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Enforcing Model Adequacy in Real-Time Optimization via Dedicated Parameter Adaptation ⋆

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Abstract: Iterative real-time optimization schemes that employ modifier adaptation add bias and gradient correction terms to the model that is used for optimization. These affine corrections lead to meeting the first-order necessary conditions of optimality *of the plant* despite plant-model mismatch. However, since the added terms do not include curvature information, satisfaction of the second-order sufficient conditions of optimality is not guaranteed, and the model might be deemed *inadequate* for optimization. In the context of modifier adaptation, this paper proposes to include a dedicated parameter-estimation step such that also the second-order optimality conditions are met at the plant optimum. In addition, we propose a procedure to select the best parameters to adapt based on a local sensitivity analysis. A simulation study dealing with product maximization in a fed-batch reactor demonstrates that the proposed scheme can both select the right parameters and determine their values such that modifier adaptation can drive the plant to optimality fast and without oscillations.

Keywords: Real-Time Optimization, Modifier Adaptation, Plant-Model Mismatch, Model Adequacy, Effective Model Adaptation, Parameter Selection.

1. INTRODUCTION

Real-time optimization (RTO) is used extensively to operate industrial processes at economically optimal operating conditions without compromising product quality and process constraints. In many RTO schemes, the successive operating points are computed via model-based optimization. However, if the process model does not predict the plant behavior accurately, the optimal operating conditions found via numerical optimization might result in sub-optimal performance and violate crucial process constraints. Hence, the optimization problem needs to be adapted based on measurements. For this purpose, there are various schemes which differ in the way in which they exploit the measurements.

In the two-step approach (Chen and Joseph, 1987), some of the model parameters are adapted so that the model outputs match the plant outputs at the current operating point as well as possible. Optimization of the steady-state operating point is then performed using the adapted model. This iterative two-step approach has the potential of converging to the plant optimum if the plant-model mismatch is of parametric nature, i.e., if the mismatch can be corrected by parameter adaptation. However, in the presence of structural plant-model mismatch, the two-step approach may perform poorly (Forbes et al., 1994).

Consequently, a modified two-step approach, called Integrated System Optimization and Parameter Estimation (ISOPE), was proposed by Roberts (1979). A parameter estimation problem is solved to make the model outputs to match the measured plant outputs. Then, assuming that the plant output gradients can be estimated experimentally, a gradient correction term is added to the cost of the optimization problem.

Alternatively, one can use measurements to compute zeroth- and first-order correction terms to the cost and constraint functions without model adaptation (Tatjewski, 2002; Gao and Engell, 2005). This approach, which has been labeled Modifier Adaptation (MA) in Marchetti et al. (2009), modifies the cost and constraint functions of the optimization problem so as to enforce the plant first-order necessary conditions of optimality known as the Karush-Kuhn-Tucker (KKT) conditions. However, plant optimality also requires satisfaction of the second-order optimality conditions at the plant optimum. Note that this property, called *model adequacy*, is a property of the model at hand, since the affine corrections that are specific to MA do not affect the Hessian of the optimization problem. One can use convex approximations to the cost and constraint functions

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to guarantee model adequacy (François and Bonvin, 2013). However, this may reduce model accuracy, and thereby the rate of convergence to the plant optimum may deteriorate.

To improve model adequacy for MA, Ahmad et al. (2017) proposed the Effective Model Adaptation (EMA) procedure in which the parameters of a first-principles model are estimated iteratively. If the updated model satisfies a certain model-adequacy criterion, the model is *adequate* for optimization. However, parameter estimation does not always result in adequate models. The present contribution focuses on improving EMA by explicitly considering the model-adequacy criterion in the parameter estimation step.

We also propose a procedure to select the best parameters to estimate for the purpose of making the model adequate. In principle, all model parameters can be estimated, but this may lead to overfitting and increased computational effort. Furthermore, the number of parameters that can be estimated accurately depends on the measurements that are available, the measurement noise, and the level of excitation. Some parameters might play a significant role in dynamic optimization, but have a negligible or no effect in steady-state optimization (Houska et al., 2015). Hence, we suggest to select the set of parameters at each RTO iteration by computing the parametric sensitivity of the Lagrangian function, thereby effectively ranking the parameters.

