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Simultaneous State and Parameter Estimation using Receding-horizon Nonlinear Kalman Filter

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Abstract: Online estimation of internal states and parameters is often required for process monitoring, control and fault diagnosis. The conventional approach to estimate drifting parameters is to artificially model them as a random walk process and estimate them simultaneously with the states. However, tuning of the random walk model is not a trivial exercise. Recently, Valluru et al. (2017) have developed a moving window based state and parameter estimator which assumes that the parameters change slowly and remain constant within the window. Also, in another development, a moving window based recursive filter, receding horizon nonlinear Kalman (RNK) filter has been proposed by Rengaswamy et al. (2013). In this work, a novel simultaneous state and parameter estimator is proposed by combining the window based parameter variation model with RNK filter formulation. The performance of the RNK based estimator is demonstrated by conducting simulation studies on the benchmark quadruple tank system and a CSTR system. The efficacy of RNK based estimator is compared with that of the conventional simultaneous EKF approach and Moving Horizon Estimator (MHE) based state and parameter approach. Analysis of the simulation results reveals that the proposed state and parameter estimation scheme is able to generate better estimation performance than that of the simultaneous EKF and closer to that of the MHE based parameter estimator with less computational efforts.

Keywords: State and Parameter Estimation, Moving window estimation, Receding-horizon Nonlinear Kalman filter, Extended Kalman Filter.

1. INTRODUCTION

In many engineering applications, online estimation of internal states and parameters of a system has become necessary for effective process monitoring and efficient control. Many of the quality related state variables are difficult to measure and often too costly to measure online. Thus, a cost effective approach is to employ dynamic model based state estimation for estimating unmeasured and/or less frequently measured variables at a fast and regular rate (Patwardhan et al. (2012)). The efficacy of the state estimation critically depends on the accuracy of the model parameters. Due to the slow drifting of the model parameters/unmeasured disturbances, efficacy of the dynamic model based soft sensors deteriorates over time. This leads to biased state estimates, which, in turn deteriorates the performance of the model based monitoring and control schemes. Thus, to continue to accrue benefits of model based monitoring and control systems, parameters/unmeasured disturbances need to be estimated simultaneously with the states (Soroush (1998)).

Various methods for parameter estimation of nonlinear models are available in the literature which are based on filtering or approximation of the likelihood based approaches. In the conventional filtering based approach, the parameters are modelled as random walk process and this model is combined with the process model. This combined model is used for developing any nonlinear Bayesian estimators, such as Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), Ensemble Kalman Filter (EnKF) or Moving Horizon Estimator (MHE), for simultaneous state and parameter estimation. The random walk model artificially assumes that the parameters vary at every sampling instant. In practice, however, the system parameters change at a significant rate. The main difficulty in this approach is in choosing appropriate distribution of the parameter noise (Patwardhan et al. (2012)). Further, the tuning of noise covariance of random walk model is non-trivial particularly for a larger dimensional system. Moreover, the choice of the parameter covariance has a significant influence on the estimator performance and a wrong choice can even destabilize the estimator (Valluru et al. (2017)). Recently, Valluru et al. (2016) and Valluru et al. (2017) has developed a moving window maximum likelihood estimator which simultaneously estimates states and parameters. Here, it is assumed that the parameters are changing slowly and are assumed to remain constant over a time window in the immediate past. The advantage of this approach is that the only tuning parameter is the length of the moving window, which is much easier to select than selecting the covariance of the random walk model. A similar simultaneous state and parameter estimation strategy has been proposed in MHE framework by Huang et al. (2010). A distinct advantage of MHE based formulation is that simultaneous smoothing of the states is carried out while es-

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timating the parameters. However, MHE is a computationally intensive approach and, as a consequence, difficult to use for large dimensional systems.

Rengaswamy et al. (2013) has introduced a new estimation approach called the Receding-horizon Nonlinear Kalman (RNK) filter for estimating states over a time window. This approach also carries out smoothing of the state estimates. It has been shown that the RNK approach has a significant computational advantages over MHE approach with tradeoff in the performance. State and parameter estimation using the random walk model approach can also be formulated choosing RNK filter as a filtering algorithm. However, this leads to the same drawback as mentioned earlier, i.e., difficulty in tuning of the parameter covariance.

In this work, taking from the motivation of Valluru et al. (2017), an RNK based estimator is developed for simultaneous estimation/smoothing of the states and estimation of slowly changing parameters/unmeasured disturbances. The performance of the proposed RNK state and parameter estimator is compared with the simultaneous EKF and MHE approach. The efficacy of the proposed formulation is demonstrated by conducting simulation studies on the benchmark quadruple tank system and a continuously stirred tank reactor (CSTR) system.

The rest of the manuscript is organized as follows. Section 2 develops the approaches for state and parameter estimation. The simulation case studies are discussed in Section 3. Main conclusions derived from the simulation studies and future work are discussed in Section 4.

