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Maximizing Fault Detectability with Closed-Loop Control ⋆

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Abstract: This paper addresses the fault detectability problem for PCA-based process monitoring methods under closed-loop control. Unlike previous research assuming the same process variation for both modeling and monitoring periods, the impact of controller on data variation is considered. The effective fault direction is given in order to describe the effect of process variation on fault detection index under different controllers. The minimum variance along the effective fault direction is developed by introducing the concept of block-lower-triangular interactor matrix and conditional minimum variance control. The sufficient condition for a fault to be detectable under any controllers is provided. The proposed method is demonstrated with the simulation examples.

Keywords: fault detection, fault detectability, time delay, minimum variance control, performance monitoring, time-series analysis

1. INTRODUCTION

Statistical process monitoring has been studied by many researchers and successfully applied in the process industries. Measurements from different sensors are collected and incorporated into a data-driven model, which is typically based on principal component analysis (PCA) (Kresta et al., 1991) or partial least squares (PLS) (MacGregor et al., 1994; Qin and Zheng, 2013). Some statistics are subsequently derived to monitor the process operation faults and sensor faults.

While fault detection and diagnosis has been researched intensively, analysis on fault detectability based on PCA is insufficient. Early work by Dunia and Qin (1998c) provides the sufficient condition for unidimensional-fault detectability with Squared Prediction Error (SPE) statistic. A more rigorous derivation is presented in (Dunia and Qin, 1998a), which is further extended to the multidimensional-fault case in Dunia and Qin (1998b) and Mnassri et al. (2013) where the necessary condition for SPE is also derived. To improve fault detectability with PCA, Wachs and Lewin (1999) have developed an improved PCA method that recursively sums the scores and increases correlations between the input-output data by time shifting. However, these research work assumes the faulty data contain a constant bias in steady state, which is parameterized as the product of fault direction ξ and magnitude f, while the distribution or variation of process data are identical during modeling and monitoring periods, which are shown as blue and red ellipses in Fig. 1.

Industrial process systems are usually operated under the regulation of feedback controllers. During the design phase, a controller is synthesized by balancing different objectives and factors, e.g., setpoint tracking, disturbance rejection, and economics. Improving fault detectability under abnormal condition is rarely taken into account, since the process is designed around a predetermined operating point. To address this problem, recently Du et al. (2016) propose an optimal tuning method seeking for a trade-off between fault detection and control performance by solving an optimization problem, where the fault detectability is expressed by a likelihood function. In the case of PCA-based monitoring method, maximizing likelihood is equivalent to reduce the process variation along the effective fault direction (Dykstra and Sun, 2017). However, in practice when a portion of faulty samples are detected by process monitoring systems, the user tends to tune the controller such that fault detectability weighs more in the control objective. Such action can be detrimental – closed-loop control performance is sacrificed while faults may never be fully detected given a confidence limit. As illustrated in Fig. 1, fault detection rate is improved by implementing a controller that generates process data represented by the magenta ellipse, but this ellipse may never collapse into a line where all faulty samples can be distinguished. Therefore, it calls for a benchmark for fault detectability with the "best" feedback controller. Users can be informed of the minimum fault magnitude that is detectable for a specific fault direction.

This paper addresses the fault detectability problem with PCA-based approaches under any feedback controllers. Since fault detection rate can be improved by variance reduction along the effective fault direction, the minimum variance of process data along this direction will be explored. While decreasing variation of manipulated variables (MV) can be easily achieved, it is generally unachievable to suppress variation in controlled variables (CV) to zero due to the presence of time delay. It is shown in the pioneering work by Harris (1989) that the minimum vari-

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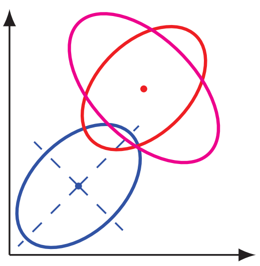

Fig. 1. Motivation. Blue ellipse: variation of normal process data; red ellipse: variation of faulty data with the same controller; magenta ellipse: variation of faulty data with a different controller.