This paper is organized as follows. Section 2 formulates the optimization problem and reviews the concepts of MA, model adequacy, and EMA. Section 3 investigates the addition of a penalty function to EMA to improve model adequacy. Section 4 proposes to use a local sensitivity analysis to select, in each iteration, the parameters to adapt. Section 5 presents a simulation study that investigates the performance of MA on the basis of an inadequate model that is then updated to make it adequate. Finally, Section 6 concludes the paper with remarks on further research.

2. PRELIMINARIES

2.1 Plant Optimization Problem

The plant optimization problem is formulated as:

$$\begin{aligned} \min_{\mathbf{u}} \quad & J_p(\mathbf{u}) := \tilde{J}_p(\mathbf{u}, \mathbf{y_p}(\mathbf{u})) \\ \text{s.t.} \quad & \mathbf{C}_p(\mathbf{u}) := \tilde{\mathbf{C}}_p(\mathbf{u}, \mathbf{y_p}(\mathbf{u})) \leq \mathbf{0}, \\ & \mathbf{u}^L \leq \mathbf{u} \leq \mathbf{u}^U, \end{aligned} \tag{1}$$

where $\mathbf{u} \in \mathbb{R}^{n_u}$ is the input vector and $\mathbf{y_p} \in \mathbb{R}^{n_y}$ is a vector of plant measurements. $J_p : \mathbb{R}^{n_u} \to \mathbb{R}$ is the cost function to be minimized, $\mathbf{C}_p : \mathbb{R}^{n_u} \to \mathbb{R}^{n_C}$ is the set of process-dependent inequality constraint functions. The solution to Problem (1) is denoted as $\mathbf{u}_p^*$.

In practice, the plant input-output mapping, $\mathbf{y_p} : \mathbb{R}^{n_u} \to \mathbb{R}^{n_y}$, is not known, but it is assumed that an approximate model is available. This model can be used in the following optimization problem:

$$\begin{aligned} \min_{\mathbf{u}} \quad & J(\mathbf{u}, \theta) := \tilde{J}(\mathbf{u}, \mathbf{y}(\mathbf{u}, \theta)) \\ \text{s.t.} \quad & \mathbf{C}(\mathbf{u}, \theta) := \tilde{\mathbf{C}}(\mathbf{u}, \mathbf{y}(\mathbf{u}, \theta)) \leq \mathbf{0}, \\ & \mathbf{u}^L \leq \mathbf{u} \leq \mathbf{u}^U, \end{aligned} \tag{2}$$

where $\theta \in \mathbb{R}^{n_\theta}$ represents the vector of model parameters and $\mathbf{y}(\mathbf{u}, \theta)$ is the vector of outputs predicted by the model. Generally, due to plant-model mismatch, the solution to Problem (2) differs from the plant optimum $\mathbf{u}_p^*$. RTO schemes for the situation with plant-model mismatch iteratively adapt the model-based optimization problem so as to reach plant optimality.

In the two-step approach, the plant measurements are used to identify some of the model parameters at the current operating point by solving the following least-squares problem:

$$\theta^{(k)} := \arg\min_{\theta} \|\mathbf{y}_p(\mathbf{u}^{(k)}) - \mathbf{y}(\mathbf{u}^{(k)}, \theta)\|^2, \tag{3}$$

with $\mathbf{y}_p(\mathbf{u}^{(k)})$ being the plant measurements at the current operating point $\mathbf{u}^{(k)}$. The updated model is then used in the optimization problem to generate the new set of inputs $\mathbf{u}^{(k+1)}$. However, this approach usually does not reach plant optimality in the presence of structural plant-model mismatch.

