A ROBUST EXTREMUM SEEKING CONTROL SCHEME USING TRANSIENT MEASUREMENTS

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Abstract

This work focuses on efficiently using transient measurements in extremum seeking control to speed up the convergence to the optimum. This is done by using a dynamic extremum seeking scheme, where local linear dynamic models are used instead of local linear static models. In particular, we propose a novel robust extremum seeking scheme, where existing domain knowledge in the form of the process dynamics is incorporated in the extremum seeking scheme. By fixing the linear dynamics, we can use real-time transient measurement data to robustly estimate the local steady-state effect of the input on the cost function. In addition, we also provide some bounds on the unmodeled dynamics to ensure robust stability of the proposed dynamic extremum-seeking scheme.

Keywords

System identification, Extremum seeking control, transient data, robust stability

Introduction

Traditional real-time optimization requires an elaborate nonlinear steady-state model of the system to compute the optimal solution. This can be challenging for many processes, where the nonlinear models may be structurally incorrect, or simply the cost of developing such models are too high.

In order to address the high cost of developing elaborate nonlinear models, there has been a surge of interest in the so-called "model-free" real time optimization (RTO) approaches such as extremum seeking control (ESC) and NCO (Necessary Condition of Optimality) tracking control, where the idea is to constantly perturb the system. The steady-state gradient from the cost to the input is then estimated using real-time process data. The estimated steady-state cost gradient is then driven to a constant setpoint of zero, thereby achieving the necessary condition of optimality.

The main disadvantage of extremum seeking control and related methods such as NCO-tracking, is the prohibitively slow convergence to the optimum. In order to estimate the steady-state cost gradient, the perturbation signal must be much slower than the plant dynamics, such that the dynamic plant can be approximated as a static map. Furthermore, the integral gain to drive the steady-state gradient to zero must be small enough such that the convergence to the optimum is much slower than the perturbation signal. In summary, this means that the overall convergence rate is about two orders of magnitude slower than the original plant dynamics (Kristic & Wang, 2000). For many process systems, which have long settling times, this leads to prohibitively slow convergence. Despite the very appealing characteristic of not requiring a detailed model, this makes extremum seeking control impractical for real-time optimization of most processes.

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The main reason for the slow convergence is the steady-state wait time, because of the simple local linear static approximation used in almost all variants of extremum seeking control. Using transient measurements leads to erroneous gradient estimates. In order to address the problem of slow convergence, one potential solution is to explicitly include the plant dynamics in the ES scheme. The use of measurements to repeatedly identify a local linear dynamic model around the current operating point for online optimization of slow chemical processes was first proposed by Bamberger and Isermann (1978) for Hammerstein plants, where ARX models were repeatedly identified online and the input was updated using an adaptation law in the same fashion as in most extremum seeking approaches. This approach can be seen as a *dynamic* variant of extremum seeking control for Hammerstein plants, where the cost measurements are used to identify a local linear ARX model, which is used to estimate the steady-state gradient, which is then driven to zero using integral action.

In simulations, local linear dynamic models such as ARX models were shown to be an effective way of taking into account the plant dynamics, enabling fast convergence to the optimum for slow dynamic processes, since this effectively removes the assumption that the plant behaves like a static map.

Following the introduction of ARX-like local linear dynamic model to estimate the gradient in the 80's (Bamberger & Isermann, 1978), this approach has remained dormant, despite the recent surge of interest in extremum seeking control. This is probably because of the robustness problems that impede practical implementation of such approaches. Identifying an ARX model online, as opposed to estimating only the steady-state component, involves estimating additional parameters. Consequently, an issue with identifying ARX models online is the need for sufficient excitation to accurately estimate the parameters of the ARX model. As the system approaches its optimum, the steady-state relation between the input and the cost *f(u)* is typically flat and there is not sufficient excitation in the measured cost to accurately estimate all the parameters of the ARX model. This leads to numerical bursting with sudden peaks in the estimated steady-state gradient, causing the control input to respond erratically close to the optimum. When identifying ARX models online in Bamberger and Isermann (1978), the optimizer was turned off once the plant reached its optimum. Therefore, this issue was not reported by Bamberger and Isermann (1978).

