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Tracking control on target signal for a class of uncertain neutral systems

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Abstract: Tracking control based on target signal for a class of uncertain neutral systems is investigated in this paper. An augmented error system is constructed by combining the control system with the target signal. Through a Lyapunov-Krasovskii function and some inequalities, a stability criterion in terms of LMIs is proposed for the auto-controlled system. Then a state feed-back control is designed for the augmented error system. And tracking control, where the target signal and error signal are utilized to help deduce the static error, is obtained for the original uncertain neutral system. A numerical example is given to illustrate the validity of our proposed method.

Keywords: uncertain neutral systems, augmented error system, Lyapunov-Krasovskii function, tracking control, LMIs.

1. INTRODUCTION

Time-delay is often encountered in various systems, such as chemical engineering systems, inferred grinding model, and manual control, neural network (see Kwon et al. (2016), Ramakrishnan et al. (2011)). It has been shown that time-delay can cause instability and poor performance of a control system. Therefore, systems with time-delay have attracted a great deal of attention over the past decades (see Fridman et al. (2016), and Wu et al. (2004)).

Neutral system is a special class of time-delay system, where delays exist not only in the state but also in the state derivative. A number of practical systems can be modeled by neutral systems, including partial element equivalent circuit (PEEC), population ecology, heat exchangers (see Ghadiri et al. (2014)). Besides, through model transformation, many time-delay systems, such as lossless transmission model and standard delay systems can be investigated as neutral systems. There has been increasing interest in analysis and synthesis of neutral systems because of their significance both in theory and application. Many studies for neutral systems have also been done; primarily on stability analysis and synthesis (see Pepe. (2016)).

In some cases, target signal is known. To help the output track the target signal better, it is necessary to make use of the target signal's information. However, target signal is less focused on in studies of neutral systems. Only few of these are aimed at H_∞ output tracking control, where a reference model is needed (see Liu et al. (2013), Zhang et al. (2010), and Xia et al. (2014)). Refs. Liu et al. (2013) and Xia et al. (2014) derived H1 tracking control respectively for switched neutral systems and uncertain delay system.

Tracking control for a class of uncertain neutral systems is proposed in this paper. Different from H_∞ control, any reference model is not used in this paper. An augmented error system is firstly constructed on basis of the properties of the system. By using a Lyapunov function and applying Jensen's inequality, we derive a criterion in terms of LMIs for the auto-controlled system. Then tracking control, which contains both the error signal and the target signal, is deduced. Similar to preview control (see Katayama et al. (1987)), where the target signal is always utilized in control design, tracking control in this paper can also help to reduce the static error. A numerical example is given to show the effectiveness of the proposed method.

Throughout this paper, $\mathbf{R}^n$ is the n-dimensional Euclidean space. $\mathbf{R}^{m\times n}$ denotes the set of $m\times n$ real matrix. $*$ represents the elements below the main diagonal of a symmetric matrix. A^{T} means the transpose of A . $P>0$ represents that P is positive definite. I and O respectively denote identity matrix and zero matrix with appropriate dimensions.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

We consider a class of uncertain neutral systems in this paper described by the following equations.

$$\begin{cases} \dot{x}(t)-G\dot{x}(t-h)=(A+\Delta A)x(t) \\ \qquad\qquad +(A_1+\Delta A_1)x(t-h)+Bu(t), \\ y(t)=Cx(t), \\ x(\theta)=\varphi(\theta), \quad \varphi(\theta)\in\mathbf{C}([-h,0],\mathbf{R}^n), \end{cases} \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state vector, $u(t)\in\mathbf{R}^m$ is the input vector, $y(t)\in\mathbf{R}^p$ is the output vector, $A\in\mathbf{R}^{n\times n}$ 、 $A_1\in\mathbf{R}^{n\times n}$, $C\in\mathbf{R}^{p\times n}$, $B\in\mathbf{R}^{n\times m}$, and $G\in\mathbf{R}^{n\times n}$ are known real parameter matrices, and $h>0$ is the time-delay constant. The initial condition function $\varphi(\theta)\in\mathbf{C}([-h,0],\mathbf{R}^n)$ is a given continuous vector valued function. The parameter uncertainties $\Delta A \in \mathbf{R}^{n\times n}$ and $\Delta A_1 \in \mathbf{R}^{n\times n}$ are assumed to be in the form of $[\Delta A \quad \Delta A_1] = DF(t)[E_0 \quad E_1]$, in which the matrices D, E_0, and E_1 are known real constant matrices with appropriate dimensions, and $F(t)$ is a real unknown matrix and satisfies $F^{\mathrm{T}}(t)F(t) \le I$.

