

Dead-Time Compensator for State-delay Stable Systems^{*}

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Abstract: This paper deals with the control of simple process models with state-delays. A pre-compensator is used to cancel the delay. Due to the cancellation of some terms, the process as well as its undelayed part should be stable. Although the approach is general, in order to simplify the notation, second order systems are considered to describe the procedure. The proposed methodology is applied to compensate the delay in a recycled reactor as well as to control a pure state-delayed academic example.

Keywords: Time delay systems, state-delay, recycling, dead-time compensation, control design

1. INTRODUCTION

Most practical processes involve the presence of time-delays at the input (if the control variable is a flow), at the output (if the sensor device has some time delay or the measurement is taken with delay) or internal, due to some kind of recirculation. State delays appear when some internal material is recycled to the process input. For instance, in continuous time chemical reactors or in distillations columns, when the product obtained in a single round is not good enough requiring to be treated once more to better extract its properties. Usually, a fraction of the outlet is fed back and mixed to the raw material to go again into the chemical process. In this way, at the process entrance, the raw material is mixed with already treated material, with the properties reached time ago during the initial treatment.

In general, the control system is designed to achieve some given controlled plant performance. As the delay is unavoidable, the purpose of the control should be to modify the dynamic behavior of the rest of the plant. Input/output delays have been extensively studied and there are many approaches to extract the undelayed part of the process and leave the delay out of the process to be studied. These subsystems are called as *dead-time compensators*. The seminal work of Smith, the so-called Smith Predictor (SP), Smith [1959], fully simplifies the problem if the plant model is linear, open-loop stable and its transfer function can be split into a fast part (undelayed) and a delay. Many extensions have been presented to deal with more general cases, including unstable and multi-delay plants (see, for

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instance Normey-Rico and Camacho [2007], Albertos and García [2009]).

For state-delay plants most efforts have been devoted to study the stability properties of the plant and the influence of the delay in these properties. When dealing with linear systems, the characteristic equation involves polynomial as well as exponential terms, and the delays cannot be represented as a factor term in the transfer function, like in the case of input/output delays. Thus, stability analysis and controller design become a difficult task in the general case Kolmanovskii et al. [1999] and a complete solution of this problem has not been reported yet. In a recent paper, Li et al. [2017], a new approach to analyse the stability of state-delayed plants in the frequency domain is presented.

In the single-input-single-output (SISO) linear case, the model of the-delayed plant can be expressed by

$$\dot{x} = Ax(t) + \sum_{i=1}^r D_i x(t - L_i) + bu(t) \quad (1)$$

$$y(t) = cx(t) \quad (2)$$

where L_i $i = 1, 2, \dots, r$ denotes each one of the state time-delays and D_i is the matrix attached to the time-delay L_i . A is the system matrix of the undelayed part, if it exists. In this case, the transfer function of the undelayed part would be

$$y(s) = G_0(s)u(s); \quad G_0(s) = c(sI - A)^{-1}b \quad (3)$$

There is a lot of theoretical research to deal with these processes Kharitonov [1998], Gu and Niculescu [2003], including the case of time varying delays Gao and Chen [2007], Mahmoud [1996], and distributed delays Kharitonov et al. [2009]. To illustrate the problem complexity, let us assume the simplest case where there is a single state time-delay

$$\dot{x}(t) = Ax(t) + Dx(t - L) + bu(t); \quad y(t) = x(t) \quad (4)$$

The classical approach to study the open-loop stability is to analyse the roots of the characteristic equation

$$|sI - A - De^{-sL}| = 0 \quad (5)$$

Moreover, if output feedback control Kimura [1975] is applied, the closed-loop characteristic equation is of the form

$$|sI - A - bKc - De^{-sL}| = 0 \quad (6)$$

and the design of K to get some performance is not straightforward.

