Preprints, 10th IFAC International Symposium on
Advanced Control of Chemical Processes
Shenyang, Liaoning, China, July 25-27, 2018

A tube feedback iterative learning control for batch processes

Jingyi Lu [*] **Zhixing Cao** [**] **Ridong Zhang** [***] **Cuimei Bo** [****]
Furong Gao [***]

[*] *Fok Ying Tung Graduate School, Hong Kong University of Science and Technology, China*
[**] *School of Biological Sciences, University of Edingurgh, UK*
[***] *Department of Chemical and Biological Engineering, Hong Kong University of Science and Technology, China, (e-mail: kefgao@ust.hk)*
[****] *College of Automation and Electrical Engineering, Nanjing Tech University, Nanjing 211816,China*

Abstract: Optimization-based iterative learning control (OILC) has been widely applied to batch processes due to its fast convergence, good control performance and ability to handle constraints. However, how to guarantee constraint satisfaction and convergence of tracking error in the presence of unknown system nonlinearity remains open in the framework of OILC. It is important to address this issue since unknown nonlinearity is common in practice and detrimental to good control performance. In this paper, we propose a tube feedback OILC to investigate the applicability of linear-model based control strategy on batch processes with an unknown nonlinear term. First, a state feedback control law is designed to stabilize the system. The stabilized system is then decomposed into two subsystems: repeatable and unrepeatable subsystems; Second, an invariant set of states corresponding to the unrepeatable subsystem is computed, based on which an OILC is further developed for the repeatable subsystem to improve the control performance. Meanwhile, the feedback controller steers the states within a tube around the trajectory of OILC. In this way, convergence and constraint satisfaction are ensured simultaneously. Compared with the currently existing methods, the proposed method has the following advantages: (1) generality covering both stable and unstable systems; (2) low computation complexity; and (3) rigorous stability. The simulation results on injection molding velocity control demonstrate that the proposed method has superior performance.

Keywords: batch process, nonlinear, iterative learning control, invariant set, convergence.

1. INTRODUCTION

Batch process plays an important role in manufacture and chemical industry with high-value added due to its versatility. Precise control of key variables in the process has a significant impact on product quality. A growing attention is devoted to batch process control in recent years (Cao et al. (2014),Cao et al. (2016b),Wang et al. (2017),Wang et al. (2018)). However, the inherent nonlinearity in batch processes introduces difficulty to control strategy design: controllers designed based on nonlinear model, such as nonlinear model predictive control, have intensive on-line computation (Nagy and Braatz (2003),Jia et al. (2013)); contrarily, controllers designed based on a linear model can efficiently lower computation complexity, but the performance is not good enough due to the existent of significant model mismatch (Yang and Gao (2000)). To reduce the model mismatch in the linear model-based design, optimization-based iterative learning control is preferred, in the framework of which model mismatch is compensated by prediction errors of previous cycles and control inputs are derived by optimizing a certain performance criteria. This idea has been widely adopted (Lee and Lee (2003),Shi et al. (2006),Lu et al. (2015b),Lu et al. (2018)).

Model mismatch in OILC design is often assumed to be cycle-wise invariant and treated as repetitive disturbances, as shown in Foss et al. (1995). However, this assumption does not hold if the model mismatch is induced by linearization. It is well known that model mismatch induced by linearization depends on system states and in general not cycle-wise invariant. Researchers later developed algorithms which are robust against bounded non-repetitive disturbances, such as Liu and Wang (2012). These methods can be extended to handle unknown nonlinearity by treating the model mismatch as a combination of repetitive and non-repetitive disturbances and assuming the non-repetitive part has a fixed upper bound. To guarantee the existence of the upper bound, cycle-wise differences on inputs and states have to be regulated into a certain region by hard constraints. This introduces a new question: if the optimization problem consistently feasible after the incorporation of those hard constraints? If not, control performance may degrade dramatically when infeasibility occurs. The key issue considered in this paper is how to design an OILC which is able to guarantee cycle-wise convergence as well as consistent feasibility despite of unknown nonlinearity. It is noted that simultaneous guarantee of convergence and constraint satisfaction was studied in Lu et al. (2015a) and Lu et al. (2016). However, both of the works are for linear systems.

In this paper, a tube feedback OILC scheme is proposed for reference tracking. The design borrows the idea of tube model predictive control, which is a well-known approach to handle bounded disturbances in MPC design (Langson et al. (2004)). First, model mismatch induced by linearization is decomposed into repeatable and unrepeatable parts, following which, the original system is decomposed into repeatable and unrepeatable subsystem accordingly. Similar to the tube MPC, a state feedback controller is then designed such that the states corresponding to the unrepeatable subsystem are regulated into an invariant tube. Second, an OILC which actually determines the center of the tube is further designed to reject the repeatable mismatch and enhance tracking performance from cycle to cycle. In this way, cycle-wise convergence and constraint satisfaction are ensured simultaneously.