The paper is intended to obtain tracking control for uncertain neutral system (1). The target signal of system (1) is assumed to be that $r(t) \in \mathbf{R}^p$, and it is a known function. The tracking error of system (1) is defined as

$$e(t) = y(t) - r(t). \tag{2}$$

To proceed further, some related assumptions are made as follows.

Assumption 1 The matrix G satisfies $G \ne 0$ and $\|\boldsymbol{G}\| < 1$.

Assumption 2 The target signal $r(t)$ is a piecewise continuously differentiable function. For its continuous points, we assumed that $r'(t)$, $r''(t)$, $\cdots$, $r^{(s-1)}(t)$ are all continuous, and $r^{(s)}(t) \equiv 0$.

Remark 1 The stability of $\dot{x}(t) - G\dot{x}(t-h)$ is guaranteed by Assumption 1 (see Duda. (2016)).

Remark 2 Assumption 2 makes that more types of signal can be contained, such as step signal. If $r(t)$ is differentiable with infinite order, it can be expanded into Taylor series with finite terms to satisfy Assumption 2 within allowable error ranges.

The following lemmas are needed to obtain the main results.

Lemma 1(Jensen's inequality, Wang. et al. (2016)) For any constant matrix $P \in \mathrm{R}^{n\times n}$, $P > 0$ and differentiable vector function $x(t)$, with appropriate dimensions, the inequality holds as follows

$$\left[\int_{t-h}^{t} x(s)\mathrm{d}s\right]^{\mathrm{T}} P\left[\int_{t-h}^{t} x(s)\mathrm{d}s\right] \le h\int_{t-h}^{t} x^{\mathrm{T}}(s)Px(s)\mathrm{d}s$$

Lemma 2(Schur Complementary, Liu. (2016)) Given constant symmetric matrices S_1, S_2, S_3 where $S_1 = S_1^{\mathrm{T}}$ and $S_2 = S_2^{\mathrm{T}} > 0$, then $S_1 + S_3^{\mathrm{T}} S_2^{-1} S_3 < 0$ if and only if

$$\begin{bmatrix} S_1 & S_3^{\mathrm{T}} \\ S_3 & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3^{\mathrm{T}} \\ S_3 & S_1 \end{bmatrix} < 0.$$

Lemma 3 3(Li. (2015)) For any matrices $Q = Q^{\mathrm{T}}$, H, E with appropriate dimensions, the inequality

$$Q + HF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}} < 0$$

holds for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$, if and only if there exists a scalar $\varepsilon > 0$, such that the following inequality holds.

$$Q + \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E < 0.$$

3. CONSTRUCTION OF AUGMENTED ERROR SYSTEM

Based on (1) and (2), the neutral dynamic equation of $e(t)$ is deduced as

$$\begin{aligned} \dot{e}(t) - \alpha\dot{e}(t-h) &= C\dot{x}(t) - \alpha C\dot{x}(t-h) \\ &\quad - \dot{r}(t) + \alpha\dot{r}(t-h), \end{aligned} \tag{3}$$

where $0 < \alpha < 1$. In order to combine $e(t)$ with $x(t)$, the state equation of system (1) is rewritten as

$$\begin{aligned} \ddot{x}(t) - G\ddot{x}(t-h) &= (A + \Delta A)\dot{x}(t) \\ &\quad + (A_1 + \Delta A_1)\dot{x}(t-h) + B\dot{u}(t). \end{aligned} \tag{4}$$

Let $z(t) = \left[\dot{x}^{\mathrm{T}}(t) \quad e^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. From (3) and (4), a new neutral system with uncertainties is constructed as

$$\begin{aligned} \dot{z}(t) - \bar{G}\dot{z}(t-h) &= (\bar{A} + \Delta\bar{A})z(t) \\ &\quad + (\bar{A}_1 + \Delta\bar{A}_1)z(t-h) + \bar{B}\dot{u}(t) \\ &\quad + N\dot{r}(t) - \alpha N\dot{r}(t-h), \end{aligned} \tag{5}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix},\ \Delta\bar{A} = \begin{bmatrix} \Delta A & 0 \\ 0 & 0 \end{bmatrix},\ \bar{A}_1 = \begin{bmatrix} A_1 & 0 \\ -\alpha C & 0 \end{bmatrix}$$

$$\Delta\bar{A}_1 = \begin{bmatrix} \Delta A_1 & 0 \\ 0 & 0 \end{bmatrix}, \bar{G} = \begin{bmatrix} G & 0 \\ 0 & \alpha I \end{bmatrix},\ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, N = \begin{bmatrix} 0 \\ -I \end{bmatrix}.$$

Remark 3 System (5) is called the augmented error system of (1). Compared with ordinary systems, the error signal is taken as a component of the state vector $z(t)$, and the target signal is contained in the new system.

