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An Efficient Model Based Control Algorithm for the Determination of an Optimal Control Policy for a Constrained Stochastic Linear System

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Abstract: In this paper, the authors have proposed an ensemble Kalman filter based stochastic model predictive control algorithm to determine an optimal control policy at every sampling time instant for a constrained stochastic linear system. To determine an optimal control policy for the constrained linear system affected by random disturbances and measurements corrupted by random noise, the authors have minimized the uncertain objective function, subject to uncertain state & output constraints and deterministic input constraints using the quantile based scenario analysis approach. In this work, ensemble Kalman filter is being employed, to generate a recursive estimate of states of the constrained stochastic linear system. The number of scenarios is considered to be equivalent to that of number of sample points used in the ensemble Kalman filter. Each scenario is viewed as one realization of the process noise, measurement noise over the prediction horizon as well as the i^{th} sample point of the state estimate at the beginning of the prediction horizon generated by the ensemble Kalman filter. Simulation studies have been carried out to assess the efficacy of the proposed control scheme on the simulated model of the constrained single-input and single-output linear stochastic system.

Keywords: Ensemble Kalman filter, Stochastic Model Predictive Control, Scenario Optimization and quantile scenario analysis.

1. INTRODUCTION

Model Predictive Control (MPC) is the most preferred multivariable control scheme at the supervisory level in the process industries because of its ability to handle systematically the multivariable interactions and constraints such as states, outputs, and inputs respectively. At each sampling time instant, the MPC computes the current and future controller outputs, by minimizing the predicted deviation of the process output from the setpoint over the prediction horizon as well as minimizing the expenditure of the control effort in driving the process output to the setpoint subject to the deterministic constraints in states, inputs, and outputs (Qin and Badgwell, 2003). It should be noted that only the current value of the controller output is applied to the plant and the whole procedure is repeated at the next sampling time instant. The aforementioned deterministic MPC formulation hasn't taken into account the model uncertainty, measurement uncertainty and probabilistic state and output constraints (Mesbah, 2016).

Depending upon the type of state space model used for prediction, the MPC can be broadly classified into deterministic MPC and stochastic MPC. The second type classification is based on the type of uncertainty and the uncertainty propagation methods (Monte Carlo simulation, Moment method and Polynomial Chaos) being used in the stochastic MPC formulation (Stochastic tube based SMPC; Scenario/Sample based SMPC and Generalized Polynomial Chaos based SMPC). Further SMPC schemes are classified also based on the control input parameterization (either open loop control or pre-stabilizing feedback control) and the type of objective function, constraints and constrained optimization algorithm being employed to numerically solve the constrained optimization problem to obtain the current and future control outputs. It may be also noted that the robust model predictive control scheme (the min-max approach described in Scokaert and Mayne, 1998, the tube-based MPC in Mayne et al. 2014) will handle only deterministic description of plant uncertainties whereas stochastic MPC can systematically handle probabilistic description of uncertainties (multi-stage or a scenario based NMPC approach in Lucia et al., 2013, Bavdekar and Mesbah, 2016). Tube-based MPC for linear systems and nonlinear systems has been proposed as an alternative to min–max approaches based robust MPC. Even though tube based MPC can guarantee stability and satisfy constraints, it does not address the issue of optimal performance in the presence of uncertainties (Mayne et al. 2014).

Several review articles have appeared which comprehensively survey the theoretical developments and industrial practices in the area of model predictive control (Qin and Badgwell 2003; Forbes et al. 2015). In recent years, the concept of stochastic MPC (Mesbah et al. 2014; Mesbah, 2016) has attracted the attention of several researchers and has been applied in many different areas, such as building climate control, power generation and distribution, chemical processes etc. Mesbah et al. 2016, while discussing future research directions in his review article on stochastic MPC, stressed that most SMPC approaches are developed under the assumption of full state feedback. Unfortunately, all states are not available for measurement in many situations. Hence, SMPC algorithms that include state estimation remain an open problem for stochastic systems. It should be noted that the state estimator based MPC formulations, which use extended Kalman filter (Ricker 1990; Subramanian et al. 2015), derivative free Kalman filter (Prakash et al. 2010), and particle filter (Sehr & Bitmead, 2017) have been reported in the process control literature.