Albeit the advantage on explicit guarantee of convergence and feasibility, the proposed method has less restrictions than other existing OILC algorithms. The only requisite is that the system should be "stabilizable', while many existing OILC algorithms have strict requirements of the controlled system. For example, a common type of OILC is designed based on an minimization of the predicted tracking error over the entire cycle, as shown in Lee et al. (2000) and Chin et al. (2004). Such an optimization is based on a lifted system model, which is only available when the system is stable. If the system subjects to unknown disturbances and constraints, this type of method can not be directly extended to unstable systems by stabilizing the system first. There are also methods (Liu and Wang (2012)) which are devised by solving linear matrix inequalities (LMIs) induced by a 2D Lyapunov function, which are limited by the feasibility of the LMIs.

The rest of the paper is arranged as follows: Section 2 gives the formulation of the problem; Section 3 details the controller design; Section 4 provides stability analysis; Section 5 presents simulation results, and Section 6 draws the conclusions.

2. PROBLEM FORMULATION

A nonlinear batch process can be represented by a linear model plus a nonlinear term (Gao et al. (2014)) as

$$\begin{aligned} x(t+1,k) =& Ax(t,k) + Bu(t,k) + g(x(t,k),u(t,k)),\\ y(t,k) =& Cx(t,k). \end{aligned}$$

Here $t \in [0, t_n - 1]$ is the time index. t_n is the length of a cycle. $k \in [1, \infty)$ is the cycle index. $x \in \mathbb{R}^{n_s}$ is the system state. $u \in \mathbb{R}^{n_i}$ is the input. $y \in \mathbb{R}^{n_o}$ is the output. States and outputs are assume to be measurable or observable (Cao et al. (2016a), Cao et al. (2016c)). System dynamic matrices A, B and C are arranged in proper dimensions. The pair (A, B) is stabilizable. In addition, it is assumed that the initial states in each cycle are the same, namely $x(0, k) = x(0, k-1) = \cdots = x(0, 1) = x_0$. The nonlinear term g satisfies Lipschiz condition as

$$\|g(u_1) - g(u_2)\|_\infty \le L\|u_1 - u_2\|_\infty, \tag{1}$$

with L a global Lipschiz constant. For simplicity of notations, it is assumed that g is solely a function of input u. Then, the system model is simplified as

$$\begin{aligned} x(t+1,k) =& Ax(t,k) + Bu(t,k) + g(u(t,k)),\\ y(t,k) =& Cx(t,k). \end{aligned} \tag{2}$$

For more general cases, the computation will become more complex but the underlying idea remains the same.

Define a polytopic set $\mathbb{U}$ in cycle k as

$$\mathbb{U}(k) = \{u : \|u(t,k) - u(t,k-1)\|_\infty \le \delta(k)\}. \tag{3}$$

An input constraint is posed as

$$u(t,k) \in \mathbb{U}(k), \tag{4}$$

such that the cycle-wise variation of the nonlinear term g is bounded as

$$\|g(u(t,k)) - g(u(t,k-1))\|_\infty \le L\delta(k) \tag{5}$$

according to (1). Define

$$\begin{aligned} d_e(t,k) = g(u(t,k-1)) = x(t+1,k-1)\\ - Ax(t,k-1) - Bu(t,k-1). \end{aligned} \tag{6}$$

and $d_\delta(t,k) = g(u(t,k)) - g(u(t,k-1))$. It is noted that d_e and d_δ split the nonlinear term into cycle-wise repeatable and unrepeatable parts. Moreover, in view of (4) and (5), d_δ is bounded in the set $\mathbb{F}(k)$ as

$$d_\delta(t,k) \in \mathbb{F}(k) \tag{7}$$

with $\mathbb{F}(k)$ defined as

$$\mathbb{F}(k) = \{f \in \mathbb{R}^{n_s} : -L\delta(k) \le f_i \le L\delta(k)\}. \tag{8}$$

Here f_i is the i-th coordinate of the vector f.

Denote the tracking reference as $y_r(t)$ with $t \in \mathbb{I}_{[0,t_n-1]}$. The control objective is to steer the output $y(t,k)$ to a given reference $y_r(t)$ despite of the unknown terms $d_e(t,k)$ and $d_\delta(t,k)$, meanwhile, maintain the input confined to $\mathbb{U}(k)$.

3. CONTROLLER DESIGN

In the section, details of the controller design are given.

3.1 System decomposition

To guarantee time-wise stability, a state feedback gain K is selected to stabilize the system, namely keep the spectral radius $\rho(A + BK) < 1$, following which the structure of the controller is fixed as

$$u(t,k) = Kx(t,k) + u_0(t,k). \tag{9}$$

The second term $u_0(t,k)$ is to be induced by ILC for reference tracking and repeatable disturbance rejection.