As previously mentioned, to deal with input/output time-delayed plants, the classical solution is to design a dead-time compensator, Normey-Rico and Camacho [2009], García et al. [2006], in such a way that an undelayed output/input model is obtained and the control is designed for this model, without paying attention to the delay which is transferred out of the control loop. But, as pointed out in (5), when there are state delays, the characteristic equation includes exponential terms and the analysis and design of the control law is not conventional.

The main goal of this paper is to suggest a dead-time compensator for state-delays plants. For the sake of clarity, SISO plants are considered and a single state delay is assumed initially. Also, the plant model is initially assumed to be a low order one (the state vector dimension is two). In a previous paper, the scalar case was considered, Albertos et al. [2017]. As it will be shown in the proposed solution, the general case defined by (1) is much more complicated. Note that the main contribution is to design a precompensator to obtain a delay-free model of the state-delayed plant. It does not deal with the stability of the state-delayed plant, which is initially assumed to be stable, neither with the design of the control. Once the plant model is reduced to a delay-free model, any suitable control design technique can be applied to the final model.

The rest of the paper is organized as follows. First, the problem is clearly stated. Then the structure of the pre-compensator is defined and some requirements are established. Two examples are considered. First, the procedure is applied to the simplified model of a chemical reactor and then, the control design options are illustrated with a pure delayed plant. Some simulations illustrate the results and, finally, some conclusions are drafted.

2. PROBLEM STATEMENT

The kind of systems to be considered are linear stable SISO plants with state time delays. In order to simplify the notation, low order models (second order as maximum) and a single time delay will be assumed first. That is, the plant model is defined by

$$\dot{x}(t) = Ax(t) + Dx(t-L) + bu(t); \quad y = cx(t) \quad (7)$$

where $x \in \mathcal{R}^2$, $\{y, u\} \in \mathcal{R}$ and A is assumed to be Hurwitz.

The plant transfer function is easily obtained as

$$G(s) = c[sI - A - De^{-sL}]^{-1}b \quad (8)$$

Moreover, the plant (8) is assumed to be stable. Although the time delay L due to a recycling could be time variant

as well as uncertain, here it would be considered constant and known.

For $D = 0$, the undelayed plant is obtained. That is

$$\dot{x}(t) = Ax(t) + bu(t); \quad y = cx(t) \quad (9)$$

whose transfer function is given by (3).

The goal is to find a precompensator $C(s)$ such that the compensated plant, $G(s)C(s)$, is the delay-free model $G_0(s)$, (3).

2.1 Plant transfer function

Denote by $a_{i,j}$ the element of the A -matrix in row i , column j , and the same for any other matrix.

The computation of (8) leads to

$$G(s) = \frac{\alpha_1 s + \alpha_2 + \beta e^{-Ls}}{p_A(s) + f(s)e^{-Ls} + de^{-2Ls}} \quad (10)$$

where

$$\alpha_1 = c_1 b_1 + c_2 b_2 \quad (11)$$

$$\alpha_2 = -c_1 b_1 a_{22} + c_1 b_2 a_{12} + c_2 b_1 a_{21} - c_2 b_2 a_{11}$$

$$\beta = -c_1 b_1 d_{22} + c_1 b_2 d_{12} + c_2 b_1 d_{21} - c_2 b_2 d_{11}$$

$$p_A(s) = s^2 - (a_{22} + a_{11})s + a_{11}a_{22} - a_{12}a_{21}$$

$$f(s) = -(d_{22} + d_{11})s + a_{11}d_{22} - a_{12}d_{21} + a_{22}d_{11} - a_{21}d_{12}$$

$$d = |D| = d_{11}d_{22} - d_{12}d_{21}$$

Note that

$$G_0(s) = \frac{\alpha_1 s + \alpha_2}{p_A(s)} = \frac{\alpha(s)}{p_A(s)} \quad (12)$$

