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# ILC Based Economic Batch-to-Batch Optimization for Batch Processes

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Abstract: The control strategies for batch processes in the past can be categorized into two levels. The higher level is economic optimization running at low frequency and the lower one tracks the given reference using MPC or PID at the higher level. The lower level regards all the disturbances as something to be rejected according to a quadratic based optimization objective. However, not all the disturbances are unfavorable to batch processes; some of them are helpful. In this paper, an economic batch-to-batch optimization method for batch processes is directly applied at the lower level. It replaces the former tracking strategy. With the information of disturbances collected from the previous batches, the iterative learning control strategy (ILC) can find out better operation profiles. ILC has the advantage of continuously improving the economic performance of the current batch with enriched information of disturbances from batch to batch. To demonstrate the potential applications of the proposed design method, a typical fed-batch bioreactor is applied.

*Keywords:* batch process; batch-to-batch; economic optimization; iterative learning control

## 1. INTRODUCTION

Batch processes, unlike continuous processes most often used for high-throughput plants, are wildly used to manufacture high-quality, low-volume products in industries, such as microelectronics, specialty chemicals, food, pharmaceuticals and biotechnology (Bonvin, 1998; Lee et al., 1999). Although several theories of process control have been significantly developed during the past few decades, most control techniques applied well to continuous processes are not suitable for batch processes because batch processes exhibit strong nonlinearity, time-varying dynamics and unsteady conditions, involving several transitions and large transient phases that cover large operating envelopes. These issues remain a big challenge in batch processes (Oh,Lee, 2016; Shi,Yang, 2016).

The features of batch processes in design, optimization, and control, like the strategy proposed in continuous processes (Ellis et al., 2016), can be represented by a block diagram of the hierarchical strategy (Fig.1). In the past few decades, more attention has been paid to the real-time optimization (RTO) layer and the advanced process control layer in batch processes. The purpose of the RTO layer is to generate a suitable reference trajectory for the next layer (Golshan et al., 2010; Godoy et al., 2016; Li et al., 2016). The traditional tracking MPC (TMPC) has been implemented to minimize the error between the process dynamic path and the reference trajectory with the objective function in a quadratic form. Because of the nature of repetitiveness in batch processes, iterative learning control (ILC) has been applied to control of batch processes for decades. The MPC technique combined with ILC for batch processes was proposed (Lee,Lee, 2003).

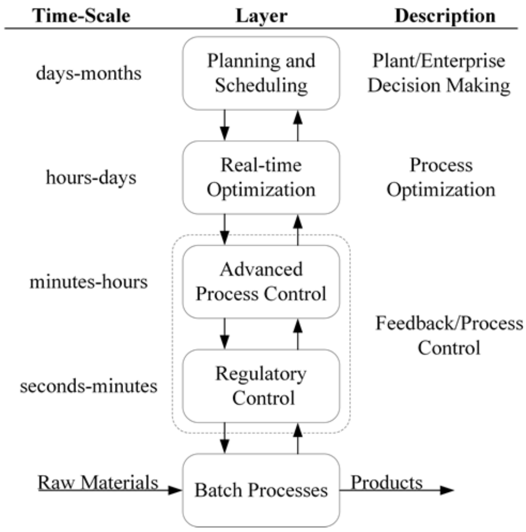


Figure 1 The traditional hierarchical paradigm employed in industries for planning/scheduling, optimization, and control of batch processes

In this technique, a linear time-varying (LTV) model was used. Also, end product properties and transient profiles of the process variable were integrated into the control structure. This control structure was extended to constrained multivariable control (Jia et al., 2016). The models often used in the ILC-based MPC strategies are linear time-invariant (LTI) or LTV because of the complexity of mechanism models. Unlike the method mentioned above, Gao et al. proposed the robust ILC based on a two-dimensional model and analyzed the behaviors of batch processes in an easier way (Wang et al., 2013; Li et al., 2016). The two-dimensional model integrates the batch index and the time index into one model for the analysis of batch processes. Also, a latent variable model combined with MPC was applied (Golshan et al., 2010; Godoy et al., 2016). To have an economic performance of batch processes, a concept from economic model predictive control (EMPC) proposed in continuous processes was directly applied to batch processes (del Rio-Chanona et al., 2016). However, it did not take the unique features of batch processes (nonlinear dynamics) into consideration.