2. STATE AND PARAMETER ESTIMATION

2.1 Process Model

Consider a continuous time nonlinear model represented by the following ODE:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\theta}) \tag{1}$$

where, $\mathbf{x} \in R^n$ denotes the state vector, $\mathbf{u} \in R^m$ denotes the manipulated inputs and $\boldsymbol{\theta} \in R^p$ denotes the unmeasured disturbance/parameter vector. Here, $\mathbf{f}(.)$ is a known non-linear function vector of dimension $(n \times 1)$. Under the assumptions considered by Valluru et al. (2017), the true system dynamics represented by (1) can be represented in discrete form as follows :

$$\mathbf{x}_{k+1} = \mathbf{F}(\mathbf{x}_k, \mathbf{u}_k, \boldsymbol{\theta}) + \mathbf{w}_{x,k} \tag{2}$$

where $\mathbf{w}_{x,k} \in R^n$ is assumed to be a zero mean Gaussian white noise process with covariance matrix $\mathbf{Q} \in R^{n\times n}$. Under the normal operating conditions, the measurements (y) are available at a regular sampling interval (T) i.e.,

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k \tag{3}$$

where, $\mathbf{C} \in R^{\nu\times n}$ denotes the measurement matrix, $\mathbf{v}_k \in R^\nu$ denotes measurement noise at instant k, which is modeled as zero mean Gaussian white noise process with covariance matrix $\mathbf{R} \in R^{\nu\times\nu}$. Further, the state disturbances, $\mathbf{w}_{x,k}$, and the measurement noise, $\mathbf{v}_k$, are assumed to be uncorrelated.

Assumption 1: It is assumed that the model parameters, $\boldsymbol{\theta}$, change slowly and remain constant over the time window in the immediate past, i.e., $[k-N,k]$.

2.2 State Estimation using Receding-horizon Nonlinear Kalman (RNK) Filter

RNK is a moving window based recursive formulation that involves prediction and update steps similar to the conventional Kalman filter (Rengaswamy et al. (2013)). To find the filtered estimates at k^{th} instant, it is assumed that the N measurements are available over a window $[k-N+1,k]$. Also, the states, $\hat{\mathbf{x}}_{k-N|k-N}$, and the covariance, $\mathbf{P}_{k-N|k-N}$, at $(k-N)^{th}$ instant are assumed to be known. The prediction and update steps for RNK are as follows:

Prediction Step
In the prediction step, the filtered estimate $\hat{\mathbf{x}}_{k-N|k-N}$ at the $(k-N)^{th}$ time instant is used to find all the predicted estimates in the interval $[k-N+1,k]$, by using the state evolution model described as

$$\hat{\mathbf{x}}_{l|k-N} = \mathbf{F}(\hat{\mathbf{x}}_{l-1|k-N}, \mathbf{u}_{l-1}, \boldsymbol{\theta}) \tag{4}$$

where, l varies from $k-N+1$ to k. In other words, the predicted estimates are obtained by performing open loop simulation of (4), under the assumption that $E[w_{x,l}] = 0$ over the interval. These predicted states are denoted as $\hat{\mathbf{x}}_{k-N+1|k-N}$, $\hat{\mathbf{x}}_{k-N+2|k-N}$ $\ldots$ $\hat{\mathbf{x}}_{k|k-N}$. At time k, if a stacked state vector is defined as

$$\mathbf{X}_k^a = [\mathbf{x}_{k-N+1}^T \;\; \mathbf{x}_{k-N+2}^T \;\; \ldots \;\; \mathbf{x}_k^T]^T \tag{5}$$

then the stacked predicted state estimates can be represented as follows

$$\hat{\mathbf{X}}_{k|k-N}^a = [\hat{\mathbf{x}}_{k-N+1|k-N}^T \;\; \hat{\mathbf{x}}_{k-N+2|k-N}^T \;\; \ldots \;\; \hat{\mathbf{x}}_{k|k-N}^T]^T \tag{6}$$

At k^{th} time instant, the error covariance matrix of the stacked predicted states is a matrix consisting of $Nn \times Nn$ elements represented as

$$\mathscr{P}_{k|k-N}^a = E[(\mathbf{X}_k^a - \hat{\mathbf{X}}_{k|k-N}^a)(\mathbf{X}_k^a - \hat{\mathbf{X}}_{k|k-N}^a)^T] \tag{7}$$

This is a block matrix consisting of $n \times n$ block elements $\mathbf{P}$. The $(ij)^{th}$ block matrix of the predicted error covariance matrix $\mathscr{P}_{k|k-N}^a$ is defined as

$$\mathscr{P}_{k|k-N}^a(i,j) = E[(\mathbf{x}_{k-N+i} - \hat{\mathbf{x}}_{k-N+i|k-N})(\mathbf{x}_{k-N+j} - \hat{\mathbf{x}}_{k-N+j|k-N})^T] \tag{8}$$

where, i and j varies from 1 to N. The diagonal matrices of the predicted error covariance matrix represents the auto-covariance matrices of the estimation errors in the open loop state estimates at same time instants and the off-diagonal matrices of the predicted error covariance matrix represents the cross-covariance matrix between the open loop estimation errors at different time instants. The error covariance matrix is calculated using the linearized state transition matrices which is defined as

$$\boldsymbol{\Phi}_l = exp\left(\frac{\partial}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\theta})_{|\hat{\mathbf{x}}_{l|k-N}} T_s\right) \tag{9}$$

for $l = k-N+1, \ldots, k$. The first block of error covariance matrix $\mathscr{P}_{k|k-N}^a$ is initialized as

$$\mathscr{P}_{k|k-N}^a(1,1) = \boldsymbol{\Phi}_{k-N}\mathbf{P}_{k-N|k-N}\boldsymbol{\Phi}_{k-N}^T + \mathbf{Q} \tag{10}$$