Another major concern is that when estimating all the parameters of the ARX model, it may also lead to robustness issues due to the unmodeled dynamics. If the plant dynamics are different from the ARX model, then the steady-state gradient estimated using the ARX model may not be correct and can lead to robustness issues. This makes the problem sensitive to the ARX model structure and in fact, it is very difficult to provide any robustness margins due to the neglected dynamics in the ARX identification problem, since the least square fitting problem may result in any value for the ARX model parameters.

To summarize, the dynamic extremum scheme may not be robust when,

- the process reaches closes to its optimum and the excitation is not sufficient to estimate all the ARX model parameters accurately,
- process includes neglected dynamics not captured by the chosen ARX model structure.

Robust Extremum seeking control using transient measurements

In order to address the robustness issues, we propose to fix the parameters of the *G(s)* with some nominal linear dynamics $G_0(s)$ in the estimation problem and use the online measured data to estimate only the unknown nonlinear steady-state component of the process. In other words, if domain knowledge in the form of time constants for the process dynamics $G_0(s)$ is known a-priori (for e.g. from step response tests), we can incorporate this knowledge in the online identification problem to estimate the steady-state effect of the process *f(u)* around the current operating point. This way, we can incorporate the "*known*" nominal system dynamics in the online gradient estimation problem which enables us to effectively use the transient measurements to estimate the steady-state gradient. Since the nominal linear dynamics $G_0(s)$ is known and fixed, we can then provide robustness margins for the neglected plant dynamics using classical robust control theory. This provides a numerically stable approach to estimate the steady-state gradient using transient measurement data.

Consider a plant, where the cost *J* is measured and can be represented as a Hammerstein model with a combination of a nonlinear, time-invariant mapping *f(u)* with proper, stable, finite-dimensional, linear, time-invariant (FDLTI) dynamics *G(s)* at its output, $J = f(u)G(s)$. Let $G_0(s)$ denote the known nominal plant dynamics, which is transformed to discrete time transfer function $G_0(q)$.

The nonlinear dynamic plant can then be approximated as a locally linear dynamic plant of the form,

$$h(u)G_0(q) \approx J_u \underbrace{\left(\frac{b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}}{1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}} \right)}_{\text{fixed}} u \qquad (1)$$

where, the steady-state gain *k* is the local linear approximation of the nonlinear steady-state component *f(u)*. In the proposed approach, we fix the parameters,

$$[a_1 \quad \cdots \quad a_{n_a} \quad b_1 \quad \cdots \quad b_{n_b}]$$

and estimate only the local steady-state gain *k* of the plant (1). To do this, we solve the linear least squares problem,

$$\hat{\theta} = arg \ \min_{\theta} \|\psi - \Phi^{\mathsf{T}}\theta\|_2^2 \qquad (2)$$

with

$$\psi = J(t) + a_1 J(t-1) + \cdots + a_{n_a} J(t - n_a)$$
$$\Phi = b_1 u(t-1) + \cdots + b_{n_b} u(t - n_b) \qquad (3)$$
$$\theta = k$$

By incorporating the a-priori knowledge about the plant dynamics, we show that the transient measurements can be used to significantly speed up the convergence of the extremum seeking scheme. In other words, we now use the transient measurement data to estimate only one unknown parameter, which is the steady-state gradient k. Reducing the number of parameters to be estimated makes the estimation problem more numerically robust.