In batch processes, the sequence of operating steps, often directly adapted from lab experimental procedures, is referred to as a recipe. Owing to differences in equipment and the scale between the laboratory and the industrial batch unit, modifications of the recipes are necessary for the industrial scale to ensure productivity, safety, quality and satisfaction of operational constraints (Jaeckle, MacGregor, 2000). Thus, the nominal optimal reference trajectories for batch processes generated from the RTO layer may be highly sensitive so that the desired economic objective and end-point quality may be infeasible. This can be attributed to the variations in the initial and operating conditions, all of which introduce disturbances in the final product quality. What is worse, it is difficult to obtain the real-time dynamic information of quality variables because of high costs, low reliability or the low sampling rate of the hardware sensor. As a result, the difficulty of design, optimization, and control of batch processes online is increasing. The solution to robust optimization is trackable for disturbances or uncertainty problems. Under such a circumstance, this method sacrifices optimality, making economic performance conservative.

There are some drawbacks in the control structure of all the past work:

- Recipes (or designed trajectories): In batch operations, because of the differences between the laboratory and the industrial scale, the operation profiles should be modified while the recipes are transferred from the laboratory to the industrial batch unit.
- Disturbances: The existence of both deterministic and stochastic disturbances will affect the dynamics of batch processes. Under such a circumstance, the given reference trajectory in the supervisory control layer would not be tracked perfectly. Even if the tracking is perfect, the generated reference trajectory may not have the optimal economic performance in reality. From the viewpoint of tracking in the control design, it is a common sense that all the disturbances are bad since they will make the process dynamic path deviate from the reference trajectory. However, from the viewpoint of the economic performance, the disturbances may be favorable since they may help enhance the operating profit or decrease the operating cost to some extent. Thus, in a good control design, what should be done is to accept favorable disturbances and to reject unfavorable disturbances to improve the economic performance.
- ILC based design: In the conventional design strategy, the RTO layer and the supervisory layer are running at different time scales. The RTO layer is updated slowly. This means there is no essential economic improvement in the traditional tracking MPC strategy until the update of the RTO layer.

Therefore, it is imperative to develop batch control strategies and enhance the economic performance. In this paper, an ILC based economic batch-to-batch optimization (ILC-EBtBO) strategy based on the economic performance of the batch operation instead of the errors from the reference trajectories in the supervisory control layer is proposed. The proposed strategy can improve the economic performance frequently by considering integrating the optimization layer and the supervisory layer. It also sufficiently uses the data from the past operating batches to estimate deterministic and stochastic disturbances. Then ILC can repetitively learn from the past batches to update the control of the next batch runs. Note that the proposed ILC-EBtBO and the ILC-TMPC strategies are different because the former concentrates on improving the economic performance and the latter only focuses on eliminating the tracking error. For the economic optimization, the ILC strategy is an indirect way as the errors are used to update the values of the states and the disturbances (Wang et al., 2009).

The rest of the paper is organized as follows. In Section 2, the optimization problem of batch processes is defined. Section 3 discusses ILC based economic batch-to-batch optimization. A case of an industrial fed-batch bioreactor is presented in Section 4. Finally, concluding remarks are given in Section 5.