In order to transfer system (5) into an ordinary neutral system in form, the target signal and its derivatives are composed as follows.

Let $R(t) = \begin{bmatrix} r(t) \\ \dot{r}(t) \\ \vdots \\ r^{(s-1)}(t) \end{bmatrix}$. Based on Assumption 2, the dynamical equation of $R(t)$ is obtained as follows.

$$\dot{R}(t) - \alpha\dot{R}(t-h) = ER(t) - \alpha ER(t-h) \tag{6}$$

where $E = \begin{bmatrix} O & I_p & O & \cdots & O \\ O & O & I_p & \cdots & O \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O & O & O & \cdots & I_p \\ O & O & O & \cdots & O \end{bmatrix}$. Then the following equation is established.

$$\dot{X}(t)-\tilde{G}\dot{X}(t-h)=(\tilde{A}+\Delta\tilde{A})X(t) \\ +(\tilde{A}_1+\Delta\tilde{A}_1)X(t-h)+\tilde{B}\dot{u}(t), \tag{7}$$

where

$$X(t)=\begin{bmatrix} z(t) \\ R(t)\end{bmatrix}, \tilde{G}=\begin{bmatrix}\bar{G} & 0 \\ 0 & \alpha I\end{bmatrix}, \tilde{A}=\begin{bmatrix}\bar{A} & \bar{N} \\ 0 & E\end{bmatrix}, \tilde{B}=\begin{bmatrix}\bar{B} \\ 0\end{bmatrix},$$

$$\tilde{A}_1=\begin{bmatrix}\bar{A}_1 & -\alpha\bar{N} \\ 0 & -\alpha E\end{bmatrix}, \Delta\tilde{A}=\begin{bmatrix}\Delta\bar{A} & 0 \\ 0 & 0\end{bmatrix}, \Delta\tilde{A}_1=\begin{bmatrix}\Delta\bar{A}_1 & 0 \\ 0 & 0\end{bmatrix}$$

$$\bar{N}=[0 \quad N \quad 0 \quad \cdots \quad 0].$$

We obtain the parameter uncertainties $\Delta\tilde{A}$ and $\Delta\tilde{A}_1$ satisfy

$$\begin{bmatrix}\Delta\tilde{A} & \Delta\tilde{A}_1\end{bmatrix}=\tilde{D}F(t)\begin{bmatrix}\tilde{E}_0 & \tilde{E}_1\end{bmatrix}$$

where

$$\tilde{D}=diag(D, O_{p\times p}, O_{sp\times sp}),$$

$$\tilde{E}_0=\begin{bmatrix} E_0 & O_{n\times(s+1)p} \\ O_{(s+1)p\times n} & O_{(s+1)p\times(s+1)p}\end{bmatrix},$$

$$\tilde{E}_1=\begin{bmatrix} E_1 & O_{n\times(s+1)p} \\ O_{(s+1)p\times n} & O_{(s+1)p\times(s+1)p}\end{bmatrix}.$$

The transformation from system (1) into (7) makes the main intend of this paper is to deduce $\dot{u}(t)$ of system (7). Then the input $u(t)$ of system (1) can be proposed. Thus the controller of system (1) contains integrators or integrations which may help the system to eliminate static error (see Katayama et al. (1987)).

4. MAIN RESULTS

4.1 The Asymptotic Stability of System (7)

In this section, we wish to design a state feedback control such that the closed-loop system of (7) is asymptotically stable. So the asymptotic stability of its following auto controlled system is needed to study.

$$\dot{X}(t)-\tilde{G}\dot{X}(t-h)=\hat{A}X(t)+\hat{A}_1X(t-h) \tag{8}$$

where $\hat{A}=\tilde{A}+\Delta\tilde{A}$, $\hat{A}_1=\tilde{A}_1+\Delta\tilde{A}_1$, $\tilde{A}$ and $\tilde{A}_1$ satisfy

$$\begin{bmatrix}\Delta\tilde{A} & \Delta\tilde{A}_1\end{bmatrix}=\tilde{D}F(t)\begin{bmatrix}\tilde{E}_0 & \tilde{E}_1\end{bmatrix}, F^{\mathrm{T}}(t)F(t)\le I.$$

A delay-dependent criterion for stabilization of system (8) is firstly presented as the following theorem.