2.2 Transfer function blocks

The plant transfer function (10) can be represented in the following blocks:

$$G(s) = \frac{\alpha_1 s + \alpha_2 + \beta e^{-Ls}}{p_A(s) + f(s)e^{-Ls} + de^{-2Ls}} \quad (13)$$

$$= F_1(s)[F_2(s) + F_3(s)] \quad (14)$$

where

$$F_1(s) = \frac{1}{1 + \frac{f(s)}{p_A(s)}e^{-Ls} + \frac{d}{p_A(s)}e^{-2Ls}} \quad (15)$$

$$F_2(s) = G_0(s) = \frac{\alpha_1 s + \alpha_2}{p_A(s)} \quad (16)$$

$$F_3(s) = \frac{\beta e^{-Ls}}{p_A(s)} \quad (17)$$

See Fig. 1 for the block structure.

Remark 1. If the plant (7) is n -dimensional, that is, $x \in \mathcal{R}^n$, the transfer function, similar to (10), would be

$$G(s) = \frac{\alpha(s) + \sum_{i=1}^{n-1} \beta_i(s)e^{-iLs}}{p_A(s) + \sum_{i=1}^n f_i(s)e^{-iLs}} \quad (18)$$

where $f_n(s) = d = |D|$.

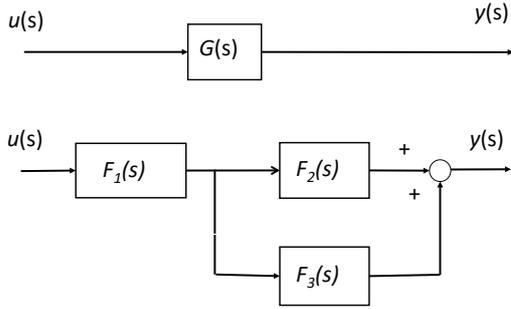


Fig. 1. Block model representation

Remark 2. For multiple time-delays in the state (1), the transfer function would be

$$G(s) = c[sI - A - \sum_{i=1}^r D_i e^{-sL_i}]^{-1} b \quad (19)$$

leading to a fraction similar to (10), such as

$$G(s) = \frac{\alpha(s) + \sum_{i=1}^m \beta_i e^{-L_i s}}{p_A(s) + \sum_{i=1}^h f_i(s) e^{-L_i s}} \quad (20)$$

where h represents the number of different combinations of the initial delays appearing in the determinant of $[sI - A - \sum_{i=1}^r D_i e^{-sL_i}]$ in (19) and m represents the number of different combinations of the initial delays appearing in its adjoint matrix. For instance, for $r = 2$ in (19), $h = 5$ and the different delays appearing in the determinant are $\{L_1, L_2, 2L_1, 2L_2, L_1 + L_2\}$.

As a conclusion from the two previous remarks, if the plant is n -dimensional and/or there are multiple state time-delays, keeping the assumption of open-loop stable plant, the approach is still applicable but the block structure depicted in Fig. 1 becomes much more complicated.

3. COMPENSATOR DESIGN

Following the ideas presented in Luan et al. [2018], in order to design the compensator, two parts are considered such that $C(s) = C_2(s)C_1(s)$.

3.1 Second order systems

Related to the model in Fig. 1, the subsystem $C_1(s)$ will deal with the $F_1(s)$ block. In order to cancel it, this part will be taken as its inverse

$$C_1(s) = \frac{1}{F_1(s)} = 1 + \frac{f(s)}{p_A(s)} e^{-Ls} + \frac{d}{p_A(s)} e^{-2Ls} \quad (21)$$

This precompensator is always realizable as the order of $p_A(s)$ is larger than those of $f(s)$ and d , and the delays are negative. Moreover, it is stable if A is Hurwitz.