The remaining blocks of $\mathscr{P}_{k|k-N}^a$ are computed as follows:

When $i = j$ (i.e., diagonal blocks)

$$\mathscr{P}_{k|k-N}^a(i,i) = \boldsymbol{\Phi}_{k-N+i-1}\mathscr{P}_{k|k-N}^a(i-1,i-1)\boldsymbol{\Phi}_{k-N+i-1}^T + \mathbf{Q} \tag{11}$$

When $i \neq j$ and $i < j$ (i.e., upper triangular blocks)

$$\mathscr{P}_{k|k-N}^a(i,j) = \mathscr{P}_{k|k-N}^a(i,j-1)\boldsymbol{\Phi}_{k-N+j-1}^T \tag{12}$$

Since $\mathscr{P}_{k|k-N}^a$ is symmetric matrices, the lower triangular blocks can be calculated by taking transpose of the upper triangular blocks.

Update Step

The update step involves merging of the predicted state estimates $\hat{\mathbf{X}}^a_{k|k-N}$ with the stacked measurements

$$\mathbf{Y}^a_k = [\mathbf{y}^T_{k-N+1} \ \mathbf{y}^T_{k-N+2} \ \ldots \ \mathbf{y}^T_k]^T \tag{13}$$

in the time window $[k-N+1,k]$. The updated state estimates

$$\hat{\mathbf{X}}^a_{k|k} = [\hat{\mathbf{x}}^T_{k-N+1|k} \ \hat{\mathbf{x}}^T_{k-N+2|k} \ \ldots \ \hat{\mathbf{x}}^T_{k|k}]^T \tag{14}$$

for linear measurement model and unconstrained case are obtained as follows

$$\mathbf{K}^a = \mathscr{P}^a_{k|k-N}\mathbf{C}^{a^T}(\mathbf{C}^a\mathscr{P}^a_{k|k-N}\mathbf{C}^{a^T} + \mathbf{R}^a)^{-1} \tag{15}$$

$$\hat{\mathbf{X}}^a_{k|k} = \hat{\mathbf{X}}^a_{k|k-N} + \mathbf{K}^a[\mathbf{Y}^a_k - \mathbf{C}^a\hat{\mathbf{X}}^a_{k|k-N}] \tag{16}$$

$$\mathscr{P}^a_{k|k} = (\mathbf{I} - \mathbf{K}^a\mathbf{C}^a)\mathscr{P}^a_{k|k-N} \tag{17}$$

$$\text{where}, \mathbf{C}^a = block\ diag(\mathbf{C},\ \mathbf{C}\ \ldots\ \mathbf{C}) \text{ and} \tag{18}$$

$$\mathbf{R}^a = block\ diag(\mathbf{R},\ \mathbf{R}\ \ldots\ \mathbf{R}) \tag{19}$$

The solutions obtained at the k^{th} instant are the filtered state estimates $\hat{\mathbf{x}}_{k|k}$ and for the other time instants $k-N+j \ \ (1 \leq j < N)$ corresponds to the smoothed estimates $\hat{\mathbf{x}}_{k-N+j|k}$. Also the updated augmented error covariance $\mathscr{P}^a_{k|k}$ gives the filtered estimates for the error covariance matrix at time k and smoothed estimates for the error covariance matrix in the interval $k-N+1$ to $k-1$.

For nonlinear systems, Rengaswamy et al. (2013) have recommended the use of smoothed state estimate and the associated covariance matrix obtained from the window $[k-N,k-1]$ for initializing the prediction step. Thus, instead of using $\hat{\mathbf{x}}_{k-N|k-N}$, $\mathbf{P}_{k-N|k-N}$, the smoothed estimates obtained from the previous $\hat{\mathbf{x}}_{k-N|k-1}$, $\mathbf{P}_{k-N|k-1}$ are used to initialize the predictions. They have also shown that these modifications works better in the constrained nonlinear state estimation problems.

MHE is the only other approach available for moving window based state estimation. In MHE formulation, however, integration of ODEs needs to be performed repeatedly within an optimization loop resulting in a large computational load. This might prohibit the MHE approach from being used in online applications particularly when the system dimension is large. On the other hand, in RNK only a single integration is required at every estimation step. This leads to a very significant computational gain over MHE.

2.3 Simultaneous State and Parameter Estimation using RNK Filter

In this work, taking motivation from Valluru et al. (2017), a state and parameter estimator is developed using RNK under the assumption that the parameter vector, $\boldsymbol{\theta}$, remains constant over the window $[k-N+1,k]$. This approach implicitly assumes that the variation of parameter occurs significantly slower rate than the rates at which the states and the manipulated inputs change over a time window, $[k-N+1,k]$, in the recent past. The proposed algorithm to find the parameter vector, $\boldsymbol{\theta}$, at time instant k over the window $[k-N+1,k]$ is as follows

$$\widehat{\boldsymbol{\theta}}_{[k-N+1,k]} = \min_{\boldsymbol{\theta}} \ \psi(\boldsymbol{\theta}) \tag{20}$$