ance of CV of single-input-single-output (SISO) systems can be estimated from routine operation data. For multi-input-multi-output (MIMO) systems, unitary interactors are adopted to derive the minimum variance of CV in terms of sum of each CV variance (Huang et al., 1997). When the variance of a CV subspace needs minimization, a block-triangular interactor matrix should be applied to describe the delay structure (Sun et al., 2011), which will be used in the current work. Based on this conditional minimum variance value, the fault detectability radius will be obtained to aid users to evaluate if it is feasible to detect a fault by re-tuning controllers.

The remaining part of this paper is organized as follows. Section 2 briefly reviews PCA-based process monitoring techniques and the sufficient condition for fault detectability. In Section 3, the conditional minimum variance is derived first followed by the development of fault detectability radius. Some simulation results are presented in Section 4. Finally conclusions are given in Section 5.

2. FAULT DETECTABILITY FOR PCA-BASED PROCESS MONITORING

2.1 PCA-Based Statistical Process Monitoring

The process monitoring method discussed in this paper relies on a PCA model which decomposes normal process data into two parts:

$$\mathbf{X} = \mathbf{T}\mathbf{P}^T + \tilde{\mathbf{X}} \tag{1}$$

where the data matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N]^T \in \mathbb{R}^{N\times m}$ consists of N data samples with each row representing a sample $\mathbf{x}_i^T \in \mathbb{R}^m$, which is scaled to zero mean and usually unit variance. $\mathbf{P} \in \mathbb{R}^{m\times l}$ is an orthonormal matrix called loading matrix, where l is the number of principle components (PC) retained. $\mathbf{T} = \mathbf{X}\mathbf{P} \in \mathbb{R}^{N\times l}$ is the score matrix. $\tilde{\mathbf{X}}$ is the residual matrix.

The principal component subspace (PCS) is represented by $\mathcal{S}_p = \text{span}\{\mathbf{P}\}$ and the residual subspace (RS) is $\mathcal{S}_r = \text{span}\{\mathbf{P}\}^{\perp}$. Therefore, a sample vector $\mathbf{x} \in \mathbb{R}^m$ can be projected onto PCS and RS by

$$\hat{\mathbf{x}} = \mathbf{C}\mathbf{x} \tag{2}$$

$$\tilde{\mathbf{x}} = (\mathbf{I} - \mathbf{C})\mathbf{x} \tag{3}$$

where $\mathbf{C} = \mathbf{P}\mathbf{P}^T$ is the projection matrix onto PCS. $\hat{\mathbf{x}}$ and $\tilde{\mathbf{x}}$ are the modeled and residual portion of $\mathbf{x}$. A geometric interpretation of PCA decomposition of $\mathbf{x}$ is shown in Fig. 2.

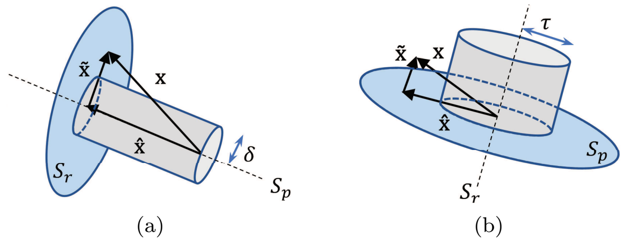

Fig. 2. A sample vector projected onto PCS and RS with (a) SPE detection cylinder, and (b) T^2 detection cylinder.

There are many existing work researching the statistics for detecting faults with PCA model. The most popular ones are SPE or Q-statistic, and Hotelling's T^2. Assuming process data observe Gaussian distribution, the control limits for either index can be derived.

