

Controller Design and Sparse Measurement Selection in Self-optimizing control

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Abstract: Self-optimizing control focuses on minimizing loss for processes in the presence of disturbances by holding selected controlled variables at constant set-points. A measurement combination can be found, using the Null-space method, which further reduces the loss. Since self-optimizing control focuses on the steady-state operation, little attention has been put on the dynamic performance when selecting measurement combinations. In this work, an iterative LMI approach is combined with the sparsity promoting weighted l_1 -norm, to find a measurement subset together with PI controllers for the Null-space method. The measurement combination and the controllers are designed such that, the dynamic response is improved when the process is facing disturbances. The proposed method is illustrated on a Petlyuk column case study.

Keywords: Self-optimizing control, Static output feedback, LMI, Petlyuk distillation column

1. INTRODUCTION

Increasing demands for efficient operation and profitability in the industry require a better understanding of the steady-state operation and the dynamics of the process. This often leads to more advanced control systems, e.g., Model Predictive Controllers (MPC) and Real-time optimizers (RTO). Depending on the model size and complexity, this can become very computational demanding.

Another approach is to use simple control structures that keep specific controlled variables (CVs) at a constant value, also known as self-optimizing control (Skogestad, 2000). The central idea of self-optimizing control is to select CVs such that in the presence of disturbances, the loss is minimized by holding them at constant set-points.

Besides using single measurements as the CVs, selecting linear combinations of measurements will further improve the self-optimizing control performance. Two methods that achieve this are the Exact local method (Halvorsen et al., 2003) and the Null-space method (Alstad and Skogestad, 2007). For a given set of measurements, both the Null-space and the Exact local method offers an infinite number of possibilities for choosing the linear measurement combinations. Using all measurements available will, in theory, result in the lowest loss but, this increases the risk of getting sensor failures and makes implementing the control structure more difficult. Usually, there exists a subset of the available measurements, that can be used without any significant increase in loss. However, selecting the optimal subset of measurements is a combinatorial optimization problem and, every possible alternative needs to be evaluated. This leads to a huge computational complexity when there is a large number of measurements.

To find the best subset of measurements Kariwala and Cao (2009) derived a branch and bound method, while in (Yelchuru and Skogestad, 2012) a mixed integer quadratic programming approach was used. However, these methods, together with most research on self-optimizing control are mainly concerned with the steady-state operation without considering the dynamic performance.

Based on (Peaucelle and Arzelier, 2001), Klemets and Hovd (2017) proposed an iterative linear matrix inequality (LMI) approach to select the measurement combination and PI controllers that improves the dynamic response while maintaining the self-optimizing control properties. In this work, this concept is expanded on by including a penalty function in the optimization problem that promotes sparsity by penalizing the number of measurements used. The sparsity promoting function is known as the weighted l_1 -norm (Candes et al., 2008), and has been used in several papers for promoting sparsity in controller design, see, e.g., (Dhingra et al., 2014, Fardad et al., 2011, and Dörfler et al., 2014). The proposed method is validated by application to a model of a Petlyuk distillation column.

The Petlyuk distillation column often referred to as the divided wall column (DWC) offers an appealing alternative for separating ternary mixtures. In comparison to the traditional configuration, where two columns are used in series, the Petlyuk column is capable of saving up to 30% in both capital and energy costs according to Triantafyllou and Smith (1992). However, despite the potential benefits of Petlyuk columns, only a few implementations exist in the industry due to their operational challenges. Adrian et al. (2004) claim that it is difficult to control using decentralized strategies and suggests an MPC should be used to achieve good controllability. Therefore, this works as

an interesting case study for designing a control structure using the proposed method.

The paper is organized as follows. Section II briefly summarises the concept of self-optimizing control and the Null-space method. The main method is presented in section III and will be applied to a Petlyuk distillation column model in section IV. Finally, a conclusion is given in section V.