## 2. PROBLEM DEFINITION

Although the LTI model is good at describing linear dynamic processes, batch processes often exhibit a significant nonlinear behavior in a wide range of operating conditions. In the RTO layer, dynamic nonlinear optimization rather than static one should be carried out, but the optimization problem with a nonlinear objective function and nonlinear constraints cannot be completely solved at high frequency. Solving the problem often takes several minutes or even a few hours, exceeding the time interval of the control adjustment in practical applications. To solve the problem, an LTV model is used for better capture of the instantaneous dynamic behaviors and tracking of the time-varying parameters of the processes. Thus, in this paper, an LTV model is adopted here to describe the nonlinear dynamic batch process. It is represented by

$$\begin{aligned} \mathbf{x}_k(t+1) &= \mathbf{A}(t)\mathbf{x}_k(t) + \mathbf{B}_u(t)\mathbf{u}(t) + \mathbf{B}_d(t)\mathbf{d}_k(t) \\ \mathbf{z}_k(t) &= \mathbf{F}(t)\mathbf{x}_k(t) + \mathbf{m}_k(t) \\ \mathbf{y}_k(t) &= \mathbf{C}(t)\mathbf{x}_k(t) + \mathbf{n}_k(t) \end{aligned} \tag{1}$$

Here, the subscript $k$ denotes the batch index while $t \in \mathbf{I}_{0:T}$ denotes the time index within a batch. $\mathbf{x}_k(t) \in \mathbf{R}^{n_x}$, $\mathbf{u}_k(t) \in \mathbf{R}^{n_u}$, $\mathbf{d}_k(t) \in \mathbf{R}^{n_d}$, $\mathbf{z}_k(t) \in \mathbf{R}^{n_z}$, and $\mathbf{y}_k(t) \in \mathbf{R}^{n_y}$ denote the state, the input, the unknown disturbance, the output of the process variables and the output of the quality variables at time $t$ of the $k$th batch, respectively. The output measurements are taken from process sensors and sampled frequently in real time. $\mathbf{A}(t)$, $\mathbf{B}_u(t)$, $\mathbf{B}_d(t)$ and $\mathbf{C}(t)$ are the corresponding time-varying matrices obtained by linearizing the mechanism model along the trajectory or system identification at each time point. The states cannot be obtained directly and only the process variables can be measured. $\mathbf{F}(t)$ is the corresponding time-varying matrix of the observations of the process variables. In batch processes, the endpoint quality is important because it provides a detailed evaluation of the reaction product, but the quality measurements are only available infrequently (often at the end of one batch process ($t = T$)). This paper denotes $\mathbf{y}_k(T)$ as the quality variable and $\mathbf{C}(T)$ is the corresponding output matrix at the end of a batch. $\mathbf{m}_k(t) \in \mathbf{R}^{n_z}$ and $\mathbf{n}_k(T) \in \mathbf{R}^{n_y}$ denote the measurement noises of the process variables and the quality variables respectively, both of which are independent. They are identically distributed white noise, $\mathbf{m}_k(t) \sim N(0, \boldsymbol{\sigma}_m^2)$ and $\mathbf{n}_k(T) \sim N(0, \boldsymbol{\sigma}_n^2)$.

Consider the outputs ($\mathbf{z}_k$) in (1) from $t = 0$ and $t = T$ for the one batch run and assemble the results of states, inputs, unknown disturbances and output of the process variables at batch $k$,

$$\begin{aligned} \mathbf{x}_k &= \boldsymbol{\Phi}\mathbf{x}_k(0) + \boldsymbol{\Psi}_u \mathbf{u}_k + \boldsymbol{\Psi}_d \mathbf{d}_k \\ \mathbf{z}_k &= \boldsymbol{\Omega}\mathbf{x}_k + \mathbf{m}_k \\ \mathbf{y}_k(T) &= \boldsymbol{\Gamma}\mathbf{x}_k + \mathbf{n}_k(T) \end{aligned} \tag{2}$$

where $\mathbf{x}_k$ is the stacked state vector, $\mathbf{u}_k$ is the input vector, $\mathbf{z}_k$ is the output vector and $\mathbf{m}_k$ is the measured noise vector. ($\boldsymbol{\Phi}$, $\boldsymbol{\Psi}_u$, $\boldsymbol{\Psi}_d$, $\boldsymbol{\Omega}$, $\boldsymbol{\Gamma}$) are the compatible matrices. $\boldsymbol{\Psi}_u$, $\boldsymbol{\Omega}$ and $\boldsymbol{\Psi}_d$ are the Hankel matrices of the input, the output, and the disturbance, respectively. Because of the space limit, they are not completely expressed in detail.