The SPE statistics evaluate the distance between PCS and sample vector $\mathbf{x}$, which is its projection onto RS.

$$\text{SPE} \equiv \|\tilde{\mathbf{x}}\|^2 = \|(\mathbf{I} - \mathbf{C})\mathbf{x}\|^2 \tag{4}$$

When correlation among process variables is broken, SPE index increases. Thus, the process is considered abnormal if

$$\text{SPE} > \delta^2$$

where the control limits for SPE is denoted by δ^2. The original expression for δ^2 is developed by Box et al. (1954). Other approximations are intensively studied, e.g., (Dunia et al., 1996). The geometric interpretation of fault detection with SPE is illustrated in Fig. 2. The process is considered to be normal when sample vector is inside the cylinder whose axis is PCS and radius is δ. A sample lies outside the cylinder can be detected by SPE index and will be labeled as faulty data.

The Hotelling's T^2 index mainly detects faults in PCS. One example is normal change of the steady-state operating point. T^2 is defined as

$$T^2 = \mathbf{x}^T\mathbf{P}\mathbf{\Lambda}^{-1}\mathbf{P}^T\mathbf{x} \tag{5}$$

where matrix $\mathbf{\Lambda}$ is a diagonal matrix with diagonal elements being the first l eigenvalues of $\Sigma = \frac{1}{N-1}\mathbf{X}^T\mathbf{X}$ in descending order. Since T^2 statistic can be approximated by a χ^2-distribution with l-degrees of freedom when N is large, the control limits τ^2 becomes $\tau^2 = \chi_l^2$. Similarly, a fault in PCS is detected when

$$T^2 > \tau^2$$

2.2 Fault Model

In this paper, a unidimensional fault is assumed whose exact source may be unknown. However, the fault can be reflected on process data by the following expression:

$$\mathbf{x} = \mathbf{x}^* + f\xi \tag{6}$$

where $\mathbf{x}$ is an abnormal sample vector, $\xi \in \mathbb{R}^m$ is a known fault direction vector, and f represents the fault magnitude. Variation in process data is denoted by $\mathbf{x}^*$, which can be different during PCA modeling and monitoring periods.

In order to make fair comparison, it is assumed in this paper that the controller does not affect fault magnitude f. This can be generally achieved by setting the steady state of process at $\mathbb{E}\{\mathbf{x}\}$ or constraining the steady-state gain of controllers.

Remark. In other literatures, $\mathbf{x}^*$ is defined as normal process variation. In this work, however, since it is possible to change the controller, the process variation under abnormal condition may be different from that of normal data.

Table 1. Summary of control limits

	SPE	T^2
Control limit Γ	δ^2	τ^2

2.3 Fault Detectability under Closed-Loop Control

Regardless of fault detection subspace, the fault index in quadratic form can always be represented by

$$\gamma = \mathbf{x}^T \mathbf{M} \mathbf{x} \tag{7}$$

where γ is the index. The semi-positive-definite matrix $\mathbf{M}$ equals $\mathbf{I} - \mathbf{C}$ for SPE and $\mathbf{P}\mathbf{\Lambda}^{-1}\mathbf{P}^{\mathbf{T}}$ for the T^2 index. Multiplying $\mathbf{M}^{\frac{1}{2}}$ on both sides of (6) gives

$$\mathbf{M}^{\frac{1}{2}}\mathbf{x} = \mathbf{M}^{\frac{1}{2}}\mathbf{x}^* + \mathbf{M}^{\frac{1}{2}} f\xi \tag{8}$$

By denoting the control limit as Γ (see Table 1), the fault is sufficiently detectable when

$$\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}\| \geqslant \|\mathbf{M}^{\frac{1}{2}} f\xi\| - \|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\| \geqslant \Gamma \tag{9}$$

Since the fault is not detectable despite the value of f when $\mathbf{M}^{\frac{1}{2}}\xi = 0$, it is assumed $\mathbf{M}^{\frac{1}{2}}\xi > 0$ thereafter. Thus, one can define the effective fault direction as

$$\xi^o \equiv \frac{\mathbf{M}^{\frac{1}{2}}\xi}{\|\mathbf{M}^{\frac{1}{2}}\xi\|} \tag{10}$$

From (9), to make the fault sufficiently detectable, the equivalent fault magnitude $f^o = \|\mathbf{M}^{\frac{1}{2}}\xi\| f$ has to satisfy

$$f^o = \|\mathbf{M}^{\frac{1}{2}} f\xi\| \geqslant \|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\| + \Gamma \tag{11}$$

$$\geqslant \|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min} + \Gamma \tag{12}$$

where $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$ is the minimum value of $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|$.

