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Passivity-based Input Observer

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Abstract: A passivity-based input observer is proposed. The problem is motivated by reaction rate and heat estimation in control of chemical reaction systems. The input observer assumes measurement of the output, and its first order time derivative. The observer gives asymptotically converging estimation when both are accurately available, the so-called ideal case. In the nonideal case, where the derivative is not available, differentiators can be used to reconstruct the derivative with some error. Simulation results show performance results using a deadbeat differentiator for derivative reconstruction.

Keywords: Input estimation, Observers, Differentiators, Measurement noise, Frequency response.

1. INTRODUCTION

The determination of the input to a system is usually considered to be a control problem. In order to achieve perfect tracking of a given time-varying reference, it is necessary that the system is stably invertible (passive) and that the time derivative of the reference is known and Lipschitz continuous. In the current paper, we apply this same idea to estimation of an unknown time-varying input parameter. In this case, the measurement assumes the role of the reference and the unknown parameters the role of the control variable. To achieve perfect estimation, the conditions are that the reference and the derivative are measured and Lipschitz continuous. The problem is relevant when we need to estimate some unknown part of the inputs. Examples include the estimation of exerted force in machine tools applications (Corless and Tu, 1998), the magnitude of unknown maneuvers in tracking problems (Lee and Tahk, 1999), fault detection (Chen et al., 1996). The proposed work is motivated by the estimation problem in chemical reaction processes, more specifically the estimation of reaction rates and reaction heat (Zhao et al., 2016). The successful estimation of such variables is helpful for process control and monitoring.

The input observer proposed by Park and Stein (1988) includes an Unknown Input Observer (UIO), estimating unknown states from output measurements. Differential equations are inverted to obtain parameter estimates. The output measurement derivatives are required in the observer calculation. Due to the possibility that output measurements could be corrupted with noise, and direct differentiating noisy signal would exaggerate the noise in the estimates, Corless and Tu (1998) propose an input observer that applies to a smaller class of system with constraints on how fast the varying inputs change. Their approach avoids the differentiation of output measurements.

The estimation of reaction heat and reaction rate is important to on-line monitoring and control of chemical reaction systems. A few works have been proposed to address this estimation problem with the classical Linear Quadratic Gaussian estimation method (Schuler and Schmidt, 1992), an inversion-based estimator (Tatiraju and Soroush, 1998), the high gain observer approach (Aguilar et al., 2002) or the integral observer approach (Aguilar-López, 2003). In this work, we propose a passivity-based input observer that can be used to obtain the estimates using conservation balances, and the approach avoids the difficulty of modeling reaction kinetics.

Derivatives of process measurements provide useful information for process monitoring and control (Preisig, 1988; Preisig and Rippin, 1993). Fundamentally, as shown in Levant (1998), perfect estimation is not possible, when the analytical expression of the signal is not known. The work shows that the Lipschitz constants of signal derivatives is helpful to reduce the inaccuracy of reconstructed derivatives. Levant (1998) also develops a robust and exact differentiator of noisy signal based on the sliding mode method.

Inclusion of measurement derivatives in the input observer algorithms is controversial due to existence of approximation errors and exaggeration of noise during differentiating. We propose to use derivative estimators producing decent estimates before using the derivatives in input observer. Mboup et al. (2007) propose the algebraic time-derivative estimation method that calculates the derivative estimates as linear combinations of finite time-integrations of the signal. Reger and Jouffroy (2009) derive the same result from the standard linear system reconstructibility theory. The Savitzky-Golay filter (Baedecker, 1985) for time derivative estimation assumes that the signal can be expressed as a polynomial. The derivatives of the regressed polynomial are estimated derivatives. Co and Ydstie (1990) apply modulating function and fast Fourier transformation method to estimate derivatives.

The paper is organized as follows. In Section 2 we introduce the problem in chemical reaction control that motivates the development of the proposed observer; in Section 3, we develop the theory of proposed passivity-based observer; example simulations are shown in Section 4, and in Section 5, we complete the paper with summaries and conclusions.