Like EMPC in the continuous system, the economic objective function of a batch is described as the sum of the stage cost function and the terminal cost function. It is defined as follows.

$$V_k(\mathbf{x}_k, \mathbf{u}_k) = \sum_{t=0}^{T-1} l(\mathbf{x}_k(t), \mathbf{u}_k(t)) + Q(\mathbf{y}_k(T)) \tag{3}$$

where $l : \mathbf{R}^{n_x} \times \mathbf{R}^{n_u} \to \mathbf{R}$ is a scalar function which is the stage cost of each time period of one batch run. $Q : \mathbf{R}^{n_y} \to \mathbf{R}$ is the scalar function which is the terminal cost of the whole batch. Apparently, it would be a negative one to describe the terminal profit of the whole batch.

In the conventional TMPC, the objective function is a quadratic form for minimizing the error of the state and the reference state. In addition to the process characteristics, the optimal trajectories of the operating batch generated from the RTO layer would be highly sensitive because of the effect of unknown disturbances from batch to batch. Even if ILC-TMPC is used to make the tracking perfect, the economic performance would become worse since ILC-TMPC only focuses on how to reject the disturbances and uncertainties in the designed system to make the states exactly reach the optimal one at the initial design. However, ILC-EBtBO pays attention to the economic performance by adjusting the input based on the disturbances and uncertainties in the designed system. Thus, ILC-EBtBO design has better economic performance than ILC-TMPC design.

Unlike ILC-TMPC design, which only focuses on eliminating the disturbances to track the desired value of the state, ILC-EBtBO considers the changes of disturbances and improves the current economic performance. Mathematically, suppose the nominal economic performance from the RTO layer is denoted as $V_{\text{nom}}(x^*, u^*, 0)$. After the nominal input ($u^*$) is applied to the process, the real economic performance is denoted as $V(x, u^*, d)$. With disturbances, the realistic economic performance may become better or worse. If $V(x, u^*, d) < V_{\text{nom}}(x^*, u^*, 0)$, it implies that the disturbance is favorable because it can enhance the economic performance; on the other hand, the disturbance could be unfavorable. However, it is quite a difficult task to directly distinguish the helpful disturbances from the unhelpful ones. Even if the favorable disturbance can improve the economic performance, the current value of $V(x, u^*, d)$ is still not optimal. One should optimize the economic performance which implicitly and automatically extracts the helpful disturbances and compensates the useless disturbances.

## 3. ILC BASED ECONOMIC BATCH-TO-BATCH OPTIMIZATION

With the augmented state-space model in (2) and the economic objective function in (3) mentioned above, one can optimize the economic performance of the current batch. Because of the process disturbances and uncertainties, the control design cannot effectively compensate for such disturbances in one batch run. In the past, ILC has been developed to successfully improve the control system for the present operating batch by feeding back the control error of the current batch to the next new batch. It was also believed to be able to produce a proper output product and reduce variability in the output product. Some model-based techniques have been developed based on system inversion to enhance ILC (Lee et al., 2000). To explore the repetitive nature of the batches and the batch-to-batch correlation of the disturbances, in this paper, with the information of the past operating batches, ILC-EBtBO is proposed to handle repetitive batch operating processes. It can gradually enhance the economic performance whenever there are batch-wise repeated disturbances. In this scheme, the best control inputs can be obtained according to the currently existing knowledge of the effects of the disturbance on the batch processes. However, in the objective function described in (3), there are no direct and explicit relationships between the objective and the disturbances, because the disturbance will directly affect the dynamic behavior of the batch processes; then the corresponding output will affect the objective function. In this situation, the best choice is to estimate the states first since they have the direct relationships between the objective function and the disturbances.

