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Parameter Identification of Train Basic Resistance Using Multi-Innovation Theory ⋆

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Abstract: Train basic resistance is important for the design of the automatic train operation, which influences the efficiency, punctuality, stop precision, energy consumption, and the safety of the train. The multi-innovation theory is a novel concept which can improve the accuracy of parameter estimation and be used to modify the traditional recursive least squares algorithm. In this paper, we derive the regularization form of the multi-innovation least squares algorithm and apply it to the train basic resistance parameter estimation. The simulation results based on the Yizhuang Line of Beijing Subway indicate that, compared with traditional least squares algorithm, the multi-innovation least squares algorithm can provide higher estimation accuracy and robustness, and can be used for online identification.

Keywords: Train basic resistance, Multi-innovation identification, Recursive identification, Parameter estimation, Urban rail transit

1. INTRODUCTION

Urban rail transit has many advantages, such as large capacity, high efficiency, punctuality, reliability and safety, so it becomes the backbone of urban public transport. Automatic train operation (ATO) plays a key role in ensuring the safety and efficient operation of urban rail transit operating with high speed and high density (see Yu and Chen (2011)). In recent years, with development of communication and control technology, the platform screen doors (PSD) has been widely used. In order to ensure that passengers are not disturbed when they get on and off on the platform with PSD, the more accurate stop precision is required (see Wang et al. (2013) and Chen et al. (2013)).

The basic resistance is one important factor of stop precision, which is crucial to the design of ATO. Many factors will effect the basic resistance, such as the wheel-rail coupling (including the friction between axle and bearing, and the friction between wheel and rail), the train's outer and air coupling (aerodynamic effect), pantograph and network coupling with electric traction, etc. Due to the complex constitution of basic resistance, a large number of experimental data are integrated according to the formation mechanism, and the classical Davis formula was proposed (see Davis (1926), Bernsteen et al. (1980), and Huang et al. (2000), and Yuan (2015)).

Currently, a domestic enterprise needs one experimental procedure called coast-down test to determine the parameter of basic resistance at the expense of time, labor, and material, so it is necessary to find a method that does not need coast-down test (see Bernsteen et al. (1980) and Yuan (2015)). The train basic resistance depends strongly on weather and track conditions, so many control decisions of ATO can be improved if the basic resistance parameters can be estimated online. The condition of track is complex and unique, and it is inevitable that the collected data is often discontinuous and abnormal because of the unexpected interruptions and measurement errors. All of these will increase the difficulty of parameter identification, especially online identification.

The development of related fields such as system identification, machine learning, and neural networks contributes to the parameter estimation of the mathematical model. Yuan (2015) applied genetic algorithm to the basic resistance parameter estimation, but the algorithm has poor real-time performance for online identification because of tremendous number of iterations. The least squares algorithm is widely used, which can realize off-line identification of both linear regression model and pseudo-linear regression model. However, every parameter of the basic resistance has its own meaning, and the least squares algo-

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rithm can not be used for constrained estimation directly, the regularization form of least squares algorithm was introduced to solve this problem (see Haykin (2009)). Furthermore, the recursive form of the least squares algorithm can be used for online identification (see Goodwin and Sin (1984)). Ding et al. (2010) proposed the multi-innovation identification theory and applied it to the recursive least squares (RLS) algorithm. The multi-innovation identification algorithm has good performance in the case of missing data and anomalous data (see Ding and Chen (2007)).

Based on the data of ATO, we preprocess the data of the train and use the regularized least squares algorithm to estimate the train basic resistance parameter. We also apply the recursive least squares (RLS) algorithm, multi-innovation least squares (MILS) algorithm, and interval-varying multi-innovation least squares (V-MILS) algorithm to the basic resistance parameter estimation. These algorithms can not only provide high estimation accuracy but also performs well in the online identification, which will reduce the expense of time, labor, and material for organizing field test.

2. MODELING AND DATA PREPROCESSING

2.1 Modeling

The factors that affect the basic resistance are very complex. An early comprehensive study of train resistance was conducted by Davis (1926). Based on the formation mechanism of basic resistance and empirical data, the aerodynamic resistance generated by aerodynamic effect is regarded as the square function of velocity, and the resistance generated by mechanical resistance (such as wheel and rail coupling, pantograph and network coupling) is considered to be a linear function of velocity. The classical Davis formula was described as

$$T_R = A + \frac{B}{(W/n)} + Cv + \frac{DX}{W}v^2,$$

where the quantity T_R is the basic resistance, W and v are the weight and velocity of the train, respectively, n is the number of axles, and X is an effective frontal cross section. The constants A, B, C, and D are empirically adjusted to fit the particular type of train considered (see Davis (1926) and Bernsteen et al. (1980)).