2. PROBLEM STATEMENT AND MOTIVATION

We have a system described by following state space model:

$$\frac{dz}{dt} = p(z, x) + \phi(z, u), \quad (1)$$

$$\frac{dx}{dt} = f(z, x) + g(x, u), \quad (2)$$

$$y = z + v. \quad (3)$$

$z \in \mathbf{R}^p$ refers to measured states, $x \in \mathbf{R}^m$ refers to unmeasured states. We are interested in estimating $p(x, z) : \mathbf{R}^{p+m} \to \mathbf{R}^p$, which is a vector of C^1 functions. $v \in \mathbf{R}^p$ is the noise vector. In this paper, we assume the noise to be absent.

In a chemical reaction system context, $\phi(z, u)$ represents the supply function, assumed to be known or measured and does not have recursive dependency on the higher order dynamics, (2). $p(x, z)$ represents the production function, that couples the dynamics of the measured states z with unmeasured x. The estimation of this time-varying term $p(x, z)$ is useful to compensate in feedback control, indicate of reaction stage, and provide history of reaction evolution. To further explain the problem, we use following chemical semi-batch reaction example.

Semi-batch reactor example Assume that we have a semi-batch reaction system with only one reaction $A + B \to C$. The dynamics of the reaction can be modeled as:

$$\frac{dC_A}{dt} = \frac{F_{in}}{V} C_{A,in} - r, \quad (4)$$

$$\frac{dC_B}{dt} = \frac{F_{in}}{V} C_{B,in} - r, \quad (5)$$

$$\frac{dC_C}{dt} = r, \quad (6)$$

$$r = k_0 e^{\frac{-E_a}{RT_r}} C_A C_B, \quad (7)$$

$$\frac{dT_r}{dt} = \frac{F_{in}}{V\rho C_p} \left(\rho_{in} C_{p,in} T_{in} - \rho C_p T_r\right) - \frac{\Delta H_r r}{C_p \rho} - \frac{UA(T_r - T_j)}{V\rho C_p}, \quad (8)$$

$$\frac{dV}{dt} = F_{in} - F_{out}. \quad (9)$$

We can measure the concentration of A, so that:

$$y = C_A, \quad (10)$$

and want to estimate the reaction rate r.

The conventional way of estimation solves the algebraic-differential equations, (4) - (9), and use a regression, for example a Kalman filter, to match the model to the measurements. Another way is to just use the differential equation of measured C_A, (4). By comparing (4) and (1), we assume to know the inlet flow information $\phi = \frac{F_{in}}{V} C_{A,in}$. The task is to estimate the reaction production term $p(C_A, C_B, T_r, V) = r(t)$.

Obviously, following the conventional approach increases the size of modeling space, or requires more measurements. Thus, we ask the question: can we just use (4) and measured C_A to achieve the estimation of $r(t)$? The solution of the question would benefit us from mainly two aspects:

(1) it saves the work of modeling the rest of the system;
(2) it frees the estimation task from knowing the reaction kinetics.

The solution seems obvious at the first glance. One would think we can compute or measure $\frac{dC_A}{dt}$, and then calculate reaction rate r algebraically from (4). The obstacles preventing us from achieving that are the following:

(1) without having the analytical expression of C_A, exact differentiation to get $\frac{dC_A}{dt}$ is not possible;
(2) if the measurements of C_A is corrupted with noise, differentiation will further exaggerate the noise in the derivatives.

The two obstacles lead us to ask for a better solution that can dampen the noise translated into the estimates of $r(t)$.

Above all, considering only (4), and assuming $F_{in} C_{A,in}$ is known, we can measure:

$$y_1 = C_A, \quad (11)$$

$$y_2 = \frac{dC_A}{dt}, \quad (12)$$

but the derivative measurement could be corrupted with some noise. We want to estimate the time-varying production term $p(z, x) = r(t)$.