Subject to:

$$\hat{\mathbf{x}}_{l|k-N}(\boldsymbol{\theta}) = \mathbf{F}(\hat{\mathbf{x}}_{l-1|k-N}, \mathbf{u}_{l-1}, \boldsymbol{\theta}) \tag{21}$$

$$\hat{\mathbf{x}}_{k-N|k-N} = \hat{\mathbf{x}}_{k-N|k-1} \tag{22}$$

$$\boldsymbol{\Phi}_l = exp\Big(\frac{\partial}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \boldsymbol{\theta})_{|\hat{\mathbf{x}}_{l|k-N}} T_s\Big) \tag{23}$$

$$l = k-N+1, \ldots, k$$

$$\left.\begin{array}{ll} \mathscr{P}^a_{k|k-N}(1,1) = \boldsymbol{\Phi}_{k-N}\mathbf{P}_{k-N|k-1}\boldsymbol{\Phi}^T_{k-N} + \mathbf{Q} & \\ \mathscr{P}^a_{k|k-N}(i,i) \ = \boldsymbol{\Phi}_{k-N+i-1}\mathscr{P}^a_{k|k-N}(i-1,i-1)\boldsymbol{\Phi}^T_{k-N+i-1} + \mathbf{Q} & \\ \mathscr{P}^a_{k|k-N}(i,j) \ = \mathscr{P}^a_{k|k-N}(i,j-1)\boldsymbol{\Phi}^T_{k-N+j-1} & \\ \qquad for \ \ i \neq j \ \ and \ \ i < j & \\ \mathscr{P}^a_{k|k-N}(j,i) \ = \mathscr{P}^a_{k|k-N}(i,j)^T & \\ \qquad i,j = 1, \ldots, N & \end{array}\right\} \tag{24}$$

$$\mathbf{K}^a = \mathscr{P}^a_{k|k-N}\mathbf{C}^{a^T}(\mathbf{C}^a\mathscr{P}^a_{k|k-N}\mathbf{C}^{a^T} + \mathbf{R}^a)^{-1} \tag{25}$$

$$\hat{\mathbf{X}}^a_{k|k}(\boldsymbol{\theta}) = \hat{\mathbf{X}}^a_{k|k-N}(\boldsymbol{\theta}) + \mathbf{K}^a[\mathbf{Y}^a_k - \mathbf{C}^a\hat{\mathbf{X}}^a_{k|k-N}(\boldsymbol{\theta})] \tag{26}$$

$$\mathscr{P}^a_{k|k} = (\mathbf{I} - \mathbf{K}^a\mathbf{C}^a)\mathscr{P}^a_{k|k-N} \tag{27}$$

$$\hat{\boldsymbol{\zeta}}^a(\boldsymbol{\theta}) = [\hat{\mathbf{X}}^a_{k|k}(\boldsymbol{\theta}) - \hat{\mathbf{X}}^a_{k|k-N}(\boldsymbol{\theta})] \tag{28}$$

$$\hat{\boldsymbol{\varepsilon}}^a(\boldsymbol{\theta}) = [\mathbf{Y}^a_k - \mathbf{C}^a\hat{\mathbf{X}}^a_{k|k}(\boldsymbol{\theta})] \tag{29}$$

$$\psi(\boldsymbol{\theta}) = \hat{\boldsymbol{\zeta}}^{a^T}(\boldsymbol{\theta})\mathscr{P}^{a\ -1}_{k|k-N}\hat{\boldsymbol{\zeta}}^a(\boldsymbol{\theta}) + \hat{\boldsymbol{\varepsilon}}^{a^T}(\boldsymbol{\theta})(\mathbf{R}^a)^{-1}\hat{\boldsymbol{\varepsilon}}^a(\boldsymbol{\theta}) \tag{30}$$

$$\boldsymbol{\theta} \ \in \ \boldsymbol{\Omega}_\theta \tag{31}$$

Here, $\boldsymbol{\Omega}_\theta$ represents a set over which $\boldsymbol{\theta}$ is constrained to take values. Thus, this approach simultaneously estimates the states using RNK while estimating for the slowly drifting parameters/unmeasured disturbances. The optimization problem formulated over the window $[k-N+1,k]$ is connected with the optimization problem formulated over the window $[k-N,k-1]$ through the use of, $\widehat{\boldsymbol{\theta}}_{[k-N,k-1]}$, as an initial guess for, $\widehat{\boldsymbol{\theta}}$ for window $[k-N+1,k]$.

The weighting matrix, $\mathscr{P}^a_{k|k-N}$, appearing in the objective function (20), are time varying and are functions of the decision variable, $\boldsymbol{\theta}$, which results in a non-convex nonlinear optimization problem. The proposed method uses only the window size N as an explicit tuning parameter.