An important contribution of this paper is the development of a state estimator based stochastic model predictive control algorithm to determine an optimal control policy for a constrained stochastic linear system. In order to determine an optimal control policy for the system affected by random disturbances and measurements corrupted by random noise, the authors have minimized the uncertain objective function, subject to uncertain constraints such as states and outputs as well as deterministic input constraints using the quantile-based scenario analysis (QSA) approach proposed by Zamar et al. 2017. The advantages of the QSA approach for stochastic optimization over the mean-based based stochastic optimization approach are outlined in Zamar et al. 2017.

It should be noted that the QSA approach minimizes a weighted average of the quantiles of the objective and constraint distributions. Hence, it will be much more robust than simply minimizing the corresponding expected values of the objective and constraint distributions, as it is typically done in all Stochastic Model Predictive Control formulations.

The organization of the paper is as follows: Section 2 presents the ensemble Kalman filter algorithm. Section 4 reports the algorithm for the determination of an optimal control policy for the stochastic linear system. Simulation studies have been reported in section 4 followed by concluding remarks in section 5.

2. ENSEMBLE KALMAN FILTER ALGORITHM

Let us assume that the stochastic linear system is represented using the state and measurement equations given below:

$$\left.\begin{aligned}\mathbf{x}(k) &= \mathbf{\Phi}\mathbf{x}(k-1) + \mathbf{\Gamma}_u\mathbf{u}(k-1) + \mathbf{\Gamma}_d\mathbf{d}(k-1) + \mathbf{w}(k)\\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k)\end{aligned}\right\}(1)$$

where $\mathbf{x}(k)\epsilon R^n$ are the state variables, $\mathbf{u}(k)\epsilon R^u$ are the manipulated inputs, $\mathbf{d}(k)\epsilon R^d$ are the disturbance variables and $\mathbf{y}(k)\epsilon R^y$ are the measured output variables. It is assumed that the state and measurement equations are affected by additive process noise and measurement noise, respectively as shown in equation 1. The system matrices (i.e. $\mathbf{\Phi}, \mathbf{\Gamma}_u, \mathbf{\Gamma}_d$ and $\mathbf{C}$) and the distribution of process noise $\{\mathbf{w}(k)\}$ and measurement noise $\{\mathbf{v}(k)\}$ are assumed to be known in this work. The determination of the optimal state estimates using an ensemble Kalman filter algorithm is as follows.

The ensemble Kalman filter (EnKF) is initialized by drawing N samples $\{\hat{\mathbf{x}}^{(i)}(0|0): i = 1, \dots N\}$ from a suitable initial state distribution$(\mathbf{p}[\mathbf{x}(0)])$. At each time step, N samples $\{\mathbf{w}^{(i)}(k), \mathbf{v}^{(i)}(k): i = 1, \dots N\}$ for $\{\mathbf{w}(k)\}$ and $\{\mathbf{v}(k)\}$ are drawn randomly using the distribution of process noise and measurement noise respectively. The computation of the optimal state estimates using an EnKF is as follows:

$$\hat{\mathbf{x}}^{(i)}(k|k-1) = [\mathbf{\Phi}\hat{\mathbf{x}}^{(i)}(k-1|k-1) + \mathbf{\Gamma}_u\mathbf{u}(k-1) + \mathbf{w}^{(i)}(k)] \quad (2)$$