Notations: Let $\mathbb{R}^{n \times m}$ denote the set of $n \times m$ real matrices. For a matrix A , its transpose is denoted A^T , and A^{-1} denotes its inverse. The symbols $He\{A\}$ indicates $A^T + A$ and A^\perp denotes any matrix of maximum rank that satisfies $A^\perp A = 0$. The identity matrix of suitable dimension is given by I . Finally, let's recall the Elimination lemma (Skelton et al., 1997):

Lemma 1. For $B \in \mathbb{R}^{n \times l}$, $C \in \mathbb{R}^{m \times n}$ and $Q = Q^T \in \mathbb{R}^{n \times n}$, the following conditions are equivalent:

- (1) $\exists X \in \mathbb{R}^{l \times m}$ such that $Q + He\{BXC\} \prec 0$.
- (2) $B^\perp Q B^{\perp T} \prec 0$ and $(C^T)^\perp Q (C^T)^{\perp T} \prec 0$.

2. SELF-OPTIMIZING CONTROL

Self-optimizing control is when an acceptable loss is achieved with constant set-points, without the need to reoptimize for disturbances (Skogestad, 2000). More precisely, the aim is to select controlled variables rather than determining optimal set-points.

The optimization problem for achieving optimal steady-state operation can be formulated as,

$$\min_u J(u, d) \quad (1)$$

where $u \in \mathbb{R}^{nu}$, and $d \in \mathbb{R}^{nd}$ are the inputs and disturbances respectively. It will be assumed that the active constraints have been satisfied and that (1) has been rewritten as an unconstrained optimization problem.

What remains is to determine which of the unconstrained variables (c) should be kept constant by using the remaining available degrees of freedom u , in order to minimize the loss caused by the disturbances d . To be able to quantify the loss for keeping the selected controlled variables at constant values, a method for calculating the worst case loss was derived in (Halvorsen et al., 2003).

2.1 The Null-space method

Rather than selecting single measurements, a further reduction in loss can be accomplished by selecting the CVs as optimal linear measurement combinations $c = Hy$. The matrix $H \in \mathbb{R}^{nu \times ny}$ defines the measurement combinations, and $y \in \mathbb{R}^{ny}$ is the available measurements.

Under the assumption that implementation error (measurement noise) can be neglected and that the number of independent measurements available is greater or equal the number of disturbances plus inputs ($ny \geq nu + nd$), then Alstad and Skogestad (2007) proposed the Null-space method for selecting a measurement combination. This results in a zero local loss by choosing H such that,

$$HF = 0 \quad (2)$$

where F is the sensitivity matrix for the optimal deviations in the measurements (∂y^{opt}) with respect to changes in the disturbances (∂d):

$$F = \frac{\partial y^{opt}}{\partial d} \quad (3)$$

The matrix F can be obtained analytically, but can also be computed numerically, by optimizing the non-linear steady-state model of the plant for selected disturbances.

The solution for the measurements combination, H in (2) is non-unique and, there exists an infinite number of possibilities in selecting H . Therefore, it would be preferable to find a combination that also improves the dynamic performance. However, the resulting closed-loop system is not just dependent on the measurement combination, but also on the feedback controllers. A method for simultaneously obtaining the H matrix and PI parameters will be presented in the next section.

3. STATIC OUTPUT FEEDBACK CONTROL

For a given set of measurements, Klemets and Hovd (2017) proposed an iterative LMI algorithm for finding H and corresponding PI controllers that minimizes the H_2 -norm of the resulting closed-loop system. This paper uses the H_∞ -norm as the performance measure but, instead of optimizing for a given set of measurements, it aims to expand on the previous work by including a sparsity promoting penalty function that penalizes the number of measurements used. Therefore, the aim is to find the PI controllers and the dynamically optimal subset of measurements when using the Null-space method.

3.1 Process model

Consider a system described by the continuous linear time-invariant state-space model,

$$\dot{x}(t) = A_x x(t) + B_u u(t) \quad (4)$$

$$y(t) = C_{yx} x(t) \quad (5)$$

where $x \in \mathbb{R}^{nx}$, $u \in \mathbb{R}^{nu}$ and $y \in \mathbb{R}^{ny}$ are the states, inputs, and measurements respectively. The aim is to find a measurement combination matrix H and design decentralized PI controllers of the form:

$$u(t) = k_p H y(t) + k_i H \int_0^t y(\tau) d\tau \quad (6)$$

The system in (4) and (5) can be augmented to include the integrating states from the decentralized PI controllers:

$$\bar{A}_x = \begin{bmatrix} A_x & B_u \\ 0 & 0 \end{bmatrix}, \quad \bar{B}_u = \begin{bmatrix} B_u & 0 \\ 0 & I \end{bmatrix}, \quad \bar{C}_{yx} = [C_{yx} \ 0]$$