For the convenience of parameter identification, we usually simplify the Davis formula as follows

$$w_0 = a + bv + cv^2,$$

where w_0 is unit basic resistance (N/kN), parameter a comprises resistances which are considered independent of speed, but variable with axle load, parameter b contains resistances which are proportional to the first power of the velocity and originates from losses caused by mechanical resistance, and parameter c comprises resistances which are proportional to the square of the velocity and originates from losses caused by aerodynamic resistance.

However, parameters a, b, and c are slow time-varying parameters due to weather, route, and train conditions. Furthermore, the aerodynamic resistance and mechanical resistance are both a source of resistance and a class of noise source, and the train is disturbed by lots of noise in the process of operation.

Thus, the Davis formula is rewritten as follows

$$w_0(t) = a(t) + b(t)v(t) + c(t)v^2(t) + \varepsilon(t), \tag{1}$$

where $v(t)$ is the velocity, $a(t)$, $b(t)$, and $c(t)$ are the parameters to be identified, $\varepsilon(t)$ is the noise term.

Rewrite (1) in the form of an identification model

$$y(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}(t) + \varepsilon(t), \tag{2}$$

where $y(t) = w_0(t)$ is the basic resistance, $\boldsymbol{\varphi}^{\mathrm{T}}(t) = \left[1, v(t), v^2(t)\right]^{\mathrm{T}}$ is the information vector consisting of the system input-output data, $\varepsilon(t)$ is a stochastic noise with zero mean, and $\boldsymbol{\theta}(t) = [a(t), b(t), c(t)]^{\mathrm{T}}$ is the parameter vector to be identified.

2.2 Data preprocessing

With the basic resistance of the train modelled, according to the model in (2), we need the basic resistance and velocity information for parameter identification.

The velocity is recorded as a function of time by ATO during the train's operation, so the decelaration is obtained by differentiation, and the retarding force is obtained by applying Newton's second law, when the train is allowed to coast on the track. Train resistance (w) is divided into basic resistance (w_0) and additional resistance (including gradient resistance (w_i), curve resistance (w_r), and tunnel resistance (w_s)), so

$$w = w_0 + w_i + w_r + w_s, \tag{3}$$

we can obtain the velocity and acceleration, then the resistance (w) can be computed. Furthermore, when the train is coasting on the track with no tunnel and no curve, we can get the equation (4),

$$w_s = 0,\ w_r = 0, \text{ and } w_i = 1000 \sin\theta, \tag{4}$$

in urban rail transit, the gradient is expressed in $\tan\theta = \mathrm{i}^0/_{00}$ which is very small, so

$$w_i \approx i.$$

Finally, we can get the basic resistance by

$$w_0 = -100\alpha - i, \tag{5}$$

where α is acceleration (m/s^2).

The Yizhuang Line of Beijing Subway is a typical urban rail transit track contains both ground and underground conditions where the train basic resistance will change significantly. The actual data we used in this paper was collected from the Yizhuang Line at the early morning of October, 2016 and January, 2017.

The data is exported directly from the ATO, containing the target distance, the speed limit, the velocity, the train's weight, the slope, and so on. The information is recorded in every 200ms. Then, we compute the acceleration by (6).

$$\alpha = \frac{v_{t+1} - v_t}{T}, \tag{6}$$

where $T = 200$ms, v_t and v_{t+1} are velocity at the sampling point t and $t+1$, respectively.

It shows in Fig. 1 that the coasting acceleration of the train can be negative, positive or zero, which is abnormal. The environment is complex, the interval time 200ms is too short to obtain accurate data, because of measurement error. In order to solve this problem, the segmented processing method is used for data preprocessing.