3. PASSIVITY-BASED INPUT OBSERVER

3.1 Ideal case

We start with a scalar, first order linear system:

$$\dot{z}(t) = az(t) + b\mu(z, t), \quad a < 0, \quad (13a)$$

$$y_1(t) = z(t), \quad (13b)$$

$$y_2(t) = \dot{z}(t). \quad (13c)$$

The system has two outputs. i.e. measured state and its time derivative. The task is to estimate the time-varying parameter $\mu(t)$. Motivated by solving the estimation problem as a control problem, we "manipulate" the estimated input $\mu(t)$ so that the estimation error of the state from the observer model (14) asymptotically declines to zero. Using Lyapunov function, $V(\tilde{z}) = \frac{1}{2}(z - \hat{z})^2$, we derive the following passivity-based input observer to solve the problem:

$$\dot{\hat{z}}(t) = a\hat{z}(t) + b\hat{\mu}(t), \quad (14)$$

$$\hat{\mu}(t) = \frac{1}{b} \left(k \left(y_1(t) - \hat{z}(t) \right) + y_2(t) - a\hat{z}(t) \right), k > 0, \quad (15)$$

where k is the proportional gain. From here, we drop the dependence of time in the notations for convenience.

Theorem 1. Given system (13a), with unknown time-varying parameter $\mu(t)$, state and time derivative measurements (13b), (13c). The passivity-based observer, (14) and (15), provides asymptotic estimates of unknown parameter.

Proof. By taking the time derivative of the update law,(15), we have:

$$\dot{\hat{\mu}} = \frac{a+k}{b}(\dot{z} - \dot{\hat{z}}) + \dot{\mu} \tag{16}$$

Now, subtract the expression of the true time derivative of μ:

$$\dot{\mu} = \frac{1}{b}(\ddot{z} - a\dot{z}), \tag{17}$$

from (16) to give:

$$\dot{\mu} - \dot{\hat{\mu}} = -\frac{a+k}{b}(\dot{z} - \dot{\hat{z}}) \tag{18a}$$

$$= \frac{(a+k)k}{b}(z - \hat{z}). \tag{18b}$$

The application of Lyapunov function $V(\tilde{z}) = \frac{1}{2}(z - \hat{z})^2 = \tilde{z}^2$ guarantees $z - \hat{z}$ converges zero. It is then obvious that $\dot{\mu} - \dot{\hat{\mu}}$ converges to zero, too. The asymptotic equilibrium point of $\mu - \hat{\mu}$ can be found by further exploring (18a). By substituting $\dot{z}$ and $\dot{\hat{z}}$ with their dynamics (13a) and (14), we get:

$$\dot{\mu} - \dot{\hat{\mu}} = -\frac{a+k}{b}\left[a(z - \hat{z}) + b(\mu - \hat{\mu})\right]. \tag{19}$$

According to previous analysis, at equilibrium,

$$\dot{\mu} - \dot{\hat{\mu}} = 0, \quad \dot{z} - \dot{\hat{z}} = 0, \tag{20}$$

forcing $\mu - \hat{\mu} \to 0$ asymptotically. □

Appendix A shows the application to the nonlinear system, (1) and (2).

3.2 Non-ideal case

In practice, it may not be possible to obtain accurate measurements of the derivatives. To study the effect of error in differentiation, we rewrite the model in the following manner:

$$\dot{z}(t) = az + b\mu(z, t), \quad a < 0, \tag{21a}$$

$$y_1 = z(t), \tag{21b}$$

$$y_2 = \dot{z}(t) + \delta(t). \tag{21c}$$

In this case, we do not measure the exact derivative, thus the second output y_2 is composed of the true time derivative of the state plus a noise term, $\delta(t)$. The noise term could result from the use of a numerical differentiator, such as the deadbeat method of Reger and Jouffroy (2009). The following result derives the frequency response of the estimated error with respect to the noise term.