Theorem 1 System (8) is asymptotically stable, if there exist $Y_i>0$ $(i=1,2,3,4)$, and a scalar $\varepsilon>0$, such that the following LMI is feasible.

$$\begin{bmatrix}\Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & \Gamma_{22} & \Gamma_{23} \\ * & * & \Gamma_{33}\end{bmatrix}<0, \tag{9}$$

where

$$\Gamma_{11}=\begin{bmatrix}\tilde{A}Y_1+Y_1\tilde{A}^{\mathrm{T}} & \tilde{A}_1Y & \tilde{G}_1Y \\ * & -Y_2 & 0 \\ * & * & -Y_3\end{bmatrix}, \Gamma_{13}=\begin{bmatrix}Y_1 & \tilde{D} & Y_1\tilde{E}_0^{\mathrm{T}} \\ 0 & 0 & Y_2\tilde{E}_1^{\mathrm{T}} \\ 0 & 0 & 0\end{bmatrix},$$

$$\Gamma_{12}=\begin{bmatrix}0 & Y_1\tilde{A}^{\mathrm{T}} & Y_1\tilde{A}^{\mathrm{T}} \\ 0 & Y_2\tilde{A}_1^{\mathrm{T}} & Y_2\tilde{A}_1^{\mathrm{T}} \\ 0 & Y_3\tilde{G}^{\mathrm{T}} & Y_3\tilde{G}^{\mathrm{T}}\end{bmatrix}, \Gamma_{22}=diag(-Y_4,-\frac{Y_3}{h^2},-Y_4),$$

$$\Gamma_{23}=\begin{bmatrix}0 & 0 & 0 \\ 0 & \tilde{D} & 0 \\ 0 & \tilde{D} & \varepsilon\tilde{E}_0^{\mathrm{T}}\end{bmatrix}, \quad \Gamma_{33}=diag(-Y_2,-I,-I).$$

Proof Based on $0<\alpha<1$ and Assumption 1, it is not hard to obtain that $\|\bar{\boldsymbol{G}}\|<1$. Thus $\|\tilde{\boldsymbol{G}}\|<1$. So $X(t)-\tilde{G}X(t-h)$ is stable. For positive definite matrices $P_i>0\,(i=1,2,3,4)$ with appropriate dimensions, we define the Lyapunov-Krasovskii function as

$$V=V_1+V_2+V_3+V_4,$$

where

$$V_1=X^{\mathrm{T}}(t)P_1X(t),\ V_2=\int_{t-h}^{t}X^{\mathrm{T}}(\theta)P_2X(\theta)\mathrm{d}\theta,$$

$$V_3=\int_{t-h}^{t}\dot{X}^{\mathrm{T}}(\theta)P_3\dot{X}(\theta)\mathrm{d}\theta,\ V_4=h\int_{-h}^{0}\int_{t+\theta}^{t}\dot{X}^{\mathrm{T}}(s)P_4\dot{X}(s)\mathrm{d}s\mathrm{d}\theta.$$

It is apparently that $V>0$. The time derivatives of $V_i(i=1,2,3,4)$ along the trajectories of (8) respectively satisfy

$$\begin{aligned}\dot{V}_1&=2X^{\mathrm{T}}(t)P_1\dot{X}(t)\\ &=2X^{\mathrm{T}}(t)P_1\left[\hat{A}X(t)+\hat{A}_1X(t-h)+\tilde{G}\dot{X}(t-h)\right]\\ &=X^{\mathrm{T}}(t)(P_1\hat{A}+\hat{A}^{\mathrm{T}}P_1)X(t)+2X^{\mathrm{T}}(t)P_1\hat{A}_1X(t-h)\\ &\quad+2X^{\mathrm{T}}(t)P_1\tilde{G}\dot{X}(t-h),\end{aligned}$$

$$\dot{V}_2=X(t)^{\mathrm{T}}P_2X(t)-X(t\text{-}h)^{\mathrm{T}}P_2X(t\text{-}h),$$

$$\dot{V}_3=\dot{X}^{\mathrm{T}}(t)P_3\dot{X}(t)-\dot{X}^{\mathrm{T}}(t\text{-}h)P_3\dot{X}(t\text{-}h),$$

$$\dot{V}_4=h^2\dot{X}^{\mathrm{T}}(t)P_4\dot{X}(t)-h\int_{t-h}^{t}\dot{X}^{\mathrm{T}}(\theta)P_4\dot{X}(\theta)\mathrm{d}\theta.$$