These transformed particles are then used to estimate the sample mean and sample covariance as follows:

$$\bar{\mathbf{x}}(k|k-1) = \frac{1}{N}\sum_{i=1}^{N}\hat{\mathbf{x}}^{(i)}(k|k-1) \quad (3)$$

$$\bar{\mathbf{y}}(k|k-1) = \frac{1}{N}\sum_{i=1}^{N}\left[\mathbf{C}\hat{\mathbf{x}}^{(i)}(k|k-1) + \mathbf{v}^{(i)}(k)\right] \quad (4)$$

$$\mathbf{P}_{\varepsilon,e}(k|k-1) = \frac{1}{N-1}\sum_{i=1}^{N}\left[\boldsymbol{\varepsilon}^{(i)}(k|k-1)\right]\left[\mathbf{e}^{(i)}(k|k-1)\right]^T \quad (5)$$

$$\mathbf{P}_{e,e}(k|k-1) = \frac{1}{N-1}\sum_{i=1}^{N}\left[\mathbf{e}^{(i)}(k|k-1)\right]\left[\mathbf{e}^{(i)}(k|k-1)\right]^T \quad (6)$$

where,

$$\boldsymbol{\varepsilon}^{(i)}(k|k-1) = \hat{\mathbf{x}}^{(i)}(k|k-1) - \bar{\mathbf{x}}(k|k-1) \quad (7)$$

$$\mathbf{e}^{(i)}(k|k-1) = \left[\mathbf{C}\hat{\mathbf{x}}^{(i)}(k|k-1) + \mathbf{v}^{(i)}(k)\right] - \bar{\mathbf{y}}(k|k-1) \quad (8)$$

The Kalman gain and updated sample points are then computed as follows:

$$\mathbf{L}(k|k-1) = \mathbf{P}_{\varepsilon,e}(k|k-1)\left[\mathbf{P}_{e,e}(k|k-1)\right]^{-1} \quad (9)$$
$$\mathbf{\Upsilon}^{(i)}(k|k-1) = \mathbf{y}(k) + \mathbf{v}^{(i)}(k) - \mathbf{C}\hat{\mathbf{x}}^{(i)}(k|k-1) \quad (10)$$
$$\hat{\mathbf{x}}^{(i)}(k|k) = \hat{\mathbf{x}}^{(i)}(k|k-1) + \mathbf{L}(k|k-1)\mathbf{\Upsilon}^{(i)}(k|k-1) \quad (11)$$
$$\mathbf{\Upsilon}^{(i)}(k|k) = \mathbf{y}(k) - \mathbf{C}\hat{\mathbf{x}}^{(i)}(k|k) \quad (12)$$

The accuracy of the estimates depends on the number of sample point (N). Evensen (2003) has indicated that sample points between 50 and 100 suffices even for large dimensional systems.

3. EnKF BASED STOCHATIC MODEL PREDICTIVE CONTROLLER

The determination of an optimal control policy for the stochastic linear system using the proposed ensemble Kalman filter based stochastic linear model predictive control scheme (EnKF - SMPC) is as follows:

Given the future set point trajectory $\mathbf{y}_{sp}(k+j|k), (j = 1 \dots P)$, the proposed EnKF - SMPC will determine the current and future controller outputs $\mathbf{U}_f = \{\mathbf{u}(k|k) \dots \mathbf{u}(k+1|k) \dots \mathbf{u}(k+M-1|k)\}$ in two stages using QSA approach:

- **Stage-1:** Compute the current and future controller outputs $\mathbf{U}_f^{(i)}$ for each scenario by minimizing the predicted deviation of the process output from the setpoint over the prediction horizon as well as minimizing the expenditure of control effort in driving the process output to setpoint subject to constraints such as states, outputs as well as inputs.
- **Stage-2:** Determine a single feasible control policy $\mathbf{U}_f$, by minimizing the mean of the objective function distribution subject to satisfying the mean of the constraint distribution.

The detailed computation procedure in each stage is as follows:

Stage-1: There are three sources of uncertainties that arise while performing predictions, they are (a) uncertainty in initial state at the beginning of the prediction $\{\hat{\mathbf{x}}^{(i)}(k|k) \forall i = 1, \dots N\}$ and (b) unmeasured random disturbances $\{\mathbf{w}^{(i)}(k+j|k) \forall j = 1, \dots P\}$ that may occur in future and that affects the state equation and (c) the unmeasured random output disturbances $\{\mathbf{v}^{(i)}(k+j|k) \forall j = 1, \dots P\}$ that may occur in future and that affects the measurement model. In this work, the number of scenarios is considered to be equivalent to that of number of sample points (N) used in the ensemble Kalman filter. Each scenario is viewed as one realization of the process noise $\mathbf{W}^{(i)} = \{\mathbf{w}^{(i)}(k+1|k)\ \mathbf{w}^{(i)}(k+2|k) \dots \mathbf{w}^{(i)}(k+P|k)\}$, measurement noise $\mathbf{V}^{(i)} = \{\mathbf{v}^{(i)}(k+1|k)\ \mathbf{v}^{(i)}(k+2|k) \dots \mathbf{v}^{(i)}(k+P|k)\}$ over the prediction horizon (P) as well as the i^{th} sample point of the state estimate at the beginning of the prediction horizon generated by the ensemble Kalman filter $\{\hat{\mathbf{x}}^{(i)}(k|k)\}$.

For each scenario, the following performance measure is minimized

$$\mathbf{J} = \min_{\mathbf{U}_f^{(i)}} \left[\sum_{j=1}^{P} \left\|\mathbf{e}_f^{(i)}(k+j|k)\right\|_{\mathbf{W}_E}^2 + \sum_{j=0}^{M-1} \left\|\Delta\mathbf{u}^{(i)}(k+j|k)\right\|_{\mathbf{W}_{\Delta u}}^2\right] \quad (13)$$

subject to the following constraints:

$$\hat{\mathbf{x}}^{(i)}(k+j+1|k) = \left[\mathbf{\Phi}\hat{\mathbf{x}}^{(i)}(k+j|k) + \mathbf{\Gamma}_u\mathbf{u}^{(i)}(k+j|k) + \mathbf{w}^{(i)}(k+j+1|k) + \mathbf{L}(k)\boldsymbol{\eta}_e^{(i)}(k+j+1|k)\right] \ \forall\, j = 0,1 \dots P-1 \quad (14)$$
$$\hat{\mathbf{y}}^{(i)}(k+j+1|k) = [\mathbf{C}\hat{\mathbf{x}}^{(i)}(k+j+1|k) + \mathbf{v}^{(i)}(k+j+1|k) + \boldsymbol{\eta}_d^{(i)}(k+j+1|k)] \forall j = 0,1 \dots P-1 \quad (15)$$
$$\mathbf{u}^{(i)}(k+j|k) = \mathbf{u}^{(i)}(k+M-1) \quad \forall j = M, M+1 \dots P-1 \quad (16)$$
$$\mathbf{x}_L \le \hat{\mathbf{x}}^{(i)}(k+j|k) \le \mathbf{x}_H \quad \forall\, j = 1 \dots P \quad (17)$$
$$\mathbf{y}_L \le \hat{\mathbf{y}}^{(i)}(k+j|k) \le \mathbf{y}_H \quad \forall\, j = 1 \dots P \quad (18)$$
$$\mathbf{u}_L \le \mathbf{u}^{(i)}(k+j|k) \le \mathbf{u}_H \quad \forall\, j = 0 \dots M-1 \quad (19)$$
$$\Delta\mathbf{u}_L \le \Delta\mathbf{u}^{(i)}(k+j|k) \le \Delta\mathbf{u}_H \quad \forall\, j = 0 \dots M-1 \quad (20)$$

where

$$\mathbf{e}_f^{(i)}(k+j|k) = \mathbf{y}_{sp}(k+j|k) - \hat{\mathbf{y}}^{(i)}(k+j|k) \quad \forall\, j = 1 \dots P \quad (21)$$
$$\Delta\mathbf{u}^{(i)}(k+j|k) = \mathbf{u}^{(i)}(k+j|k) - \mathbf{u}^{(i)}(k+j-1|k) \quad \forall\, j = 0 \dots M-1 \quad (22)$$