The closed loop system with self-optimizing control and decentralized PI controllers can thus be given by,

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}_x \bar{x}(t) + \bar{B}_u \bar{u}(t) + \bar{B}_w \bar{w}(t) \\ \bar{z}(t) = \bar{C}_{zx} \bar{x}(t) + \bar{D}_{zu} \bar{u}(t) + \bar{D}_{zw} \bar{w}(t) \\ \bar{y}(t) = \bar{C}_{yx} \bar{x}(t) + \bar{D}_{yw} \bar{w}(t) \end{cases} \quad (7)$$

where $\bar{x} \in \mathbb{R}^{(nx+nu)}$, $\bar{u} \in \mathbb{R}^{2nu}$, $\bar{y} \in \mathbb{R}^{ny}$, $\bar{w} \in \mathbb{R}^{nw}$, and $\bar{z} \in \mathbb{R}^{nz}$ are the augmented state, control input, measurement output, disturbance, and controlled output vectors respectively. For $K = \text{diag}(k_{p1}, \dots, k_{pnu}, k_{i1}, \dots, k_{inu})$, and $\Gamma = [I \ I]^T$, the control input $\bar{u}(t)$ can be given by:

$$\bar{u}(t) = K \Gamma H \bar{C}_{yx} \bar{x}(t) + K \Gamma H \bar{D}_{yw} \bar{w}(t). \quad (8)$$

3.2 H_∞ static output feedback in self-optimizing control

The H_∞ optimal control problem consists of minimizing the H_∞ -norm of the closed-loop system from exogenous disturbance signals $\bar{w}(t)$ to the controlled output signals $\bar{z}(t)$. Defining the closed-loop matrix as,

$$T_{w,z} := \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix} \quad (9)$$

with the closed loop matrices,

$A_{cl} = \bar{A}_x + \bar{B}_u K \Gamma H \bar{C}_{yx}$, $B_{cl} = \bar{B}_d + \bar{B}_u K \Gamma H \bar{D}_{yw}$, $C_{cl} = \bar{C}_{zx} + \bar{D}_{zu} K \Gamma H \bar{C}_{yx}$, $D_{cl} = \bar{D}_{zw} + \bar{D}_{zu} K \Gamma H \bar{D}_{yw}$. The objective is to find K and H such that $\|T_{w,z}\|_\infty$ is minimized. The H_∞ -norm has several interpretations regarding performance. One is that it minimizes the peak of the singular value of $T_{w,z}(j\omega)$. Alternatively, from a time domain interpretation, it can be considered as the worst-case 2-norm:

$$\|T_{w,z}\|_\infty = \max_{\bar{w}(t) \neq 0} \frac{\|\bar{z}(t)\|_2}{\|\bar{w}(t)\|_2} \quad (10)$$

Finding an H_∞ optimal controller can be difficult and therefore, in practice, it is often easier to design a sub-optimal one, by obtaining the minimum upper bound γ that satisfies $\|T_{w,z}\|_\infty < \gamma$. According to the well-known Bounded Real Lemma, $T_{w,z}$ is asymptotically stable and $\|T_{w,z}\|_\infty < \gamma$ if and only if there exists a symmetric matrix $P \succ 0$ such that the following inequality holds:

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} + C_{cl}^T C_{cl} & P B_{cl} + C_{cl}^T D_{cl} \\ B_{cl}^T P + D_{cl}^T C_{cl} & D_{cl}^T D_{cl} - \gamma^2 I \end{bmatrix} \prec 0 \quad (11)$$

A new parametrization for H_2 static output feedback (SOF) control was introduced in (Peaucelle and Arzelier, 2001) where the conservativeness was reduced by including slack variables. An extension to the parametrization for the H_∞ SOF case was given in (Arzelier and Peaucelle, 2002) and (Ebihara et al., 2015) from which the following linear function can be defined.

