TKP4140 Process Control

Department of Chemical Engineering NTNU

Autumn 2018 - Midterm Exam

12. October 2018

Student number: SOLUTION

- Write your student number on **every** page in the indicated space.
- Write your answers on the enclosed pages.
- Use the last page for details if you have too little space.
- Do not separate the enclosed pages.
- Time: **100** minutes

Problem 1 (20 points)
Consider two flash tanks in series, as shown in Fig. 1, where the feed stream F is separated into gas product G_1 and liquid product L_1 in the first tank at pressure P_1. The liquid product of tank 1 is further heated, and then flashed in the second tank to gas product G_2 and liquid product L.

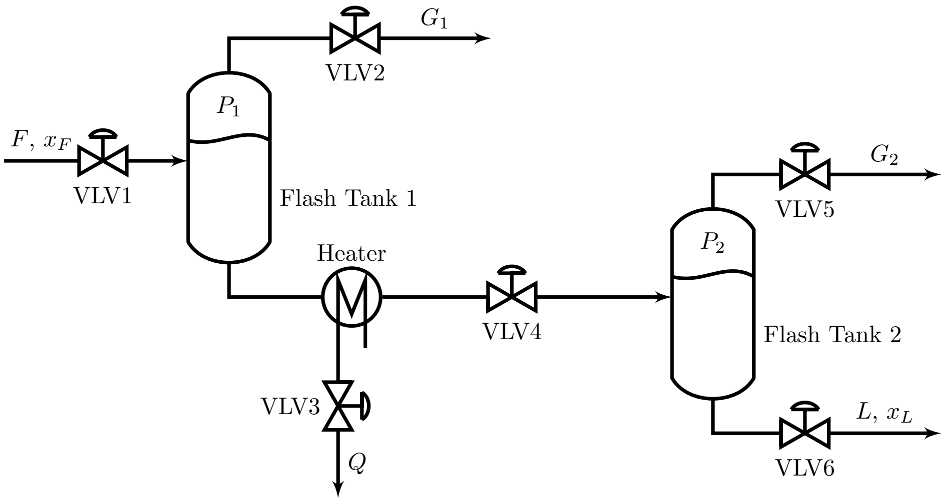

Figure 1: Two flash tanks in series

The control objectives are:

- Keep the composition of light component in the liquid product L at $x_L^s = 0.05$.
- Keep the process stable.
- Keep the pressures in both tanks at their setpoints: $P_1^s = 10$ bar and $P_2^s = 2$ bar respectively.
- Keep the liquid product flow L at a given value (this should be controlled tightly).

The available measurements are:

- Pressure in flash tank 1 and 2: P_1 and P_2.
- Level in flash tank 1 and 2: h_1 and h_2.
- Temperature of liquid product.
- Composition of liquid product L: x_L (with a large delay and unreliable).
- All flows (if needed).

(a) Identify:

CVs:
- pressure in flash tank 1 and 2: P_1, P_2 (to stabilize the process)
- level in flash tank 1 and 2: h_1, h_2 (to stabilize the process)
- liquid product flow, L (this is the TPM)
- liquid product composition x_L (valuable product)

MVs: 6 valves VLV1 to VLV6.

(b) Propose a control structure based on feedback. Draw the proposed control structure in Fig. 1 (Show the MVs-CVs pairing on the flowsheet).
We follow the rule pair CVs with MVs that are close

- control L using VLV6 (for tight control)
- control $P1$ using VLV2
- control $P2$ using VLV5
- control $h2$ using VLV4 (only option since VLV5 and VLV6 are already used)
- control $h1$ using VLV1 (only option since VLV4 and VLV5 are already used)
- Only VLV3 is left to control x_L. Because measurement of x_L, we use a cascade control structure for the composition. The outer loop (master controller-slow) is a composition controller that gives the setpoint to a temperature controller (slave controller-fast), which in turn manipulates VLV3.

Remark. A more detailed explanation to the pairing will be provided later in the semester, in time for the final exam.

Problem 2 (5 points)
Draw the block diagram for the transfer function given by Eq. 1. Indicate signals $y_s(s)$, $e(s)$, $u(s)$, $y(s)$ and $d(s)$, and blocks $c(s)$, $g(s)$, $g_m(s)$ and $g_d(s)$ on your figure.

$$y(s) = \frac{g(s)c(s)y_s(s) + g_d(s)d(s)}{1 + g(s)g_m(s)c(s)} \quad (1)$$

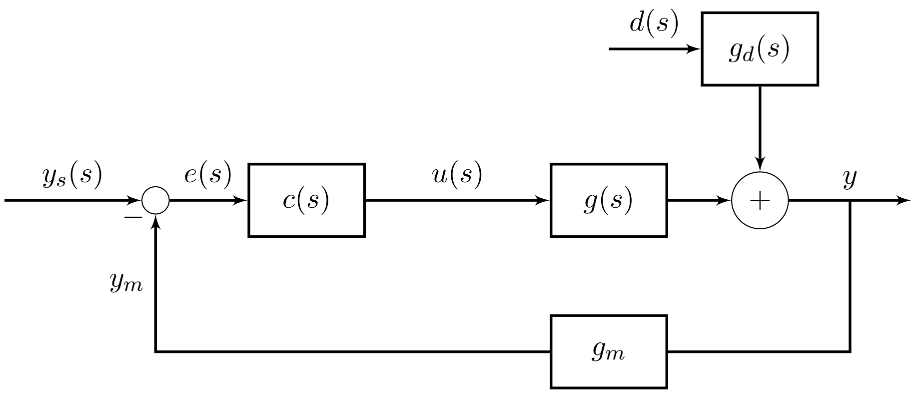

Figure 2: Block Diagram

Problem 3 (5 points)
SIMC tuning rules.

