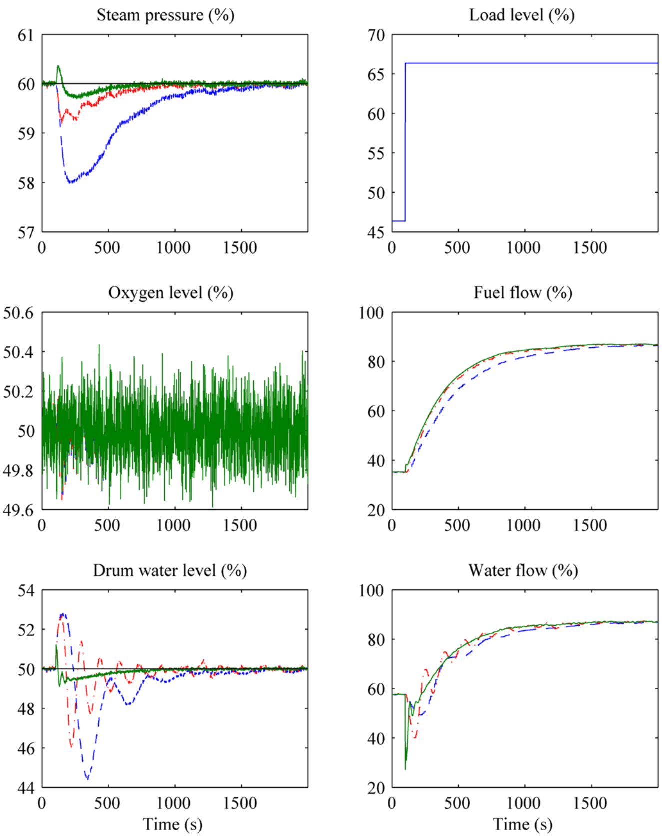


Fig. 5. Comparative standard test (Proposed control: green solid line; Reference control 1: blue dashed line; Reference control 2: red dashed-dotted line).

The simulation results for the test type 1 are shown in Fig. 6. In this case, there is a variant load level. The proposed control achieves the smallest deviations of steam pressure and water level from their respective set-points. The global performance index is 0.5621, less than the unit too.

In both previous experiments, there are load level changes, so the feedforward compensation should have improved the response. If this compensation is not used, good results can be also achieved, obtaining better performance indexes than those of the reference cases. However, the performance index associated to the error in the first output ($RIAE_1$) is considerable increased in comparison with the control scheme that uses feedforward compensation. For instance, when feedforward is not used, $RIAE_1$ index would be equal to 0.2457 in the standard test, and equal to 0.2387 in the test type 1. With the proposed feedforward compensation, the first output response is improved, with more than two times lower $RIAE_1$ values.