$$M(P, \gamma) = \begin{bmatrix} I & 0 & 0 \\ \bar{A}_x & \bar{B}_w & \bar{B}_u \end{bmatrix}^T \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ \bar{A}_x & \bar{B}_w & \bar{B}_u \end{bmatrix} \quad (12)$$

$$+ \begin{bmatrix} \bar{C}_{zx} & \bar{D}_{zw} & \bar{D}_{zu} \\ 0 & I & 0 \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \bar{C}_{zx} & \bar{D}_{zw} & \bar{D}_{zu} \\ 0 & I & 0 \end{bmatrix}$$

An H_∞ optimal solution for K and H that satisfies the self-optimizing control principle for the Null-space method, while ensuring a stable closed-loop system for (9) can be obtained from the following theorem.

Theorem 1. There exist PI controllers and a measurement combination $H \in \mathbb{R}^{nu \times ny}$ that gives a stable closed-loop system and minimizes γ while achieving $\|T_{w,z}\|_\infty \leq \gamma$, if there exists matrices $K_1 \in \mathbb{R}^{2nu \times (nx+nu)}$, $K_2 \in \mathbb{R}^{2nu \times (nx+nu)}$, a diagonal matrix $X = K^{-1} \in \mathbb{R}^{2nu \times 2nu}$ and a matrix $P = P^T \in \mathbb{R}^{(nx+nu) \times (nx+nu)}$ that solves the following non-convex optimization problem:

$$J_k = \min_{K_1, K_2, P, X, H} \gamma^2 \quad (13)$$

$$\text{subject to: } P \succ 0 \quad (14)$$

$$X = \text{diag}(x_1 \dots x_{2nu}) \quad (15)$$

$$HF = 0 \quad (16)$$

$$M(P, \gamma) + He \left\{ \begin{bmatrix} K_1 \\ K_2 \\ -I \end{bmatrix} \begin{bmatrix} \Gamma H \bar{C}_{yx} & \Gamma H \bar{D}_{yw} & X \end{bmatrix} \right\} \prec 0 \quad (17)$$

Proof. Similar to the proof in (Arzelier and Peaucelle, 2002), the inequality in (11) can be rewritten as:

$$\begin{bmatrix} I & 0 & \bar{C}_{yx}^T H^T \Gamma^T K^T \\ 0 & I & \bar{D}_{yw}^T H^T \Gamma^T K^T \end{bmatrix} M(P, \gamma) \begin{bmatrix} I & 0 & \bar{C}_{yx}^T H^T \Gamma^T K^T \\ 0 & I & \bar{D}_{yw}^T H^T \Gamma^T K^T \end{bmatrix}^T \prec 0. \quad (18)$$

According to Lemma 1, (18) is equivalent to

$$M(P, \gamma) + He \left\{ \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \begin{bmatrix} -K \Gamma H \bar{C}_{yx} & -K \Gamma H \bar{D}_{yw} & I \end{bmatrix} \right\} \prec 0, \quad (19)$$

with the matrices $Z_1 \in \mathbb{R}^{(nx+nu) \times 2nu}$, $Z_2 \in \mathbb{R}^{(nx+nu) \times 2nu}$ and, $Z_3 \in \mathbb{R}^{2nu \times 2nu}$. Factorizing Z_3 gives,

$$M(P, \gamma) + He \left\{ \begin{bmatrix} Z_1 Z_3^{-1} \\ Z_2 Z_3^{-1} \\ I \end{bmatrix} \begin{bmatrix} -Z_3 K \Gamma H \bar{C}_{yx} & -Z_3 K \Gamma H \bar{D}_{yw} & Z_3 \end{bmatrix} \right\} \prec 0. \quad (20)$$

Defining $K_1 := Z_1 Z_3^{-1}$, $K_2 := Z_2 Z_3^{-1}$, and $Z_3 := -X$ (where $X = K^{-1}$) gives the expression in (17). ■

The optimization problem in (13)-(17) requires solving a bilinear matrix inequality (BMI), and thus an optimal global solution can't be guaranteed. However, an iterative algorithm can be used to find a local optimum, following the procedure described in Algorithm 1:

Algorithm 1

- (1) Initialize, choose stabilizing state feedback gains K_1 , and K_2 .
- (2) For fixed K_1 , and K_2 solve the LMI:

$$J_{k,1} = \min_{P, X, H} \gamma^2 \quad (21)$$

Subject to: (14), (15), (16), and (17).