Theorem 2. Given system (21a), with time-varying parameter $\mu(t)$, we assume that state is perfectly measured, (21b), but derivative measurement is corrupted with noise, (21c). Assume that we can model the noise as $\delta(t) = \delta_0 + A_\delta \sin(\omega_\delta t + \phi_\delta)$. The observer (14) - (15) provides parameter estimates with bounded error $\tilde{\mu}(t)$ as $t \to +\infty$:

$$\tilde{\mu}(t \to +\infty) = \frac{a}{bk}\delta_0 + \frac{(\sqrt{(a^2 + \omega_\delta^2)k^2 + a^2\omega_\delta^2 + \omega_\delta^4} sin(\omega_\delta t + \phi'_\delta)}{b(k^2 + \omega_\delta^2)}A_\delta. \tag{22}$$

Proof. First we differentiate $\tilde{\mu}$ and $\tilde{z}$, combining the results with equations (14) and (15), then we can obtain following relationships:

$$\dot{\tilde{z}} = -k\tilde{z} - \delta, \tag{23a}$$

$$\dot{\tilde{\mu}} = \frac{(a+k)k}{b}\tilde{z} + \frac{a+k}{b}\delta - \frac{1}{b}\dot{\delta}, \tag{23b}$$

$$\tilde{\mu} = -\frac{a+k}{b}\tilde{z} - \frac{1}{b}\delta. \tag{23c}$$

The solution to (23a) and (23b) are:

$$\tilde{z}(t) = e^{-kt}\tilde{z}_0 - e^{-kt}\int_0^t e^{k\tau}\delta(\tau)d\tau, \tag{24}$$

$$\tilde{\mu}(t) = -\frac{a+k}{b}e^{-kt}\tilde{z}_0 + \frac{a+k}{b}e^{-kt}\int_0^t e^{k\tau}\delta(t)d\tau - \frac{1}{b}\delta(t). \tag{25}$$

The noise $\delta(t)$ in the derivative is modeled as:

$$\delta(t) = \delta_0 + A_\delta \sin(\omega_\delta t + \phi_\delta), \tag{26}$$

which can be plugged in the solutions, and for $\tilde{z}(t)$ we have:

$$\begin{aligned}\tilde{z}(t) &= e^{-kt}\tilde{z}_0 - e^{-kt}\int_0^t e^{k\tau}\delta(\tau)d\tau \\ &= e^{-kt}\left(\tilde{z}_0 + \frac{\omega_\delta\cos(\phi_\delta) - k\sin(\phi_\delta)}{k^2 + \omega^2} - \frac{1}{k}\right) \\ &+ \frac{A_\delta\left(k\sin(\omega_\delta t + \phi_\delta) - \omega_\delta\cos(\omega_\delta t + \phi_\delta)\right)}{k^2 + \omega_\delta^2} + \frac{\delta_0}{k} \\ &= e^{-kt}\left(\tilde{z}_0 + \frac{\omega_\delta\cos(\phi_\delta) - k\sin(\phi_\delta)}{k^2 + \omega^2} - \frac{1}{k}\right) \\ &+ A_\delta\frac{\sqrt{k^2 + \omega_\delta^2}\sin(\omega_\delta t + \phi''_\delta)}{k^2 + \omega_\delta^2} + \frac{\delta_0}{k},\end{aligned} \tag{27}$$

where

$$\phi''_\delta = \phi_\delta + \arctan(-\frac{\omega_\delta}{k}). \tag{28}$$

In (27), except for the exponential decaying term, the magnitude of the periodic signal amplitude and modeled average are both dampened by the observer gain k. For $\tilde{\mu}$, we have:

$$\begin{aligned}&\tilde{\mu}(t) \\ &= \frac{a+k}{b}e^{-kt}\left(-\tilde{z}_0 + \frac{\omega_\delta\cos(\phi_\delta) - k\sin(\phi_\delta)}{k^2 + \omega_\delta^2} - \frac{1}{k}\right) \\ &+ \frac{a+k}{bk}\delta_0 - \frac{1}{b}\left(\delta_0 + A_\delta\sin(\omega_\delta t + \phi_\delta)\right) \\ &+ \frac{a+k}{b}\frac{A_\delta\left(k\sin(\omega_\delta t + \phi_\delta) - \omega_\delta\cos(\omega_\delta t + \phi_\delta)\right)}{k^2 + \omega_\delta^2} \\ &= \frac{a+k}{b}e^{-kt}\left(-\tilde{z}_0 + \frac{\omega_\delta\cos(\phi_\delta) - k\sin(\phi_\delta)}{k^2 + \omega_\delta^2} - \frac{1}{k}\right) + \\ &\frac{a}{bk}\delta_0 + \frac{\sqrt{(a^2 + \omega_\delta^2)k^2 + a^2\omega_\delta^2 + \omega^4}\sin(\omega_\delta t + \phi'_\delta)}{b(k^2 + \omega_\delta^2)}A_\delta,\end{aligned} \tag{29}$$

where

$$\phi'_\delta = \phi_\delta + \arctan\left(\frac{-(a+k)\omega_\delta}{ak - \omega_\delta^2}\right). \tag{30}$$