Define $A_k = A + BK$. The repeatable subsystem is constructed with respect to $d_e(t,k)$ as

$$\bar{x}(t+1,k) = A_k\bar{x}(t,k) + Bu_0(t,k) + d_e(t,k). \tag{10}$$

By taking difference between (2) and (10), the unrepeatable subsystem is constructed as

$$\hat{x}(t+1,k) = A_k\hat{x}(t,k) + d_\delta(t,k), \tag{11}$$

with $\hat{x}(t,k) = x(t,k) - \bar{x}(t,k)$. Set $\bar{x}(0,k) = x(0,k) = x_0$, $\hat{x}(0,k) = 0$.

Accordingly, the inputs can also be decomposed into two parts. Define $\bar{u}(t,k) = K\bar{x}(t,k) + u_0(t,k)$ as the input corresponding to the repeatable subsystem and

$$\hat{u}(t,k) = K\hat{x}(t,k) \tag{12}$$

as the input corresponding to the non-repeatable subsystem. Furthermore, the tracking error satisfies

$$e(t,k) = y_r(t) - y(t,k) = y_r(t) - C\bar{x}(t,k) - C\hat{x}(t,k). \tag{13}$$

Thus, define

$$\text{Repeatable:} \quad \bar{e}(t,k) = y_r(t) - C\bar{x}(t,k) \tag{14}$$

and

$$\text{Unrepeatable:} \quad \hat{e}(t,k) = -C\hat{x}(t,k). \tag{15}$$

To this end, the original system has been decomposed into two subsystems.

3.2 Preliminaries of invariant sets

Note from (7) that $d_\delta(t,k) \in \mathbb{F}(k)$ for any t and k. Based on (11), according to Theorem 4.1 in Kolmanovsky and Gilbert (1998), it can be concluded that

Lemma 1. Given that the spectral radius of $A + BK$ less than 1, $d_\delta(t,k)$ contained in the polyhedral set $\mathbb{F}(k)$ and $\hat{x}(0,k)$ stays at the origin, there exists a polyhedral robust positively invariant set $\Omega_x(k)$ on $\hat{x}(t,k)$ in cycle k such that
(i) for any $\hat{x}(t,k) \in \Omega_x(k)$, $\hat{x}(t+1,k)$ satisfying (11) stays in $\Omega_x(k)$;
(ii) the set $\Omega_x(k)$ contains the origin $\mathbf{0}$.

In view of the item (ii) in Lemma 1, we have $\hat{x}(0,k) \in \Omega_x(k)$. Moreover, by recursively applying (i), it can be proved that $\hat{x}(t,k) \in \Omega_x(k)$ for any t.

Remark 1. Generally, the invariant sets in Lemma 1 are not unique. In order to reduce conservativeness, the minimal robust positively invariant (mRPI) set is adopted. Methods on computing mRPIs are provided by Kolmanovsky and Gilbert (1998). Briefly speaking, a mRPI is computed based on sequential Minkowski summation:

$$\Omega_x(k) = \sum_{i=0}^{\infty} A_k^i B\mathbb{F}(k). \tag{16}$$

Toolboxes shown in Herceg et al. (2013) can be used to compute the set.

According to (12), the set of $\hat{u}(t,k)$ can be characterized by a linear mapping as

$$\hat{\mathbb{U}}(k) = \{\hat{u} = K\hat{x} : \hat{x} \in \Omega_x(k)\}. \tag{17}$$

For any t and k it can be easily proved that $\hat{u}(t,k)$ is bounded in the set $\hat{\mathbb{U}}(k)$ since $\Omega_x(k)$ is a compact set and the mapping in (12) is continuous. Similarly, according to (15), $\hat{e}(t,k)$ stays in $\hat{\mathbb{E}}(k)$ which is a polyhedral set defined as

$$\hat{\mathbb{E}}(k) = \{\hat{e} = C\hat{x} : \hat{x} \in \Omega_x(k)\}. \tag{18}$$

Since $\bar{u}(t,k)$ and $\hat{u}(t,k)$ are governed by $u(t,k) = \bar{u}(t,k) + \hat{u}(t,k)$, to guarantee input constraint fulfillment, $\bar{u}(t,k)$ should be steered to a set $\bar{\mathbb{U}}(k)$ according to Lemma 2.