Here, by utilizing Lemma 1, we obtain

$$-\int_{t-h}^{t}\dot{X}^{\mathrm{T}}(\theta)P_4\dot{X}(\theta)\mathrm{d}\theta\le-\frac{1}{h}\left[\int_{t-h}^{t}\dot{X}(\theta)\mathrm{d}\theta\right]^{\mathrm{T}}P_4\left[\int_{t-h}^{t}\dot{X}(\theta)\mathrm{d}\theta\right].$$

So

$$\dot{V}_4\le h^2\dot{X}^{\mathrm{T}}(t)P_4\dot{X}(t)-\left[\int_{t-h}^{t}\dot{X}(\theta)\mathrm{d}\theta\right]^{\mathrm{T}}P_4\left[\int_{t-h}^{t}\dot{X}(\theta)\mathrm{d}\theta\right].$$

Then, the derivative of V is given by

$$\begin{aligned}\dot{V} &= \dot{V}_1+\dot{V}_2+\dot{V}_3+\dot{V}_4\\ &\le X^{\mathrm T}(t)(P_1\hat{A}+\hat{A}^{\mathrm T}P_1+P_2)X(t)+2X^{\mathrm T}(t)P_1\tilde{G}\dot{X}(t-h)\\ &\quad+2X^{\mathrm T}(t)P_1\hat{A}_1X(t-h)-X(t\text{-}h)^{\mathrm T}P_2X(t\text{-}h)\\ &\quad-\dot{X}^{\mathrm T}(t\text{-}h)P_3\dot{X}(t\text{-}h)+\dot{X}^{\mathrm T}(t)(P_3+h^2P_4)\dot{X}(t)\\ &\quad-\left[\int_{t-h}^{t}\dot{X}(\theta)\mathrm{d}\theta\right]^{\mathrm T}P_4\left[\int_{t-h}^{t}\dot{X}(\theta)\mathrm{d}\theta\right]\end{aligned}$$

Substituting $\dot{X}(t)=\hat{A}X(t)+\hat{A}_1X(t-h)+\tilde{G}\dot{X}(t-h)$ into $\dot{V}$ obtains

$$\dot{V}\le\zeta^{\mathrm T}(t)\Omega\zeta(t),$$

where

$$\zeta^{\mathrm T}(t)=\begin{bmatrix}X^{\mathrm T}(t) & X^{\mathrm T}(t-h) & \dot{X}^{\mathrm T}(t-h) & \int_{t-h}^{t}\dot{X}^{\mathrm T}(\theta)\mathrm{d}\theta\end{bmatrix},$$

$$\Omega=\begin{bmatrix}P_1\hat{A}+\hat{A}^{\mathrm T}P_1+P_2 & P_1\hat{A}_1 & P_1\tilde{G}_1 & 0\\ * & -P_2 & 0 & 0\\ * & * & -P_3 & 0\\ * & * & * & -P_4\end{bmatrix}+\delta\Pi\delta^{\mathrm T},$$

$$\Pi=P_3+h^2P,\ \delta^{\mathrm T}=\begin{bmatrix}\hat{A} & \hat{A}_1 & \tilde{G} & 0\end{bmatrix}.$$

If $\Omega<0$, we can obtain $\dot{V}<0$. Therefore, system (8) is asymptotically stable.

Pre- and post-multiplying $\Omega<0$ by $diag(P_1^{-1},P_2^{-1},P_3^{-1},P_4^{-1})$. And let $X_i=P_i^{-1}\ (i=1,2,3,4)$. We obtain the following result.

$$\begin{bmatrix}\Upsilon & \hat{A}_1X_2 & \tilde{G}_1X_3 & 0\\ * & -X_2 & 0 & 0\\ * & * & -X_3 & 0\\ * & * & * & -X_4\end{bmatrix}+\upsilon\Pi\upsilon^{\mathrm T}<0,$$

where $\Upsilon=\hat{A}X_1+X_1\hat{A}^{\mathrm T}+X_1P_2X_1$,

$$\upsilon^{\mathrm T}=\begin{bmatrix}\hat{A}X_1 & \hat{A}_1X_2 & \tilde{G}X_3 & 0\end{bmatrix}.$$