2.4 Simultaneous State and Parameter Estimation using Moving Horizon Estimator (MHE)

Moving horizon estimation is an optimization approach that uses a series of measurements over a time window and estimates states along with unknown disturbances. A formulation of MHE for simultaneous state and parameter estimation was proposed by Huang et al. (2010). In this version of MHE, the assumptions regarding the variation of parameters/unmeasured disturbances are similar to that of Valluru et al. (2017). By this approach, the simultaneous state and parameter estimation algorithm is formulated as follows:

$$\min_{\mathbf{x}_{k-N},\ldots,\mathbf{x}_k,\boldsymbol{\theta}} \boldsymbol{\Phi}(\mathbf{x}_{k-N}) + \frac{1}{2}\sum_{j=k-N}^{k}\mathbf{v}^T_j\mathbf{R}^{-1}\mathbf{v}_j + \frac{1}{2}\sum_{j=k-N}^{k-1}\mathbf{w}^T_{x,j}\mathbf{Q}^{-1}\mathbf{w}_{x,j} \tag{32}$$

Subject to:

$$\mathbf{w}_{x,j} = \mathbf{x}_{j+1} - \mathbf{f}(\mathbf{x}_j, \mathbf{u}_j, \boldsymbol{\theta}) \tag{33}$$

$$\mathbf{v}_j = \mathbf{y}_j - \mathbf{g}(\mathbf{x}_j) \tag{34}$$

$$\boldsymbol{\theta} \ \in \ \boldsymbol{\Omega}_\theta, \ \ j = k-N, ..., k \tag{35}$$

Here, $\boldsymbol{\Phi}(\mathbf{x}_{k-N})$ is known as the arrival cost. In the present work, the arrival cost is estimated as follows

$$\boldsymbol{\Phi}(\mathbf{x}_{k-N}) = \frac{1}{2}(\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N|k-N})^T\boldsymbol{\Pi}^{-1}_{k-N|k-N}(\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N|k-N}) \tag{36}$$

where, $\hat{\mathbf{x}}_{k-N|k-N}$ is the updated state estimates and $\boldsymbol{\Pi}_{k-N|k-N}$ is the updated covariance generated using the EKF.

2.5 State and Parameter Estimation using Simultaneous EKF

The conventional observer based approach for estimation of the unmeasured disturbances/parameters is to assume that their variation can be captured using the random walk model, i.e.

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \mathbf{w}_{\theta,k} \quad (37)$$

where, $\mathbf{w}_{\theta,k}$, is assumed to be zero mean Gaussian white noise process with covariance matrix $\mathbf{Q}_\theta$. The matrix $\mathbf{Q}_\theta$ is assumed to be diagonal and the individual variances are treated as the tuning parameters. The state dynamics given by vector, $\mathbf{x}_k$ is augmented with the random walk model for the parameters and the resulting augmented model is used for simultaneous estimation of the states and the parameters/disturbances. Consider the augmented state vector $\mathbf{X}_k = [\mathbf{x}_k^T \;\; \boldsymbol{\theta}_k^T]^T$, and an augmented noise vector, $\mathbf{W}_k = [\mathbf{w}_{x,k}^T \;\; \mathbf{w}_{\theta,k}^T]^T$ with covariance $\mathbf{Q}_a = Cov(\mathbf{W}_k^T) = diag[\mathbf{Q}_x \;\; \mathbf{Q}_\theta]$. The augmented dynamic model is represented as follows:

$$\mathbf{X}_{k+1} = \mathscr{F}(\mathbf{X}_k, \mathbf{u}_k) + \mathbf{W}_k \quad (38)$$

$$\mathbf{y}_k = \mathbf{C}_a \mathbf{X}_k + \mathbf{v}_k \quad (39)$$

$$\mathscr{F}[.] = \begin{bmatrix} \mathbf{F}(\mathbf{X}_k, \mathbf{u}_k, \boldsymbol{\theta}_k) \\ \boldsymbol{\theta}_k \end{bmatrix} \quad (40)$$

where, $\mathbf{C}_a = [\mathbf{C}_x \;\; [\mathbf{0}]]$
The above augmented model is used for developing simultaneous EKF which estimate states and parameters.

3. SIMULATION STUDIES

3.1 Quadruple Tank (QT) System

To demonstrate the effectiveness of the proposed RNK based state and parameter estimator, estimation performance of the proposed approach is compared with performances of the conventional simultaneous EKF approach and MHE approach described in section 2.4 by performing simulation studies on the benchmark quadruple tank system (Johansson (2000)). Depending on the operating point, the quadruple tank system can behave as a minimum phase or a non-minimum phase system. In simulation studies, the system is operated in the open loop and under the non-minimum phase operating conditions. The dynamics of quadruple tank system is governed by the set of ODEs that can be referred from Johansson (2000).

The manipulated variables are the pump voltages ranges from 0 to 5 V, denoted as $\mathbf{U} = [\, v_1 \;\; v_2 \,]^T$ and the measured outputs are the levels of bottom two tanks (Tank 1 and Tank 2) ranges from 0 to 20 cm. The nominal values of model parameters and relevant steady state operating conditions can be found in Johansson (2000).

For simulation studies sampling interval is chosen as 5 seconds. The manipulated inputs are perturbed in the neighborhood of their steady state values by introducing a Pseudo Random Binary Signal (PRBS) of amplitude $0.5V$ and frequency range $[0, 0.05\pi/T_s]$. The state and the measurement noise are simulated as zero mean Gaussian random signals with the covariance matrices $\mathbf{Q} = 0.05 \times \mathbf{I}_{4\times 4}$ and $\mathbf{R} = 0.05 \times \mathbf{I}_{2\times 2}$, respectively.