- (3) Fix X and H at the values obtained in step 2 and solve the LMI:

$$J_{k,2} = \min_{K_1, K_2, P} \gamma^2 \quad (22)$$

Subject to: (14) and (17).

- (4) If $J_{k,1} - J_{k,2} < \epsilon$ stop, else update K_1 , and K_2 and repeat step 2 and 3.

The controller parameters can be obtained from $K = X^{-1}$.

The initialization variables K_1 and K_2 can be interpreted as a suboptimal solution to a convex, full information H_∞ -control problem (with $\bar{C}_{yx} = [I \ 0]^T$, and $\bar{D}_{yw} = [0 \ I]^T$) and can be obtained using, e.g., (Ebihara et al., 2015).

3.3 Sparse static output feedback design

In the previous section, the problem of finding the H_∞ -optimal PI controllers and measurement combination, H was considered for a given set of measurements. This section aims to find the best measurement subset by including a column-wise sparsity promoting function in the optimization problem. The problem can be formulated as,

$$J_k = \min_{K_1, K_2, P, X, H} \gamma^2 + \lambda \text{card}(H) \quad (23)$$

subject to (14), (15), (16), and (17). The cardinality of the measurement matrix H is defined:

$\text{card}(H) :=$ the number of non-zero columns of H .

The cardinality function is non-convex and non-smooth, making the optimization formulation in (23) a combinatorial problem, which is difficult to solve.

To address this issue, several convex relaxations like the l_1 -norm and the weighted l_1 -norm have been proposed (Candes et al., 2008). By using the weighted l_1 -norm, the cardinality function can be replaced with:

$$f(H) = \sum_{i,j} W_{i,j} |H_{i,j}| \quad (24)$$

Candes et al. (2008) noted that if the weights $W_{i,j}$ are chosen to be inversely proportional to the l_1 -norm, then there is an exact correspondence between the l_1 -norm and the cardinality function. However, this requires *a priori* knowledge of the H matrix, and therefore, a re-weighted scheme needs to be implemented, where the weights are updated after every iteration (k) as,

$$W_{i,j}^{(k+1)} = \frac{1}{|H_{i,j}^{(k)}| + \kappa} \quad (25)$$

where $1 \gg \kappa > 0$ ensures the update is well-defined.

The weighted l_1 -norm in (24) promotes element-wise sparsity. However, it can easily be modified to promote column (or row) sparsity as, e.g., shown in (Argha et al., 2017) by revising it as,

$$f(H) = \sum_{i,j} W_j^{(k)} |H_{i,j}^{(k)}| \quad (26)$$

with the update rule:

$$W_j^{(k+1)} = \frac{1}{\sum_i |H_{i,j}^{(k)}| + \kappa} \quad (27)$$

The following iterative algorithm can then be used to find a subset of the available measurements:

Algorithm 2

- (1) Initialize: Using algorithm 1, obtain K_1 , K_2 , and H for all measurements and compute $W^{(k)}$ using (27).
- (2) For the fixed K_1 , and K_2 solve the LMI:

$$J_k = \min_{P,X,H} \gamma^2 + \lambda \sum_{i,j} W_j^{(k)} |H_{i,j}^{(k)}| \quad (28)$$

Subject to: (14), (15), (16), and (17).

- (3) If $\|H^{(k-1)} - H^{(k)}\| < \epsilon$ stop, else update $W^{(k+1)}$ using (27) and repeat step 2.
- (4) Remove the measurements that correspond to the zero columns in H and repeat algorithm 1.

By varying the value λ there will be a trade-off between the H_∞ performance and number of measurements used. Additional weights may also be included to penalize certain control structures, e.g., if some measurement links are unattractive due to high implementation cost.