Now, let us consider the remaining two blocks.

$$F_2(s) + F_3(s) = G_0(s) \left[1 + \frac{\beta e^{-Ls}}{\alpha(s)} \right] \quad (22)$$

Thus, the second block of the precompensator will be

$$C_2(s) = \left[1 + \frac{\beta e^{-Ls}}{\alpha(s)} \right]^{-1} \quad (23)$$

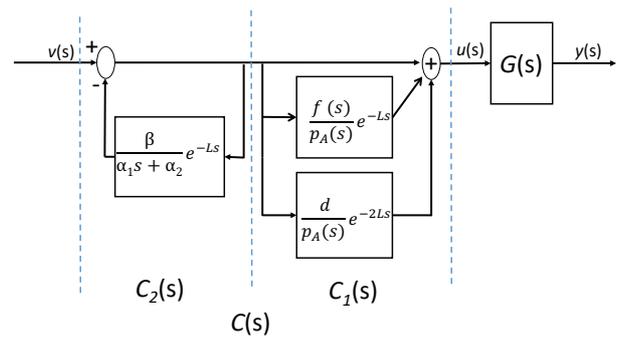


Fig. 2. Precompensator representation

which can be easily implemented as a loop with unitary forward path and $\frac{\beta e^{-Ls}}{\alpha(s)}$ in the feedback path. Again, this term is always realizable as the relative degree is one, but it should be stable.

As a result, the resulting compensated system will be the undelayed part, $G_0(s)$. This result can be expressed by the following theorem.

Theorem 1. Given a stable second order system, as described in (7), with a Hurwitz undelayed system matrix A , and a state delay L , the contribution of the delayed part can be cancelled by using a precompensator $C_1(s)C_2(s)$ as defined in (21),(23) if the zero of the polynomial $\alpha(s)$ (12) is negative.

In summary, the following conditions are required to directly apply the precompensator design:

- (1) The delayed plant should be stable
- (2) The A matrix should be Hurwitz. This condition will be relaxed later on just requiring it to have non positive eigenvalues.
- (3) The zeros of $\alpha(s)$ should be out of the RHP ($\frac{\alpha_2}{\alpha_1} \geq 0$).

The last two conditions require that the undelayed plant should be stable and minimum phase.

The structure of the precompensator can be seen in Fig. 2.

3.2 Pure delayed plant

Let us assume that the system matrix A is null. So, the plant model is

$$\dot{x}(t) = Dx(t - L) + bu(t); \quad y = cx(t) \quad (24)$$

The plant transfer function as well as the precompensator parameters can be computed following the expressions in (10) and (11), assuming that $a_{i,j} = 0, \forall \{i, j\}$.

Thence, the precompensator parameters are:

$$\alpha_1 = c_1 b_1 + c_2 b_2 \quad (25)$$

$$\alpha_2 = 0; \quad \alpha(s) = \alpha_1 s \quad (26)$$

$$\beta = -c_1 b_1 d_{22} + c_1 b_2 d_{12} + c_2 b_1 d_{21} - c_2 b_2 d_{11} \quad (27)$$

$$p_A(s) = s^2; \quad f(s) = -(d_{22} + d_{11})s \quad (28)$$

$$d = |D| = d_{11} d_{22} - d_{12} d_{21} \quad (29)$$

leading to the precompensator components

$$C_1(s) = 1 + \frac{d_{11} + d_{22}}{s} e^{-Ls} + \frac{d}{s} e^{-2Ls} \quad (30)$$

$$C_2(s) = \left[\frac{\alpha_1 s + \beta e^{-Ls}}{\alpha_1 s} \right]^{-1} \quad (31)$$

Note that the precompensated plant is reduced to

$$G_0(s) = \frac{\alpha_1 s}{p_A(s)} = \frac{\alpha_1}{s} \quad (32)$$

that is, an integrator.