Geometrically, the effective fault direction ξ^o is parallel to the radial direction of the cylinder in Fig. 2, while f^o represents the distance between the fault vector and longitudinal axis of the cylinder. A fault is considered to be detectable if f^o is at least the sum of the control limit Γ and the minimum process variation along the radial axis $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$.

More specifically, for SPE index shown in Fig. 2(a), ξ^o is parallel to $\tilde{\mathbf{x}}$; a detectable fault should be a least $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$ away from the cylindrical surface. Similarly, for T^2 index illustrated in Fig. 2(b), ξ^o aligns with $\hat{\mathbf{x}}$ with the same distance between a detectable fault and the control limit surface.

The value of $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$ can be determined by the following cases.

Case 1. All nonzero elements of ξ^o correspond to the controlled variables (CV) of the process. Since digital sampling normally introduces a unit time delay, $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$ is generally greater than zero. Its value is computed by the method shown in the next section.

Case 2. All nonzero elements of ξ^o correspond to the manipulated variables (MV) of the process. In this case, reducing the degree-of-freedom of the controller can force $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min} = 0$

Case 3. Not all nonzero elements of ξ^o correspond to either the MV or the CV. The projected process variance $\mathbf{x}^{*T}\mathbf{M}\mathbf{x}^*$ becomes a combination of MV and CV variance, whose minimum value may not be obtained unless the model of true process is known. This case will not be discussed in the current work.

3. CONDITIONAL MINIMUM VARIANCE

In this section, the conditional minimum variance control is discussed. The corresponding minimum variance of CV in a subspace will be derived, which is used to obtain CV fault detectability.

3.1 Block-Lower-Triangular-Interactor Matrix

The time delay of a SISO system can be characterized by q^{-d} where q^{-1} is the backshift operator and d denotes the delay. However, the time-delay structure of a MIMO system usually takes a more complicated form called interactor. Assume a MIMO process can be modeled as

$$\mathbf{y}_k = G(q)\mathbf{u}_k + N(q)\mathbf{a}_k \tag{13}$$

where $\mathbf{y}$, $\mathbf{u}$, and $\mathbf{a}$ represents process output, input, and Gaussian noise vectors with corresponding sizes, respectively. Transfer function matrices $G(q)$ and $N(q)$ are the process and disturbance models.

Definition 1. (Sun et al. (2011)). Given a proper and rational transfer-function matrix $G(q)$ with size $n \times n$, $D(q)$ is known as block-lower-triangular interactor matrix if

$$G(q) = D(q)^{-1}\tilde{G}(q)$$

$$\lim_{q^{-1}\to 0} \tilde{G}(q) = \lim_{q^{-1}\to 0} D(q)G(q) = K$$

and

$$D(q) = \begin{bmatrix} D_{11}(q) & 0 & \cdots & 0 \\ D_{21}(q) & D_{22}(q) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ D_{m,1}(q) & D_{m,2}(q) & \cdots & D_{m,m}(q) \end{bmatrix} \tag{14}$$

where K is a full rank finite and non-zero matrix, and $D_{i,i}$ ($i = 1, \ldots, m$) are the unitary interactor matrices.

In the context of this article, it is sufficient that $m = 2$ since there are only two types of CVs – the CVs relevant and CVs irrelevant to the fault subspace.

