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Modified Hankel Interaction Index Array for Input-Output Pairing with Improved Characteristics ⋆

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Abstract: In this study, a modified version of Hankel Interaction Index Array (HIIA) for control configuration selection is presented which can overcome some of its shortcomings, like e.g. scaling dependency, or not relating to closed loop system properties. Inspired by the relative gain array approach, the HIIA is reformulated in the relative gain thinking by considering the effect of closing loops. The ratio of the Hankel norm of the subsystems in closed and open loop are used to state a modified version of HIIA, which has improved characteristics compared to the original HIIA. Properties of the modified HIIA are discussed and benchmarked with established methods on three example cases.

Keywords: Control configuration selection, Interaction measure, Hankel Interaction Index Array, System Gramians.

1. INTRODUCTION

Decentralized control systems are the most used control configurations for industrial processes (Scattolini, 2009). Its attractiveness in comparison to multivariable control structures stems from the ease of controller synthesis and implementation. Moreover, these control systems are often associated with a low cost and robustness to uncertainties and vulnerabilities. Such control systems are usually designed in two steps:

(1) Selecting the appropriate control configuration in terms of input-output pairing.
(2) Designing single-input single-output (SISO) controllers for the selected input-output pairs.

There are numerous methods available for addressing the input-output pairing problem, initiated with the introduction of the Relative Gain Array (RGA) by Bristol (1966) as a so-called interaction measure. Bristol defined relative gains as a ratio of the open loop gain from one input to one output and the loop gain when the other loops are closed with the requirement of perfect control at steady-state. Alongside its simple computation based on the DC gain of the system, the RGA provides information on integrity and closed-loop stability for an input-output pairing based on open-loop information which makes it very attractive and useful from an engineering perspective. However, these properties of the RGA are only valid at steady state, and in order to overcome this drawback a number of solutions have been proposed, like e.g. the Dynamic RGA (DRGA) (Mc Avoy et al., 2003) expanding the interpretation to all frequencies, the Effective RGA (ERGA) (Xiong et al., 2006) adding a weight to the effective frequency range, the effective relative energy array (EREA) (Monshizadeh-Naini et al., 2009) and relative response array (RRA) (Jain and Babu, 2015).

In general, all relative gain related approaches do not give any insight in more complex control configurations, where several outputs are related to an input, which is a limitation. Control configuration selection methodologies for linear multivariable plants can be classified in two categories, transfer function based relative gain related methods and Gramian based methodologies for state space realisations (Khaki-Sedigh and Moaveni, 2009). At this point it is important to note that Gramian based methods can be used for the input-output pairing problem and for the selection of more complex control configurations. The initial method for Gramian-based measure was the Participation matrix suggested by Salgado and Conley (2004). In the same context the Hankel interaction index array (HIIA) (Wittenmark and Salgado, 2002) and the Σ_2 array (Birk and Medvedev, 2003) were proposed as alternatives with somewhat different characteristics, like e.g. how time delays affect the outcome. The methods were later extended in (Arranz and Birk, 2012) and (Shaker and Tahavori, 2014) to be more widely applicable.

Gramian-based measures aim at quantifying the system dynamic in terms of controllability and observability using a state space model of multivariable plants. There, state controllability and state observability is quantified by Gramian matrices to evaluate the joint controllability and observability for specific input-output connections. The main critic on Gramian based methodologies is that the methods do not provide any insight on closed loop properties like stability and integrity. In addition the methods are scaling dependent, requiring proper scaling of the system model prior to the analysis. In other words, Gramian based methods just consider open loop properties of a multivariable plant for control configuration selection.

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In this paper, a modified version of the HIIA is suggested where closed loop properties are considered. Since such properties are not integral to Gramian based measures, the outset is found in the RGA definition and a subsequent translation for state space realisations. The paper provides an interpretation of the perfect control requirement for state space realisations and re-uses the ratio calculation between open-loop and closed-loop in the definition of the modified HIIA.