3.3 General case

Now, let us consider the general case of multiple state time-delays in a multidimensional system (1). As developed in Remark 1 and Remark 2, the global transfer function has the same structure

$$G(s) = F_1(s)[F_2(s) + F_3(s)]$$

but each block has several elements in parallel, being

$$F_1(s) = \frac{1}{1 + \frac{\sum_{i=1}^h f_i(s) e^{-L_i s}}{p_A(s)}} \quad (33)$$

$$F_2(s) = G_0(s) = \frac{\alpha(s)}{p_A(s)} \quad (34)$$

$$F_3(s) = \frac{\sum_{i=1}^m \beta_i e^{-L_i s}}{p_A(s)} \quad (35)$$

Thence, the following theorem can be stated

Theorem 2. Given a state delay plant with several delays, as described in (1), it can be reduced to the undelayed part of the plant, if it exists, by means of a precompensator similar to the one shown in Fig. 2 with h feedback paths (33) in $C_2(s)$ and m parallel paths (35) in $C_1(s)$. The same assumptions summarized after the previous theorem are required.

Remark 3. If the undelayed part of the plant is null, that is, $A = 0$, some precompensator's elements will exhibit integrative behavior.

4. EXAMPLES

Two examples are considered. First, a simplified model of a continuous stirred tank reactor (CSTR), typical in many chemical processes, is used to illustrate the design procedure. Then, an academic example for a pure delayed system is used to show the control design options for integrative plants.

4.1 Example 1. A CSTR

A CSTR where a simple irreversible reaction $A \rightarrow B$ occurs is a typical process representing many processes involving catalytic reactors and hydrolysis reactions, among others. The issues about the dynamic behavior of CSTRs have been treated in various papers (Teymour [1997], Soroush [1997]) and books (Luyben [1990], Marlin [1995]). Non-isothermal CSTR can show three steady states. In order to apply the proposed methodology, a stable operating point will be selected. In practice, the reaction is not complete

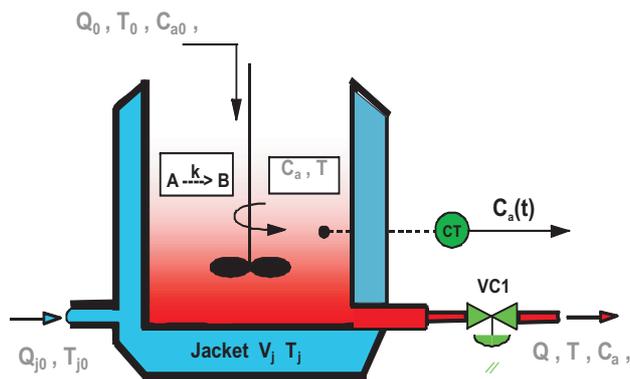


Fig. 3. CSTR with recycling

and in order to increase the overall conversion, reducing costs and use the reactant at most, part of the outlet flow is recycled to the reactor input. But there is a delay in the material feedback. This leads to a CSTR model with internal delays.

Following Pérez and Albertos [2004], let us consider a first order, exothermic, irreversible reaction, as shown in Fig. 3. It is refrigerated by a cooling jacket. Suppose now that the outlet product has a recycling flow rate $\lambda q(t)$; $0 \leq \lambda \leq 1$, where $q(t)$ is the total reactor flow, and L is the transport delay. The limits 0 and 1 correspond to no recycling stream and to a complete recycle, respectively. Then the material and energy balances are described by a dynamical system, including delays of the form:

$$\begin{aligned} \frac{dC_a(t)}{dt} &= \frac{1}{V} [(1 - \lambda)q(t)C_{a,0} + \lambda q(t)C_a(t - L)] \\ &\quad - \frac{1}{V} [q(t)C_a(t) - \alpha C_a(t) e^{-\frac{E}{RT(t)}}] \\ \frac{dT(t)}{dt} &= \frac{1}{V} [(1 - \lambda)q(t)T_0 + \lambda q(t)T(t - L)] \\ &\quad - \frac{1}{V} q(t)T(t) + \frac{H\alpha}{\rho c_p} C_a(t) e^{-\frac{E}{RT(t)}} - \frac{US}{\rho c_p V} [T - T_J] \\ \frac{dT_J(t)}{dt} &= \frac{q_J(t)}{V_J} [T_{J,0} - T_J(t)] + \frac{US}{\rho_J c_{p,J} V_J} [T(t) - T_J(t)] \end{aligned}$$