Whenever there is a disturbance at each batch run, $\mathbf{d}_k = [\mathbf{d}_k(0), \mathbf{d}_k(1), \ldots \mathbf{d}_k(T-1)]^T$, it is generally understood that the disturbance consists of two inseparable parts:

$$\mathbf{d}_k = \overline{\mathbf{d}}_k + \mathbf{v}_k \tag{4}$$

where $\overline{\mathbf{d}}_k = [\overline{\mathbf{d}}_k(0), \overline{\mathbf{d}}_k(1), \ldots \overline{\mathbf{d}}_k(T-1)]^T$ is the unknown deterministic part, and $\mathbf{v}_k = [\mathbf{v}_k(0), \mathbf{v}_k(1), \ldots \mathbf{v}_k(T-1)]^T$ is the inherent variability because of random signals during each batch. Besides, as the change of the initial condition is usually random, the disturbances in two successive batches can be represented by

$$\overline{\mathbf{d}}_{k+1} = \overline{\mathbf{d}}_k + \mathbf{w}_k \tag{5}$$

where $\mathbf{w}_k = [\mathbf{w}_k(0), \mathbf{w}_k(1), \ldots \mathbf{w}_k(T-1)]^T$ denotes the variance between two adjacent batches. In (4) and (5), both $\mathbf{w}_k(t),\ t \in \mathbf{I}_{0:T-1}$ and $\mathbf{v}_k(t),\ t \in \mathbf{I}_{0:T-1}$ are independent. They are identically distributed white noises, $\mathbf{w}_k(t) \sim N(0, \boldsymbol{\sigma}_w^2)$ and $\mathbf{v}_k(t) \sim N(0, \boldsymbol{\sigma}_v^2)$, respectively.

Since the disturbances contain the deterministic/repetitive part and the stochastic/non-repetitive part, the state in (2) can naturally be divided into the deterministic part of the state and the stochastic part of the state. The deterministic part of the state of the $k$th batch is

$$\overline{\mathbf{x}}_k = \mathbf{\Phi}\mathbf{x}_k(0) + \mathbf{\Psi}_u \mathbf{u}_k + \mathbf{\Psi}_d \overline{\mathbf{d}}_k \tag{6}$$

Similarly, the deterministic part of the state of the $k+1$th batch is

$$\overline{\mathbf{x}}_{k+1} = \mathbf{\Phi}\mathbf{x}_{k+1}(0) + \mathbf{\Psi}_u \mathbf{u}_{k+1} + \mathbf{\Psi}_d \overline{\mathbf{d}}_{k+1} \tag{7}$$

With (6) and (7), the incremental form of the deterministic part of the process state can be easily obtained

$$\overline{\mathbf{x}}_{k+1} = \overline{\mathbf{x}}_k + \mathbf{\Psi}_u \Delta\mathbf{u}_{k+1} + \mathbf{\Psi}_d \mathbf{w}_k \tag{8}$$

where $\Delta\mathbf{u}_{k+1} = \mathbf{u}_{k+1} - \mathbf{u}_k$

With (4), the stochastic part of the states at the $k$th and the $k+1$th batches can be given by

$$\begin{aligned} \mathbf{x}_k &= \overline{\mathbf{x}}_k + \mathbf{\Psi}_d \mathbf{v}_k \\ \mathbf{x}_{k+1} &= \overline{\mathbf{x}}_{k+1} + \mathbf{\Psi}_d \mathbf{v}_{k+1} \end{aligned} \tag{9}$$

From (8)-(9), the incremental form of the state-space model can be obtained,

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{\Psi}_u \Delta\mathbf{u}_{k+1} + \mathbf{\Psi}_d (\mathbf{w}_k + \mathbf{v}_{k+1} - \mathbf{v}_k) \tag{10}$$

Now the disturbance in (2) is replaced with the state of two successive batches in (10). (10) is primarily concerned with the changes of the state of the current batch from its state of the previous batch. Thus, the state-space model of the $k$th batch in Eq.(2) can be rewritten,