The paper is arranged as follows. First, some notation and the original HIIA are reviewed in the Preliminaries in section 2. Then, the modified HIIA and its theoretical foundation is presented in section 3. Using some well established benchmarking examples are then used in section 4 to evaluate and compare the modified HIIA with other methods. Finally some conclusions and an outlook on future work is given in section 5.

2. PRELIMINARIES

Consider the following $m \times m$ linear time invariant (LTI) multivariable system, which is assumed to be stable, controllable and observable:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

with $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{m\times n}$, $u \in \mathcal{R}^m$, and $y \in \mathcal{R}^m$. The realisation of (1) as a transfer function matrix of size $m \times m$ is given as

$$G(s) = C(sI - A)^{-1}B \tag{2}$$

where I denotes the identity matrix of appropriate size and an elementary transfer function in $G(s)$ is referred to as $g_{ij}(s)$. Further, the Laplace operator s, will be dropped for the sake of simplicity.

In the sequel, to express the decomposition of B into column vectors and C into row vectors, the following notation is used

$$\begin{aligned} B &= [b_{*1}, b_{*2}, ..., b_{*m}] \\ C^T &= [c_{1*}^T, c_{2*}^T, ..., c_{m*}^T] \end{aligned}$$

The HIIA was originally proposed in (Wittenmark and Salgado, 2002) and is henceforth denoted as Σ_H. Using (1) and the transfer function realisation of a general state space system from (2), the Σ_H is defined as follows

$$[\Sigma_H]_{ij} = \frac{\|g_{ij}\|_H}{\Sigma \|g_{ij}\|_H} \tag{3}$$

where, g_{ij} is the transfer function of elementary subsystem (i, j), corresponding to the 3-tuple (A, b_{*j}, c_{j*}) in accordance with (2) , and $\|\bullet\|_H$ denotes the Hankel norm.

According to Wittenmark and Salgado (2002) an appropriate input-output pairing would be constituted by a permutation matrix $\mathcal{P}$ of size $m \times m$ which corresponds to the largest elements of Σ_H. Essentially, this selection can be stated as the following optimization problem

$$\mathcal{P}^* := \arg\max_{\mathcal{P}\in\mathbb{P}} ||\Sigma_H \circ \mathcal{P}||_{sum} \tag{4}$$

where $\mathbb{P}$ is the set of all possible $m \times m$ permutation matrices, and $\circ$ denotes the Hadamard product.

Already in the initial work by Wittenmark and Salgado (2002) on the HIIA, some important properties of the HIIA are discussed. The HIIA is scaling dependent, which means that input and output scaling affects the array and in turn the pairing decision. Hence, it is important to properly scale the process prior to the application of the HIIA. Moreover, time delays affect the HIIA such that interconnections with large time delays are favored for control. In that sense, the pairing decision might not be feasible. Further, the HIIA is not considering closed loop properties, which is often used as an argument to discard the method. It is due to these reasons that an improved variant of the HIIA should be derived, addressing the above shortcomings.

3. MODIFIED HIIA

From (3) it becomes clear that only open loop properties of the plant G are considered for I/O pairing, and just evaluates the effect of input variables on output variables. While the HIIA considers controllability and observability aspects of the individual interconnections in G, closed loop properties like stability or integrity are not reflected.

In case of the RGA, closed loop properties are considered through the assumption of perfect control, which relates to the inverse of the transfer function matrix G. The basic idea of the RGA lies in the evaluation of the gain ratio between open loop and a perfectly controlled closed. Closeness to one simply indicates small gain changes when loops are closed or opened. The idea is now to recast this concept into the Gramian based interaction measure framework.

In this section, the theoretical foundation for the Modified HIIA is introduced to transform the HIIA methodology into an RGA-like interaction measure.