where $C_a(t)$ is the concentration of the component A in the reactor and also at the exit, ($C_{a,0}$ is the input concentration), $T(t)$ is the temperature in the reactor (T_0 is the reactant input temperature) and similarly for the temperature in the jacket, $T_J(t)$. Volumes in the reactor (V) as well as in the jacket (V_J) are considered constant. E , α and R are chemical constants, H is the reaction heat, ρ and c_p are the density and specific thermal coefficient of the liquid and the same for the refrigerant. U is the heat transmission global coefficient and S is the heat transmission surface area.

Using the parameters of a typical CSTR, as described in Pérez and Albertos [2004], a third order model is obtained. To further simplify the model, the following assumptions are taken:

- The model is linearized around an operating point defined by the reactor and jacket outflows $q = 1.13 \text{ m}^3/\text{s}$, $q_J = 1.41 \text{ m}^3/\text{s}$, leading to $C_{a,e} = 4.031$; $T_e = 333.6$; $T_{J,e} = 330$.
- The dynamics related to the jacket temperature is much faster than that related to the reactor temperature. Thus, the jacket temperature time constant is

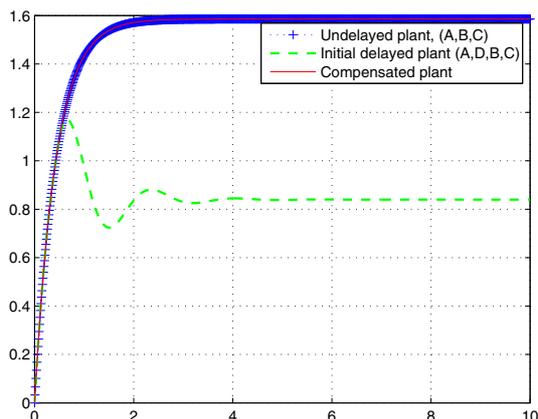


Fig. 4. Reactor step response of the undelayed plant (blue), the initial delayed plant (green) and the compensated plant (red).

negligible and a simplified (second order) model can be derived.

- The refrigerator flow is maintained constant at the operating point. Thus, a single input model is obtained.
- The output is the component A measured composition at the exit.

Thus, the simplified model is given by

$$\begin{bmatrix} \dot{C}_a(t) \\ \dot{T}(t) \end{bmatrix} = \begin{bmatrix} -1.9076 & -0.1834 \\ 161.0772 & -71.0777 \end{bmatrix} \begin{bmatrix} C_a(t) \\ T(t) \end{bmatrix} + \begin{bmatrix} 2.07 & 0 \\ 0 & 2.07 \end{bmatrix} \begin{bmatrix} C_a(t-L) \\ T(t-L) \end{bmatrix} + \begin{bmatrix} 3.1693 \\ -199.5326 \end{bmatrix} q(t) \quad (36)$$

$$u(t) = q(t); \quad y(t) = C_a(t)$$

First, the constraints fulfilment is verified: A is Hurwitz, the plant is stable and $z_\alpha < 0$. Then, according to (21), (23), the compensator components are computed as:

$$C_1(s) = 1 + \frac{143.2e^{-0.5s} - 4.285e^{-s}}{s^2 + 72.99s + 165.1}$$

$$C_2(s) = \left[1 + \frac{-6.56e^{-0.5s}}{3.169s + 261.9} \right]^{-1}$$

The effect of the precompensator is shown in Fig. 4.