The filtered innovation signals are computed as follows:

$$\left.\begin{aligned}\boldsymbol{\eta}_e^{(i)}(k+j+1|k) &= \boldsymbol{\eta}_e^{(i)}(k+j|k)\\ \boldsymbol{\eta}_e^{(i)}(k|k) &= \boldsymbol{\gamma}_f^{(i)}(k|k-1)\end{aligned}\right\} \forall\, j = 0,1 \dots P-1 \quad (23)$$
$$\left.\begin{aligned}\boldsymbol{\eta}_d^{(i)}(k+j+1|k) &= \boldsymbol{\eta}_d^{(i)}(k+j|k)\\ \boldsymbol{\eta}_d^{(i)}(k|k) &= \boldsymbol{\gamma}_f^{(i)}(k|k)\end{aligned}\right\} \forall\, j = 0,1 \dots P-1 \quad (24)$$
$$\boldsymbol{\gamma}_f^{(i)}(k|k-1) = [\mathbf{\Phi}_x\mathbf{\Upsilon}_f^{(i)}(k-1|k-2) + [\mathbf{I} - \mathbf{\Phi}_x]\ \mathbf{\Upsilon}^{(i)}(k|k-1)] \quad (25)$$
$$\boldsymbol{\gamma}_f^{(i)}(k|k) = [\mathbf{\Phi}_y\mathbf{\Upsilon}_f^{(i)}(k-1|k-1) + \left[\mathbf{I} - \mathbf{\Phi}_y\right]\mathbf{\Upsilon}^{(i)}(k|k)] \quad (26)$$

It may be noted that $\boldsymbol{\gamma}_f^{(i)}(k|k-1)$ and $\boldsymbol{\gamma}_f^{(i)}(k|k)$ are filtered values of innovation signals $\mathbf{\Upsilon}^{(i)}(k|k-1)$ and $\mathbf{\Upsilon}^{(i)}(k|k)$, respectively, which are defined by equation (10) and equation (12). The $\mathbf{\Phi}_x$ and $\mathbf{\Phi}_y$ matrices are parameterized as follows,

$\mathbf{\Phi}_x = \mathrm{diag}\{\alpha_1\ \alpha_2 \dots \alpha_y\}$ and $\mathbf{\Phi}_y = \mathrm{diag}\{\beta_1\ \beta_2 \dots \beta_y\}$ where $0 \le \alpha_i \le 1$ and $0 \le \beta_i \le 1\ \forall\, i = 1,2, \dots y$ can be chosen to shape the response of the stochastic model predictive controller in the presence of unmeasured disturbance. Equation (16) states that no future control moves are planned beyond the control horizon of M steps.

Stage-2: The QSA method developed by Zamar et al. (2017), is used to find a single, feasible, and robust control policy. That is, each scenario solution $\mathbf{U}_f^{(i)}$ is evaluated across all sampled scenarios as shown below.

for i:1:N
{
for j:1:N
{
$$\mathbf{h}^{(i,j)} = \mathbf{J}(\mathbf{U}_f^{(i)}, \{\mathbf{W}^{(j)}, \mathbf{V}^{(j)}, \hat{\mathbf{x}}^{(j)}(k|k)\})$$
for r:1:m
{
$$\mathbf{C}^{(i,j,r)} = \mathbf{\Psi}_r(\mathbf{U}_f^{(i)}, \{\mathbf{W}^{(j)}, \mathbf{V}^{(j)}, \hat{\mathbf{x}}^{(j)}(k|k)\})$$
}
}
}

Here, $\mathbf{\Psi}_r(.)$ represents the probabilistic constraints. Next, the cumulative distribution functions (CDF) of the objective function ($\mathbf{F}_{\mathbf{U}_f^{(i)}}(z)$) and constraints ($\mathbf{G}_{\mathbf{U}_f^{(i)},r}(\zeta)$) of each solution are obtained as follows,

for i:1:N
{
$$\mathbf{F}_{\mathbf{U}_f^{(i)}}(z) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{I}(\mathbf{h}^{(i,j)} \le z)$$
for r:1:m
{
$$\mathbf{G}_{\mathbf{U}_f^{(i)},r}(\zeta) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{I}(\mathbf{C}^{(i,j,r)} \le \zeta)$$
}
}