3.1 Quantifying the effect of input variables on output variables in open loop condition

The effect of input variable u_j on output variable y_i can be quantified using the Hankel norm of G_{ij}, $\sigma_{H,o,ij} = \|G_{ij}\|_H$. Thus, for multivariable system (1), $\bar{\Sigma}_{H,o}$ is defined as (5) to quantify the effect of the input variables on out variables, when all loops are open.

$$\bar{\Sigma}_{H,o} = \left[\sigma_{H,o,ij} = \|G_{ij}\|_H\right] \tag{5}$$

3.2 Quantifying the effect of input variables on output variables under the tight control

The state space model of the MIMO plant (1) can be rewritten as :

$$\begin{cases} \dot{x} = A_{n\times n}x + B^j u^j + b_{*j}u_j \\ y^i = C^i x \\ y_i = c_{i*}x \end{cases} \tag{6}$$

where B^j and C^i denote matrices B and C after removing the j^{th} column and i^{th} row, respectively. Also, u^j and y^i denote input and output vectors after removing the j^{th} and i^{th} element, respectively.

Assuming all loops except $u^j - y^i$ are under tight control, the following condition is fulfilled

$$\dot{y}^i = C^i A x + C^i B^j u^j + C^i b_{*j} u_j = 0 \tag{7}$$

If $C^i B^j$ has full rank, then using (7) the control signal u^j to obtain the tight control condition is given by

$$u^j = -\left(C^i B^j\right)^{-1} C^i A x - \left(C^i B^j\right)^{-1} C^i b_{*j} u_j \tag{8}$$

Using (6) and (8), the loop gain for the interconnection $u^j - y^i$ is found as

$$\bar{g}_{ij}(x) := \begin{cases} \dot{x} = A_t x + B_t u_j \\ y_i = c_{i*} x \end{cases} \tag{9}$$

with

$$\begin{aligned} A_t &= \left(I_n - B^j \left(C^i B^j\right)^{-1} C^i\right) A \\ B_t &= \left(I_n - B^j \left(C^i B^j\right)^{-1} C^i\right) b_{*j} \end{aligned} \tag{10}$$

If $C^i B^j$ has not full rank, the control signal u^j fulfilling the tight control condition can not be described using (7). In this case, a higher order derivative of y^i can be used instead

$$\ddot{y}^i = C^i A^2 x + C^i A B^j u^j + C^i A b_{*j} u_j + C^i B^j \dot{u}^j + C^i b_{*j} \dot{u}_j \tag{11}$$

Thus, if $C^i A B^j$ in (11) has full rank, and assuming all loops except $u^j - y^i$ are under tight control, then $\ddot{y}^i = 0$, $\dot{u}^j = 0$, $\dot{u}_j = 0$, and we have

$$u^j = -\left(C^i A B^j\right)^{-1} C^i A^2 x - \left(C^i A B^j\right)^{-1} C^i A b_{*j} u_j \tag{12}$$

Using (6) and (12), the loop gain of the interconnection $u^j - y^i$ becomes

$$\bar{g}_{ij}(x) := \begin{cases} \dot{x} = A_t x + B_t u_j \\ y_i = c_{i*} x \end{cases} \tag{13}$$

where

$$\begin{aligned} A_t &= \left(I_n - B^j \left(C^i A B^j\right)^{-1} C^i A\right) A \\ B_t &= \left(I_n - B^j \left(C^i A B^j\right)^{-1} C^i A\right) b_{*j} \end{aligned} \tag{14}$$

Now, using the Hankel norm of the system (9) or (13) the effect of input variable u_j on output variable y_i when all loops except $u^j - y^i$ are under tight control can be quantified as follows

$$\sigma_{H,T,ij} = \|\bar{g}_{ij}\|_H \tag{15}$$

Thus, $\|\bar{g}_{ij}\|_H$ is the Hankel norm counterpart of the corresponding element in the inverse of the G as used in the relative gain array context.