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{\Psi}_u \Delta\mathbf{u}_k + \mathbf{\Psi}_d (\mathbf{w}_{k-1} + \mathbf{v}_k - \mathbf{v}_{k-1}) \\ \mathbf{z}_k &= \mathbf{\Omega}\mathbf{x}_k + \mathbf{m}_k \\ \mathbf{y}_k(T) &= \mathbf{\Gamma}\mathbf{x}_k + \mathbf{n}_k(T) \end{aligned} \tag{11}$$

To improve the current economic performance of batch processes, more knowledge of the disturbances should be obtained. The indirect way is to improve the accuracy of the estimated state in the unknown disturbances. Thus, the problem becomes a general problem of estimating the state from the noisy sensor measurements and the disturbances in the Kalman filter. The Kalman filter is a recursive filter; only the estimated state from the previous time step and the current measurement are needed to estimate the current state. Thus, the estimation at each time point in continuous processes is reformulated as the estimation at each batch run in batch processes. Now the notation $\hat{\mathbf{x}}_{n|m}$ represents the estimate of $\mathbf{x}$ at batch $n$ with the given observations up to batch $m$, $m \le n$. $\hat{\mathbf{x}}_{k|k}$ is a posteriori state estimated at the $k$th batch with the given observations up to batch $k$. With the state-space model in (11), both the expected value of the state and the covariance matrix can be predicted by

$$\hat{\mathbf{x}}_{k|k-1} = \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{\Psi}_u \Delta\mathbf{u}_{k-1} \tag{12}$$

$$\begin{aligned} \mathbf{P}_{k|k-1} &= \mathbf{P}_{k-1|k-1} + \mathbf{Q}_k \\ &= \mathbf{P}_{k-1|k-1} + \mathbf{\Psi}_d \begin{bmatrix} \mathbf{R}_w + 2\mathbf{R}_v & 0 & \cdots & 0 \\ 0 & \mathbf{R}_w + 2\mathbf{R}_v & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{R}_w + 2\mathbf{R}_v \end{bmatrix} \mathbf{\Psi}_d^T \end{aligned} \tag{13}$$

where $\mathbf{R}_w = \mathbf{w}_k(t)\mathbf{w}_k^T(t)$ and $\mathbf{R}_v = \mathbf{v}_k(t)\mathbf{v}_k^T(t)$. With the new measurements, the predicted state and the covariance matrix should be updated. The error between the measurement and the prediction in the process variable ($\mathbf{z}_k$) and the quality variable ($\mathbf{y}_k(T)$) is

$$\mathbf{e}_k = \begin{bmatrix} \mathbf{e}_k^z \\ \mathbf{e}_k^y \end{bmatrix} = \begin{bmatrix} \mathbf{z}_k \\ \mathbf{y}_k(T) \end{bmatrix} - \begin{bmatrix} \mathbf{\Omega} \\ \mathbf{\Gamma} \end{bmatrix} \hat{\mathbf{x}}_{k|k-1} = \begin{bmatrix} \mathbf{z}_k \\ \mathbf{y}_k(T) \end{bmatrix} - \mathbf{L}_k \hat{\mathbf{x}}_{k|k-1} \tag{14}$$

where $\mathbf{L}_k = \begin{bmatrix} \mathbf{\Omega} \\ \mathbf{\Gamma} \end{bmatrix}$ and $\mathbf{e}_k$ contains the measurement residual of the process variables and the quality variables. The residual covariance can be calculated in the following equation,

$$\begin{aligned} \mathbf{S}_k &= \mathbf{L}_k \mathbf{P}_{k|k-1} \mathbf{L}_k^T + \mathbf{R}_k \\ &= \mathbf{L}_k \mathbf{P}_{k|k-1} \mathbf{L}_k^T + \begin{bmatrix} \mathbf{R}_m & 0 \\ 0 & \mathbf{R}_n \end{bmatrix} \end{aligned} \tag{15}$$

where $\mathbf{R}_m = \mathbf{m}_k \mathbf{m}_k^T$ and $\mathbf{R}_n = \mathbf{n}_k(t)\mathbf{n}_k(t)^T$. The optimal Kalman gain is