Subsequently, the optimal values of the current and future controller outputs are computed by solving the following coordination model.

$$\mathbf{U}_f = \underset{i \in \mathbf{I}}{\operatorname{argmin}} \int_0^1 \mathbf{F}^{-1}_{\mathbf{U}_f^{(i)}}(t)\mathbf{\Omega}_0(t)dt \quad \mathbf{I} = [1,2, \dots N] \tag{27}$$

subject to

$$\int_0^1 \mathbf{G}^{-1}_{\mathbf{U}_f^{(i)},r}(t)\mathbf{\Omega}_r(t)dt \le \boldsymbol{\gamma}_r \quad r \epsilon R \tag{28}$$

The above formulation of the problem attempts to minimize the weighted average of the quantiles of the objective function, subject to satisfying a weighted average of the quantiles of the constraint performance functions, also called a risk spectrum (Zamar et al. (2017)). $\mathbf{\Omega}_0(t)$ & $\mathbf{\Omega}_r(t)$ are positive weighting functions that integrate to unity over the range 0-1. In the present work all quantiles have been given equal weights.

The desired closed loop performance of the proposed SMPC scheme can be achieved by appropriately selecting the prediction horizon P, control horizon M, the error weighting matrix ($\mathbf{W}_E$) input weighting matrix($\mathbf{W}_{\Delta u}$) and other parameters. Further, the SMPC scheme is implemented in a receding horizon framework. That is, only the current controller output $\mathbf{u}(k|k)$ is implemented on the plant and the constrained optimization problem is reformulated at the next sampling instant based on the updated information from the plant.

4. SIMULATION STUDY

The efficacy of the proposed control scheme has been validated on the constrained single-input and single-output linear system given by

$$\mathbf{x}(k+1) = 0.5\mathbf{x}(k) + \mathbf{u}(k) + d(k) + \mathbf{w}(k)$$
$$\mathbf{y}(k) = \mathbf{x}(k) + \mathbf{v}(k);\ -2 \le \mathbf{u} \le 2;\ -1 \le \mathbf{x} \le 1$$

TABLE 1. PARAMETER ASSOCIATED WITH EnKF and MPC

Parameter	Value	Parameter	Value
R	0.001	Q	0.01
No. of Scenarios	200	$\mathbf{\Phi}_x$ & $\mathbf{\Phi}_y$	0
P(0|0)	0.01	$\hat{x}(0\|0)$	0.5
P	10	M	1
$\mathbf{w}_E$	1000	$\mathbf{w}_{\Delta u}$	1000
$\mathbf{\Gamma}_u$	1	$\mathbf{\Gamma}_d$	1

The random disturbances $\{\mathbf{w}(k)\}$ and measurement noise $\{\mathbf{v}(k)\}$are assumed to be zero mean Gaussian white noise sequences with covariance matrices Q & R respectively. The disturbance term $\mathbf{d}(k)$ is assumed to deterministic in this work. It is assumed that the system is controllable and also observable. The servo-regulatory performance of the system with the proposed EnKF-SMPC and EnKF-min-max MPC in the presence of model-plant mismatch (MPM), which is 50 % increase in the system matrix ($\mathbf{\Phi}$) are reported in Fig. 1. The controller computations, however, are based on the nominal model parameters. The parameters associated with the EnKF and MPC are reported in Table 1. The evolution of true and estimated state variables of the system with EnKF-SMPC is reported in Fig. 2. It can be inferred from Fig.2 that the EnKF is able to generate fairly accurate filtered estimate of the state variable. The evolution of controller outputs is reported in Fig.3.

The inferences drawn from the simulation studies are as follows:

- It may be noted that both control schemes approached the desired setpoint, as shown in Fig. 1, during the discrete time interval between 1 and 49. This part of the simulation demonstrates the ability to transfer the system from the initial state $\mathbf{x}(0) = 0.5$ to the desired setpoint (i.e. the origin).
- A step change disturbance (d) of magnitude 0.5 is introduced at discrete time instant 50 and both schemes are able to reject the disturbance. As a result, the output reaches the setpoint, as shown in Fig. 1, during the discrete time interval between 50 and 99.
- With the disturbance being persistent, a step change in the setpoint of magnitude 0.8 (See Fig. 1) is introduced at the 100th sampling instant. Both schemes are able to maintain the output at the desired setpoint, as evident from Fig.1, during the discrete time interval between 100 and 300.