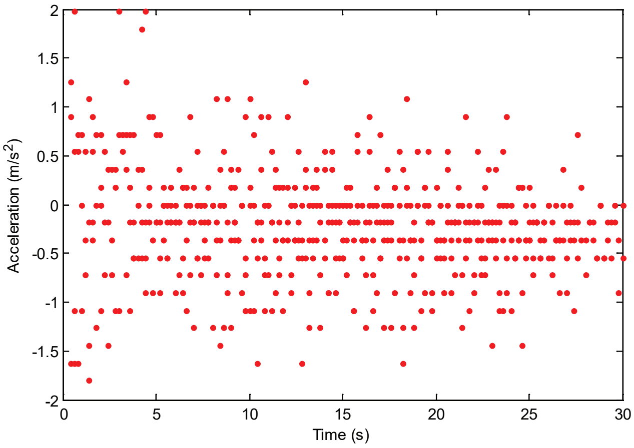

Fig. 1. Unpreprocessed data from ATO system

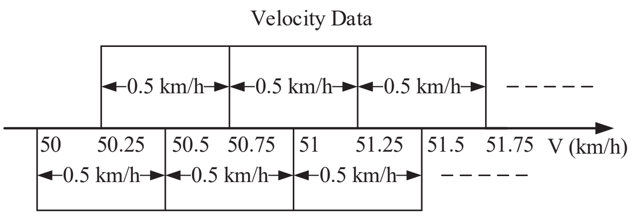

Fig. 2. Method of segmenting data

As shown in Fig. 2, the segmentation interval is 0.5km/h, and the velocity change is 0.5km/h at the end of the segmentation, then the second section starts from the middle of the first section and extends 0.5km/h. The velocity data of each segment is obtained, and the acceleration of each section is computed by the least squares estimation method. The basic resistance is calculated by the (5), then we can identify the parameters.

3. PARAMETER IDENTIFICATION

Rewrite the identification model in section 2 as

$$y(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}(t) + \varepsilon(t),$$

where $y(t) = w_0(t)$ is the basic resistance, $\boldsymbol{\varphi}^{\mathrm{T}}(t) = \left[1, v(t), v^2(t)\right]^{\mathrm{T}}$ is the information vector consisting of the system input-output data, $\varepsilon(t)$ is a stochastic noise with zero mean, and $\boldsymbol{\theta}(t) = [a(t), b(t), c(t)]^{\mathrm{T}}$ is the parameter vector to be identified. This model is a linear parameter model.

Least squares (LS) algorithm is effective for linear and nonlinear parametric systems identification. We can get the LS algorithm by minimizing the Euclidean distance.

$$\hat{\boldsymbol{\theta}} = \left[\sum_{t=1}^{L} \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t)\right]^{-1} \sum_{t=1}^{L} \boldsymbol{\varphi}(t)\, y(t).$$

We use this algorithm to identify the basic resistance parameter, and obtain the results

$$a = 23.74,\ b = -6.99 \times 10^{-1},\ c = 7.23 \times 10^{-3}.$$

As the estimation result and the Fig. 3 show, the parameter b is negative, but the basic resistance can not be negative. Every parameter of the basic resistance has its own meaning, and the LS algorithm can not be used for

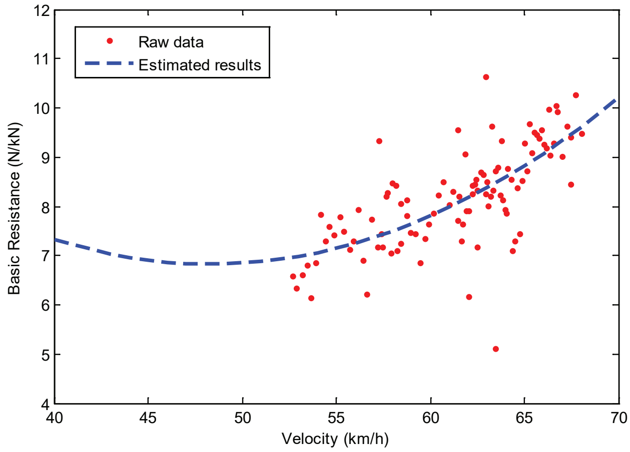

Fig. 3. Traditional least squares algorithm

constrained identification directly. The regularization form of LS algorithm can be used to solve this problem (see Haykin (2009)), the new criterion function is

$$J(\boldsymbol{\theta}) = \sum_{t=1}^{N} \left(y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}\right)^2 + \frac{\lambda}{2}\,\|\boldsymbol{\theta}\|_2^2,$$

where $\sum_{t=1}^{N} \left(y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}\right)^2$ is the criterion of traditional least squares erstimation, $\|\boldsymbol{\theta}\|_2$ is the L_2 norm of $\boldsymbol{\theta}$, and N is the length of data, λ is restraint constant, so we obtain the regularized least squares (LS) algorithm

$$\hat{\boldsymbol{\theta}} = \left[\sum_{t=1}^{N} \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t) + \lambda \boldsymbol{I}\right]^{-1} \sum_{t=1}^{N} \boldsymbol{\varphi}(t)\, y(t). \tag{7}$$

We use this method to enhance the generalization ability of the algorithm, and obtain the following results:

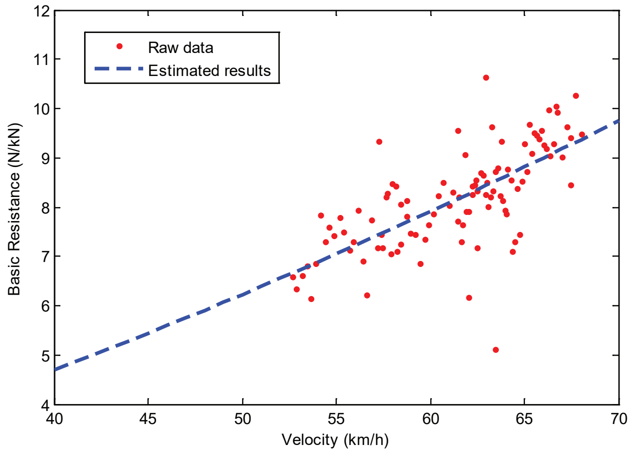

Fig. 4. Regularized least squares algorithm

$$w_0 = 5.15 \times 10^{-1} + 8.597 \times 10^{-2} v + 7.512 \times 10^{-4} v^2. \tag{8}$$

From the Fig. 4 and the equation (8), it can be seen that the identification results are more consistent with the actual situation.

The least squares algorithm performs well in the off-line estimation, but the basic resistance changes with the line conditions, the weather, and the train's own conditions.

Therefore, it is necessary to estimate the train basic resistance parameter online so that the ATO system can adjust its control with more precise information.

The recursive least squares (RLS) algorithm can be used for online identification (see Goodwin and Sin (1984)). Compared with the traditional least squares algorithm, it is more efficient computationally if we update the estimates as new data becomes available online. The recursive least squares algorithm is given by

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)\left[y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1)\right], \quad (9)$$

$$\boldsymbol{L}(t) = \boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)\left[1 + \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)\right]^{-1}, \quad (10)$$

$$\boldsymbol{P}(t) = [\boldsymbol{I} - \boldsymbol{L}(\mathrm{t})\boldsymbol{\varphi}^{\mathrm{T}}(t)]\boldsymbol{P}(t-1), \boldsymbol{P}(0) = p_0\boldsymbol{I}, \quad (11)$$

where $\hat{\boldsymbol{\theta}}(t)$ is the estimate of $\hat{\boldsymbol{\theta}}$ at time t, $\hat{\boldsymbol{\theta}}(0) = \mathbf{1}_3/p_0$, $\mathbf{1}_3$ is a 3-dimensional column vector whose elements are 1, $\boldsymbol{P}(t) \in \mathbb{R}^{n\times n}$ is the covariance matrix, p_0 is taken to be a large positive number (e.g. $p_0 = 10^6$), $\boldsymbol{I}$ is identity matrix of suitable order (here, the order of $\boldsymbol{I}$ is 3), and the scalar value $e(t) = y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1) \in \mathbb{R}^1$ was defined as innovation. Equation (9) updates the estimates at each step using the innvation $e(t)$.

Based on (7) and the derivation process of RLS algorithm, in this paper, we derive the regularization form of RLS algorithm (equation (12)-(16)).

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)e(t) - \frac{\lambda}{N}\boldsymbol{P}(t)\hat{\boldsymbol{\theta}}(t-1), \quad (12)$$

$$\boldsymbol{L}(t) = \boldsymbol{Q}(t)\boldsymbol{\varphi}(t)\left[1 + \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{Q}(t)\boldsymbol{\varphi}(t)\right]^{-1}, \quad (13)$$

$$\boldsymbol{Q}(t) = \frac{N}{\lambda}\boldsymbol{P}(t-1)\left[\frac{N}{\lambda}\boldsymbol{I} + \boldsymbol{P}(t-1)\right]^{-1}, \quad (14)$$

$$\boldsymbol{P}(t) = [\boldsymbol{I} - \boldsymbol{L}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t)]\boldsymbol{Q}(t), \quad \boldsymbol{P}(0) = p_0\boldsymbol{I}. \quad (15)$$

$$e(t) = y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1). \quad (16)$$

The flowchart of the algorithm is shown in Fig. 5. We apply this algorithm to the parameter identification of the train basic resistance.