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{L}_k^T\mathbf{S}_k^{-1} \tag{16}$$

The updated state estimation ( $\hat{\mathbf{x}}_{k|k}$ ) and the updated covariance matrix ( $\mathbf{P}_{k|k}$ ) are given by

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{e}_k \tag{17}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{L}_k)\mathbf{P}_{k|k-1} \tag{18}$$

With the posterior estimation of the state of the previous batch and the change of the input of the previous batch, the economic optimization problem in (3) can be rewritten as

$$\min_{\mathbf{u}_{k+1}} V_{k+1}(\hat{\mathbf{x}}_{k+1|k}, \mathbf{u}_{k+1}) = \min_{\mathbf{u}_{k+1}} \sum_{t=0}^{T-1} l(\hat{\mathbf{x}}_{k+1|k}(t), \mathbf{u}_{k+1}(t)) + Q(\hat{\mathbf{y}}_{k+1|k}(T)) \tag{19}$$

$$\begin{aligned} \hat{\mathbf{x}}_{k+1|k} &= \hat{\mathbf{x}}_{k|k} + \mathbf{\Psi}_u \Delta\mathbf{u}_{k+1} \\ \mathbf{u}_{k+1} &= \mathbf{u}_k + \Delta\mathbf{u}_{k+1} \\ \hat{\mathbf{y}}_{k+1|k}(T) &= \mathbf{\Gamma}\hat{\mathbf{x}}_{k+1|k} \end{aligned} \tag{20}$$

The optimal solution obtained by optimizing the optimization problem of (19) and (20) is denoted as $(\hat{\mathbf{x}}_{k+1|k}^*, \mathbf{u}_{k+1}^*)$. In batch-to-batch optimization, the control loop in the batch direction is closed-loop because the control sequence will be put into the next batch directly. The real state of this batch and the posterior estimation of the state can be obtained by the Kalman filter at the end of the batch. With the new state estimation, the next batch is designed using (19)-(20). Thus, batch-to-batch optimization can be done recursively.

## 4. CASE STUDY

The proposed ILC-EBtBO strategy will be illustrated and compared with the traditional tracking method in this section through the simulated operation of a fed-batch bioreactor in the presence of disturbances. This example shows a penicillin fermentation process described in (Srinivasan et al., 2003) with the variables defined like this: $X$ defined as the concentration of biomass, $S$ as the concentration of substrate, $V$ as the volume, $u$ as the feed flow rate, and $S_{in}$ as the inlet substrate concentration. The economic objective of the process is to obtain the product of specific biomass concertation as much as possible and to minimize the cost of the substrate. The economic objective function in this simulation case is described as follows.

$$V = -0.2e^{-0.1(X(T)-X_s)^2} X(T)V(T) + 0.01\sum_{t=0}^{T-1} u^2(t)S_{in} \tag{21}$$

The disturbance here is the substrate inlet concentration $S_{in}$, which may deviate from the assumed value. Two simulation cases are studied, including the tracking batch-to-batch case and the economic batch-to-batch case. Two different deterministic disturbances of the substrate concentration, $+5$g/L and $-5$g/L, are separately applied to the fermentation bioreactor process. The disturbances occur at each batch run. In the $+5$g/L case, the realistic substrate concentration is higher than the assumed designed value. In order to keep the specific biomass concentration, the feed flowrate needs a lower value than the designed one so that the economic cost can decrease. This implies that the disturbance is useful here because it can help increase the economic performance.