4. CASE STUDY: PETLYUK DISTILLATION

The Petlyuk distillation column consists of six sections as illustrated in Fig. 1, with the model information seen in Table 1. Each section is arranged in the same column shell with eight stages for each section. The ternary feed is located between section 1 and 2, and consists of components A , B , and C with the mole fractions z_A , z_B , and z_C . Three product streams are drawn off in the

Table 1. Data for Petlyuk column

Column data	
Relative volatilities	$\alpha^T = [9 \ 3 \ 1]$
Liquid time constant	$\tau_L = 0.063 \text{ min}$
Holdup top and bottom	$M_B = M_D = 20M_i$
Holdup stages	$M_i = 1 \text{ kmol}$
Boiling points A,B,C	$T_B^T = [299.3 \ 342.15 \ 399.3] \text{ K}$
Antoine's parameters	$[2.86 \ -1143 \ -0.349]$
Feed	
Flow	$F = 1 \text{ kmol/min}$
Composition	$z_A = z_B = z_C = 0.33$
Liquid fraction	$q_F = 0.477$
Product specifications	$x_{A,D} = x_{B,S} = x_{C,D} = 0.97 \pm 0.01$
Measurement delays	
Compositions	5 min
Temperatures	1 min

sections 3 - 6, where the light component A dominates the distillate stream (D), component B dominates in the side-stream (S) and the heavy component C dominates the bottom stream (B). For a more detailed description of the model, the reader is referred to (Alstad, 2005).

The distillate boilup (D) and bottom flow rate (B) are used to stabilize the levels in the condenser and the reboiler, respectively. Furthermore, the vapor split R_V will be kept constant as it is difficult to use in practice. Halvorsen et al. (2000) found that good self-optimizing control can still be achieved when R_V is fixed. Therefore, the remaining available degrees of freedom are,

$$u = [L \ V \ S \ R_L]^T \quad (29)$$

corresponding to the reflux, boilup, side-stream flow and liquid split, respectively.

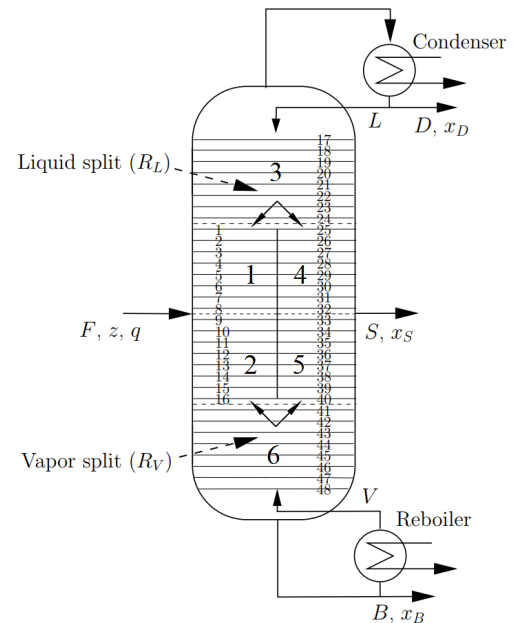


Fig. 1. Petlyuk distillation column

For the Petlyuk column, three product specifications should be kept during operation; distillate purity ($x_{A,D}$), bottom purity ($x_{C,B}$) and side-stream purity ($x_{B,S}$), where $x_{i,j}$ denotes the mole fraction of component i in stream j . The operational objective is, to minimize energy cost (V) while maintaining the product purity specifications.

Controlled variables	PI Parameters
$c_{ind,7} = \begin{bmatrix} -0.1907T_4 + 0.0440T_9 + 1.0000T_{19} - 0.0035T_{24} - 0.4325T_{38} + 0.2973T_{41} + 0.0664T_{47} \\ 0.2699T_4 - 0.0694T_9 - 1.0000T_{19} - 0.0777T_{24} + 0.5713T_{38} - 0.3289T_{41} - 0.3060T_{47} \\ -0.5350T_4 + 0.1889T_9 + 1.0000T_{19} + 0.2622T_{24} - 0.6944T_{38} + 0.4433T_{41} + 0.0636T_{47} \\ 0.2171T_4 + 0.0085T_9 - 0.8688T_{19} - 0.1867T_{24} + 1.0000T_{38} - 0.6313T_{41} - 0.5101T_{47} \end{bmatrix}$	$k_p = 0.4745, k_i = 0.0035$ $k_p = 0.1908, k_i = 0.1563$ $k_p = 0.1763, k_i = 0.0012$ $k_p = 0.0495, k_i = 0.0007$

Table 2. Controlled variables and PI parameters.