Here the leaks occurring at the bottom of the Tank 1 and Tank 2 have been investigated, which are modelled as changes in model parameters, a_1 and a_2. For sampling instant $k = 16$ to $k = 115$, a_1 is set to 115% of its nominal value, keeping a_2 constant at the nominal value. Subsequently, from sampling instant $k = 166$ to $k = 265$, a_2 is set to 115% keeping a_1 constant at the nominal value. From $k = 316$ to $k = 415$, both a_1 and a_2 are set to 110% of their nominal value. Other than these instants, at every other instant a_1 and a_2 are maintained at their nominal values. Thus, initially the leaks are introduced one at a time. Later, leaks are introduced in both tanks simultaneously. The bounds on the unmeasured parameters is taken to be $0.01 \leq a_i \leq 0.1$, where $i = 1, 2$.

The effectiveness of the estimators are assessed using the average sum of squared estimation error (ASSEE) defined as

$$(ASSEE)_i = \frac{1}{N_r} \sum_{j=1}^{N_r} \sum_{k=1}^{N_s} (x_{k,i,j} - \hat{x}_{k|k,i,j})^2 \quad (41)$$

where $x_{k,i}$ represents the true value of i^{th} state at the k^{th} instant, $\hat{x}_{k|k,i}$ represents the estimated value of i^{th} state at the k^{th} instant, N_s represents the simulation size and N_r represents number of realizations. For simulation of quadruple tank system, $N_s = 570$ and $N_r = 10$ have been considered. The performance of the estimators is also assessed by computing the average computation time.

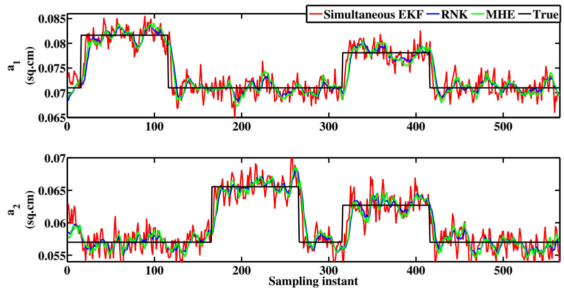

Fig. 1. QT System: Comparison of parameters a_1 and a_2 using Simultaneous EKF, RNK based estimator and MHE

Table 1. QT System: Comparison of ASSEE values of states and parameters

Variables	EKF	RNK	MHE
x_1	18.9781	21.2041	15.7950
x_2	14.5438	15.3674	10.7292
x_3	7.6947	6.9997	8.3680
x_4	14.4290	10.1617	11.2090
a_1	0.0034	0.0027	0.0027
a_2	0.0048	0.0019	0.0020

Table 2. QT System: Comparison of computational times

Method	EKF	RNK	MHE
Computational time (s)	0.0037	3.91	179.35

The proposed RNK based estimator with window size $N = 10$, MHE estimator with horizon length $N = 10$ and simultaneous EKF with covariance matrix $\boldsymbol{Q_\theta} = 10^{-3} diag[a_1 \;\; a_2]$ are compared for the estimated parameters in Figure (1). Also, comparison of performance of these filters in terms of ASSEE values is presented in Table 1. For measured states x_1 and x_2, the MHE estimator generates better results when compared to simultaneous EKF and RNK based estimator. However, for unmeasured states x_3 and x_4, RNK based estimator generates better estimates when compared to both simultaneous EKF and MHE estimators. For parameters a_1 and a_2, the RNK based estimator generates significantly better estimates when compared to that of simultaneous EKF estimates, whereas, it gives similar estimates as that of MHE estimator.

Table 2 reports the average computation time for implementation of simultaneous EKF, RNK and MHE estimators using a 3.3 GHz Intel Xeon E3 processor and Matlab (version R2013a) function, *fmincon*. The computational cost associated with proposed RNK estimator is high compared to simultaneous EKF approach but it is significantly less when compared to MHE estimator. Thus, RNK based estimator is able to give close results as that of MHE based parameter estimator with much less computational time.

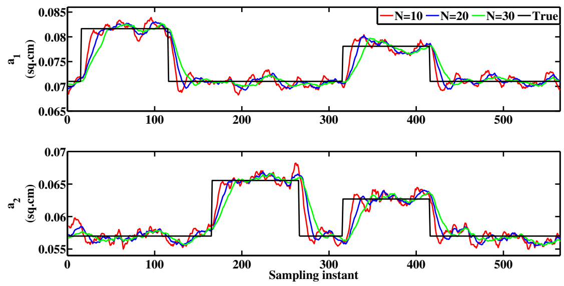

Fig. 2. QT System: Comparison of parameters a_1 and a_2 using RNK based estimator for different horizon lengths

Table 3. QT System: Comparison of ASSEE values of states and parameters for RNK based estimator

Variables	RNK		
	$N = 10$	$N = 20$	$N = 30$
x_1	21.2041	22.8669	23.6301
x_2	15.3674	16.7255	17.5996
x_3	6.9997	7.1363	8.2493
x_4	10.1617	10.1962	10.9561
a_1	0.0027	0.0032	0.0040
a_2	0.0019	0.0023	0.0028

To examine the effect of choice of horizon length, performances of the RNK based estimator is compared for three different choices of horizon lengths i.e., $N = 10$, 20 and 30 which are presented in Figure (2). As the length of the horizon increases in the RNK estimator, the estimates of unmeasured parameters and states become smoother. However, the ASSEE values reported in Table 3 for the states and parameters obtained using RNK estimator increases with increase in the horizon size due to increase in the delay.