Ding et al. (2010) extended the RLS algorithm from the view point of innovation modification and proposed the multi-innovation least squares (MILS) algorithm. The essential idea is to expand the scalar innovation $e(t)$ to an innovation matrix

$$\boldsymbol{E}(p,t) = \begin{bmatrix} y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\widehat{\boldsymbol{\theta}}(t-1) \\ y(t-1) - \boldsymbol{\varphi}^{\mathrm{T}}(t-1)\widehat{\boldsymbol{\theta}}(t-1) \\ \cdot \\ \cdot \\ \cdot \\ y(t-p+1) - \boldsymbol{\varphi}^{\mathrm{T}}(t-p+1)\widehat{\boldsymbol{\theta}}(t-1) \end{bmatrix} \in \mathbb{R}^p,$$

where the length of the innovation is p, the input and output are also changed to the vector form

$$\boldsymbol{\Phi}(p,t) = [\boldsymbol{\varphi}(t), \boldsymbol{\varphi}(t-1), ..., \boldsymbol{\varphi}(t-p+1)] \in \mathbb{R}^{n\times p},$$

$$\boldsymbol{Y}(p,t) = [y(t), y(t-1), ..., y(t-p+1)]^{\mathrm{T}} \in \mathbb{R}^p.$$

Then,

$$\boldsymbol{E}(p,t) = \boldsymbol{Y}(p,t) - \boldsymbol{\Phi}^{\mathrm{T}}(p,t)\widehat{\boldsymbol{\theta}}(t-1).$$

The MILS algorithm with the innovation length p was given as follows

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)\left[\boldsymbol{Y}(p,t) - \boldsymbol{\Phi}^{\mathrm{T}}(p,t)\hat{\boldsymbol{\theta}}(t-1)\right],$$

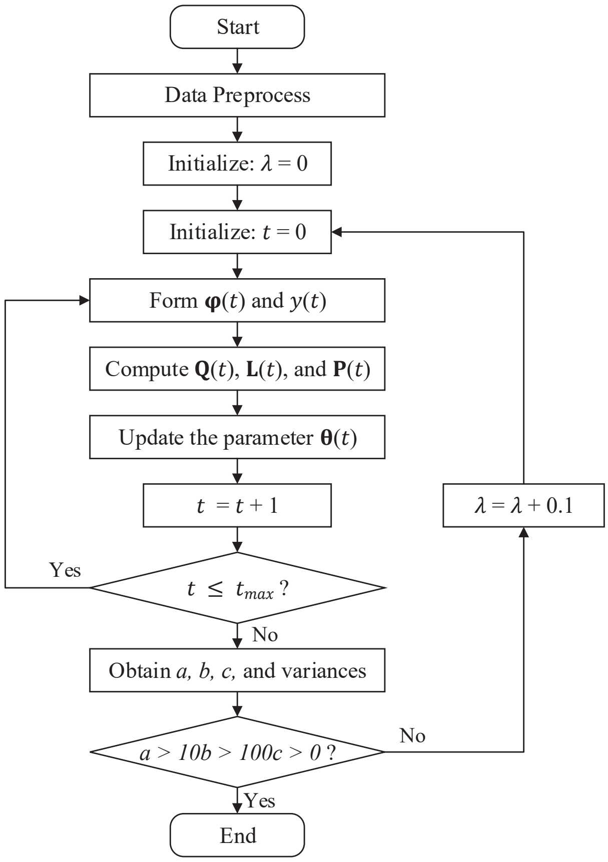

Fig. 5. The flowchart of the regularized RLS algorithm.

$$\boldsymbol{L}(t) = \boldsymbol{P}(t-1)\boldsymbol{\Phi}(p,t)\left[\boldsymbol{I}_p + \boldsymbol{\Phi}^{\mathrm{T}}(p,t)\boldsymbol{P}(t-1)\boldsymbol{\Phi}(p,t)\right]^{-1},$$

$$\boldsymbol{P}(t) = \boldsymbol{P}(t-1) - \boldsymbol{L}(t)\boldsymbol{\Phi}^{\mathrm{T}}(p,t)\boldsymbol{P}(t-1),$$

$$\boldsymbol{\Phi}(p,t) = [\boldsymbol{\varphi}(t), \boldsymbol{\varphi}(t-1), ..., \boldsymbol{\varphi}(t-p+1)],$$

$$\boldsymbol{Y}(p,t) = [y(t), y(t-1), ..., y(t-p+1)]^{\mathrm{T}},$$

where $\boldsymbol{I}_p$ denotes an identity matrix of order p. When $p = 1$, the MILS algorithm reduces to RLS algorithm.

We derive the regularization form of MILS algorithm for train basic resistance parameter estimation.