Then, on basis of Lemma 2, we get

$$\begin{bmatrix}\Theta & \hat{A}_1X_2 & \tilde{G}_1X_3 & 0 & X_1\hat{A}^{\mathrm T} & X_1\hat{A}^{\mathrm T} & X_1\\ * & -X_2 & 0 & 0 & X_2\hat{A}_1^{\mathrm T} & X_2\hat{A}_1^{\mathrm T} & 0\\ * & * & -X_3 & 0 & X_3\tilde{G}^{\mathrm T} & X_3\tilde{G}^{\mathrm T} & 0\\ * & * & * & -X_4 & 0 & 0 & 0\\ * & * & * & * & -\frac{1}{h^2}X_3 & 0 & 0\\ * & * & * & * & * & -X_4 & 0\\ * & * & * & * & * & * & -X_2\end{bmatrix}<0. \tag{10}$$

where $\Theta=\hat{A}X_1+X_1\hat{A}^{\mathrm T}$.

Substitute $\hat{A}=\tilde{A}+\Delta\tilde{A}$, $\hat{A}_1=\tilde{A}_1+\Delta\tilde{A}_1$ and

$$\begin{bmatrix}\Delta\tilde{A} & \Delta\tilde{A}_1\end{bmatrix}=\tilde{D}F(t)\begin{bmatrix}\tilde{E}_0 & \tilde{E}_1\end{bmatrix}$$

into (11), we get that

$$\begin{bmatrix}\Theta & \tilde{A}_1X_2 & \tilde{G}_1X_3 & 0 & X_1\tilde{A}^{\mathrm T} & X_1\tilde{A}^{\mathrm T} & X_1\\ * & -X_2 & 0 & 0 & X_2\tilde{A}_1^{\mathrm T} & X_2\tilde{A}_1^{\mathrm T} & 0\\ * & * & -X_3 & 0 & X_3\tilde{G}^{\mathrm T} & X_3\tilde{G}^{\mathrm T} & 0\\ * & * & * & -X_4 & 0 & 0 & 0\\ * & * & * & * & -\frac{1}{h^2}X_3 & 0 & 0\\ * & * & * & * & * & -X_4 & 0\\ * & * & * & * & * & * & -X_2\end{bmatrix} \tag{11}$$

$$+\xi\tilde{F}(t)\eta+\eta^{\mathrm T}\tilde{F}^{\mathrm T}(t)\xi^{\mathrm T}<0,$$

where $\xi^{\mathrm T}=\begin{bmatrix}\tilde{D}^{\mathrm T} & 0 & 0 & 0 & \tilde{D}^{\mathrm T} & \tilde{D}^{\mathrm T} & 0\end{bmatrix}$,

$$\eta=\begin{bmatrix}\tilde{E}_0X_1 & \tilde{E}_1X_2 & 0 & 0 & 0 & \tilde{E}_1 & 0\end{bmatrix}.$$

Let Q represents the left side of (11).Based on Lemma 3, (11) holds if and only if there exists a scalar $\varepsilon>0$, such that the following inequality holds.

$$Q+\varepsilon^{-1}\xi\xi^{\mathrm T}+\varepsilon\eta\eta^{\mathrm T}<0.$$

i.e. $\qquad \varepsilon Q+\xi\xi^{\mathrm T}+(\varepsilon\eta)(\varepsilon\eta)^{\mathrm T}<0.$

Using Schur complmentary, we have

$$\begin{bmatrix}\varepsilon\tilde{A}X_1+\varepsilon X_1\tilde{A}^{\mathrm T} & \varepsilon\tilde{A}_1X_2 & \varepsilon\tilde{G}_1X_3 & 0 & \varepsilon X_1\tilde{A}^{\mathrm T} & \varepsilon X_1\tilde{A}^{\mathrm T} & \varepsilon X_1 & \tilde{D} & \varepsilon X_1\tilde{E}_0^{\mathrm T}\\ * & -\varepsilon X_2 & 0 & 0 & \varepsilon X_2\tilde{A}_1^{\mathrm T} & \varepsilon X_2\tilde{A}_1^{\mathrm T} & 0 & 0 & \varepsilon X_2\tilde{E}_1^{\mathrm T}\\ * & * & -\varepsilon X_3 & 0 & \varepsilon X_3\tilde{G}^{\mathrm T} & \varepsilon X_3\tilde{G}^{\mathrm T} & 0 & 0 & 0\\ * & * & * & -\varepsilon X_4 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\frac{\varepsilon}{h^2}X_3 & 0 & 0 & \tilde{D} & 0\\ * & * & * & * & * & -\varepsilon X_4 & 0 & \tilde{D} & \varepsilon\tilde{E}_0^{\mathrm T}\\ * & * & * & * & * & * & -\varepsilon X_2 & 0 & 0\\ * & * & * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & * & * & -I\end{bmatrix}<0.$$

Let $Y_i=\varepsilon X_i\,(i=1,2,3,4)$, inequality (9) is established.