Similar situation with (29), the not exponential decaying terms can be dampened by large observer gain. □

The derivation above shows how the magnitude $|\tilde{\mu}_\infty|$ change as a function of the noise parameter, and it is summarized as follows:

$$|\tilde{\mu}_\infty| = |\frac{a}{bk}\delta_0|, \text{ as } A_\delta = 0 \text{ or } \omega_\delta = 0;$$

$$|\tilde{\mu}_\infty| \to |\frac{A_\delta}{b}|, \text{ as } \omega_\delta \to \infty.$$

The dampening effect of observer in the estimates of the derivative noise can be shown through a Bode diagram. We first derive the transfer function relations between $\tilde{z}(t)$ vs. $\delta(t)$ and $\tilde{\mu}(t)$ vs. $\delta(t)$. From (23a) and (23b), the transfer functions of these two pairs of outputs and inputs are:

$$G_1(s) = \frac{\tilde{z}(s)}{\delta(s)} = -\frac{1}{s+k} \tag{31}$$

$$G_2(s) = \frac{\tilde{\mu}(s)}{\delta(s)} = -\frac{s(s-a)}{bs(s+k)} \tag{32}$$

Assume $a = -1, b = 1$. The Bode diagrams of $G_1(s)$ and $G_2(s)$ are shown in Figures 1 and 2, respectively. Responses of high gain observer, $k = 10$, and low gain observer , $k = 2$, are compared. The error bound can be reduced by increasing the observer gain k while the noise frequency is small. As shown in both magnitude diagrams of $G_1(s)$ and $G_(s)$, the noise δ is much more dampened in $\tilde{z}$ and $\tilde{\mu}$ by the high gain observer than low gain observer while the noise frequency is small. As the frequency increases, the differences between using high gain and low gain grows smaller. However, from Figure 2, we can see the observer is not able to dampen the noise in $\tilde{\mu}$ when noise frequency is very high.

4. SIMULATION RESULTS

A simple scalar linear system with one varying parameter is used to test the developed passivity-based observer:

$$\dot{z}(t) = -z(t) + \mu(t), \tag{33a}$$
$$\mu(t) = 0.1\sin(0.5t), \tag{33b}$$
$$y_1(t) = z(t), \tag{33c}$$
$$y_2(t) = \dot{z}_d(t), \tag{33d}$$

where $\dot{z}_d$ is an estimate of the derivative of z.

Ideal case ($\dot{z}_d = \dot{z}$) Both the state and derivative is measured continuously and accurately, more specifically $\dot{z}_d = \dot{z}$. (14) and (15) are used to estimate the parameter $\mu(t)$. The results of the estimation are shown in Figures 3 and 4. We also compare the passivity based input observer (PBIO) result with inversion based (IBO) (Tatiraju and Soroush, 1998) result. The inversion based observer also treats the time-varying parameter estimation problem, but does not use information of deriavtive of measurement in the estimation equation (15). Figure (3a) shows that the observers starts with the same wrong initial condition, which also results in the deviation of the estimated parameter, $\hat{\mu}$ from the true value, Figure (4a). Because of the asymptotic stability of PBIO observer, the PBIO estimation errors converge asymptotically to zero in Figures (3b) and (4b) as shown in Theorem 1. In comparison, the IBO is shown to be marginal stable, due to fail to capture the dynamic change of the measurement.