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)\boldsymbol{E}(p,t) - \frac{\lambda}{N}\boldsymbol{P}(t)\hat{\boldsymbol{\theta}}(t-1),$$

$$\boldsymbol{L}(t) = \boldsymbol{Q}(t)\boldsymbol{\Phi}(p,t)\left[\boldsymbol{I}_p + \boldsymbol{\Phi}^{\mathrm{T}}(p,t)\boldsymbol{Q}(t)\boldsymbol{\Phi}(p,t)\right]^{-1},$$

$$\boldsymbol{Q}(t) = \frac{N}{\lambda}\boldsymbol{P}(t-1)\left[\frac{N}{\lambda}\boldsymbol{I} + \boldsymbol{P}(t-1)\right]^{-1},$$

$$\boldsymbol{P}(t) = [\boldsymbol{I} - \boldsymbol{L}(t)\boldsymbol{\Phi}^{\mathrm{T}}(p,t)]\boldsymbol{Q}(t),$$

$$\boldsymbol{E}(p,t) = \boldsymbol{Y}(p,t) - \boldsymbol{\Phi}^{\mathrm{T}}(p,t)\widehat{\boldsymbol{\theta}}(t-1),$$

$$\boldsymbol{\Phi}(p,t) = [\boldsymbol{\varphi}(t), \boldsymbol{\varphi}(t-1), ..., \boldsymbol{\varphi}(t-p+1)],$$

$$\boldsymbol{Y}(p,t) = [y(t), y(t-1), ..., y(t-p+1)]^{\mathrm{T}}.$$

The MILS algorithm extends the length of the innovation. Because the MILS algorithm use not only the current data but also the past data at each recursive step, parameter estimation accuracy can be improved, and the algorithm can also be used for online estimation.

We can obtain the results of the RLS algorithm (or the MILS algorithm with $p=1$) and the MILS algorithm (with $p = 2$, $p = 4$, and $p = 6$) from Fig. 6. For comparison,

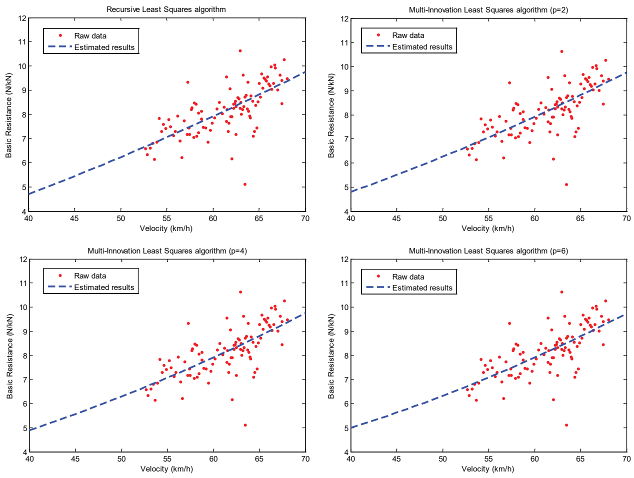

Fig. 6. MILS algorithm with $p = 1$, $p = 2$, $p = 4$, $p = 6$.

table 1 lists the differences between the results of LS algorithm, RLS algorithm and the MILS algorithm.

Table 1. Parameter identification results

Algorithms		a (10^0)	b (10^{-2})	c (10^{-3})	σ^2 (10^{-1})	Online Yes/No
LS		0.515	8.597	0.751	5.632	No
RLS		0.783	8.511	0.758	5.632	Yes
	$p = 2$	0.932	5.749	0.981	5.623	Yes
MILS	$p = 4$	1.638	3.591	1.422	5.618	Yes
	$p = 6$	2.194	1.941	1.262	5.613	Yes

In table 1, a, b, and c are basic resistance parameters, and σ^2 represents the variance.

The given results show that with the increase of the length of the innovation, the variance of the identification results is decreasing, so the MILS algorithm can achieve better performance than RLS algorithm. Both RLS and MILS algorithms can be used for online identification.

4. ANALYSIS AND VERIFICATION

4.1 Verification of results

We use a set of data for validation. The basic resistance is calculated using the identification results obtained by the LS and MILS algorithms, and then we use the basic resistance to calculate the acceleration and the velocity by (5) and (6). We compare the velocities calculated from LS algorithm and MILS algorithm with the actual velocity and obtain the Fig. 7 and table 2.

Table 2. Comparison of different algorithms

Algorithms	Variances ($\sigma^2 = 10^{-2}$)
LS	0.99
MILS	0.88

The simulation results shows that the estimation results become more accurate by applying the MILS algorithm to the basic resistance parameter estimation.

Parameter online identification results are depicted in Fig. 8, it can be observed from this figure that after about 70 recursions, the value of a, b, and c no longer change significantly, the stabilization of parameters demonstrate the real-time characteristic and validity of the algorithm.