2.2 Modifier Adaptation

In MA, the model-based optimization Problem (2) is modified by the introduction of zeroth-order (bias) and first-order (gradient) correction terms as follows:

$$\begin{aligned} \min_{\mathbf{u}} \quad & J_m^{(k)}(\mathbf{u}, \theta) := J(\mathbf{u}, \theta) + \varepsilon_J^{(k)} + \lambda_J^{(k)} \left(\mathbf{u} - \mathbf{u}^{(k)}\right) \\ \text{s.t.} \quad & \mathbf{C}_m^{(k)}(\mathbf{u}, \theta) := \mathbf{C}(\mathbf{u}, \theta) + \varepsilon_C^{(k)} + \mathbf{\Lambda}_C^{(k)} \left(\mathbf{u} - \mathbf{u}^{(k)}\right) \leq \mathbf{0}, \\ & \mathbf{u}^L \leq \mathbf{u} \leq \mathbf{u}^U, \end{aligned} \tag{4}$$

where the correction terms are described by:

$$\varepsilon_J^{(k)} = J_p(\mathbf{u}^{(k)}) - J(\mathbf{u}^{(k)}, \theta) \tag{5}$$

$$\lambda_J^{(k)} = \left(\nabla J_p(\mathbf{u}^{(k)}) - \nabla J(\mathbf{u}^{(k)}, \theta)\right)^T \tag{6}$$

$$\varepsilon_C^{(k)} = \mathbf{C}_p(\mathbf{u}^{(k)}) - \mathbf{C}(\mathbf{u}^{(k)}, \theta) \tag{7}$$

$$\mathbf{\Lambda}_C^{(k)} = \left(\nabla \mathbf{C}_p(\mathbf{u}^{(k)}) - \nabla \mathbf{C}(\mathbf{u}^{(k)}, \theta)\right)^T, \tag{8}$$

with the scalar $\varepsilon_J^{(k)}$ and the n_C-dimensional vector $\varepsilon_C^{(k)}$ representing zeroth-order corrections, and the n_u-dimensional row vector $\lambda_J^{(k)}$ and the $n_C \times n_u$ matrix $\mathbf{\Lambda}_C^{(k)}$ being the first-order modifiers. Note that the model parameters θ are not adapted.

To prevent excessive corrective action, the inputs can be filtered as follows:

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{*(k+1)} + \mathbf{K}(\mathbf{u}^{(k)} - \mathbf{u}^{*(k+1)}), \tag{9}$$

where $\mathbf{u}^{*(k+1)}$ is the solution to Problem (4), and $\mathbf{K}$ is a diagonal matrix of filter gains in the interval $[0, 1)$. In this contribution, the experimental gradients are calculated as in Gao and Engell (2005) by using finite-difference approximation if needed with additional perturbations around $\mathbf{u}^{(k)}$.

2.3 Model Adequacy

If the process model used in an RTO algorithm is capable of producing a fixed point that is a local minimum at the plant optimum $\mathbf{u}_p^*$, then the model is said to be adequate for optimization (Forbes and Marlin, 1996).

By using MA, $\mathbf{u}_p^*$ satisfies the first- and second-order optimality conditions of the modified optimization Problem (4). Since MA corrects only the first-order KKT conditions, it is however not guaranteed that $\mathbf{u}_p^*$ will satisfy the second-order optimality conditions.

Concretely, model adequacy requires that the reduced Hessian of the model-based optimization Problem (4) at $\mathbf{u}_p^*$, $\nabla^2\mathcal{L}_{m,r}\left(\mathbf{u}_p^*, \theta, \mu^*\right)$, is positive definite (Marchetti et al., 2016), where $\mathcal{L}_m$ represents the Lagrangian function:

$$\mathcal{L}_m\left(\mathbf{u}, \theta, \mu\right) := \quad J_m\left(\mathbf{u}, \theta\right) + \mu^T \mathbf{C}_m\left(\mathbf{u}, \theta\right), \tag{10}$$

with μ being the n_C-dimensional vector of Lagrange multipliers.

The vector of active constraints at $\mathbf{u}_p^*$ is denoted by $\mathbf{C}_m^a\left(\mathbf{u}_p^*, \theta\right) \in \mathbb{R}^{n_C^a}$, where n_C^a is the cardinality of the active constraints at $\mathbf{u}_p^*$. Assuming that Linear Independence Constraint Qualification (LICQ) holds at $\mathbf{u}_p^*$, one can write:

$$\frac{\partial \mathbf{C}_m^a\left(\mathbf{u}_p^*, \theta\right)}{\partial \mathbf{u}} \mathbf{Z} = \mathbf{0}, \tag{11}$$