Nonideal case ($\dot{z}_d = \dot{z} + \delta$) Again, we consider the example: (33a) - (33d). The difference from the ideal case

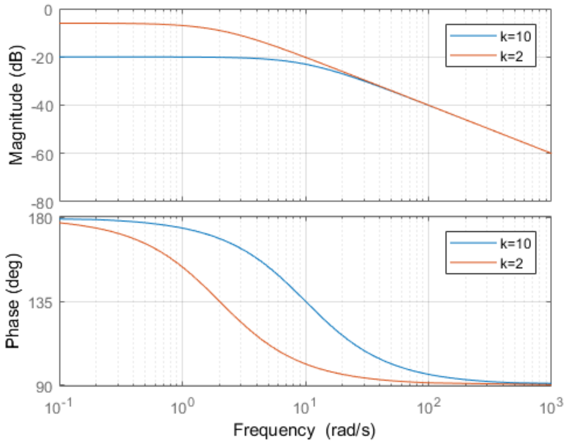

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Fig. 1. Bode diagram of $G_1(s) = \frac{\tilde{z}(s)}{\delta(s)}$

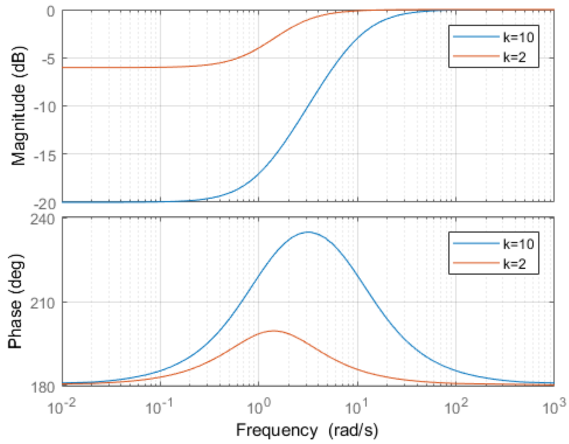


Fig. 2. Bode diagram of $G_2(s) = \frac{\tilde{\mu}(s)}{\delta(s)}$

is the second output $y_2(t)$ is calculated by using deadbeat differentiator,

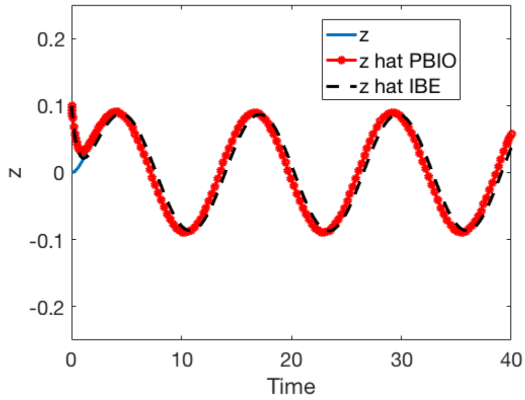
$$\dot{z}_d(t) = \dot{z}(t) + \delta(t), \tag{34}$$

where $\delta(t)$ is the differentiation error. Any type of differentiator would cause error in the reconstructed derivatives. Here, we pick the deadbeat differentiation technique proposed by Reger and Jouffroy (2009) as an example. The work re-derives the derivative estimation scheme in Mboup et al. (2007) based on the reconstructibility Gramian. Deadbeat differentiation treats the signal as a polynomial signal of time within the differentiation moving horizon T, here 0.1, and the signal $z(t)$ is a degree-one polynomial. First order derivative can be reconstructed using following formula from the paper:

$$\dot{z}_d(t) = \frac{6}{T^2}\int_{t-T}^{t} z(\tau)d\tau + \frac{12}{T^3}\int_{t-T}^{t}(\tau - t)z(\tau)d\tau, \tag{35}$$

The assumption of a degree-one polynomial is required to derive the weighting factors of the integration terms.