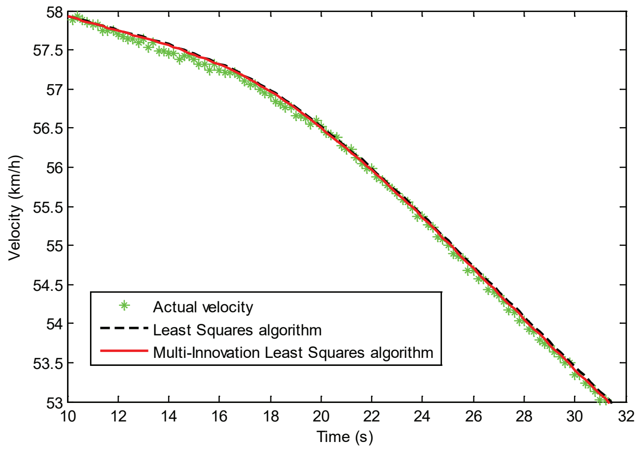

Fig. 7. Comparison of LS and MILS algorithms.

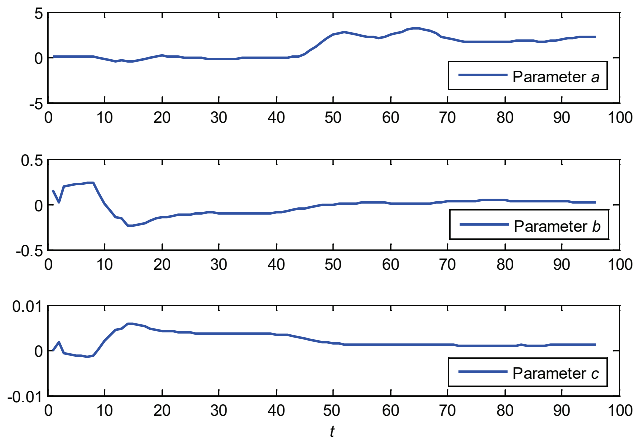

Fig. 8. Parameter online identification.

Generally, the least squares algorithm and RLS algorithm have fast convergence properties and high precision, and the usage of the data is efficient. Thus, sometimes we use the MILS algorithm, the improvement of the parameter estimation accuracy is limited, and we can use LS algorithm for off-line identification and RLS algorithm for online identification. However, the condition of the train and the track is complex, so it is inevitable that the collected data is often discontinuous and abnormal because of the unexpected interruption and measurement error. The multi-innovation theory based identification algorithm has strong robustness, and can improve the accuracy of identification greatly when we encounter this abnormal situation.

4.2 Analysis of robustness

There are strict restrictions on the data used for parameter estimation. The results of parameter identification can be more accurate only when the train coasts on the level tangent track. Therefore, the data used for basic resistance parameter estimation collected by ATO usually discontinuous. The train operating on track with complex conditions, ATO often collects discontinuous data and anomalous data because of the unexpected interruption and measurement error. These phenomena are inevitable and will increase the difficulty of parameter identification especially online identification.

The multi-innovation theory based identification algorithm also has good performance in the case of anomalous data and missing-data and can provide fast convergence and high parameter estimation accuracy, the interval-varying multi-innovation least squares (V-MILS) algorithm was given by Ding et al. (2010),

$$\begin{aligned}
\hat{\boldsymbol{\theta}}(t_s) &= \hat{\boldsymbol{\theta}}(t_{s-1}) + \boldsymbol{L}(t_s)\left[\boldsymbol{Y}(p,t_s) - \boldsymbol{\Phi}^{\mathrm{T}}(p,t_s)\,\hat{\boldsymbol{\theta}}(t_{s-1})\right],\\
\mathbf{L}(t_s) &= \mathbf{P}(t_{s-1})\,\boldsymbol{\Phi}(p,t_s)\\
&\quad\times\left[\mathbf{I}_p + \boldsymbol{\Phi}^{\mathrm{T}}(p,t_s)\,\mathbf{P}(t_{s-1})\,\boldsymbol{\Phi}(p,t_s)\right]^{-1},\\
\boldsymbol{P}(t_s) &= \boldsymbol{P}(t_{s-1}) - \boldsymbol{L}(t_s)\,\boldsymbol{\Phi}^{\mathrm{T}}(p,t_s)\,\boldsymbol{P}(t_{s-1}),\\
\boldsymbol{\Phi}(p,t_s) &= [\boldsymbol{\varphi}(t_s), \boldsymbol{\varphi}(t_s-1), ..., \boldsymbol{\varphi}(t_s-p+1)],\\
\boldsymbol{Y}(p,t_s) &= [y(t_s), y(t_s-1), ..., y(t_s-p+1)]^{\mathrm{T}},
\end{aligned}$$

where $0 = t_0 < t_1 < t_2 < ...$, and $1 \leq t_s^* = t_s - t_{s-1}$. The parameter estimate $\hat{\boldsymbol{\theta}}(t)$ is updated only at instant $t = t_s$, and so is the convariance matrix $\boldsymbol{P}$. The V-MILS algorithm computes the parameter estimates using the interval-varying iteration, so it can overcome the affect of bad data on the parameter estimates.