3.2 CSTR System

The aim of this example is to demonstrate applicability of the proposed approach to a chemical process. The CSTR system which is considered undergoes a reversible exothermic reaction of the type $A \rightleftharpoons B$. The dynamic model, the nominal parameters and the optimal operating steady state conditions used in the simulation studies of CSTR system can be found in Deshpande et al. (2009). Inlet temperature (T_i) and inlet flow rate (F_i) are the manipulated inputs for the system. Kinetic forward rate constant (k_f) is treated as a slowly varying model parameter.

The sampling time T_s for the process is chosen as 0.4 min. The state dynamics is added with the state noise as given in (2) and has zero mean with covariance $\mathbf{Q} = diag[0.005^2\ 0.005^2\ 0.15^2\ 0.002^2]$. The composition C_b and reactor level (h) are assumed to be measured. The noise in these measurements has zero mean with covariance $\mathbf{R} = diag[0.005^2\quad 0.001^2]$. The estimator is initialized with $\hat{\mathbf{x}}_{0|0} = [0.4912\quad 0.5088\quad 438.49\quad 0.16]^T$ and $\mathbf{P}_{0|0} = 5 \times \mathbf{Q}$. For simulation of CSTR system, $N_s = 2445$ and $N_r = 5$ have been considered for calculating ASSEE values.

The manipulated input variables, F_i and T_i are perturbed in the neighborhood of their steady state values by introducing a Pseudo Random Binary Signal (PRBS) in the frequency range of $[0, 0.05\pi/T_s]$. Also, the unmeasured parameter, k_f, is assumed to change as follows: at 250^{th} sampling instant, a ramp disturbance is given in the parameter, k_f, and is held till the magnitude of the parameter reaches -20% of its nominal base value (see Figure (3) and Figure (4)).

Here, the performance of the proposed RNK based parameter estimator is compared with the simultaneous EKF approach. Initially, the performance of simultaneous EKF approach is studied for different parameter covariance matrices, $\boldsymbol{Q_\theta}$. The covariance matrix, $\boldsymbol{Q_\theta}$, is chosen in proportional to the nominal value of the parameter k_f. The estimates are generated by simultaneous EKF for the following high, medium and low choices of $\boldsymbol{Q_\theta}$ represented as

$$\left.\begin{aligned} \boldsymbol{Q_\theta}, high &= 10^{-4} \times \bar{k_f}^2 \\ \boldsymbol{Q_\theta}, medium &= 10^{-6} \times \bar{k_f}^2 \\ \boldsymbol{Q_\theta}, low &= 10^{-8} \times \bar{k_f}^2 \end{aligned}\right\} \tag{42}$$

From Figure (3), it can be seen that the choice of covariance matrix $\boldsymbol{Q_\theta}$ has a significant influence on the parameters. For high $\boldsymbol{Q_\theta}$, the estimates of simultaneous EKF are noisy and for medium $\boldsymbol{Q_\theta}$, the estimates are able to track the true states with a little bias in the transient region. However, for the choice of low $\boldsymbol{Q_\theta}$, simultaneous EKF is unable to track the true parameter variation during the transient. The ASSEE values for states and parameters using simultaneous EKF are presented in Table 4. The ASSEE values for medium $\boldsymbol{Q_\theta}$ using the simultaneous EKF gives better performance compare to other choices of covariance matrix $\boldsymbol{Q_\theta}$.

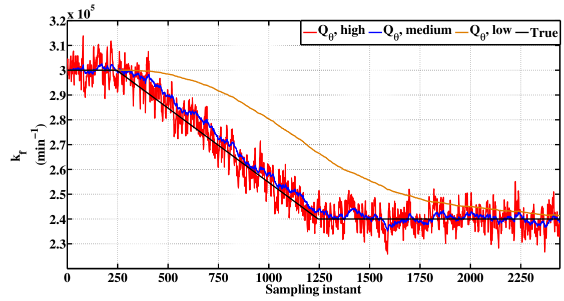

Fig. 3. CSTR System: Comparison of parameter k_f using Simultaneous EKF for different parameter covariance matrices

Table 4. CSTR System: Comparison of ASSEE values of states and parameters for Simultaneous EKF

Variables	Simultaneous EKF		
	$\boldsymbol{Q_\theta}, high$	$\boldsymbol{Q_\theta}, medium$	$\boldsymbol{Q_\theta}, low$
C_a	0.1606	0.1063	0.9419
C_b	0.1225	0.1062	0.1332
T	730.76	732.04	755.90
h	0.0172	0.0172	0.0172
$k_f \times 10^{10}$	5.1581	1.2084	52.958

To get better insight into the effect of horizon lengths, the performance of the RNK based estimator is compared with three choices of horizon lengths i.e., $N = 10$, 20 and 30. In Figure (4), it can be seen that as the size of the window length (N) increases, variability in the parameter estimates decreases and the estimates become smoother. It can be seen that, the estimates of parameter, k_f, has zero mean and the variance keeps decreasing with the increase of horizon length. The ASSEE values for states and parameter for different horizon lengths using RNK estimator are presented in Table 5. It is observed that, the ASSEE values for both states and parameter decreases with the increase in the horizon length.