Lemma 2. Given a $\hat{u}(t,k) \in \hat{\mathbb{U}}(k)$, if $\bar{u}(t,k)$ satisfies $\bar{u}(t,k) \in \bar{\mathbb{U}}(k)$ with $\bar{\mathbb{U}}(k) = \mathbb{U}(k) \sim \hat{\mathbb{U}}(k)$, then it can be guaranteed that

$$u(t,k) = \hat{u}(t,k) + \bar{u}(t,k) \in \mathbb{U}(k).$$

Here '$\sim$' denotes Pontryagin difference. This lemma can be directly proved by Theorem 2.1 in Kolmanovsky and Gilbert (1998). In addition, as stated in Section 3.1.2 in Blanchini and Miani (2007), C-set and 0-symmetric are defined as:

Definition 1. (C-set): A C-set is a convex and compact subset of $\mathbb{R}^n$ including the origin as an interior point.

Definition 2. (0-symmetric): A C-set $\mathcal{S}$ is 0-symmetric if $x \in \mathcal{S} \Rightarrow -x \in \mathcal{S}$.

According to Definition 1 and 2, it can be checked that the set $\mathbb{F}(k)$ is a 0-symmetric C-set. Moreover, it can also be proved that

Proposition 1. The set $\Omega_x(k)$, $\hat{\mathbb{U}}(k)$ and $\hat{\mathbb{E}}(k)$ are all 0-symmetric C-sets.

3.3 Design of OILC

Section 3.2 tells that the states, inputs and tracking error corresponding to the unrepeatable subsystem are bounded in the set Ω_x, $\hat{\mathbb{U}}$ and $\hat{\mathbb{E}}$ respectively, based on which an OILC can be applied to the repeatable subsystem and devised by a quadratic optimization.

Problem 1.

$$\min_{\delta(k),\Delta_k \mathbf{u}_0(t,k),\bar{\mathbf{e}}(k)} \|\bar{\mathbf{e}}(k)\|_2^2 + \|\sqrt{R_w}\Delta_k \mathbf{u}_0(k)\|_2^2$$

$$\begin{aligned}
\text{s.t.} \quad & \bar{x}(t+1,k) = A_k\bar{x}(t,k) + Bu_0(t,k) \\
& + d_e(t,k),\ t = 0,1,\ldots,t_n, && (19)\\
& \bar{e}(t,k) = y_r(t) - C\bar{x}(t,k) && (20)\\
& K\bar{x}(t,k) + u_0(t,k) - u(t,k-1) \in \bar{\mathbb{U}}(\delta(k)), && (21)\\
& \|\bar{\mathbf{e}}(k)\|_2 + \sqrt{t_n}\|\hat{e}(k)\|_\infty \le \|\bar{\mathbf{e}}(k-1)\|_2 \\
& + \sqrt{t_n}\|\hat{e}(k-1)\|_\infty, && (22)\\
& \bar{\mathbf{e}}(k) = [\bar{e}(1,k)^T\ \bar{e}(2,k)^T\ \ldots \bar{e}(t_n,k)^T]^T, \\
& \mathbf{u}_0(k) = [u_0(0,k)^T,\ \ldots u_0(t_n-1,k)^T]^T, \\
& \Delta_k \mathbf{u}_0(k) = \mathbf{u}_0(k) - \mathbf{u}_0(k-1).
\end{aligned}$$

The first term in the objective function is to minimize the predicted tracking error corresponding to the repeatable subsystem over the entire cycle. The matrix $R_w \in \mathbb{R}^{n_i \times n_i}$ is positive definite working as a penalty weight on the input term to regulate its variation. Eq. (19) and (20) give the system equations. The constraint in (21) is used to ensure input constraint satisfaction. The inequality (22) guarantees a monotonic decrease on $\|\bar{\mathbf{e}}(k)\|_2 + \sqrt{t_n}\|\hat{e}(k)\|_\infty$ which is an upper bound of the tracking error in cycle k. The variable $\delta(k)$ is used to determine the range of variation on $\Delta_k u(t,k)$ such that the convergence condition in (22) can be satisfied.

3.4 Analysis of computation complexity

In this section, we analyze the computation complexity of Problem 1.

Proposition 2. Problem 1 is a quadratic constrained quadratic programming.

Proof: According to Proposition 1, the set $\mathbb{U}(k)$, $\hat{\mathbb{U}}(k)$ and $\hat{\mathbb{E}}(k)$ are polyhedral C-sets, they can be denoted by linear inequalities as

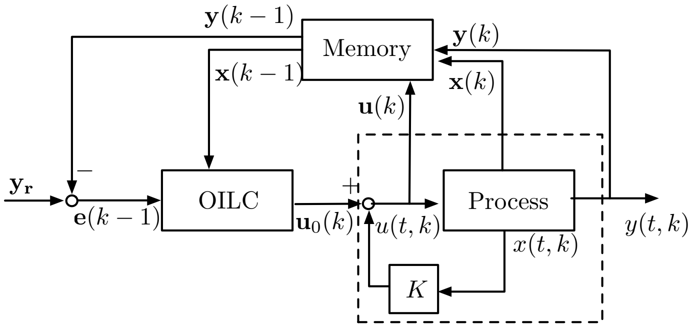

Fig. 1. Illustration of the control scheme.