where $\mathbf{Z} \in \mathbb{R}^{n_u \times (n_u - n_C^a)}$ is a null-space matrix. The reduced Hessian of the Lagrangian on this null space, $\nabla^2\mathcal{L}_{m,r}\left(\mathbf{u}_p^*, \theta, \mu^*\right) \in \mathbb{R}^{(n_u - n_C^a)\times(n_u - n_C^a)}$, is given by Gill et al. (1981) as:

$$\nabla^2\mathcal{L}_{m,r}\left(\mathbf{u}_p^*, \theta, \mu^*\right) := \mathbf{Z}^T \left[\frac{\partial^2 \mathcal{L}_m\left(\mathbf{u}_p^*, \theta, \mu^*\right)}{\partial \mathbf{u}^2}\right] \mathbf{Z}. \tag{12}$$

Note that, if the Hessian of the Lagrangian function is positive definite, then the reduced Hessian of the Lagrangian is also positive definite.

2.4 Effective Model Adaptation

Model adequacy upon convergence can be achieved iteratively by updating the model parameters θ in the modified optimization Problem (4). EMA exploits this idea and adapts the model parameters via the least-squares Problem (3) as detailed in Ahmad et al. (2017). EMA accepts the new parameters $\theta^{(k+1)}$ only if the following two conditions are satisfied:

- $\nabla^2 J_m^{(k)}(\mathbf{u}, \theta^{(k+1)})$ is positive definite,
- $\left|J_m^{(k)}(\mathbf{u}^{(k+1)}, \theta^{(k+1)}) - J_p(\mathbf{u}^{(k+1)})\right| < \left|J_m^{(k)}(\mathbf{u}^{(k+1)}, \theta^{(k)}) - J_p(\mathbf{u}^{(k+1)})\right|$.

The first condition forces the cost term of the Lagrangian function (10) to be positive definite at the k^{th} iteration. Note that $\nabla^2 J_m^{(k)}(\mathbf{u}, \theta^{(k+1)})$ being positive definite implies that $\nabla^2 J(\mathbf{u}, \theta^{(k+1)})$ is positive definite.

The second condition can be analyzed via a Taylor's series expansion of $J_m^{(k)}(\mathbf{u}, \theta) - J_p(\mathbf{u})$ at $\mathbf{u}^{(k)}$. Upon neglecting the terms higher than second order, one can write:

$$J_m^{(k)}(\mathbf{u}, \theta) - J_p(\mathbf{u}) \approx \frac{1}{2}\left(\mathbf{u} - \mathbf{u}^{(k)}\right)^T \left(\nabla^2 J^{(k)} - \nabla^2 J_p^{(k)}\right)\left(\mathbf{u} - \mathbf{u}^{(k)}\right), \tag{13}$$

from which it can be inferred that the second condition reflects the gap between the Hessian matrices of the adapted model and of the plant. Hence, enforcing the second condition ensures that the adapted model exhibits a Hessian that approximates the plant Hessian.

The parameters $\theta^{(k+1)}$ that result from EMA are then used in the modified optimization Problem (4).

3. EMA WITH PENALTY FUNCTION

The least-squares parameter estimation Problem (3) that is used in EMA minimizes the output prediction error. However, it does not guarantee that the adapted model will satisfy model adequacy even when there exist parameter values that make the model adequate. Furthermore, since MA aims at meeting the first-order KKT conditions, the parameter adaptation problem should be designed to primarily search for an adequate model by meeting the second-order optimality conditions. Therefore we propose to add a penalty term to the least-squares Problem (3) as follows:

$$\theta^{(k)} := \arg\min_{\theta} \|\mathbf{y}_p(\mathbf{u}^{(k)}) - \mathbf{y}(\mathbf{u}^{(k)}, \theta)\|^2 + \alpha^T \mathbb{P}^{(k)}(\theta)$$
$$\text{s.t.} \quad \alpha > 0,$$

with

$$\mathbb{P}^{(k)}(\theta) = \left[min\left(0, eig\left(\nabla^2\mathcal{L}_m(\mathbf{u}^{(k)}, \theta, \mu^{(k)})\right)\right)\right]^2, \tag{14}$$

where α is a penalty parameter and eig represents a vector of eigenvalues. The added penalty term penalizes the cost function of the identification problem when the eigenvalues of the Hessian of the Lagrangian function are not positive for a given value of the parameters θ. Positive eigenvalues imply that the Hessian of the model Lagrangian is positive definite, which also makes the reduced Lagrangian $\mathcal{L}_{m,r}$ positive definite.