3.3 Quantifying the interaction

The basic idea of the relative gain array approach is to identify elements which are close to one, meaning that opening and closing loops has little effect on the loop gains. Similarly, the element-wise ratio of the open loop Hankel norm with the Hankel norm of the closed loop could be used as an indicator of interaction.

Now the Hankel Ratio can be defined, which compares $\sigma_{H,o,ij}$ and $\sigma_{H,T,ij}$ to quantify the interaction

$$\delta_{ij} = \frac{\sigma_{H,o,ij}}{\sigma_{H,T,ij}} \tag{16}$$

Consequently, using (16), the modified HIIA for the system in (1) can be defined as

$$\Delta = \left[\delta_{ij} = \frac{\sigma_{H,o,ij}}{\sigma_{H,T,ij}}\right] \tag{17}$$

Based on the definition of the Hankel Ratio, δ_{ij}, the following interpretations can be stated:

- $\delta_{ij} \approx 1$ implies $\sigma_{H,o,ij} \approx \sigma_{H,T,ij}$, which means that the Hankel norm of the elementary system corresponding to $u^j - y^i$ is not changed by closing the other loops.
- $\delta_{ij} < 1$ indicates that $\sigma_{H,o,ij} < \sigma_{H,T,ij}$. In other words, the Hankel norm of the elementary system corresponding to $u^j - y^i$ is increased by closing the other loops.
- $\delta_{ij} > 1$ means that $\sigma_{H,o,ij} > \sigma_{H,T,ij}$ and thus, the Hankel norm of the elementary system corresponding to $u^j - y^i$ is decreased by closing the other loops.

In contrast to the relative gain array, sign changes due to opening and closing loops can not be assessed by the modified HIIA, as the Hankel norm is always positive. Thus, integrity and stability conditions can not be derived from the modified HIIA. As such the modified HIIA has similar properties as the HIIA in that context.

Moreover, it can be shown that the modified HIIA is not sensitive to scaling of the input and output signals of the system. The proof is straight forward by introducing the scaled variables $y^S = \mathcal{D}_y y$ and $u^S = \mathcal{D}_u u$, and thereafter subsequently replacing the u and y by its scaled versions. There scaling matrices $\mathcal{D}_y$ and $\mathcal{D}_u$ are diagonal matrices.

Using the scaling property of the modified HIIA it can be seen that time delays which can be represented as input and output delays, will not affect the modified HIIA. On the contrary, internal time delays which do affect the internal structure of the state space system, yield a change in the modified HIIA.

It should also be noted that the modified HIIA and the original HIIA consider the complete dynamics of the process.

3.4 Selection of a pairing

In order to perform the selection of the I/O pairs, the closeness to one is the main indicator for selecting a pair. Nevertheless, the overall selection would need to be performed such that the overall distance for all the pairs to one is minimized.

In the context of the RGA, Skogestad and Postlethwaite (2007) suggest the RGA number as indicator for the selection of the overall I/O pairing for the decentralized control structure, which is defined as follows

$$\text{RGA number} := ||\Lambda(G) - I||_{sum} \tag{18}$$

The RGA number can be easily adapted to the modified HIIA by substituting $\Lambda(G)$ with $\Delta(G)$ in (18). The new indicator will be denoted modified HIIA number.

4. NUMERICAL EXAMPLES

We will now make use of three examples to benchmark the modified HIIA with the RGA, DRGA, HIIA and ERGA. Differences and similarities in the indications will be discussed and analyzed in the following. The modified HIIA will be referred to as mHIIA.

Example 1. Consider a 2 input-2 output discrete time transfer function matrix as presented in (Salgado and Conley, 2004).

$$G_1(z) = \begin{bmatrix} \dfrac{0.5}{z-0.5} & \dfrac{0.15}{(z-0.8)\,z^l} \\ \dfrac{0.1}{(z-0.5)\,(z-0.8)} & \dfrac{0.3}{z-0.7} \end{bmatrix} \tag{19}$$

The main challenge in this example is considering the effect of internal time delays on the I/O pairing.