$$\hat{\mathbb{U}}(k) = \{u : M_0 u \leq \delta(k) N_0\},$$
$$\mathbb{U}(k) \sim \hat{\mathbb{U}}(k) = \{u : M_1 u \leq \delta(k) N_1\},$$
$$\hat{\mathbb{E}}(k) = \{e : M_2 e \leq \delta(k) N_2\},$$

Thus, (21) is equivalent to $M_1(K\bar{x}(t,k)+u_0(t,k)-u(t,k-1)) \leq \delta(k)N_2$. It is linear on the variable $\delta(k)$, $\bar{x}(t,k)$ and $u_0(t,k)$. Given $\hat{e}(k) \in \hat{\mathbb{E}}(k)$, we have

$$\|\hat{e}(k)\|_\infty = \sup_{j\in[1,n_o],\ e\in\hat{\mathbb{E}}(k)} I^j e.$$

It can be further checked that

$$\|\hat{e}(k)\|_\infty = \delta(k) \sup_{j\in[1,n_o],\ e\in\{e:M_2e\leq N_2\}} I^j e.$$

Note that the term $c_1 = \sup_{j\in[1,n_o],\ e\in\{e:M_2e\leq N_2\}} I^j e$ is fixed after K is determined and therefore can be considered as a constant. Similarly, the term $\|\bar{\mathbf{e}}(k-1)\|_2 + \sqrt{t_n}\|\hat{e}(k-1)\|_\infty$ is also a constant since it is fixed when cycle $k-1$ is finished. Denote it as c_2. The inequality in (22) is equivalent to

$$\|\mathbf{e}_p(k)\|_2 + \delta(k)c_1 \leq c_2.$$

which is a quadratic constraint. Since the objective function of Problem 1 is a quadratic function and the constraints are either linear or quadratic functions, Problem 1 is a quadratically constrained quadratic programming, which is convex and therefore can be efficiently solved by methods such as interior point method (Mehrotra (1992)).

The overall control scheme is shown in Fig. 1. The state feedback control (dotted block) is executed at each sample time while the outer loop is conducted once a cycle. Thus, it is noted that the method enjoys a low computation complexity.

4. STABILITY ANALYSIS

Similar to many MPC algorithms, feasibility can imply stability in our problem. Thus, we study the consistent feasibility of Problem 1 first.

Theorem 1. (Feasibility to constraints): If the constraint in (21) is satisfied, then satisfaction of (4) is guaranteed.

Proof: The proof is similar as Lemma 2. Since $\hat{u}(t,k) \in \hat{\mathbb{U}}(k)$ and $K\bar{x}(t,k) + u_0(t,k) - u(t,k-1) \in \mathbb{U}(k) \sim \hat{\mathbb{U}}(k)$, we have $u(t,k)-u(t,k-1) = \hat{u}(t,k)+K\bar{x}(t,k)+u_0(t,k)-u(t,k-1) \in \mathbb{U}_1 \sim \hat{\mathbb{U}}_1$. Thus, (4) is ensured.

Theorem 2. (Recursive feasibility): If Problem 1 is feasible in cycle $k-1$, then it is also feasible in cycle k.

Proof: Assume Problem 1 is feasible in cycle $k-1$. Then, in cycle k, taking $\delta(k) = 0$, it can be derived that $u_0(t,k) = u_0(t,k-1)$, $\bar{x}(t,k) = \bar{x}(t,k-1)$ and $e(t+1,k) = e(t+1,k-1)$ for any $t \in [0, t_n-1]$, which provide a group of feasible solution to the constraint in (21). Moreover, in view of (13), (14) and (15), the tracking error satisfies

$$\mathbf{e}(k) = \mathbf{e}(k-1) = \bar{\mathbf{e}}(k-1) + \hat{\mathbf{e}}(k-1).$$

According to the triangle inequality, it is further proved that

$$\begin{aligned}\|\mathbf{e}(k-1)\|_2 \leq& \|\bar{\mathbf{e}}(k-1)\|_2 + \|\hat{\mathbf{e}}(k-1)\|_2 \\ \leq& \|\bar{\mathbf{e}}(k-1)\|_2 + \sqrt{t_n}\|\hat{\mathbf{e}}(k-1)\|_\infty,\end{aligned} \tag{23}$$

which ensures that the inequality (22) holds. By explicitly giving a group of feasible solution to Problem 1 in cycle k, its feasibility is proved.

Next, we can prove the boundedness of tracking error based on its feasibility.

Theorem 3. (Bounded tracking error): By applying the control law in (9), the tracking error $\|\mathbf{e}(k)\|_2$ is bounded in each cycle.