The proposed passivity-based observer (14) - (15) with proportional gain $k = 10$ is used. The simulation results are shown in Figure 6. The observer starts with a wrong initial condition with an error of 0.1 as shown in Figure (5a), then converge close to the true profile. The same



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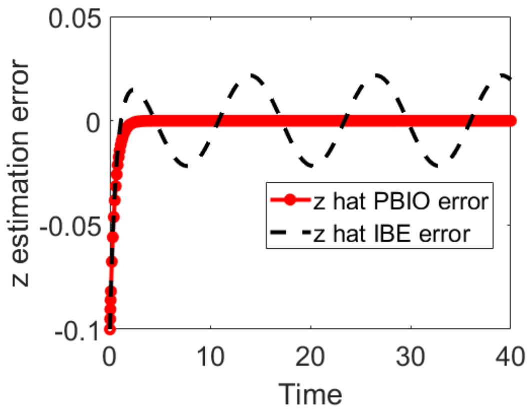
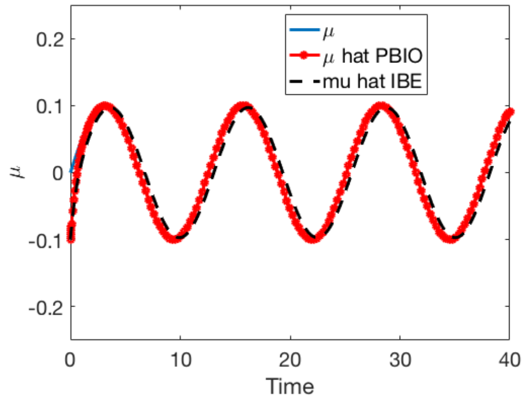
(a) Profiles of z and $\hat{z}$ $(k = 2)$

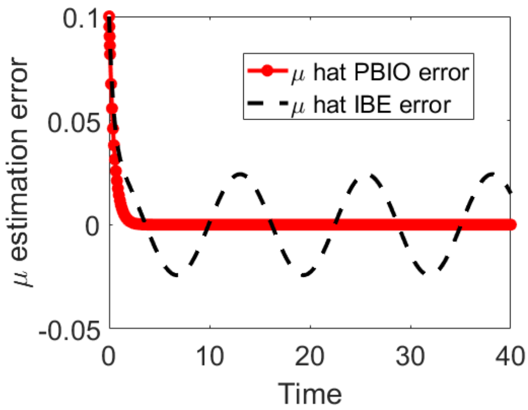


(b) Profiles of $\tilde{z}$ $(k = 2)$

Fig. 3. Observer performance in ideal case (z and $\tilde{z}$)



(a) Profiles of μ and $\hat{\mu}$ $(k = 2)$



(b) Profiles of $\tilde{\mu}$ $(k = 2)$

Fig. 4. Observer performance in ideal case(μ and $\tilde{\mu}$)

performance can be found for the parameter estimate in Figure (5b). The estimation error of μ though very small, but still exists in Figure (6b). However, we can see that the magnitude of μ error is very much dampen by the observer, compared with the magnitude of the differentiator estimated derivative error. Also, in this figure, we can observer the magnitudes of noise reduction with different k. As shown in the theory, with higher k, the bounds of estimation error are smaller.

5. CONCLUSIONS

In this work, we developed a passivity-based input observer for scalar linear system with a time-varying parameter to be estimated. The observer is derived from Lyapunov stability perspective, and requires the use of a measured or estimated output derivative. In the ideal case, the derivative is perfectly measured, and the observer estimates have asymptotic convergence to the true values. In the nonideal case, the derivative is not perfectly measured. Instead it is obtained through a differentiation technique, such as a deadbeat differentiator. We showed that the proposed observer could dampen the magnitude of the derivative error in the parameter estimates by using large observer gain. Illustrative examples are simulated to show the proposed observer performance for ideal and nonideal cases. Potential applications include production estimation in chemical reaction systems for process control and monitor. Inclusion of the proposed observer in a model based control scheme can reduce control model size and save modeling cost.