The pairing analysis using RGA, ERGA, HIIA, mHIIA and DRGA is summarized in Table 1. There, it can be seen that RGA and ERGA result in the same matrices for $l = 0$ and $l = 10$, since these two indicators do not consider the effect of time delays in the pairing analysis. In Fig. 1 the DRGA is displayed and recommends the diagonal I/O pairing in the bandwidth range for both $l = 0$ to $l = 10$. It is worthwhile noting that the DRGA is minorly affected by the internal time delays. The HIIA results in two different matrices and different pairing decision for to $l = 10$. It is clear that without any change in the system dynamics, the recommended I/O pair by HIIA is changed since the time delay in element (1,2) is increased. This shows the main deficiency of the HIIA, as elements with large time delay are usually preferred for I/O pairing, which it is not desirable from a closed loop perspective.

Table 1. Pairing analysis using RGA, HIIA, Modified HIIA and DRGA for G_1

Method	$l = 0$	$l = 10$
RGA	$\Lambda = \begin{bmatrix} 4.00 & -3.00 \\ -3.00 & 4.00 \end{bmatrix}$	$\Lambda = \begin{bmatrix} 4.00 & -3.00 \\ -3.00 & 4.00 \end{bmatrix}$
ERGA	$\Gamma = \begin{bmatrix} 1.15 & -0.15 \\ -0.15 & 1.15 \end{bmatrix}$	$\Gamma = \begin{bmatrix} 1.15 & -0.15 \\ -0.15 & 1.15 \end{bmatrix}$
HIIA	$\Sigma_H = \begin{bmatrix} 0.29 & 0.18 \\ 0.28 & 0.25 \end{bmatrix}$	$\Sigma_H = \begin{bmatrix} 0.263 & 0.251 \\ 0.253 & 0.232 \end{bmatrix}$
mHIIA	$\Delta = \begin{bmatrix} 1.07 & 0.42 \\ 0.54 & 1.23 \end{bmatrix}$	$\Delta = \begin{bmatrix} 0.84 & 0.57 \\ 1.00 & 0.79 \end{bmatrix}$
DRGA	see Fig. 1	see Fig. 1

On the other hand, mHIIA results result in different matrices for $l = 0$ and $l = 10$, but in both cases, mHIIA

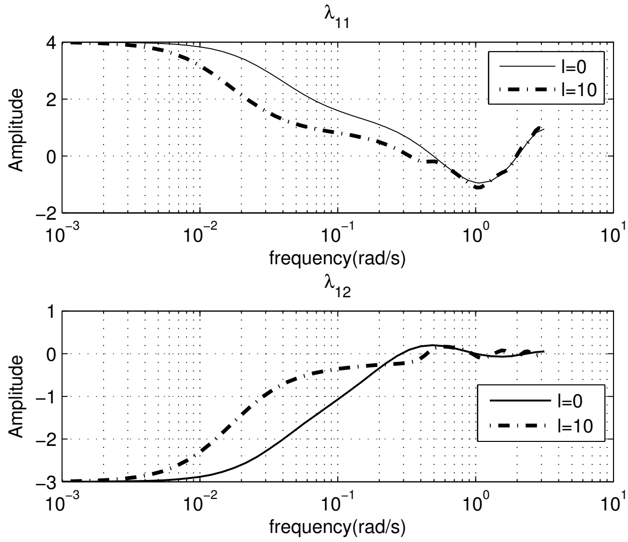

Fig. 1. Real part of the frequency responses of the elements of DRGA for G_2 in two cases: $l = 0$ and $l = 10$.

recommends the diagonal pairing. Clearly, the mHIIA is also affected by internal time delays, but to a much lower degree than the HIIA. Preliminarily, this indicate that mHIIA considers the effect of internal time delays in I/O pairing analysis more appropriately.