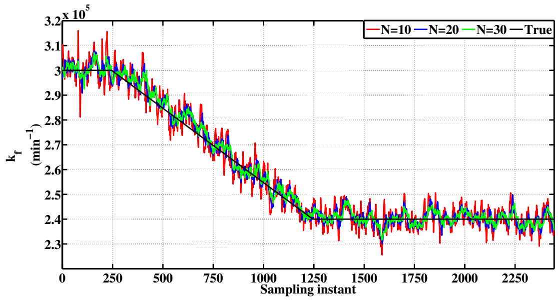

Fig. 4. CSTR System: Comparison of parameter k_f using RNK based estimator for different horizon lengths

Table 5. CSTR System: Comparison of ASSEE values of states and parameters for RNK based estimator

Variables	RNK		
	$N = 10$	$N = 20$	$N = 30$
C_a	0.1572	0.1191	0.1081
C_b	0.1132	0.1087	0.1074
T	475.34	476.32	476.22
h	0.0172	0.0172	0.0172
$k_f \times 10^{10}$	4.9535	2.4605	1.6536

Depending on the choice of $\boldsymbol{Q_\theta}$, simultaneous EKF estimator is able to track the true parameter, but with a bias during the transient region or with high variance in the estimates (refer to Figure 3). On the other hand, the proposed RNK based estimator generates unbiased estimates for all the choices of window length. The bias observed in the simultaneous EKF is a function of the choice of the tuning parameter, i.e., covariance of the random walk model. In simulation studies, since variation of the true parameters is known, it is possible to judge which tuning parameter is suitable for simultaneous EKF. In practice, however, true variation of the parameters is unknown. Thus, it is difficult to decide which tuning parameter is best for simultaneous EKF. The proposed RNK based estimator, on the other hand, produces qualitatively similar performance for in terms of tracking the true value. The only change with the change in the window length is variance of the estimates.

4. CONCLUSION

The conventional approach to estimate drifting parameters is to model them as random walk process and estimate simultaneously with the states using any recursive Bayesian estimator. The main difficulty in this approach is tuning of the covariance of random walk model. Recently, Valluru et al. (2017) have developed a moving window based state and parameter estimator which assumes that the parameters change slowly and remain constant within the window. Also, in another development, a moving window based recursive filter, receding horizon nonlinear Kalman (RNK) filter has been proposed by Rengaswamy et al. (2013). In this work, a novel simultaneous state and parameter estimator is proposed by combining the window based parameter variation model with RNK filter formulation. In this approach, the only tuning parameter is the length of the moving window which is easier to select. The effectiveness of the proposed approach is demonstrated by carrying out simulation studies on the benchmark quadruple tank system and a CSTR system. RNK based parameter estimator is carried out to identify the leaks occurring in the quadruple tank and it is compared with the conventional simultaneous EKF and the MHE approach. From the analysis of simulation studies, it is observed that RNK estimator gives a better performance when compared to that of simultaneous EKF and similar performance as that of MHE. The computation time associated with MHE implementation is significantly large, while the computation burden associated with RNK based estimator is relatively less when compared to that of MHE. The effectiveness of RNK estimator is also demonstrated by conducting simulation studies on a CSTR system and compared with the simultaneous EKF approach. RNK based estimator results in unbiased estimates. Further, it is also shown that as the window size increases the estimates obtained using RNK based parameter estimator become more accurate and smoother. However, the improved accuracy is achieved at the cost of increased computational burden. Currently research is in progress on demonstrating the proposed approach on experimental studies and for larger dimensional systems.

REFERENCES

Deshpande, A.P., Patwardhan, S.C., and Narasimhan, S.S. (2009). Intelligent state estimation for fault tolerant nonlinear predictive control. *Journal of Process Control*, 19(2), 187–204.

Huang, R., Beigler, L.T., and Patwardhan, S.C. (2010). Fast offset-free nonlinear model predictive control based on moving horizon estimation. *Industrial and Engineering Chemistry Research*, 49(17), 7882–7890.

Johansson, K.H. (2000). The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on Control Systems Technology*, 8(3).

Patwardhan, S.C., Narasimhan, S., Jagadeesan, P., Gopaluni, B., and Shah, S.L. (2012). Nonlinear bayesian state estimation: A review of recent developments. *Control Engineering Practice*, 20, 933–953.

Rengaswamy, R., Narasimhan, S., and Kuppuraj, V. (2013). Receding-horizon nonlinear Kalman (RNK) filter for state estimation. *IEEE Transactions on Automatic Control*, 58(8).

Soroush, M. (1998). State and parameter estimations and their applications in process control. *Computers and Chemical Engineering*, 23, 229–245.

Valluru, J., Lakhmani, P., Patwardhan, S.C., and Biegler, L.T. (2017). Development of moving window state and parameter estimators under maximum likelihood and bayesian frameworks. *Journal of Process Control*, 60, 48–67.

Valluru, J., Purohit, J.L., Patwardhan, S.C., and Biegler, L.T. (2016). Development of a moving window maximum likelihood parameter estimation and its application on ideal reactive distillation system. *IFAC-PapersOnLine*, 49(7), 484–489.