The penalty parameter α drives the identification problem to search for parameter values that enforce model adequacy. Provided that the set of model-adequate parameters is non-empty, the penalty term in Eq. (14) enforces model adequacy for a sufficiently large penalty weight. Note that, instead of the penalty term, one could have added a constraint to the identification problem. However, the identification problem becomes infeasible when there are no parameter values that result in model adequacy. For example, if the Lagrangian function is concave in the parameters, it is not possible to satisfy model adequacy via parameter adaptation.

4. PARAMETER SELECTION FOR EMA

In principle, EMA can be designed to adapt all model parameters. However, since adapting all parameters may result in identifiability issues and overfitting, it is often desirable to adapt only a subset of the parameters. Hille et al. (2017) proposed to select the parameters based on a local sensitivity analysis performed on the outputs and the gradients of the cost and constraint functions. The parameters are then ranked and selected according to their sensitivity.

Since the identification step in EMA with penalty function aims at obtaining a positive-definite Hessian of the Lagrangian function that guarantees model adequacy, we propose to select the parameters based on the parametric sensitivity of the Lagrangian function. At the k^{th} RTO

iteration, the sensitivity of the Lagrangian with respect to the model parameter θ_i is defined as:

$$\mathbb{S}^{\theta_i}_{\mathcal{L}_m} = \frac{\partial \mathcal{L}_m\left(\mathbf{u}^{(k)}, \theta_i, \mu^{(k)}\right)}{\partial \theta_i}. \tag{15}$$

The parameters with the largest sensitivities have the strongest influence on the Lagrangian function and, thus, also on the eigenvalues of the corresponding Hessian. Ideally, n_u parameters could be selected to independently adjust the n_u eigenvalues of the Hessian. However, if the Hessian is already convex along certain input directions, the number of parameters that need to be estimated can be reduced.

Note that for parameter selection we do not consider the sensitivity of the output error in the cost function of Problem (14), since reducing the output error is not the primary goal of the identification step. Nevertheless, keeping the output error small helps to find parameter values that match the reality well, in particular when a large set of parameter values satisfy the model adequacy criterion.

5. SIMULATION STUDY

We test the performance of modifier adaptation combined with (a) EMA with penalty function (EMA-PF), and (b) EMA with penalty function and parameter selection (EMA-PF-PS) on a fed-batch reactor for penicillin production. This process has been used to evaluate RTO methodologies e.g. in Ahmad et al. (2017) and Mandur and Budman (2015).

5.1 Problem Definition

The plant dynamics are described by the following equations:

$$\frac{dX}{dt} = \frac{\mu_x\, S\, X}{K_x\, X + S} - \frac{X}{V}\frac{dV}{dt} \tag{16}$$

$$\frac{dP}{dt} = \frac{\mu_p\, S\, X}{K_p + S + S^2/K_I} - \frac{P}{V}\frac{dV}{dt} - K_H\, P \tag{17}$$

$$\frac{dS}{dt} = -\frac{\mu_x\, S\, X}{Y_{x/s}(K_x\, X + S)} - \frac{\mu_p\, S\, X}{Y_{p/s}(K_p + S + S^2/K_I)} - m_x\, X + \frac{S_F\, F}{V} - \frac{S}{V}\frac{dV}{dt} \tag{18}$$

$$\frac{dV}{dt} = F - \beta\, V, \tag{19}$$

where X, S and P are the concentrations of biomass, substrate and penicillin, respectively. V is the volume of the culture medium and F is the feed flowrate of substrate. The parameter values used in the simulation study are given in Table 1.

Plant-model mismatch is considered by ignoring the rate of consumption of penicillin by hydrolysis in the model used for optimization. The penicillin dynamics in the optimization model then reads:

$$\frac{dP}{dt} = \frac{\mu_p\, S\, X}{K_p + S + S^2/K_I} - \frac{P}{V}\frac{dV}{dt}. \tag{20}$$

The operational objective is to maximize the concentration of penicillin at the end of the batch, while satisfying a terminal constraint on the culture volume. The manipulated variables are the feed flowrate of substrate F and the initial substrate concentration S_0.