Example 2. Distillation columns are important unit operations in chemical process plants. In this example a binary distillation column with pressure variation including 8 plates as Fig. 2 has been considered (Davison, 1967) (Davison, 1990). The system matrices of the linearized state space model of the binary distillation column with 3 inputs, 3 outputs and 11 state variables is formulated by Davison (1967) as follows:

$$A = 10^{-3}\cdot \begin{bmatrix} -14 & 4.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9.5 & -13.8 & 4.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 9.5 & -14.1 & 6.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 9.5 & -15.8 & 11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.5 & -31.2 & 15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20.2 & -35 & 22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20.2 & -42.2 & 28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20.2 & -48.2 & 37 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20.2 & -57 & 42 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20.2 & -48.3 & 0.5 \\ 25.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25.5 & -18.5 \end{bmatrix}$$

$$B^T = 10^{-3} \begin{bmatrix} 0 & 0.005 & 0.002 & 0.001 & 0 & 0 & -0.005 & -0.01 & -0.04 & -0.02 & 0.46 \\ 0 & -0.04 & -0.02 & -0.01 & 0 & 0 & 0.01 & 0.03 & 0.005 & 0.002 & 0.46 \\ 0 & 2.5 & 5 & 5 & 5 & 5 & 5 & 5 & 2.5 & 2.5 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 2 shows the I/O pairing analysis and the recommended I/O pairing using RGA, ERGA, HIIA, mHIIA and DRGA for the Binary Distillation Column. Also, Fig. 3 shows the rows of the DRGA. Based on the RGA pairing rule, $y_{1,2,3} - u_{1,3,2}$ has been recommended for I/O pairing, while the ERGA recommends $y_{1,2,3} - u_{3,2,1}$. The DRGA shows that pairing recommended by ERGA is not correct, since λ_{13} and λ_{22} of DRGA have negative signs. The main reason why the ERGA does not result in the correct I/O

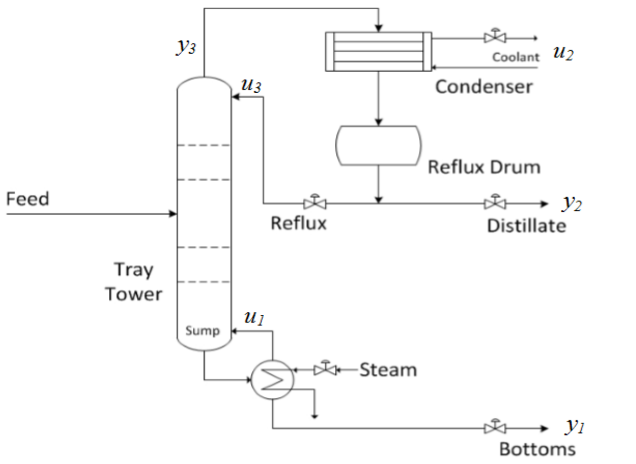

Fig. 2. Binary Distillation Column with pressure variation

pairing can be found in the deviation of the frequency response of the distillation column form the typical assumed frequency response used in the definition of the ERGA (Xiong et al., 2006). The HIIA shows the high interaction between some elements and the corresponding recommended pairing is $y_{1,2,3} - u_{2,1,3}$. Neither RGA nor DRGA support this pairing result. On the contrary, the mHIIA recommends $y_{1,2,3} - u_{2,1,3}$, which is the same recommendation as provided by RGA and DRGA. The mHIIA thereby combines the benefits of RGA based and Gramian based interaction measures.