Due to the system nonlinearity, there could be unmodeled dynamics that are not included in the nominal dynamics $G_0(s)$. We show that unmodeled dynamics, (including inverse responses) can be treated as multiplicative uncertainty $(1 + w_I(s)\Delta_I(s))$ (Skogestad and Postlethwaite, 2007), where $|\Delta_I(j\omega)| \leq 1 \forall\omega$ represents the complex perturbations and w_I denotes the multiplicative weight. In this case, we show that robust stability of the proposed extremum seeking scheme can be ensured as long as the unmodeled dynamics are bounded by,

$$|w_I| < \frac{1}{|T_0|}, \quad \forall\omega \tag{4}$$

with $T_0 = (1 + kG_0K)^{-1}kG_0K$ being the nominal complementary sensitivity function with the integral control action $K = K_I/s$.

Illustrative example

We demonstrate the proposed robust extremum seeking control using a simple example, where the process is given by $J = f(u)G_0(s)g(s)$. Here, f(u) is the nonlinear steady-state effect, $G_0(s) = 1/(174s + 1)$ is the known nominal linear dynamics and $g(s) = (-5s + 1)/(10s + 1)$ is the neglected dynamics. Note the neglected dynamics has a RHP-zero leading to inverse response.

We first apply the dynamic extremum seeking scheme from Bamberger and Isermann (1978) (denoted by BI-approach), where we identify a first order ARX model while neglecting the dynamics $g(s)$. This is shown using gray lines in Fig.1(b), where we see that the by identifying all the parameters of the ARX model, the closed-loop response is clearly not robust due to the neglected inverse response in the process measurements. This leads to erratic behavior of the closed-loop system. The robust stability margins (4) are verified as shown in Fig.1(a). We then apply the proposed robust extremum seeking scheme by fixing the nominal dynamics $G_0(s)$ and estimating only the steady-state gain from the process measurements using (2). This is shown in black curves, where we see that the closed-loop response is stable and is able to smoothly drive the process to its optimum, despite the neglected dynamics $g(s)$.

We also simulate the case with a small time delay uncertainty, $g(s) = e^{-0.01s}$. The robust stability margins and the simulation results are shown in Fig.2, where we see that the proposed method is robust to the neglected time delay, whereas the Bamberger and Isermann approach performs erratically.

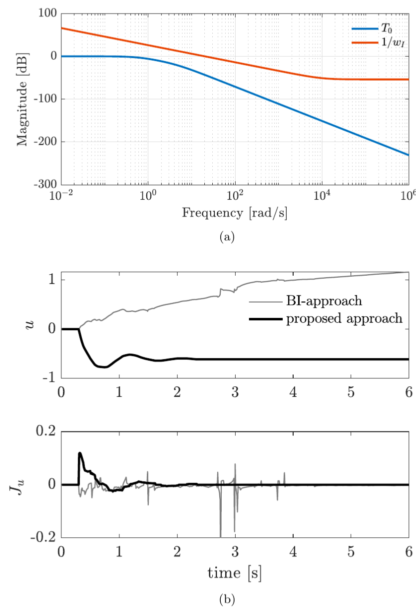

Figure 1. Simulation results using the proposed robust extremum seeking scheme, with neglected RHP-zero.

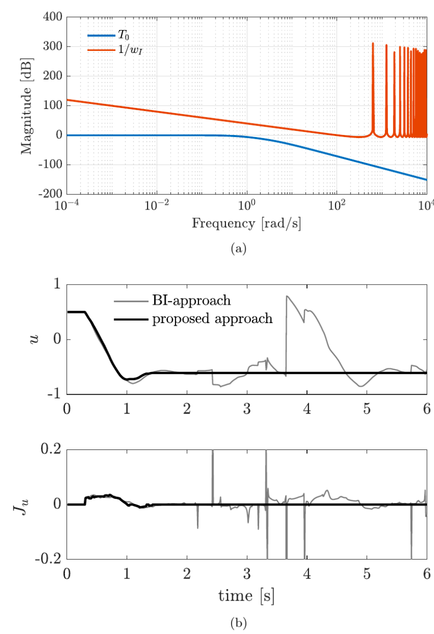

Figure 2. Simulation results using the proposed robust extremum seeking scheme, with neglected time delay.

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