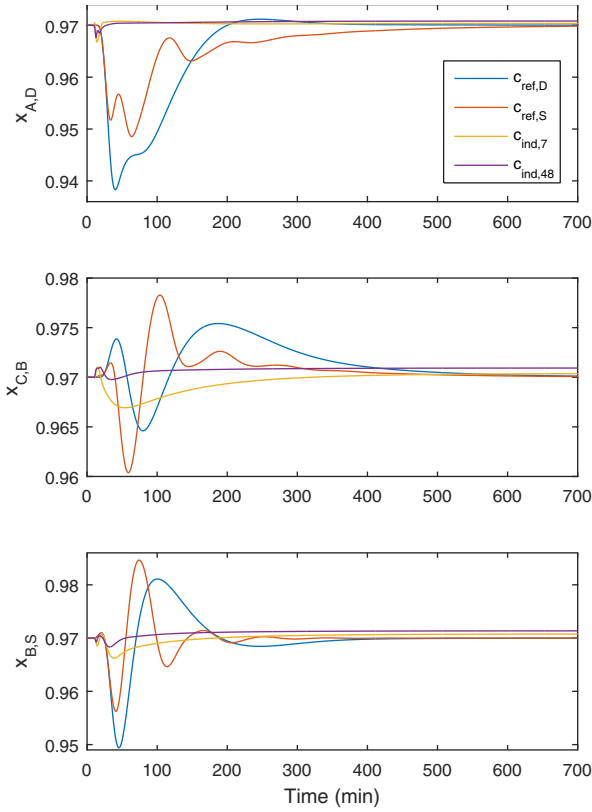


Fig. 2. Step disturbance of -0.1 in z_A .

4.1 Previous control structures

The control structure design for the Petlyuk column model has previously been studied by Alstad (2005), Halvorsen et al. (2000) and Zumoffen et al. (2013). Most of the work has mainly been focused on the steady-state operation of the column. However, decentralized PI controllers were designed by Alstad (2005) for CVs consisting of three composition measurements and a linear combination of temperatures obtained using the Null-space method. The Petlyuk column was also investigated by Zumoffen et al. (2013). For the same CVs, the authors proposed using sparse PI controllers (with partial interactions between the controllers), that further improved the transient performance. These control structures will be used for comparison in the dynamic simulations and are denoted $c_{ref,D}$, and $c_{ref,S}$, with the subscripts, D and S representing the decentralized and sparse control structures respectively.

4.2 Indirect Control

Indirect control is when the primary variables are kept close to their desired value by controlling secondary variables at constant set-points (Hori et al., 2005). The proposed control structures in $c_{ref,D}$ and $c_{ref,S}$, both use the

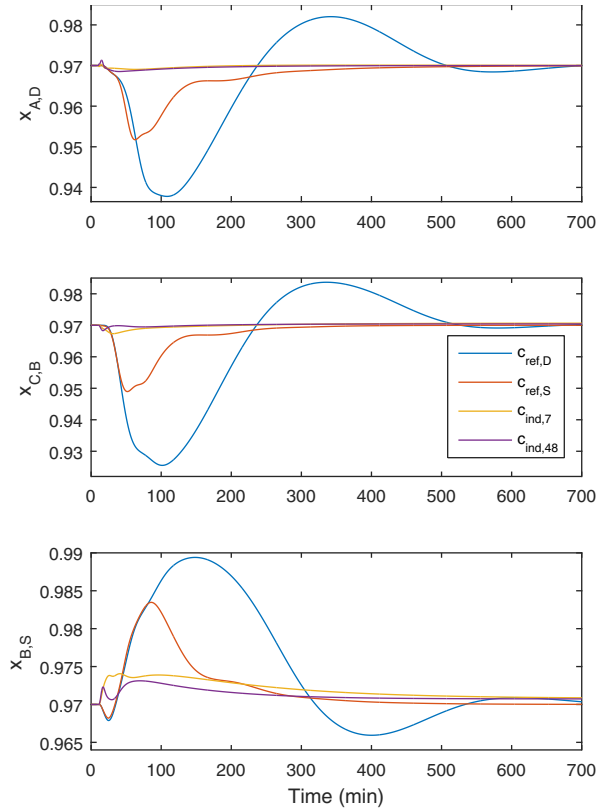


Fig. 3. Step disturbance of $+0.1$ in z_B .