The optimization problem thus reads:

$$\begin{aligned} \max_{F, S_0} \quad & P(t_f) \\ \text{s.t.} \quad & V(t_f) \le V_{max}, \end{aligned} \tag{21}$$

where $V_{max} = 120\,L$ is the maximal volume of the reactor. The batch time t_f is taken to be 192 h. The measurements of X, P and S that are used for parameter estimation are available every 6 h. The initial conditions are given in Table 2.

5.2 Performance without Iterative Parameter Selection

This section compares three RTO approaches, namely:

(1) Standard MA without parameter adaptation,
(2) MA with EMA,
(3) MA with EMA-PF.

In this section, we do not use the iterative parameter selection procedure described in Section 4, which will be investigated later. Here, in MA with EMA and EMA-PF, the model parameters K_x and K_I are adapted after each batch. The performances of the three RTO approaches are compared in Figure 1. The following remarks can be made:

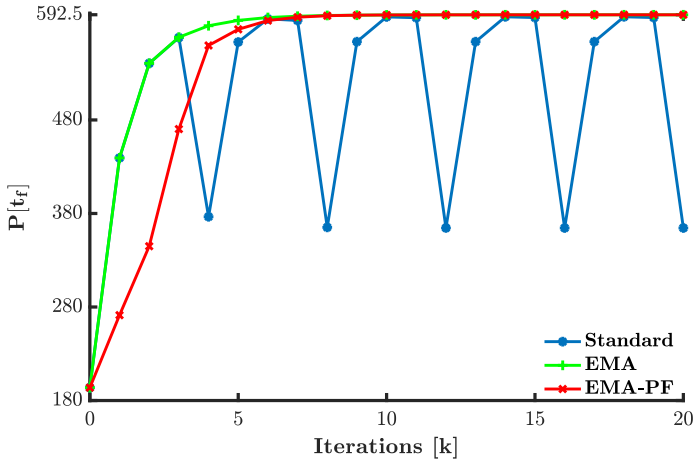
(1) Without parameter adaptation, standard MA moves quickly toward the optimal penicillin concentration but then starts oscillating since the model used is inadequate as it does not satisfy the second-order optimality conditions.
(2) In MA with EMA, although not guaranteed, the parameter identification step finds parameter values that make the model adequate after three iterations. Hence, MA with EMA converges to plant optimality.
(3) MA with EMA-PF also converges to plant optimality. Although MA with EMA and EMA-PF give similar final results, MA with EMA-PF (i) finds adequate model parameters after a single iteration, and (ii) is

Table 1. Parameters of the penicillin production process.

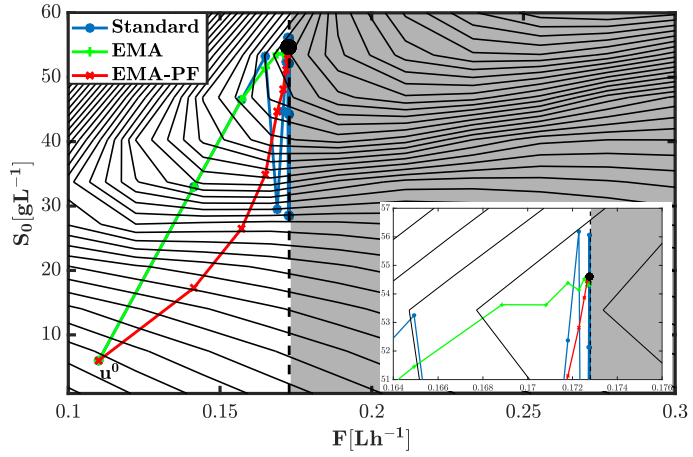
Parameters	Description	Value
μ_x	Specific growth rate of biomass	0.092
μ_p	Specific rate of penicillin production	0.005
K_x	Saturation constant	0.15
K_p	Saturation constant	0.0002
K_I	Substrate inhibition constant	0.1
K_H	Rate of consumption of penicillin by hydrolysis	0.04
$Y_{x/s}$	Biomass yield per unit mass of substrate	0.45
$Y_{p/s}$	Penicillin yield per unit mass of substrate	0.9
m_x	Substrate consumption rate needed to maintain the biomass	0.014
S_F	Concentration of substrate in the feed	600
β	Evaporative loss during fermentation	0.000 62

Table 2. Initial condition for the states and inputs.