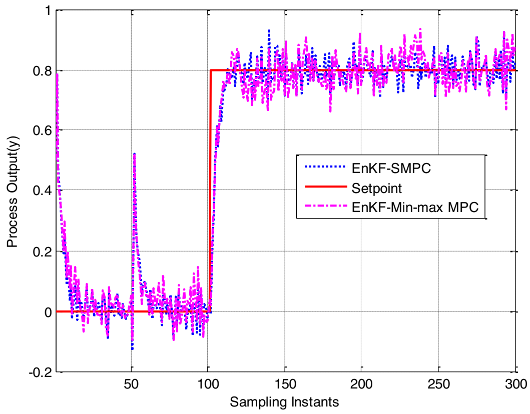

Fig.1. Servo-Regulatory Response of a Stochastic Linear System with EnKF-SMPC and EnKF based min-max based MPC

The performance of the proposed control scheme has been assessed through stochastic simulation studies. A simulation run consisting of N_{TR}= 25 trials with the length of each simulation trail, L, being equal to 300 is conducted. The sum of squared output error (SSOE), defined $\mathrm{SSOE} = \sum_{k=1}^{L}\left[\left(\mathbf{y}_{sp}(k) - \mathbf{y}(k)\right)^2\right]$ is used as a performance index, where $\mathbf{y}_{sp}(k)$ denotes the setpoint at time step k. Statistics of SSOE computed for each simulation run is used to assess the efficacy of the control scheme. The mean and standard deviation of SSOE values based on the $N_{TR} = 25$ trials for EnKF-SMPC and EnKF-min-max MPC are reported in Table 2. As expected, the state estimates generated by EnKF are found to be biased after the introduction of step change in the unmeasured disturbance at discrete time instant 50 (Fig.2). It should be noted that even if the states are biased the proposed EnKF-SMPC scheme and EnKF-min-max MPC scheme are able to achieve offset free servo-regulatory performance. From Table 2, it can be inferred that the average SSOE is found to be less for the proposed control scheme. The results of a t-test comparing the mean difference in SSOE between the two control schemes revealed that EnKF-SMPC obtained a statistically significant improvement in the SSOE compared to that of the EnKF-min-max MPC. The improvement is estimated to be 0.1518 with a standard error of 0.0477.

The efficacy of the proposed control scheme has been also validated on the constrained single-input and single-output linear system with non-negative constraints on process noise as shown below:

$$\mathbf{x}(k+1) = 0.5\mathbf{x}(k) + \mathbf{u}(k) + |\mathbf{w}(k)|$$

The servo-regulatory performance of the linear system in the presence of non-negative constraints on the process noise is reported in Fig. 4. It can be inferred from Fig.4, that the EnKF-SMPC is able to reject the non-negative random disturbances (Fig.5b) and achieve offset-free servo-regulatory performance. The evolution of controller output is shown in Fig.5a.