measurements of the three product compositions to keep the products at their targeted values. While this ensures that the purity specifications are kept, the long delays associated with the composition measurements imposes limitations on the closed-loop performance and makes fast control difficult. Therefore, an indirect control structure is proposed, that uses only the temperature measurements.

Perfect indirect control can be achieved using the Null-space method assuming, $ny \geq nd + nu$. In this example, only changes in z_A , z_B , and q_F will be considered as disturbances. Therefore, at least seven independent measurements are required. Using the proposed method, two control structures were computed, using 48 (all), and 7 temperature measurements. These are denoted $c_{ind,48}$, and $c_{ind,7}$ respectively. The values for $c_{ind,7}$ are shown in Table 2, with T_i being the temperature at stage i .

4.3 Dynamic simulation

Dynamic simulations were performed on the non-linear model of the Petlyuk distillation column. In the Figs. 2, 3, and 4, the proposed $c_{ind,48}$, and $c_{ind,7}$ are compared to $c_{ref,D}$, and $c_{ref,S}$ for step disturbances in z_A , z_B , and q_F . Since the sensitivity matrix F in (3) is based on a local linear model, it causes some steady-state loss for $c_{ind,48}$

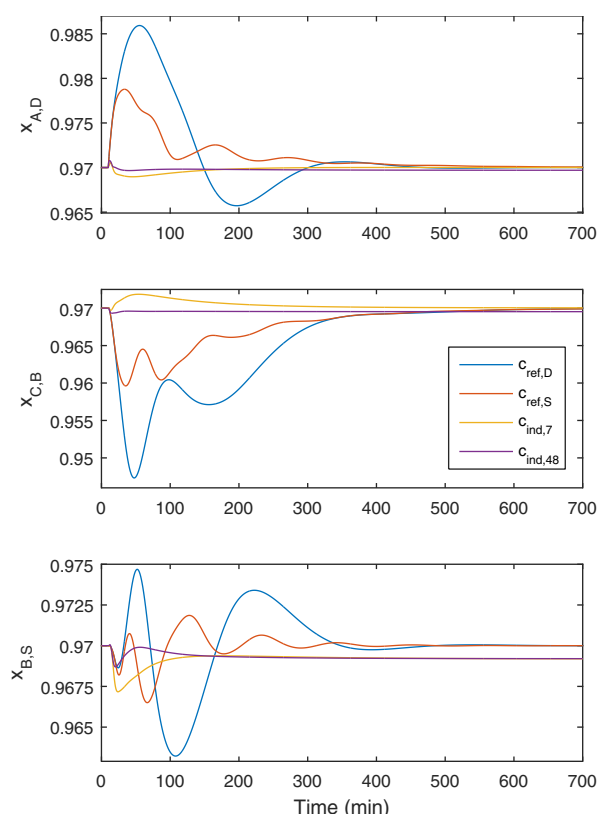


Fig. 4. Step disturbance of +0.1 in q_F .

and $c_{ind,7}$. However, there is a significant improvement in the transient behavior compared to $c_{ref,D}$, and $c_{ref,S}$. The steady-state loss can also easily be removed by, e.g., including an outer loop that adjusts the set-points, using a slow integral action from the composition measurements. As expected, using all 48 temperature measurements gives a better response compared to using 7, since algorithm 2 gives a trade-off between the dynamic performance (H_∞ -norm) and the number of measurements used.

5. CONCLUSION

In this work, the transient behavior for the Null-space method in self-optimizing control was considered. The main objective is to find a subset of available measurement combinations together with PI controllers that minimize the dynamic impact from disturbances. The proposed method was successfully applied to a Petlyuk distillation column model. It was demonstrated that the obtained control structures improve the transient response compared to other existing control structures.

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