

# BAYESIAN STATISTICAL LEARNING AND STOCHASTIC PROGRAMMING FOR OPTIMAL ENERGY MARKET PARTICIPATION

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## *Abstract Overview*

We explore the deep connection between data-driven Bayesian statistical inference and stochastic programming paradigms for decision-making under uncertainty. As a motivating example, we consider optimal participation strategies in multiscale electricity markets. We use Gaussian Process (GP) regression to develop probabilistic models for energy prices. Time series price forecasts are then sampled from the GP predictive distribution. These forecasts enable two modes of optimal market participation: i) for self-scheduling, the resource (e.g. generators, consumers, storage) determines when to buy/sell energy and takes the market price; ii) alternately, a resource can submit bidding curves. Both participation modes are also formulated as stochastic programs. We compare these two modes of market participation using historical day-ahead market data from CAISO for both energy storage and thermal generators. We show that the self-schedule mode is less robust to market uncertainty but allows a resource to ensure feasible operation. In contrast, bidding into the market is more robust to uncertainty but does not guarantee feasible operation.

## *Keywords*

*Stochastic Programming, Gaussian Process Regression, Energy Markets*

## **Introduction**

As part of the emerging smart grid paradigm, many energy intensive industrial systems now modulate their operations to better align with fluctuations in energy market prices (Chmielewski, 2014). A standard technique to estimate these energy economic opportunities is to calculate the maximum possible revenue in retrospect (Walawalkar et al., 2007). An important limitation of this technique is that the use of perfect information, which does not capture market uncertainty and thus only provides an upper bound on revenue opportunity. In reality, resources participate in wholesale energy markets under uncertainty via two modes: self-schedule and bidding.

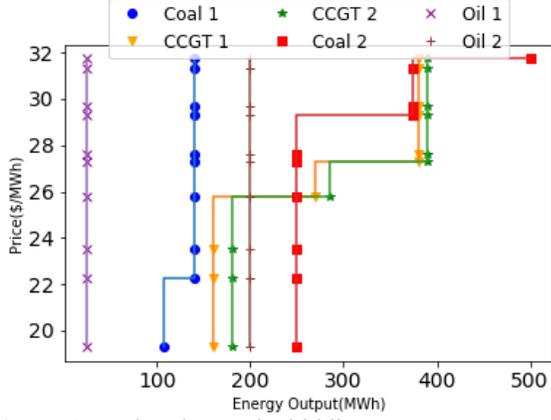
If a resource self-schedules in the market, they determine when and how much electricity to consume/produce and take the market price. This market participation mode can be formulated as a single or multi-stage stochastic program. Resources seek to minimize their expected operational cost or maximize their revenue. In their seminal work, Ierapetritou et al. (2002) analyzed energy-intensive processes like air separation units (ASU) using 2-stage stochastic programs. Recently, Kumar et al. (2018) proposed a multi-stage stochastic model predictive control (MPC) framework for stationary battery systems.

See the literature reviews of Dowling et. al. (2017), Dowling & Zavala (2018) and Sorourifar et. al. (2018) for additional examples.

Alternatively, an energy resource can also submit bidding curves. Bidding curves are time-varying piece-wise constant price and energy pairs, as shown in Fig. 1. A bidding curve communicates to the market the resources' flexibility and marginal costs. Calculation of bidding curves for the day-ahead and real-time market is also formulated as stochastic programs, and again the expected operational cost (revenue) is minimized (maximized). Non-decreasing constraints, an analog to non-anticipativity constraints, enforce the shape. For example, stochastic programs have been applied to derive bidding curves for wind power systems (Dai and Qiao, 2015), concentrating solar power plant (Dominguez et al., 2012), aluminum smelters (Zhang and Hug, 2015), thermal generators (Plazas et al., 2005), and virtual power plants (Pandžić et al., 2013).

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**Figure 1.** Each point on the bidding curve corresponds to a price forecast (scenario) in the stochastic program. When the market clears and the energy clearing price is set, the resource is scheduled to deliver/consume the corresponding power specified on the bidding curve.

We emphasize that both participation modes use energy price forecasts to construct the scenarios for the stochastic programs. Autoregressive integrated moving average (ARIMA) is one of the most popular forecasting methods (Dai and Qiao, 2015; Plazas et al., 2005; Dominguez et al., 2012). We argue that Bayesian methods offer advantages over these traditional timeseries forecasting techniques. Bayesian formalism naturally provides posterior and predictive distributions for which to compute an expectation (or risk metrics) over. In this work, we use Gaussian Process (GP) regression to model uncertain energy prices. Time series price forecasts are then sampled from the GP predictive distribution. Using these forecasts, we compare both modes of market participation for stand-alone energy storage and thermal generators.

