Exam 03 Dec 2007

**Control Problem (20%)**

Consider controlling the air temperature (T) in a room using an electric heater (Q).

1. Someone has proposed the following model,

$$m\ cP\ dT/dt = Q + UA\ (Tout - T) + w\ cP\ (Tout - T)$$

Where does this equation come from? Explain in detail what the different terms and quantities are (with units), and make a flowsheet that shows the process. What assumptions have been made?

2. Find the time constant $\tau$ for the process by writing the model on the form $\tau\ dy/dt = -y + b$, where $y = T$ = room temperature. Use symbols only.

3. Find the numerical value of the time constant $\tau$ using the following information:

At steady-state the room temperature is 20C, the outdoor temperature is 0C, and the supplied electric heat is 2 kW. Data: Heat capacity of air = 1. 0 kJ/kg, K. Volume of air in room = 50 m3. Mole weight of air = 29 g/mol.

4. Assume that the outdoor temperature increases by 2C (step change). Sketch the time response for the indoor temperature when there is no control (if you did not find $\tau$ then you can assume it is 10 min).

5. How do you propose to control the temperature in the room? Consider both feedback and feedforward control, and show the proposed control structure on the flow sheet. Is feedforward necessary in this case?

6. Extra credit: Propose settings (tunings) for your controllers.

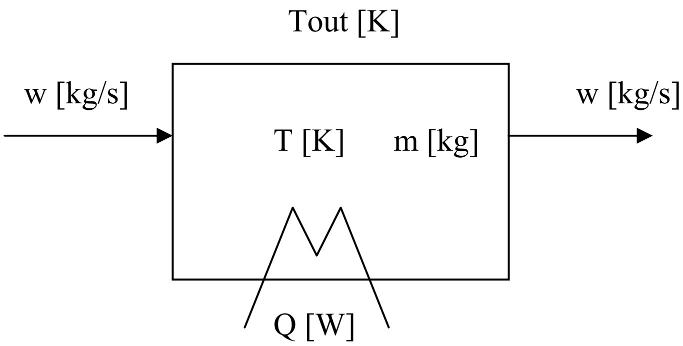
**Solution:**

1. This is the energy balance (Acc = In – Out [J/s]) for the air in the room in [W] = [J/s].

| | |
|---|---|
| m cP dT/dt | – accumulation of energy in room |
| Q | – supplied heat from heater |
| UA (Tout – T) | – heat loss through walls and windows |
| w cP (Tout – T) | - Heat supplied – heat removed by air flow |

m [kg] = mass of air in room (assumed constant)
w [kg/s] = massflow of air in and out of the room
UA [W/K] = overall heat transfer coffecient for heat loss through walls and windows
cP [J/ kg, K] = heat capacity of gas (assumed constant)

Assumptions made: Neglect heat capacity of walls (or rather, neglect temperature changes in the walls). Perfect mixing of air. Constant pressure




2. Introducing y = T and rewriting gives $\tau = m\,c_P / (UA + w\,c_P)$

3. The steady-state energy balance gives
$0 = Q + UA\,(T_{out} - T) + w\,c_P\,(T_{out} - T)$

And we find

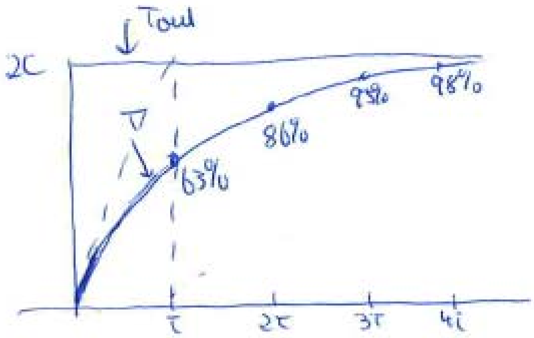
$0 = 2000 + (UA + w\,c_P)\,(0 - 20)$

And

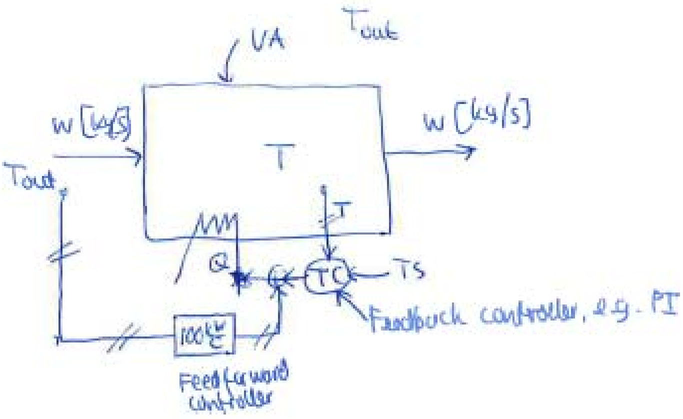
$(UA + w\,c_P) = 2000/20 = 100$ W/K

The mass of air is $m = \rho V$.
Assume ideal gas at 1 atm. Then at 20 C (use SI units):
$\rho = p M / RT = 1.013\text{e}5 * 29\text{e-}3 / 8.31 * 293 = 1.21$ kg/m3
$m = \rho V = 50 * 1.21 = 60$ kg
Then (again SI units for everything)
$\tau = m\,c_P / (UA + w\,c_P) = 60 * 1000 / 100 = 600$ s (= 10 min).

4. The response from d=Tout to y=T is first-order with a gain k=1 and time constant $\tau$ = 10 min (the gain k=1 because, from the steady-state energy balance, (Tout – T) must be a constant if everything else is constant, so the change in T at steady-state is the same as the steady-state change in Tout). Thus, the steady-state change in T is 2C and the increase in T after $\tau$ = 10 min is 0.63*2C = 1.26 C, and after $3\tau$ = 30 min it is 0.95*2C = 1.9 C




5. We use a feedback control (TC) where Q is adjusted to keep y=T constant at its setpoint Ts. A PI-controller is proposed. Feedforward is based on measuring d = Tout. It is probably not very important to have feedforward in this case, because the expected changes in the outdoor temperature are rather slow and not very large.




4. Extra:

Tuning of feedforward controller:

The steady-state gain from d=Tout to y=T is kd=1, and from u=Q to y=T is k=1/(UA + w cP) = 0.01 K/W (with no control). Thus, the feedforward controller from d to y should have a gain of kd/k = 100 W/K. No dynamics are needed in the feedforward controller since the dynamic effect of d=Tout and u=Q on y=T are the same (first-order with time constant $\tau$).

Tuning of PI feedback controller.

From the SIMC tuning rules, the two PI-parameters are $K_c = (1/k)\,\tau / (\tau_c + \theta)$, $\tau_I = \min(\tau, 4(\tau_c + \theta))$. We have no delay, so $\theta = 0$. The time constant for the closed-loop response, $\tau_c$, (with control) is a tuning parameter. Assume we want to "speed up" the response by factor 2, so $\tau_c$ = 5 min. We then get: $K_c = (1/0.01)\, 10/5 = 0.02$ W/K, $\tau_I = \tau = 10$ min.