3.2 Derivation of Conditional Minimum Variance

To obtain $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$, the coordinate needs to be rotated such that the first CV of the process in the new coordinate aligns with ξ^o. One of the solutions is to perform QR decomposition $\xi^o = QR$ which provides the rotation matrix Q^T. Thus, by left multiplying Q^T on both sides of (13) the new system becomes

$$\mathbf{y}'_k = G'(q)\mathbf{u}_k + N'(q)\mathbf{a}_k \tag{15}$$

where $\mathbf{y}'_k = Q^T\mathbf{y}_k$, $G'(q) = Q^T G(q)$, and $N'(q) = Q^T N(q)$.

Next, the rotated CV vector $\mathbf{y}'_k$ is divided into two groups:

$$\mathbf{y}'_k = \begin{bmatrix} \mathbf{y}'_{1,k} \\ \mathbf{y}'_{2,k} \end{bmatrix} \tag{16}$$

where $\mathbf{y}'_{1,k}$ is the first CV of rotated system which is aligned with ξ^o while the rest of CVs are included in

$\mathbf{y}'_{2,k}$. According to (14), the corresponding block-lower-triangular interactor can be expressed by

$$D = \begin{pmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{pmatrix}.$$

Hence, the rotated process (15) is rewritten as

$$\mathbf{y}'_k = D^{-1}\tilde{G}'\mathbf{u}'_k + N'\mathbf{a}'_k \quad (17)$$

where the delay-free transfer function matrix $\tilde{G}' = DG'$ satisfies that $\lim_{q^{-1}\to 0}\tilde{G}'$ is nonzero, finite, and full rank. Note that the time delay d of the MIMO process is defined as the order of matrix D.

By introducing the filtered output $\tilde{\mathbf{y}}'_k = q^{-d}D\mathbf{y}'_k$, further simplification of (17) leads to

$$\tilde{\mathbf{y}}'_{t+d} = \tilde{G}'\mathbf{u}_k + DN'\mathbf{a}_k. \quad (18)$$

Since $N'(q)$ is a rational model of the delay-free disturbance, DN' can be factorized as

$$DN' = \underbrace{F_d q^d + F_{d-1}q^{d-1} + \cdots + F_1 q}_{F(q)} + R(q).$$

Therefore, Equation (18) becomes

$$\tilde{\mathbf{y}}'_{k+d} = \tilde{G}'\mathbf{u}_k + F\mathbf{a}_k + R\mathbf{a}_k. \quad (19)$$

Due to causality, it is impossible to eliminate $F\mathbf{a}_k$. Therefore, the conditional minimum variance control law can be subsequently derived:

$$\mathbf{u}_k = -\tilde{G}'^{-1}R\mathbf{a}_k \quad (20)$$

since $\tilde{G}'$ is invertible. We have the following lemma for conditional minimum variance control law under block-lower-triangular interactor.

Lemma 2. The conditional minimum variance control law (20) minimizes

(1) the variance of first rotated CV; and
(2) the sum of variances of the rest CV when 1) is achieved.

Proof. See Sun et al. (2011).

Based on (20), the conditional minimum variance is obtained.

$$\min \mathbb{E}\{\mathbf{y}'^2_{1,k}\} = \mathrm{tr}(e_1^T F\Sigma_a F^T e_1) \quad (21)$$

where $e_1 = [1, 0, \ldots, 0]^T$ and Σ_a represents the noise covariance.

3.3 Fault Detectability with Conditional Minimum Variance

Since the plant and disturbance model of process is linear and noise source is Gaussian, projection of process variation $\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*$ is also multivariate Gaussian. Therefore, given a confidence interval, $\|\mathbf{M}^{\frac{1}{2}}\mathbf{x}^*\|_{\min}$ can be determined from Student's t-distribution with mean being $f\xi$ and variance calculated by (21).

Note that the derivation of conditional minimum variance (21) does not require true process model. Only the delay-structure, i.e., the lower-block-triangular interactor matrix, along with disturbance dynamics $N(q)$ needs to be known.