## Gaussian Process (GP) Regression

We start by learning a Gaussian Process regression model for stochastic energy prices (Bishop, 2006). Immediately previous  $d$  historical prices are used as a vector input. We use a Gaussian distribution with a zero mean and a covariance ( $\mathbf{K}$ ) constructed by the radial-based function (RBF) kernel as the energy price ( $\mathbf{y}$ ) prior distribution:

$$p(\mathbf{y}) = \mathcal{N}(\mathbf{y}|\mathbf{0}, \mathbf{K}) \quad (1)$$

We also use a Gaussian likelihood function with Gaussian noise for historical prices ( $\mathbf{t}$ ):

$$p(\mathbf{t}|\mathbf{y}) = \mathcal{N}(\mathbf{y}|\mathbf{t}, \beta^{-1}\mathbf{I}_N) \quad (2)$$

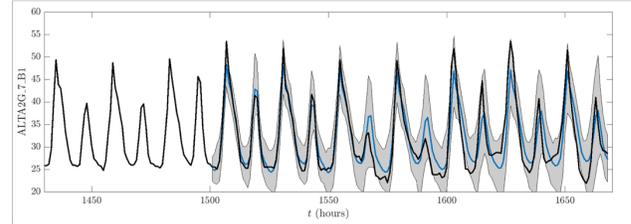
Using the properties of linear Gaussian models, we can derive the marginal likelihood function (Eq. (3)) and the

predictive distribution (Eq. (4)), where  $m$  and  $\sigma^2$  are functions of the inputs, i.e. immediately previous prices.

$$p(\mathbf{t}) = \mathcal{N}(\mathbf{t}|\mathbf{0}, \mathbf{K} + \beta^{-1}\mathbf{I}_N) \quad (3)$$

$$p(t^*|\mathbf{t}) = \mathcal{N}(t^*|m, \sigma^2) \quad (4)$$

The kernel hyperparameters are learned by maximizing Eq. (3) the marginal likelihood function (Type II ML). Finally, with the learned hyperparameters, price forecasts can be sampled from the predictive distribution. For example, an ensemble of forecasts for the 1-hour day-ahead market is shown in Fig. 2.



**Figure 2.** The first 72 prices (hour 1429 - 1500) are used as a vector input. The predicted mean energy price is shown in blue along with a 95% prediction interval (grey area). The true prices are shown in black.

## Stochastic Programming Models

Linear stochastic programs (Shapiro et al., 2009) are applied in both market participation modes. We minimize the expected operation cost function  $f(\mathbf{x}, \mathbf{t}^*)$ , where  $\mathbf{x}$  is the set of control variables of the energy resource and  $\mathbf{t}^*$  is the uncertain energy price. We formulate the stochastic programs as follows:

$$\min. \mathbb{E}_{\mathbf{t}^*}[f(\mathbf{x}, \mathbf{t}^*)]$$

$$\text{s.t. } \mathbf{Ax} = \mathbf{b}, \quad \mathbf{x} \geq \mathbf{0} \quad (5)$$

where the constraints include the system physics and market participation rules. This formula is general and can be augmented to model either market participation mode. For self-scheduling, a set of non-anticipativity constraints is enforced to reach unanimous operational decisions across the uncertain space for some part of the planning horizon. Whereas, for computing bidding curves, a set of constraints is introduced to ensure higher energy production at higher prices, which makes the bidding curves non-decreasing and consistent with bidding rules in most of the markets. An example of day-ahead market bidding curves of thermal generators is shown in Fig. 1.

## Results

We compare two types of energy systems. We consider the energy storage model from Dowling et al. (2017), where the stand-alone system is able to both buy and sell energy

subject to a round-trip efficiency. Second, we consider the thermal generator model from Plazas et al., (2005) which can only sell energy to the market. We use the energy prices from CAISO in 2015 to perform all the case studies. The descriptive statistics of the dataset are reported by Dowling & Zavala (2018).

We also compare the two modes of market participation. Our preliminary results show that the self-schedule mode is less robust to market uncertainty but allows a resource to insure feasible operation. In contrast, bidding into the market is more robust to uncertainty (i.e., the resource submits several contingencies via their bid curve) but does not guarantee feasible operation. This is especially important for energy storage systems, where the amount of stored energy may prevent a resource from satisfying the cleared market schedule, incurring a penalty. Our ongoing work focuses on implementing a receding horizon framework for simultaneous participation in both day-ahead market and (real-time) fifteen-minute market while ensuring a feasible operation.

## Conclusions

We argue that there is a great opportunity for statistical learning to incorporate forecasting and uncertainty modeling. We especially advocate for Bayesian perspectives, as it provides posterior and predictive distributions after observations which is well-suited for stochastic programming, which seeks to optimize an expected value or other risk metric defined over of probability space. As an example, we review the different energy market participation modes and show how they leverage stochastic programming, a popular paradigm in process system engineering. A Gaussian Process regression model is developed for energy price forecasting. Finally, using energy storage and thermal generator models, we compare the two market participation modes: self-scheduling and bidding.

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