4.2 Example 2

In order to illustrate the use of the precompensator in the control design stage for undelayed integrative behavior plants, let us assume a pure state delay plant such as (24), where the following parameters are assumed:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -2 & -0.1 \\ 10 & -0.4 \end{bmatrix} \begin{bmatrix} x_1(t-0.5) \\ x_2(t-0.5) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \quad (37)$$

$$y(t) = x_1(t)$$

the control goal being to design a controller stabilizing the plant and providing an over-damped step response characterized by a time constant $\tau = 1.0$ s. Obviously, other requirements could be demanded. Observe that this plant is stable (see the step response in dotted line and

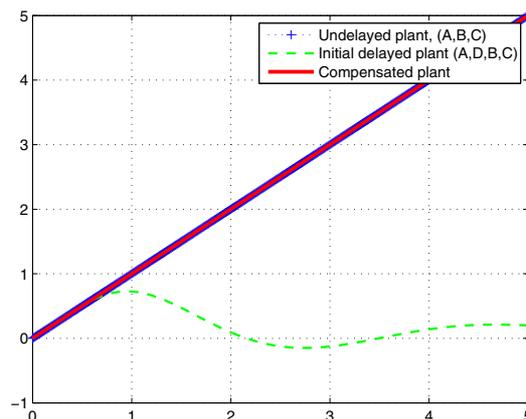


Fig. 5. Step response of the undelayed plant (blue), the initial pure delayed plant (green) and the compensated plant (red), for the plant in Example 2.

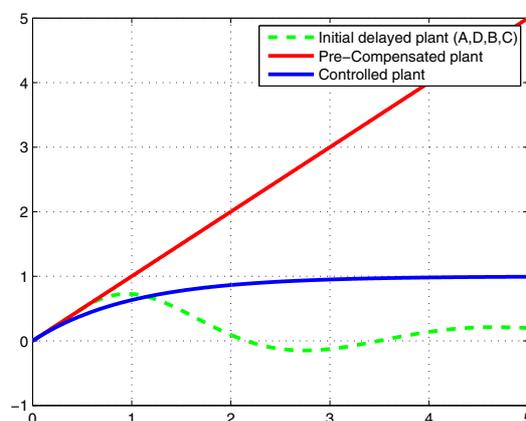


Fig. 6. Step response of the delayed plant (green), the compensated plant (red) and the P-controlled plant (blue), as for Example 2.

green in Fig. 5). First, the precompensator is computed following the approach proposed in section 3.2 to cancel the delay. The result is:

$$C_1(s) = 1 + \frac{-2.4}{s}e^{-0.5s} + \frac{1.8}{s}e^{-s}$$

$$C_2(s) = \left[\frac{s + 0.2e^{-0.5s}}{s} \right]^{-1}$$

leading to a pure integrator, as shown in the Fig. 5. Then to achieve the desired controlled plant behavior, a simple P-controller (unitary gain) is implemented by feed backing the output signal to be compared with the reference. The closed loop response to a unit step reference change is shown in Fig. 6. The gain of the P-controller can be adjusted to achieve the desired transient response.

Note that, if the initial plant would be unstable (for instance, taking $d_{2,1} = -10$ instead of 10), the design procedure could be applied but the controlled plant will loose the internal stability due to the cancellation by the precompensator $C_1(s)$ of the characteristic polynomial of

the initial plant $[1 + \frac{f(s)}{p_A(s)}e^{-Ls} + \frac{d}{p_A(s)}e^{-2Ls}]$. The same kind of instability could appear if the delay is changed ($L = 1.5$), making the initial system (37) also unstable.

5. CONCLUSION

The main and original contribution of this paper is the proposal of a DTC for stable and minimum phase plants with state time-delay. The main development has been presented for second order plants, but it can be easily extended for any order plant, with multiple state time delays. The proposed algorithm allows to reduce the plant to the undelayed part, if it exists. If the plant is a pure delay, the compensated plant is reduced to an integrator, for a second order plant.

Two examples have been presented to illustrate the procedure. First, a simplified model of a typical reactor with recycling has been transformed into a non-delayed plant. Then, an academic example to show the control design options for state-delay plants has been developed.

The proposed approach is based on process cancellations. Thus, the plant should be stable, minimum phase and the non-delayed part should be also stable. In any case, the internal stability of the design should be always verified.

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