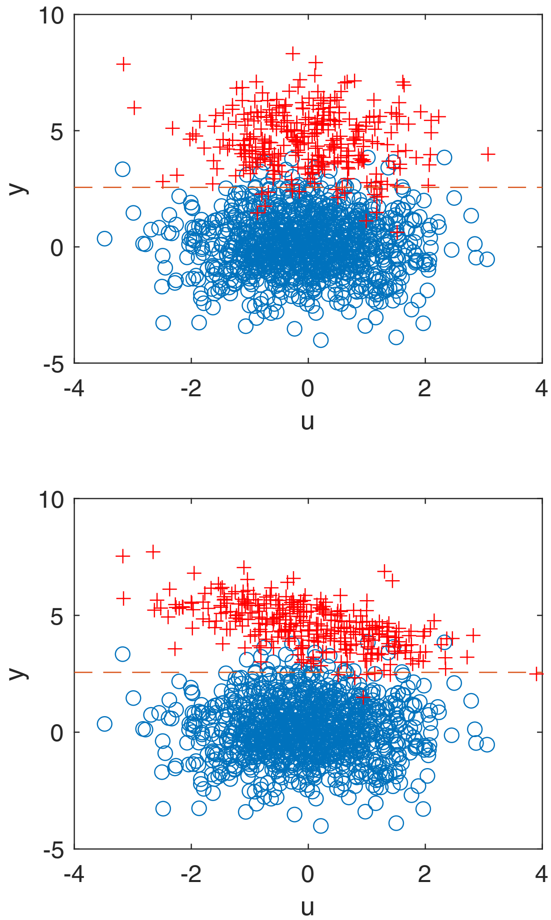

Fig. 3. CV fault in Example 4.1. Blue dots are normal data; red crosses are faulty samples. Abnormal data in the upper and bottom figure are regulated by the original controller and the minimum variance controller, respectively. Control limit is represented by the dashed line.

4. SIMULATION EXAMPLES

A motivating simulation example is presented first to provide an intuition to the method, which is followed by a more realistic case of MIMO systems.

4.1 A Motivating Example

Let a SISO process be

$$y_k = G(q)u_k + N(q)a_k + b \quad (22)$$

where $G(q) = q^{-1}$, $N(q) = 1 + 0.8q^{-1} + q^{-3}$, the noise source term $a_k \sim N(0,1)$ and b is the bias term. Under normal condition, the setpoint is chosen as $[r_y, r_u] = [0, 0]$ with zero bias $b = 0$. The controller is designed to be

$$u_k = -\frac{q^{-2}}{1 + 0.8q^{-1}}(r_y - y_k) \quad (23)$$

Substituting (23) into (22) suggests

$$y_k = a_k + 0.8a_{k-1} \quad (24)$$

and

$$u_k = -a_{k-2} \quad (25)$$

Stacking y_k and u_k gives the sample vector for fault detection:

$$\mathbf{x}_k = [u_k, y_k]^T = [-a_{k-2},\ a_k + 0.8a_{k-1}]^T \tag{26}$$

It is obvious that the components of $\mathbf{x}_k$ are independent. Thus, performing PCA on $\mathbf{X}$ results in the first PC being the MV u and the second PC being the CV y as shown in Fig. 3. Setting the number of PC $l = 1$, the SPE and T^2 indices simply become y^2 and u^2, respectively, with the 95% control limits $\delta^2 = 6.56$ and $\tau^2 = 4$. A total of 1000 samples are collected to build the PCA model.

Faults in CV Suppose there exists a fault on y that shifts the steady-state value of y from 0 to 4.56, which is reflected in the model that $r_y = b = 4.56$. Simple calculation suggests that $\xi = \xi^o = [0, 1]^T$, and $f = f^o = 4.56$. If the controller still takes the form of (23), the fault is not fully detectable. In fact, f needs to be at least $2\delta = 5.12$ to ensure detectability. However, since the process is SISO with unit time-delay, the minimum variance of y_k is $\sigma^2_{\text{MVC}} = 1$ which suggests that $f \geqslant \delta + 2\sigma_{\text{MVC}} = 4.56$ can be detected. The corresponding minimum variance controller is

$$u_k = (0.8 + q^{-2})(r_y - y_k) \tag{27}$$

Some visual clues are provided in Fig. 3. A total of 300 faulty samples are generated. Simulation results show that the fault detection rates for faulty samples under same controller and minimum variance controller are 94.3% and 97.7%, respectively.