Table 2. Pairing analysis using RGA, ERGA, HIIA, mHIIA and DRGA for Binary Distillation Column

Method	Qualification	Recommended pairing
RGA	$\Lambda = \begin{bmatrix} \mathbf{1.17} & 0.95 & -1.12 \\ -0.44 & -0.59 & \mathbf{2.03} \\ 0.28 & \mathbf{0.63} & 0.09 \end{bmatrix}$	$y_{1,2,3} - u_{1,3,2}$
ERGA	$\Gamma = \begin{bmatrix} 0.0003 & -0.34 & \mathbf{1.34} \\ 0.0012 & \mathbf{1.34} & -0.34 \\ \mathbf{0.999} & -0.0008 & 0.0022 \end{bmatrix}$	$y_{1,2,3} - u_{3,2,1}$
HIIA	$\Sigma_H = 10^{-3} \begin{bmatrix} 0.1 & \mathbf{0.2} & 27 \\ \mathbf{0.1} & 0.1 & 148.2 \\ 0.5 & 0.5 & \mathbf{579.6} \end{bmatrix}$	$y_{1,2,3} - u_{2,1,3}$
mHIIA	$\Delta = \begin{bmatrix} \mathbf{0.88} & 1.08 & 121.9 \\ 0.57 & 0.89 & \mathbf{2.00} \\ 0.05 & \mathbf{0.82} & 6.92 \end{bmatrix}$	$y_{1,2,3} - u_{1,3,2}$
DRGA	see Fig. 3	$y_{1,2,3} - u_{1,3,2}$

Example 3. Consider the Wood and Berry distillation column with following transfer function matrix (Wood and Berry 1973):

$$G_3(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-s}}{14.4s+1} \end{bmatrix} \tag{20}$$

This example has been extensively used in literature to benchmark different indicators. The result of the I/O pairing analysis is presented in Table 3. There, it can be seen that the decision on the pairing is not consistent between the different indicators.

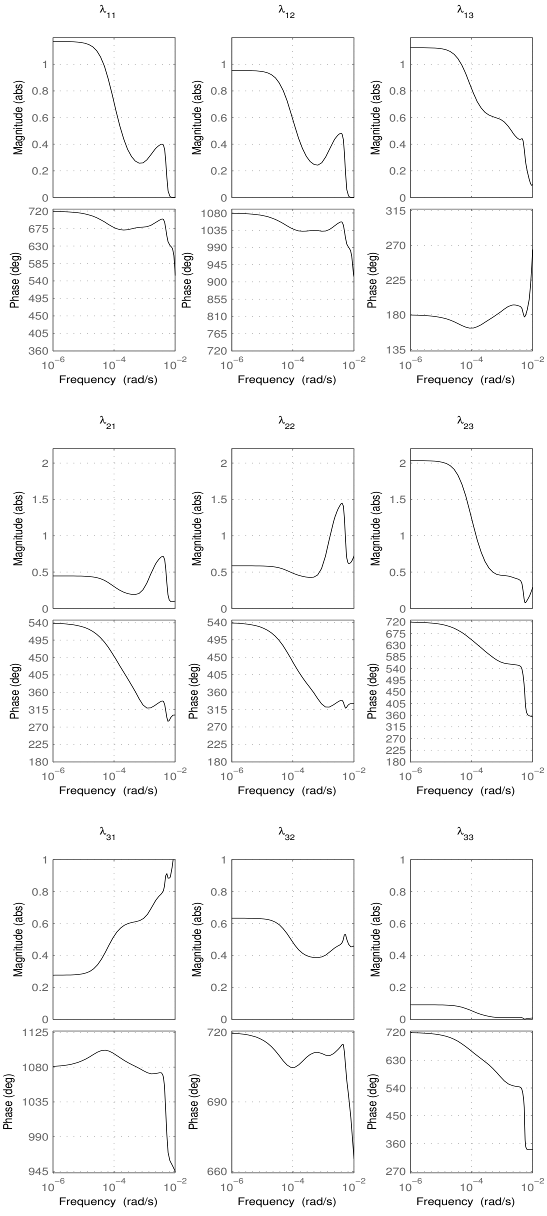

Fig. 3. Frequency responses of the elements of the DRGA for Binary Distillation Column.