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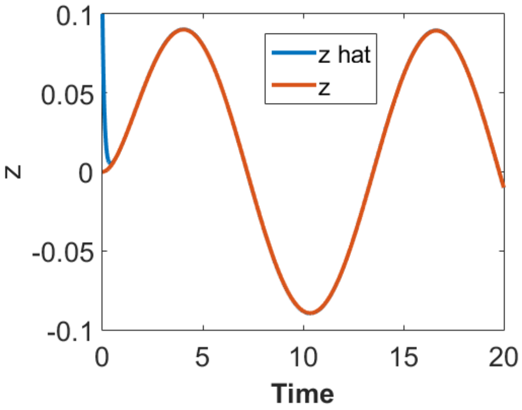

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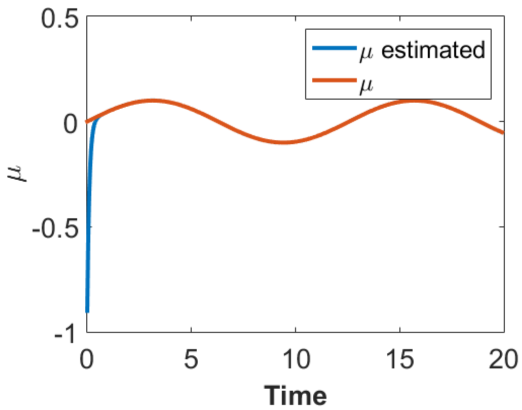
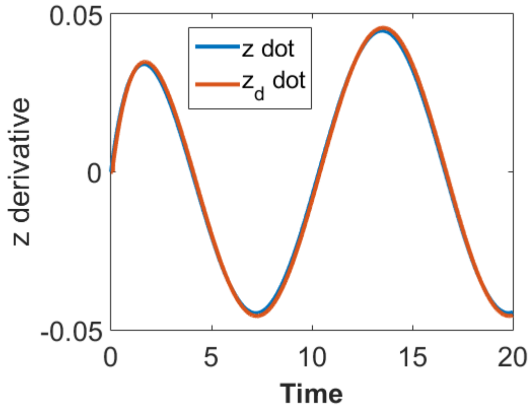
(a) Profiles of z and $\hat{z}$ ($k = 10$)

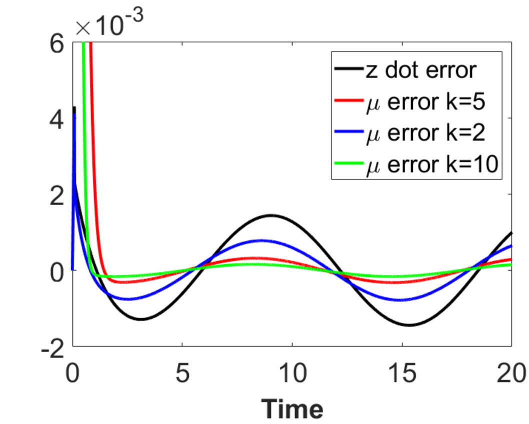


(b) Profiles of μ and $\hat{\mu}$ ($k = 10$)

Fig. 5. Observer performance in nonideal case (z and μ)



(a) Profiles of $\dot{z}$ and $\dot{z}_d$ ($k = 10$)



(b) Profiles of δ and $\tilde{\mu}$

Fig. 6. Observer performance in nonideal case ($\dot{z}$ and $\tilde{\mu}$)

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Appendix A. OBSERVER OF NONLINEAR SYSTEMS

The observer can be used for the nonlinear system:

$$\frac{dz}{dt} = p(z, x) + \phi(z, u), \tag{A.1}$$

$$\frac{dx}{dt} = f(z, x) + g(x, u), \tag{A.2}$$

$$y_1 = z, \tag{A.3}$$

$$y_2 = \dot{z}, \tag{A.4}$$

when $z \in \mathbf{R}^p$, are measured states, and $x \in \mathbf{R}^m$ are unmeasured states.$p : \mathbf{R}^{p+m} \rightarrow \mathbf{R}^p$ are a vector of C_1 functions. In this case, the observer is

$$\frac{d\hat{z}}{dt} = \hat{p}(t) + \phi(\hat{z}, u), \tag{A.5}$$

$$\hat{p}(t) = y_2 + K(y_1 - \hat{z}) - \phi(\hat{z}, u). \tag{A.6}$$

$\mathbf{K} \in diag(k_1, k_2, k_3, ..., k_p)$ is positive definite.