4.2 Controller Design

In 4.1, the asymptotical stability condition for system (8) is obtained. Thus, the feedback controller of (7) can be derived.

As (7) is the augmented error system of (1), the control designed for (1) can also be obtained.

Suppose $\dot{u}(t) = KX(t)$ is the controller of system (7), where K is the feed-back matrix with appropriate dimensions. The closed-loop system of (7) is as follows.

$$\dot{X}(t) - \tilde{G}\dot{X}(t-h) = (\tilde{A} + \tilde{B}K + \Delta\tilde{A})X(t) + (\tilde{A}_1 + \Delta\tilde{A}_1)X(t-h)$$

Substitute $\tilde{A}$ of (9) with $\tilde{A} + \tilde{B}K$, and let $W = KY_1$. The matrix block Γ_{11} and Γ_{12} of (9) change while others keep invariant. The feedback controller of augmented error system (7) is shown in the following theorem.

Theorem 2 The closed-loop system of (7) is asymptotically stable by $\dot{u}(t) = KX(t)$, if there exist a scalar $\varepsilon > 0$, some matrices $Y_i > 0 (i = 1,2,3,4)$, and a matrix W, such that the following LMI is feasible:

$$\begin{bmatrix} \Gamma'_{11} & \Gamma'_{12} & \Gamma_{13} \\ * & \Gamma_{22} & \Gamma_{23} \\ * & * & \Gamma_{33} \end{bmatrix} < 0, \tag{12}$$

where

$$\Gamma'_{11} = \begin{bmatrix} \tilde{A}Y_1 + Y_1\tilde{A}^{\mathrm{T}} + \tilde{B}W + W^{\mathrm{T}}\tilde{B}^{\mathrm{T}} + Y_1\tilde{A}^{\mathrm{T}} & \tilde{A}_1 Y & \tilde{G}_1 Y \\ * & -Y_2 & 0 \\ * & * & -Y_3 \end{bmatrix},$$

$$\Gamma'_{12} = \begin{bmatrix} 0 & Y_1\tilde{A}^{\mathrm{T}} + W^{\mathrm{T}}\tilde{B}^{\mathrm{T}} & Y_1\tilde{A}^{\mathrm{T}} + W^{\mathrm{T}}\tilde{B}^{\mathrm{T}} \\ 0 & Y_2\tilde{A}_1^{\mathrm{T}} & Y_2\tilde{A}_1^{\mathrm{T}} \\ 0 & Y_3\tilde{G}^{\mathrm{T}} & Y_3\tilde{G}^{\mathrm{T}} \end{bmatrix},$$

other matrix blocks $\Gamma_{13}, \Gamma_{22}, \Gamma_{23}, \Gamma_{33}$ are the same as that of (9). Moreover, the stabilizing feedback control gain is obtained by $K = WY_1^{-1}$.

If there exists K to satisfy (12), we discompose K as

$$K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix},$$

where $k_3 = \begin{bmatrix} k_{3,0} & k_{3,1} & \cdots & k_{3,s-1} \end{bmatrix}$. Then

$$\dot{u}(t) = k_1\dot{x}(t) + k_2 e(t) + \sum_{j=0}^{s-1} k_{3,j} r^{(j)}(t).$$

We get that

$$u(t) - u(-h) = k_1\int_{-h}^{t}\dot{x}(v)\mathrm{d}v + k_2\int_{-h}^{t} e(v)\mathrm{d}v + k_{3,0}\int_{-h}^{t} r(v)\mathrm{d}v + \sum_{j=1}^{s-1} k_{3,j} r^{(j-1)}(t).$$

Thus,

$$\begin{aligned} u(t) &= k_1 x(t) - k_1\varphi(-h) + k_2 C\int_{-h}^{0}\varphi(\theta)\mathrm{d}\theta \\ &\quad + k_2\int_0^t e(\theta)\mathrm{d}\theta + k_{3,0}\int_0^t r(\theta)\mathrm{d}\theta + \sum_{j=1}^{s-1} k_{3,j} r^{(j-1)}(t) \end{aligned} \tag{13}$$

From (13), we find that not only the state feedback, but also the integration of the desired output are contained in the control design for neutral system (1). Furthermore, the error integrator is also considered as we deduce the input of (1) from the augmented error system. Actually, the error integrator can help reduce the static error.