Faults in MV Assume a fault occurs on the MV such that the steady-state value of MV has a bias of 2.05 while the process bias term is $b = -2.05$. Hence the setpoint becomes $[r_y, r_u] = [0, -2.05]$. If the controller (23) remains unchanged, nearly half of the faulty data cannot be distinguished (Fig. 4). However, when operated open-loop (or equivalently, the controller gain is zero), all abnormal samples are correctly identified as shown in Fig. 4.

4.2 A MIMO Process Example

The Wood-Berry distillation column model is simulated. With 1 minute sampling time, the process model is discretized as

$$G(q) = \begin{bmatrix} q^{-2}\dfrac{0.7665}{1-0.9419q^{-1}} & q^{-4}\dfrac{-0.9000}{1-0.9535q^{-1}} \\ q^{-8}\dfrac{0.6055}{1-0.9123q^{-3}} & q^{-4}\dfrac{-1.3472}{1-0.9329q^{-1}} \end{bmatrix}.$$

with the following disturbance model being selected:

$$N(q) = \begin{bmatrix} \dfrac{1}{1-0.8q^{-1}} & \\ & \dfrac{1-0.5q^{-1}}{1-q^{-1}} \end{bmatrix} \tag{28}$$

whose associated $\mathbf{a}_k \sim N(\mathbf{0}, \mathbf{I}_2)$.

An unconstrained model predictive controller (MPC) is designed to control the process under normal condition. The prediction and control horizons are set to 100 and 10, respectively. The MPC has the output weight $W_y = \text{diag}\{100, 1\}$ and input-move weight $W_{\Delta u} = \text{diag}\{0.01, 0.01\}$. The setpoint of MPC is selected as $r_y =$ [95%, 5%]. Since (28) is nonstationary, the sample vector is composed of CV data only. There are 1000 samples collected for PCA modeling with $l = 1$. The loading matrix is found to be

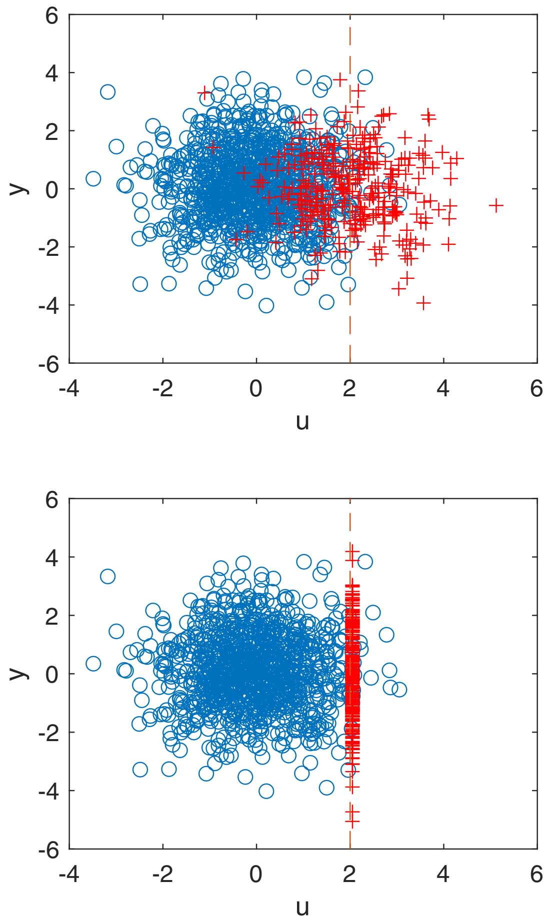

Fig. 4. MV fault in Example 4.1. Blue dots are normal data; red crosses are faulty samples. Abnormal data in the upper and bottom figure are regulated by the original controller and the zero-gain controller, respectively. Control limit is represented by the dashed line.

$$\mathbf{P} = [0.707, -0.707]^T \tag{29}$$

Suppose there exists a fault in CV adding a bias term $\Delta r_y = [3.3\%, 3.3\%]$ to the setpoint, i.e., $\xi = [0.707, 0.707]$. Since the fault lies in the RS, SPE index is applied during the monitoring period, and hence $\xi^o = \xi$. The 95% control limit is calculated to be $\delta^2 = 4.13$.