Variable	Initial conditions	Unit
Biomass concentration (X_0)	0.1	$\mathrm{g\,L^{-1}}$
Penicillin concentration (P_0)	0.0	$\mathrm{g\,L^{-1}}$
Fed-batch culture volume (V_0)	100.0	l
Substrate concentration (S_0)	6.0	$\mathrm{g\,L^{-1}}$
Input feed (F)	0.11	$\mathrm{L\,h^{-1}}$



(a)



(b)

Fig. 1. Comparison of iterative batch optimization using modifier adaptation: standard MA, MA with EMA, and MA with EMA-PF. (a) Convergence of the plant cost, and (b) successive operating points in the input space. The contour lines represent the plant cost. The shaded region is infeasible due to the volume constraint. The black dot represents the plant optimum.

slower as model accuracy is lower since the parameters are estimated primarily to make the model adequate rather than accurate.

To demonstrate the benefit of using the penalty function, we ran the simulation for different values of the initial inputs. The comparison is shown in Figure 2, where it can be observed that, in MA with EMA-PF, all trajectories converge smoothly to the plant optimum, whereas large oscillations occur in MA with EMA for some of the initial inputs. Clearly, MA with EMA is not always capable of reaching model adequacy, whereas MA with EMA-PF does a good job for all tested initial operating conditions.

5.3 Performance with Iterative Parameter Selection

The performance of MA with EMA-PF can still be improved by carefully selecting the model parameters that are adapted, for example by using the scheme proposed in Section 4. Here, we consider K_x, K_I and μ_x as potentially adjustable parameters. Since there are only two Hessian eigenvalues to be adjusted, we select two parameters from the set of three at each iteration.

Figure 3a presents a comparison between a fixed choice of the model parameters K_x and K_I and the selection resulting from the scheme proposed in Section 4. The ranking of parameters at each iteration is shown in Figure 3b, the most influential parameter being ranked 1. At each iteration, the two best-ranked parameters are selected for parameter estimation. As seen in Figure 3a, a significant improvement in performance is observed. Initially, the parameters μ_x and K_x are selected for adaptation. As a result, not only is the updated model adequate, but also the optimum can be reached faster. The proposed real-time optimization scheme completes 90% of the improvement in three iterations, whereas the adaptation of a fixed set of parameters reduces this value to 58%.

6. CONCLUSIONS

This contribution proposes to add a penalty term to the objective of the parameter estimation problem so as to increase the reliability of MA with EMA. This penalty term guarantees meeting the model-adequacy condition, provided that such parameter values exist. Furthermore, we have proposed a parameter selection procedure based on a local sensitivity analysis. The method selects the most influential parameters from a given set in order to improve the MA performance.

It is interesting to note that the goal of parameter estimation is not better model prediction, but rather model adequacy. Furthermore, MA with EMA attempts to improve the performance of real-time optimization in two distinct ways:

(1) by meeting the first-order KKT conditions via the bias and gradient modifiers, and
(2) by meeting the second-order optimality conditions by an adapted parameter estimation by EMA.

The effect of the measurement noise on the identification of an adequate model will be studied in future work.