Proof: This property is guaranteed by the inequality (22). According to (23), $\|\bar{\mathbf{e}}(k)\|_2 + \sqrt{t_n}\|\hat{\mathbf{e}}(k)\|_\infty$ is an upper bound of $\|\mathbf{e}(k)\|_2$. The constraint in (23) guarantees that the upper bound of each cycle is non-increasing. Thus, $\|\mathbf{e}(k)\|_2$ for each cycle is bounded.

Remark 2. According to Theorem 3, the controller guarantees the decrease of an upper bound of the tracking error, instead of the tracking error itself. If a monotonic decrease on tracking error is desired, (23) can be replaced by $\|\bar{\mathbf{e}}(k)\|_2+\sqrt{t_n}\|\hat{e}(k)\|_\infty \leq \|\mathbf{e}(k-1)\|_2$. However, this constraint brings conservativeness, which means the variable $\delta(k)$ may quickly go to 0. In this case, control performance can hardly get improved even though monotonic decrease of tracking error is guaranteed. Problem 1 uses (23) to guarantee the robust stability and optimize the tracking performance based on the nominal system. Shown by Xu et al. (2012), this approach can help to avoid conservativeness.

5. SIMULATIONS

In this section, control of injection velocity in an injection molding process, which is an important polymer processing technique, is taken as an example. In velocity control, ram velocity is controlled by hydraulic pressure. The mechanism model is given in Cao et al. (2015) as

$$\begin{aligned}\dot{P}_h =& \beta_h/V_h(q_h - A_h v_z),\\ \dot{v}_z =& 1/M(P_h A_h - P_n A_n - f_v),\\ \dot{P}_n =& \beta_p/V_n(A_n v_z - q_p).\end{aligned}$$

Here P_h denotes hydraulic pressure. P_n denotes nozzle pressure. V_h and V_n are volumes in cylinder and nozzle respectively. A_h and A_n are cross-section areas of hydraulic system and nozzle. f_v is the friction force. q_h is hydraulic flow rate and q_p is polymer melt flow rate. β_h and β_p are hydraulic oil and polymer melt bulk modulus. M is ram mass. In Cao et al. (2015), it was shown that β_h/V_h and β_p/V_n can be treated as constants. Under the air shot operation, P_n is also a constant. Thus, the map between q_h and v_z is linear by taking P_nA_n and f_v as exogenous disturbances. This has been verified by experiments (Yang (2004)) and the following state space model is used in this work.

$$x(t+1,k) = \begin{bmatrix} 1.6 & -0.5916 \\ 1 & 0 \end{bmatrix} x(t,k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t,k) + n(t,k),$$
$$y(t,k) = [\,1.69 \;\; 1.419\,]\, x(t,k).$$

In this simulation, the nonlinear term $n(k)$ is set as

$$n(t,k) = \left[\, 0.2\sqrt{u(t,k)^2+1}^{\,T} \;\; 0.2|u(t,k)|^T \,\right]^T.$$

The reference to be tracked is set as

$$y_r(t) = \begin{cases} 15, & t \in [1,20] \\ 30, & t \in [21,50]. \end{cases}$$

The eigenvalue of linear part is 1.02 and 0.58, which shows the system unstable. A state feedback gain $K = [\,-1.6 \;\; 0.5916\,]$ is selected to stabilize the system, based on which, the set $\hat{\mathbb{U}}$ and $\mathbb{U} - \hat{\mathbb{U}}$ can be computed. Then, take $Q_w = \mathrm{diag}(1,1)$ and $R_w = \mathrm{diag}(0.01, 0.01)$. System inputs for each cycle can be derived by Problem 1.

For comparison, the method in Shi et al. (2006) is implemented. This robust control strategy was designed by solving linear matrix inequalities induced by a two-dimensional Lyapunov function. Parameters are taken as $\epsilon = 0.1$, $\gamma = 2.5$, $\rho_k = 0.1$ and $\rho_t = 0.1$. To compare the cycle-wise convergent rate, the method in Shi et al. (2006) is also adopted to control the first cycle when the proposed method is simulated such that the outputs of the two methods are the same in cycle 1.

Fig. 2 and Fig. 3 gives the outputs in cycle 1, 5 and 10 of the two methods, showing that the proposed method has a superior performance with less oscillations. This is further verified by Fig. 4 in which the mean square tracking errors (MSE) of the two methods are plotted. The MSE of the proposed method is significantly reduced by the proposed method.