In the meanwhile, Eq. (29) suggests that the system needs to be rotated 45° so that the first rotated CV aligns with f^o. The block-lower-triangular interactor can be obtained

$$D = \begin{bmatrix} q^2 & 0 \\ q^4 & q^4 \end{bmatrix} \tag{30}$$

with which the conditional minimum variance along ξ^o in the RS space is $\sigma_{\text{MVC}} = 1.72$. Thus, the fault is closed-loop detectable if $f^o \geqslant \delta + 2\sigma_{\text{MVC}} = 4.655$. The lower bound is achieved by conditional minimum variance controller which can be approximated by setting

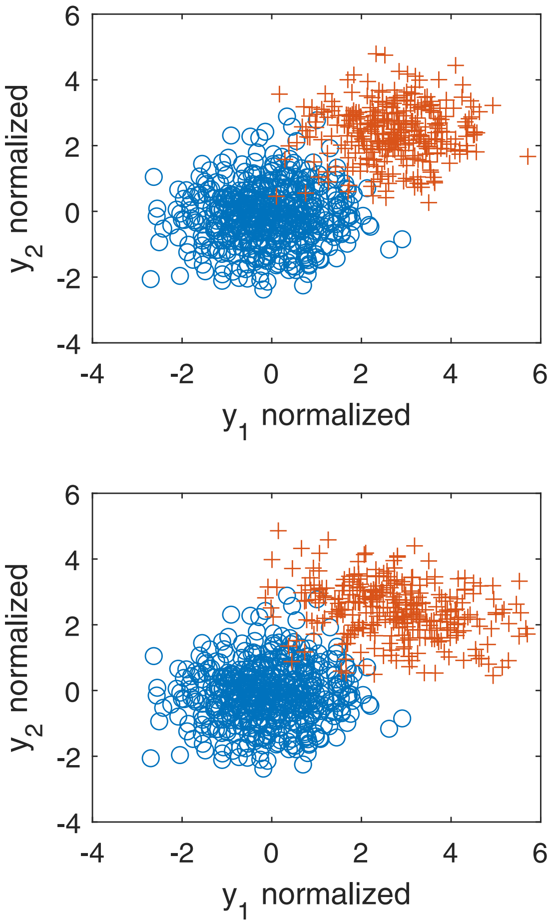

Fig. 5. Simulation results in Example 4.2. Blue dots represents normal data; red crosses are faulty samples. Abnormal data in the upper and bottom figure are regulated by the original and the modified MPC, respectively.

MPC weights $W_y = [1, 0.99; 0.99, 1]$ and $W_{\Delta u} = \mathbf{0}$ with no constraints. Simulated results are demonstrated in Fig. 5. It can be seen that with the modified controller variance along fault direction is reduced at the expense of variance along the orthogonal direction. Fault detectability is therefore improved.

5. CONCLUSIONS

In this paper, fault detectability of process with arbitrary controller is discussed. It is shown that minimum magnitude for a fault to be sufficiently detectable is the sum of control limit and conditional minimum variance in the equivalent fault direction. The tool of block-lower-triangular interactor is introduced to aid the derivation of minimum variance. The results are supported by simulation results.

The results of this paper can be used to enhance fault detectability for processes under closed-loop. For instance, when a fault is detected with only a portion of samples in a time window, one can determine if the fault is detectable under closed-loop conditions and then alter the controller to reduce uncertainty in the fault direction to enhance fault detectability.

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