We also derive the regularization form of V-MILS algorithm for the train basic resistance parameter estimation

$$\begin{aligned}
\hat{\boldsymbol{\theta}}(t_s) &= \hat{\boldsymbol{\theta}}(t_{s-1}) + \boldsymbol{L}(t_s)\boldsymbol{E}(p,t_s) - \frac{\lambda}{N}\boldsymbol{P}(t_s)\hat{\boldsymbol{\theta}}(t_{s-1}),\\
\boldsymbol{L}(t_s) &= \boldsymbol{Q}(t_s)\boldsymbol{\Phi}(p,t_s)[\boldsymbol{I}_p + \boldsymbol{\Phi}^{\mathrm{T}}(p,t_s)\boldsymbol{Q}(t_s)\boldsymbol{\Phi}(p,t_s)]^{-1},\\
\boldsymbol{Q}(t_s) &= \frac{N}{\lambda}\boldsymbol{P}(t_{s-1})\left[\frac{N}{\lambda}\boldsymbol{I} + \boldsymbol{P}(t_{s-1})\right]^{-1},\\
\boldsymbol{P}(t_s) &= [\boldsymbol{I} - \boldsymbol{L}(t_s)\boldsymbol{\Phi}^{\mathrm{T}}(p,t_s)]\boldsymbol{Q}(t_s),\\
\boldsymbol{E}(p,t_s) &= \boldsymbol{Y}(p,t_s) - \boldsymbol{\Phi}^{\mathrm{T}}(p,t_s)\hat{\boldsymbol{\theta}}(t_{s-1}),\\
\boldsymbol{\Phi}(p,t_s) &= [\boldsymbol{\varphi}(t_s), \boldsymbol{\varphi}(t_s-1), ..., \boldsymbol{\varphi}(t_s-p+1)],\\
\boldsymbol{Y}(p,t_s) &= [y(t_s), y(t_s-1), ..., y(t_s-p+1)]^{\mathrm{T}}.
\end{aligned}$$

We chose a set of data that contain anomalous data collected from the Yizhaung Line for verification.

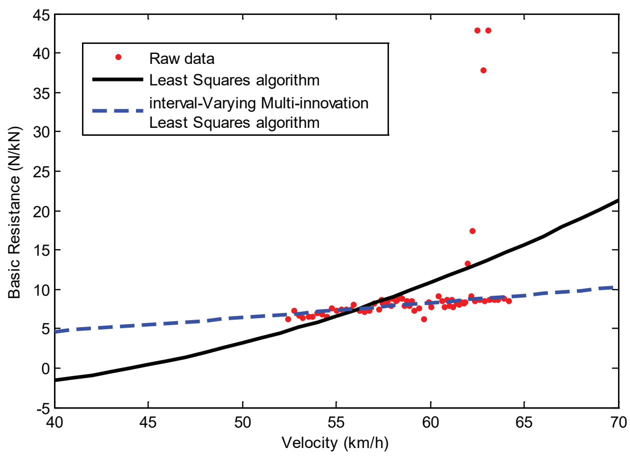

Fig. 9. Comparison of LS and V-MILS algorithms.

The given results in Fig. 9 show that the performance of traditional LS algorithm is poor in the case anomalous data, but the V-MILS algorithm can skip the anomalous data so that we can get more reliable parameters.

5. CONCLUSION

The multi-innovation theory based identification algorithm can improve the parameter estimation accuracy, we applied it to the basic resistance parameter estimation using the data collected from Yizhaung Line of Beijing Subway. By comparing the MILS algorithm and V-MILS algorithm with traditional least squares algorithm, we conclude that the multi-innovation least squares algorithm can not only provide high estimation accuracy but also perform well in the case of missing data and anomalous data.

The MILS algorithm and V-MILS algorithm can also be used for online identification, which will make the train basic resistance parameter estimation simpler and more accurate, and will reduce the expense of time, labor, and material for organizing field test.

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