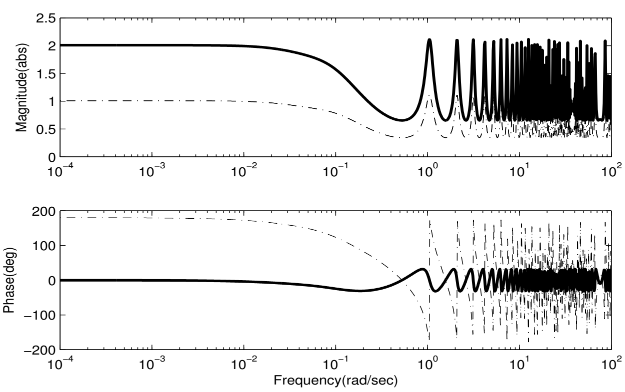

Fig. 4. The frequency responses of the elements of DRGA for G_3 (λ_{11}: solid line, λ_{12}: dash-dot line).

The RGA, ERGA and the HIIA promote the diagonal pairing. In case of the RGA, DRGA and ERGA, λ_{11} reaches a value of 2, which is usually associated with robustness issues. The HIIA decision is based on a value of 0.558 over 0.442, which is a quite small separation between diagonal and off-diagonal pairing. Fig. 4 shows the frequency responses of two elements of the DRGA, λ_{11} and λ_{12}. It is clear that in all frequencies λ_{11} has positive dominant value and diagonal pairing is recommended. The mHIIA has a strong overweight for the off-diagonal pairing which might be a result of stability and integrity aspects are not considered in the indicator, in addition to the fact that the mHIIA considers the complete frequency range and for higher frequencies the effect of the time delays becomes dominant. Since the recommended pairing is the diagonal pairing, further investigation is needed to understand properties of the mHIIA in relation to such cases. As a result, these aspects require further investigation and development of the mHIIA approach. More specifically, it need to be studied how closed loop properties are reflected in the mHIIA, besides the perfect control condition at steady state. Here it would be necessary to derive relationship between the mHIIA and stability as well as integrity of the closed loop system.

5. CONCLUSION

This paper proposes the modified Hankel Interaction Index Array (mHIIA) as a new control configuration selection methodology based on the original HIIA methodology. The mHIIA combines the benefits of RGA based methodologies with Gramian based I/O pairing methods by considering the closed-loop effects. Thereby, the mHIIA solves some of the shortcomings of HIIA, especially when it comes the analysis of plants with time delays. The effectiveness of mHIIA is evaluated on three well known benchmarking examples. In the evaluation established methods like the RGA, ERGA, DRGA and the original HIIA are used for comparison.

The mHIIA methodology enables Gramian-based interaction measures to consider the effect of closing loops with a tight control condition, which has not been possible before. Despite this novelty and improvement of the HIIA approach, some of the advantages of the relative gain approaches are not yet realized, like the prediction of stability and integrity properties of the closed-loop system. Accordingly, further research need to investigate stability and integrity conditions which can be integrated into the mHIIA methodology. Moreover, the performance of the mHIIA approach in larger systems need to be studied and benchmarked.

Table 3. I/O Pairing decision for the Wood and Berry distillation column

Method	Quantification	Pairing
RGA	$\Lambda = \begin{bmatrix} 2.0094 & -1.0094 \\ -1.0094 & 2.0094 \end{bmatrix}$	Diagonal
ERGA	$\Gamma = \begin{bmatrix} 2.1175 & -1.1175 \\ -1.1175 & 2.1175 \end{bmatrix}$	Diagonal
HIIA	$\Sigma_H = \begin{bmatrix} 0.2218 & 0.3276 \\ 0.1144 & 0.3362 \end{bmatrix}$	Diagonal
mHIIA	$\Delta = \begin{bmatrix} 1.9612 & 0.9685 \\ 0.9531 & 1.9572 \end{bmatrix}$	Off-diagonal
DRGA	see Fig. 4	Diagonal

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