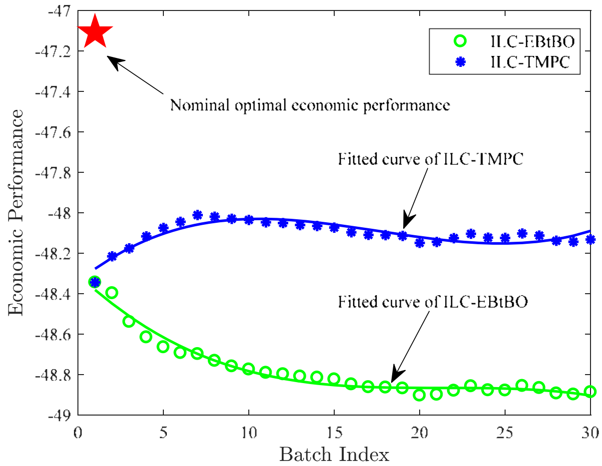


Figure 2 The comparison of the economic performance with the favorable disturbance between ILC-TMPC and ILC-EBtBO strategies

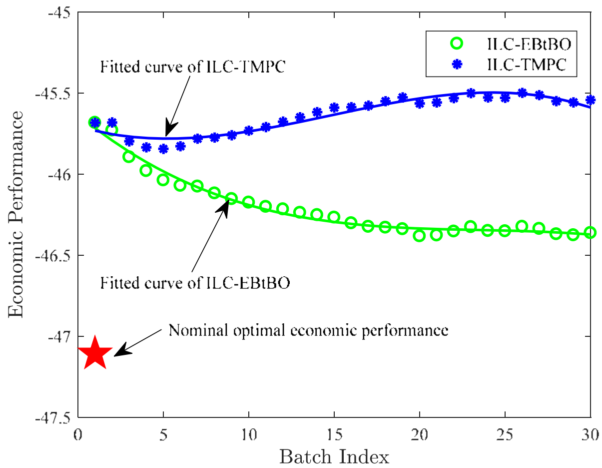


Figure 3 The comparison of the economic performance with the unfavorable disturbance between ILC-TMPC and ILC-EBtBO strategies

In Fig.2, the nominal optimal economic performance obtained from the higher optimization layer is marked with a red star. The economic performances of ILC-TMPC and ILC-EBtBO are also shown in Fig.2. The fitted curves of the two performance profiles are plotted, too. It is easy to see that both ILC-TMPC and ILC-EBtBO have better economic performance than the nominal optimal one (marked with a red star). With the increase of the batch runs, both ILC-TMPC and ILC-EBtBO converge to steady economic performances, respectively, but ILC-EBtBO has better economic performance. In the −5g/L case (Fig. 3), the concentration of the inlet substrate is lower than the assumed designed value. In order to keep the specific biomass concentration, the feed flowrate needs a higher value than the designed one. Thus, the disturbance is unfavorable. It can be seen clearly in Fig.3 that both strategies (ILC-TMPC and ILC-EBtBO) have worse economic performance than the nominal optimal counterpart. However, even the disturbance is unfavorable, ILC-EBtBO can still find better economic performance.

## 5. CONCLUSIONS

In this paper, ILC-EBtBO for batch processes is presented. The objective of the algorithm is to obtain better economic performance by learning unknown repetitive disturbances along the batch horizon because the disturbances in the initial design may be different from that in the implementation. An incremental state-space model combined with the unknown disturbance model is constructed in this paper. The proposed ILC-EBtBO has an advantage over ILC-TMPC in view of treating disturbances because ILC-TMPC always rejects disturbances as the undesired things while ILC-EBtBO enhances the economic performance based on the extracted useful information of favorable or unfavorable disturbances. The convergence of the algorithm is found to be successfully achieved in the presented industrial case. The case studies also show the effectiveness and the economic superiority of the proposed method.

In this paper, an LTV system is used for practical consideration because the proposed ILC-EBtBO can be solved easily. The extension work will focus on online economic within-batch optimization. Also, it is assumed that the model used here is completely accurate. In reality, there is always a model-plant mismatch. The economic performance design with a model-plant mismatch for batch processes will be considered in the future work.

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