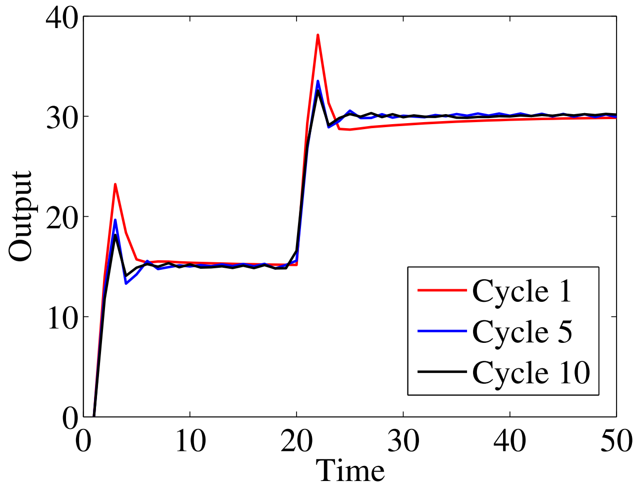

Fig. 2. Proposed method: Outputs of cycle 1, 5 and 10.

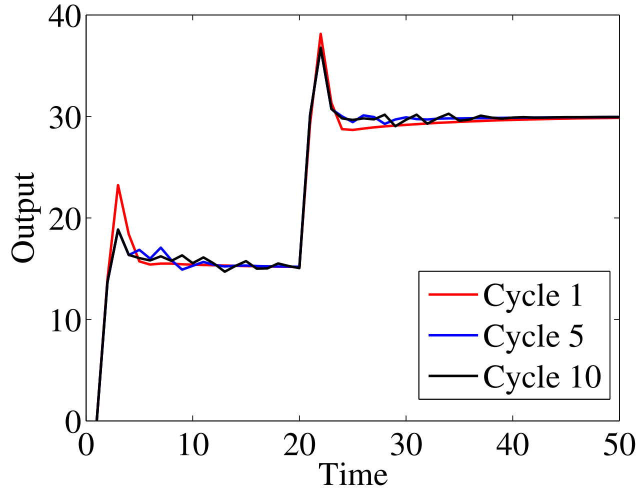

Fig. 3. The method in Shi et al. (2006): Outputs of cycle 1, 5 and 10

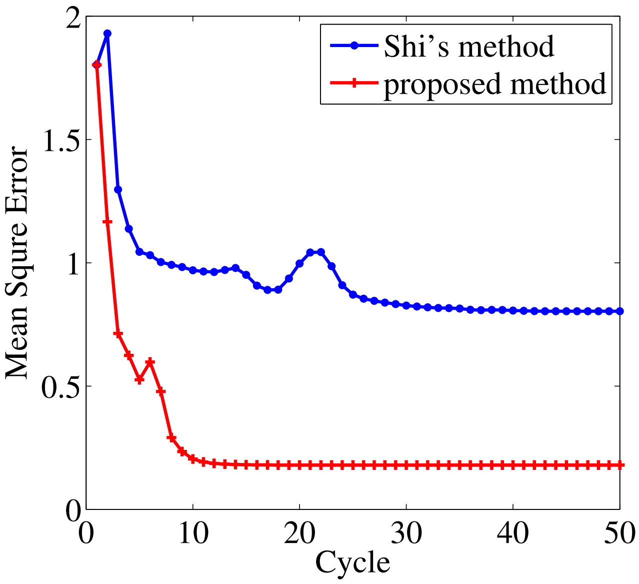

Fig. 4. Comparison on mean square error of the proposed method and Shi's method in Shi et al. (2006)

6. CONCLUSIONS

In this paper, a tube feedback iterative learning control strategy is designed for batch processes with unknown nonlinearity. Intrinsically, the control input is composed of a feedforward term and a feedback term. The feedforward term, derived by optimal iterative learning control, improves the tracking performance. The feedback term, induced by a state feedback controller, stabilizes the entire system and regulates the states within a tube around the trajectory determined by the OILC. In this way, constraint fulfillment and cycle-wise tracking error decrease are ensured simultaneously. In sum, the proposed method has low online computation complexity and wide applications covering both stable and unstable systems.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support from Hong Kong Research Grants Council(No.16233316) and the National Natural Science Foundation of China project (No. 61433005).

REFERENCES

Blanchini, F. and Miani, S. (2007). *Set-theoretic methods in control*. Springer Science & Business Media.

Cao, Z., Lu, J., Zhang, R., and Gao, F. (2016a). Iterative learning kalman filter for repetitive processes. *Journal of Process Control*, 46, 92–104.

Cao, Z., Yang, Y., Lu, J., and Gao, F. (2014). Constrained two dimensional recursive least squares model identification for batch processes. *Journal of Process Control*, 24(6), 871–879.

Cao, Z., Yang, Y., Lu, J., and Gao, F. (2015). Two-time-dimensional model predictive control of weld line positioning in bi-injection molding. *Industrial & Engineering Chemistry Research*, 54(17), 4795–4804.