5. ILLUSTRATIVE EXAMPLE

Consider the following neutral-type system with parameter uncertainties given by

$$\begin{cases} \dot{x}(t) - G\dot{x}(t-h) = (A + DF(t)E_0)x(t) \\ \qquad\qquad + (A_1 + DF(t)E_1)x(t-h) + Bu(t), \\ y(t) = Cx(t), \\ x(\theta) = \varphi(\theta), \quad \varphi(\theta) \in \mathbf{C}([-h,0], \mathbf{R}^n), \end{cases} \tag{14}$$

where

$$G = \begin{bmatrix} -0.05 & -0.03 \\ 0.2 & -0.1 \end{bmatrix}, A = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & -0.2 \end{bmatrix}, B = \begin{bmatrix} 0.03 \\ -0.04 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -0.02 & -0.05 \\ -0.01 & -0.3 \end{bmatrix}, C = \begin{bmatrix} -0.11 & 0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.04 \end{bmatrix}, F(t) = \sin t, \ E_0 = \begin{bmatrix} -0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.02 \end{bmatrix}, \ h = 0.1, \ \varphi(\theta) = \begin{bmatrix} \sin\theta/\theta \\ 1/\theta^2 \end{bmatrix}.$$

Our aim here is solving the gain matrix K in Theorem 2 such that the closed-loop system is asymptotically stable and the control law is obtained.

The target signal is chosen as the following stair step signal:

$$r(t) = \begin{cases} 0, & t < 10 \\ 1, & t \geq 10 \end{cases}.$$

Based on Theorem 2 and through Matlab, the gain matrix K is obtained as follows

$$K = \begin{bmatrix} -2.9325 & 2.2007 & 0.9733 & 0.4132 & -0.2931 & 0.2878 \end{bmatrix}.$$

And the output response of system (13) is shown in Fig.1. When the compensation of the desired output is deleted from Theorem 2, the output response of system (13) is shown as Fig.2.

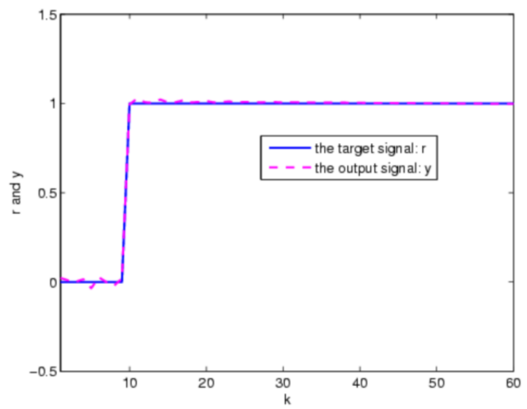

Fig. 1. Response of (13) with the target compensation

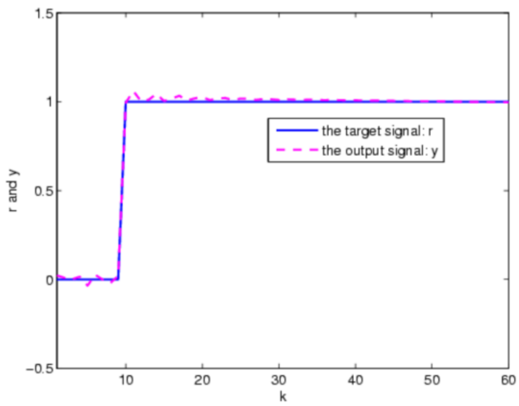

Fig. 2. Response of (13) without the target compensation

From the above two figures, we can find that the system output track the desired output better when there is target signal compensation. Under the usual control law without the target signal compensation, the time that system (13) becomes stable is longer.

6. CONCLUSION

In the paper, based on a class of uncertain neutral systems, we construct an augmented error system which is combined with the target signal and the error signal. By applying some inequalities and LMIs, the asymptotical stability of the auto-controlled system is studied in term of a Lyapunov-Krasovskii function. Then a feedback controller is given for the augmented error system. We analyse the construction of the control design for the uncertain neutral system. Not only the state feedback, but also the target signal is contained in the controller. Especially, the error integrator is also considered in the control design, which can help to reduce the static error and track the target signal better

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