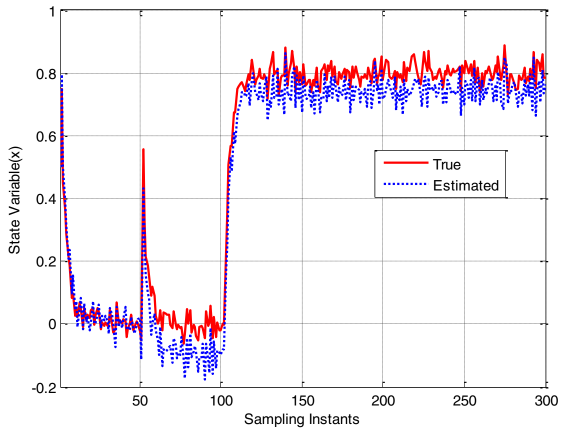

Fig.2. EnKF-SMPC: Evolution of true and estimated state variable using EnKF

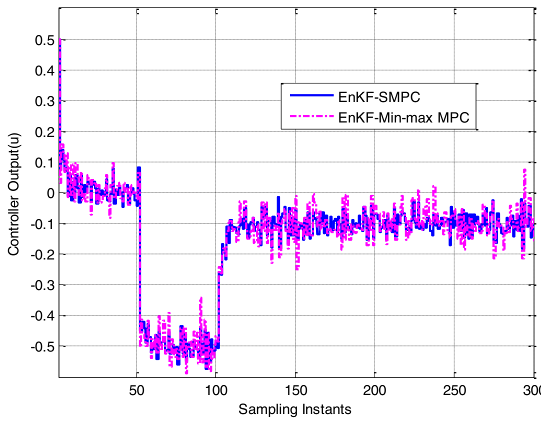

Fig.3. Evolution of Controller Outputs

TABLE 2. AVERAGE SSOE VALUES FOR 25 TRIALS

Control Scheme	SSOE in the Presence of Model Plant Mismatch
EnKF – MPC	3.1713 (0.0668)
EnKF-Min – Max MPC	3.3220 (0.2291)

5. CONCLUSIONS

The quantile based scenarios analysis (QSA) approach was used to determine an optimal control policy for a constrained stochastic linear system in an elegant manner. Monte Carlo simulation analyses found that the proposed ensemble Kalman filter based stochastic model predictive control scheme, EnKF, can reject step like disturbances by bringing the process variable back to the setpoint and exhibits offset free performance. The average SSOE of the proposed control scheme was found to be less compared to the min-max MPC scheme in the presence of model-plant mismatch. It should be noted that, both EnKF schemes were able to generate accurate state estimates. Since, the EnKF-SMPC obtained a statistically significant improvement in the SSOE, compared to that of the EnKF-min-max MPC in the presence of mode-plant mismatch, its performance can be considered to be efficient. It should be noted that with the help of parallel computing, it is possible to reduce the computational time of the control scheme proposed in this work. Further work is in progress to extend the proposed control scheme for a stochastic non-linear system in the presence of probabilistic state constraints.

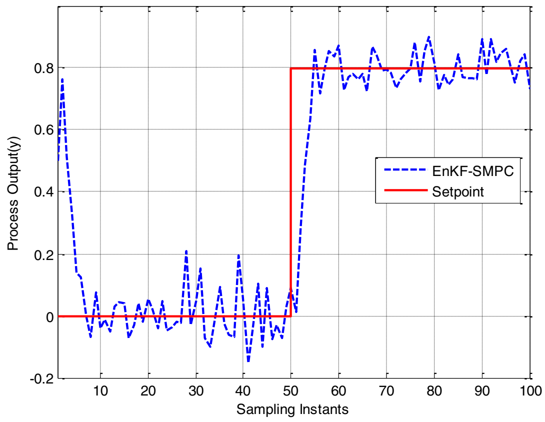

Fig.4. Servo-Regulatory Response of a Stochastic Linear System with EnKF-SMPC in the presence of non-negative constraints on the process noise

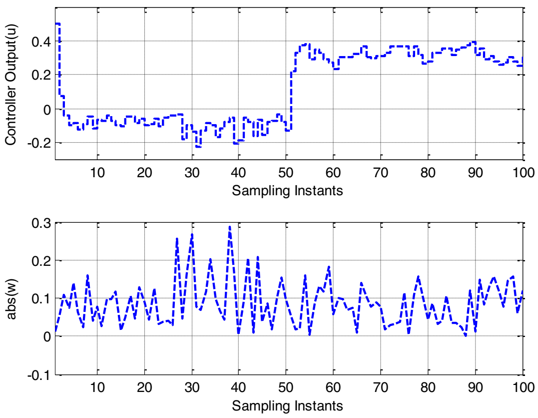

Fig.5. Evolution of Controller Output (EnKF-SMPC) & non-negative Random Disturbance

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