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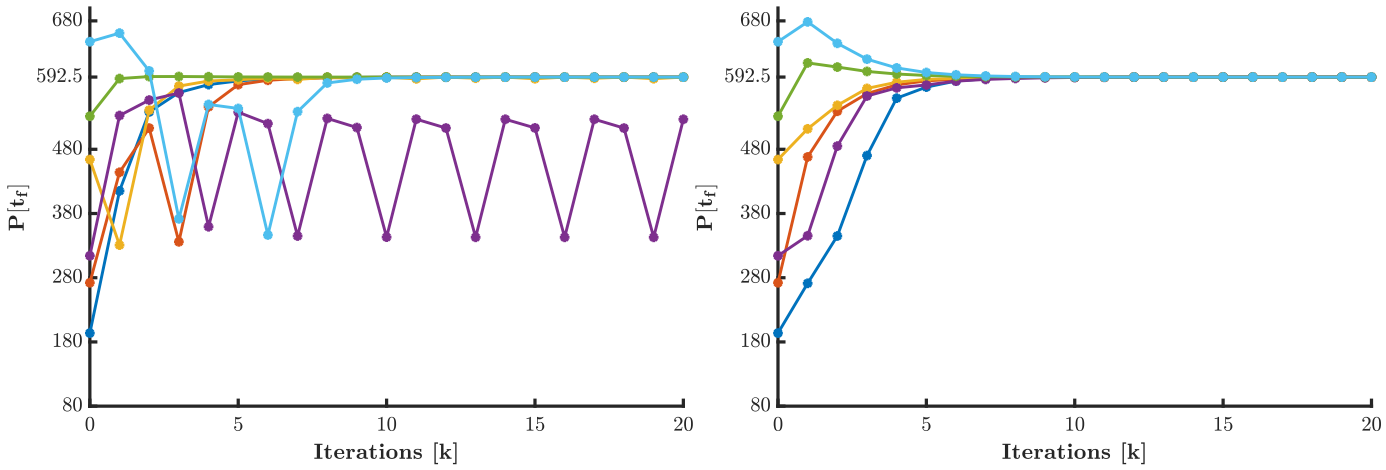

Fig. 2. Evolution of the plant cost for different initial operating conditions. (a) MA with EMA, (b) MA with EMA-PF.

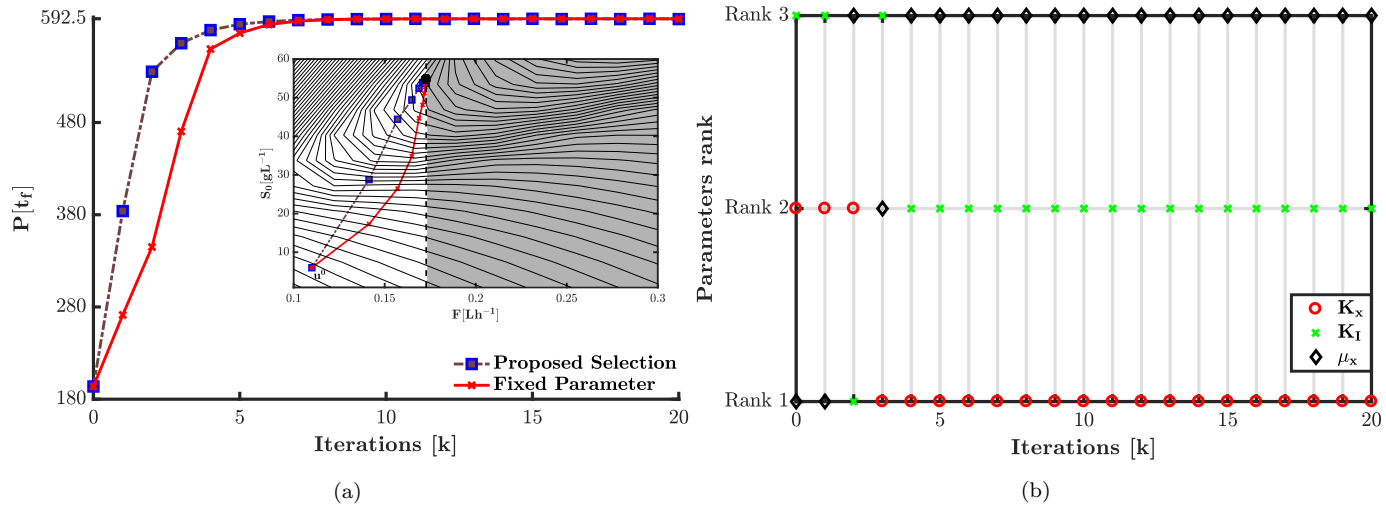

Fig. 3. Performance of MA with EMA-PF using a fixed and an iteratively selected set of parameters for adaptation. (a) Convergence of the plant cost, and (b) ranking of the parameters over the iterations. The inset presents successive operating points in the input space. The contour lines indicate the plant cost. The shaded region is infeasible due to the volume constraint. Black dot represents the plant optimum.

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