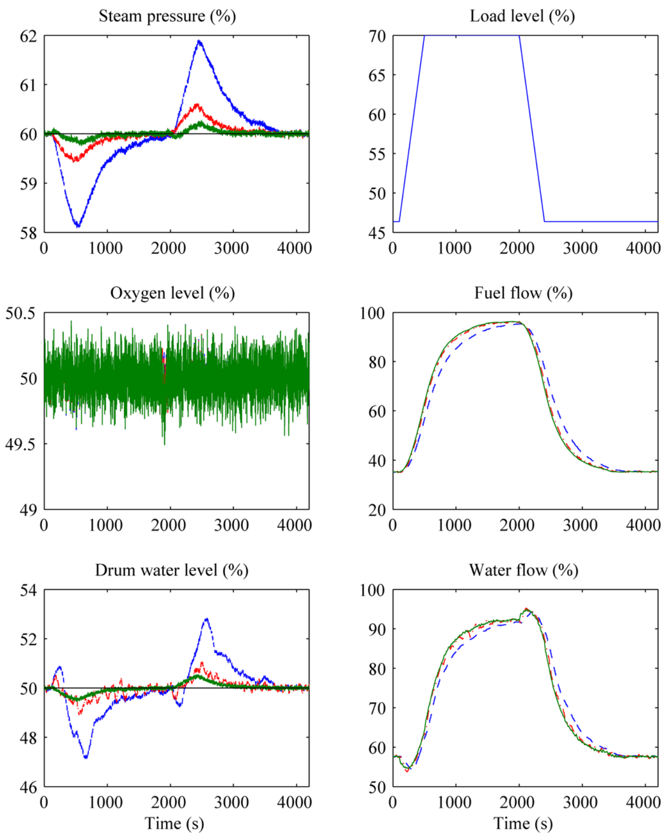


Fig. 6. Comparative test type 1 (Proposed control: green solid line; Reference control 1: blue dashed line; Reference control 2: red dashed-dotted line).

Finally, Figure 7 shows the simulation results for the test type 2, which includes a 5% step change in the steam pressure reference. The proposed control reaches the new steam pressure set-point without oscillations and very fast in comparison with the reference controls. In addition, the water level is almost decoupled from this reference change. Nevertheless, the other reference controllers show great interactions in this output. Moreover, the lowest peak in the indirect controlled variable of oxygen level is obtained with the proposed control. Most of performance indexes are smaller than those of the reference cases, obtaining a global index of 0.5378.

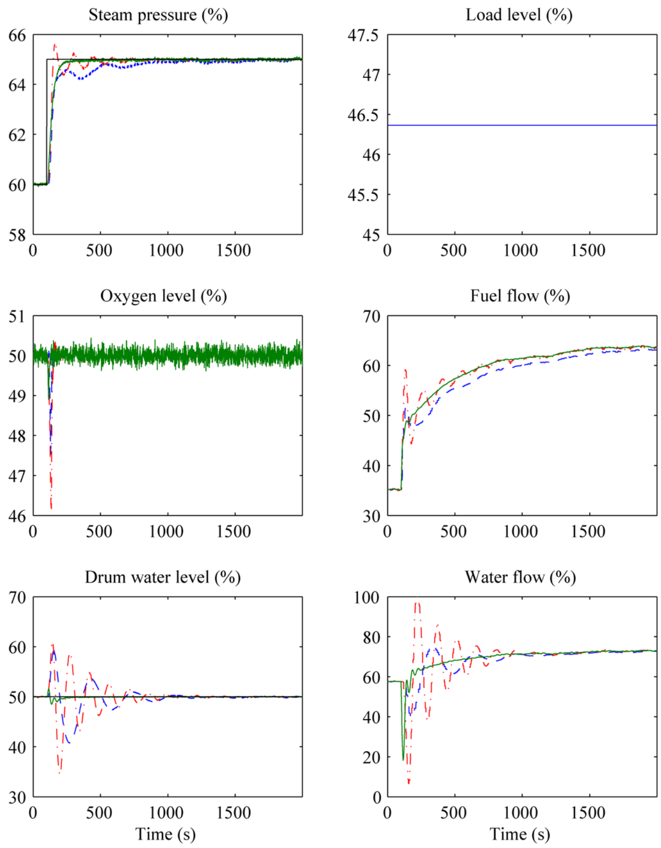


Fig. 7. Comparative test type 2 (Proposed control: green solid line; Reference control 1: blue dashed line; Reference control 2: red dashed-dotted line).

## 5. CONCLUSIONS

In this work, a boiler control problem, proposed as a benchmark, has been approached using a PID control by inverted decoupling with feedforward compensation. The methodology of this new centralized decoupling strategy has been explained. And then, it has been applied to the process under review. This methodology makes possible an easy design. In addition, and thanks to the structure of the proposed decoupling scheme, other problems, like feedforward compensation and anti-windup, can be dealt as in the monovariable case. This is not so simple for other centralized methods. After simulation, the effectiveness of the proposed design is verified obtaining smaller global performance indexes than those of the two reference cases.

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