Cao, Z., Zhang, R., Lu, J., and Gao, F. (2016b). Online identification for batch processes in closed loop incorporating priori controller knowledge. *Computers & Chemical Engineering*, 90, 222–233.

Cao, Z., Zhang, R., Yang, Y., Lu, J., and Gao, F. (2016c). Discrete-time robust iterative learning kalman filtering for repetitive processes. *IEEE Transactions on Automatic Control*, 61(1), 270–275.

Chin, I., Qin, S.J., Lee, K.S., and Cho, M. (2004). A two-stage iterative learning control technique combined with real-time feedback for independent disturbance rejection. *Automatica*, 40(11), 1913–1922.

Foss, B.A., Johansen, T.A., and Sørensen, A.V. (1995). Nonlinear predictive control using local modelsapplied to a batch fermentation process. *Control Engineering Practice*, 3(3), 389–396.

Gao, Y., Gray, A., Tseng, H.E., and Borrelli, F. (2014). A tube-based robust nonlinear predictive control approach to semiautonomous ground vehicles. *Vehicle System Dynamics*, 52(6), 802–823.

Herceg, M., Kvasnica, M., Jones, C.N., and Morari, M. (2013). Multi-parametric toolbox 3.0. In *Control Conference (ECC), 2013 European*, 502–510. IEEE.

Jia, L., Yang, T., and Chiu, M. (2013). An integrated iterative learning control strategy with model identification and dynamic r-parameter for batch processes. *Journal of Process Control*, 23(9), 1332–1341.

Kolmanovsky, I. and Gilbert, E.G. (1998). Theory and computation of disturbance invariant sets for discrete-time linear systems. *Mathematical problems in engineering*, 4(4), 317–367.

Langson, W., Chryssochoos, I., Raković, S., and Mayne, D.Q. (2004). Robust model predictive control using tubes. *Automatica*, 40(1), 125–133.

Lee, J.H., Lee, K.S., and Kim, W.C. (2000). Model-based iterative learning control with a quadratic criterion for time-varying linear systems. *Automatica*, 36(5), 641–657.

Lee, K.S. and Lee, J.H. (2003). Iterative learning control-based batch process control technique for integrated control of end product properties and transient profiles of process variables. *Journal of Process Control*, 13(7), 607–621.

Liu, T. and Wang, Y. (2012). A synthetic approach for robust constrained iterative learning control of piecewise affine batch processes. *Automatica*, 48(11), 2762–2775.

Lu, J., Cao, Z., and Gao, F. (2015a). A stable two-time dimensional (2d) model predictive control with zero terminal state constraints for constrained batch processes. *IFAC-PapersOnLine*, 48(8), 514–519.

Lu, J., Cao, Z., and Gao, F. (2016). Ellipsoid invariant set-based robust model predictive control for repetitive processes with constraints. *IET Control Theory & Applications*, 10(9), 1018–1026.

Lu, J., Cao, Z., Wang, Z., and Gao, F. (2015b). A two-stage design of two-dimensional model predictive iterative learning control for nonrepetitive disturbance attenuation. *Industrial & Engineering Chemistry Research*, 54(21), 5683–5689.

Lu, J., Cao, Z., Zhang, R., and Gao, F. (2018). Nonlinear monotonically convergent iterative learning control for batch processes. *IEEE Transactions on Industrial Electronics*, 65(7), 5826–5836.

Mehrotra, S. (1992). On the implementation of a primal-dual interior point method. *SIAM Journal on optimization*, 2(4), 575–601.

Nagy, Z.K. and Braatz, R.D. (2003). Robust nonlinear model predictive control of batch processes. *AIChE Journal*, 49(7), 1776–1786.

Shi, J., Gao, F., and Wu, T.J. (2006). Robust iterative learning control design for batch processes with uncertain perturbations and initialization. *AIChE journal*, 52(6), 2171–2187.

Wang, L., Liu, B., Yu, J., Li, P., Zhang, R., and Gao, F. (2018). Delay-range-dependent-based hybrid iterative learning fault-tolerant guaranteed cost control for multi-phase batch processes. *Industrial & Engineering Chemistry Research*.

Wang, L., Sun, L., Yu, J., Zhang, R., and Gao, F. (2017). Robust iterative learning fault-tolerant control for multiphase batch processes with uncertainties. *Industrial & Engineering Chemistry Research*, 56(36), 10099–10109.

Xu, Z., Zhao, J., Yang, Y., Shao, Z., and Gao, F. (2012). Robust iterative learning control with quadratic performance index. *Industrial & engineering chemistry research*, 51(2), 872–881.

Yang, Y. (2004). *Injection molding control: From process to quality*. PhD thesis in Hong Kong University of Science and Technology.

Yang, Y. and Gao, F. (2000). Adaptive control of the filling velocity of thermoplastics injection molding. *Control Engineering Practice*, 8(11), 1285–1296.