(a) Write the SIMC PI tuning rules for a first-order plus delay process ($g(s)$).

$$g(s) = \frac{ke^{-\theta s}}{\tau_1 s+1}$$

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_C+\theta} \qquad \tau_I = \min(\tau_1, 4(\tau_C+\theta))$$

(b) Write the SIMC PID tuning rules (cascade form) for a second-order plus delay process ($g(s)$).

$$g(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_C+\theta} \qquad \tau_I = \min(\tau_1, 4(\tau_C+\theta)) \qquad \tau_D = \tau_2$$

Problem 4 (5 points)
PID Controller

Two commonly used representations of a PID controller are

- Series/cascade form: $c(s) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)(\tau_D s+1) = \frac{K_c}{\tau_I s}\left(\tau_I\tau_D s^2 + (\tau_I+\tau_D)s + 1\right)$
- Ideal/parallel form: $c(s) = K_c'\left(\frac{1}{\tau_I' s} + \tau_D' s + 1\right) = \frac{K_c'}{\tau_I' s}\left(\tau_I'\tau_D' s^2 + \tau_I' s + 1\right)$

Given the controller settings K_c, τ_I and τ_D for a PID controller in series form, what are the corresponding values of K_c', τ_I' and τ_D' for the ideal form? Derive the relationship.

See Exercise 6 solution.

Problem 5 (25 points)
Given the transfer function in Eq. 2:

$$g(s) = \frac{2}{(5s+1)(3s+1)} \tag{2}$$

(a) Find the first order plus delay (FOPD) approximation using the Half rule.

$$k = 2$$
$$\tau_1 = 5 + \frac{3}{2} = 6.5$$
$$\theta = \frac{3}{2} = 1.5$$
$$g(s) \approx \frac{2e^{-1.5s}}{6.5s+1}$$

(b) Find PI controller tuning parameters using the SIMC rules. Select $\tau_C = 2$.

$$K_c = \frac{1}{2}\frac{6.5}{1.5+2} = 0.93$$
$$\tau_I = \min(6.5, 4(2+1.5)) = \min(6.5, 14) = 6.5$$

(c) Find PID controller tuning parameters using the SIMC rules. Select $\tau_C = 1.5$.

$$K_c = \frac{1}{2}\frac{5}{1.5+0} = 1.67$$
$$\tau_I = \min(5, 4(1.5+0)) = \min(5, 6) = 5$$
$$\tau_D = 3$$

(d) Sketch in Fig. 3 the closed-loop response for a step in setpoint for both the PI and PID controllers with the obtained tuning parameters. We start by deriving $T(s)$, the transfer function from the setpoint $y_s(s)$ to the output $y(s)$, considering:

$$T(s) = \frac{L(s)}{1+L(s)}$$
$$\text{with,} \quad L(s) = g(s)c(s)$$

For a PI controller, the closed loop transfer function $L(s)$ is:

$$\begin{aligned} L(s) &= g(s)c(s) \\ L(s) &= \frac{2}{(5s+1)(3s+1)} \cdot 0.93\frac{6.5s+1}{6.5s} \\ &= 0.286\frac{6.5s+1}{s(5s+1)(3s+1)} \end{aligned}$$

And the transfer function $T(s)$ is:

$$T(s) = 0.078788\frac{s+0.1538}{(s+0.1305)(s^2+0.4028s+0.09289)} \tag{3}$$

which is a rather tedious function to sketch. To avoid calculating a sixth order transfer function and simplify T(s), we can also use the half-rule to approximate $L(s)$ to a first order plus delay model, with the rule

$$\frac{Ts+1}{\tau s+1} \approx \frac{T}{\tau}$$

, where T and τ are close. Thus, we get:

$$\frac{6.5s+1}{5s+1} \approx \frac{6.5}{5} \tag{4}$$

$$\text{and,} \quad L(s) = 0.286\frac{6.5}{5}\frac{1}{s(3s+1)} \tag{5}$$

$$L(s) = 0.37\frac{1}{s(3s+1)} \tag{6}$$

$$\tag{7}$$

With this approximation for $L(s)$, the transfer function $T(s)$ for PI becomes:

$$T(s) = \frac{0.37\frac{1}{s(3s+1)}}{1+0.37\frac{1}{s(3s+1)}} \tag{8a}$$

$$= \frac{0.37}{3s^2+s+0.37} \tag{8b}$$

$$= \frac{1}{8.1s^2+2.7s+1} \tag{8c}$$

For PID controller, the closed loop transfer function is:

$$\begin{aligned} L(s) &= g(s)c(s) \\ &= \frac{2}{(5s+1)(3s+1)} \cdot 1.67\frac{5s+1}{5s}(3s+1) \\ &= \frac{0.67}{s} \end{aligned}$$

And the transfer function $T(s)$ for a PID controller is:

$$
\begin{aligned}
T(s) &= \frac{\frac{0.67}{s}}{1+\frac{0.67}{s}} \\
&= \frac{1}{1.5s+1}
\end{aligned}
$$

Thus, the closed loop transfer function for this PID controller is a first order plus delay function.

The results for a unit step change in the set point are shown in Figure 3, both for PI with $T(s)$ from Eq.3(black)and with approximated $T(s)$ from Eq. 8 (dotted), and PID (blue) controllers. The approximated PI (dotted line) is not that accurate, due to using the half rule twice (once for approximating $g(s)$ to obtain $c(s)$, and second for approximating $L(s)$). However, for the purpose of sketching the response in this exercise, it is good enough.

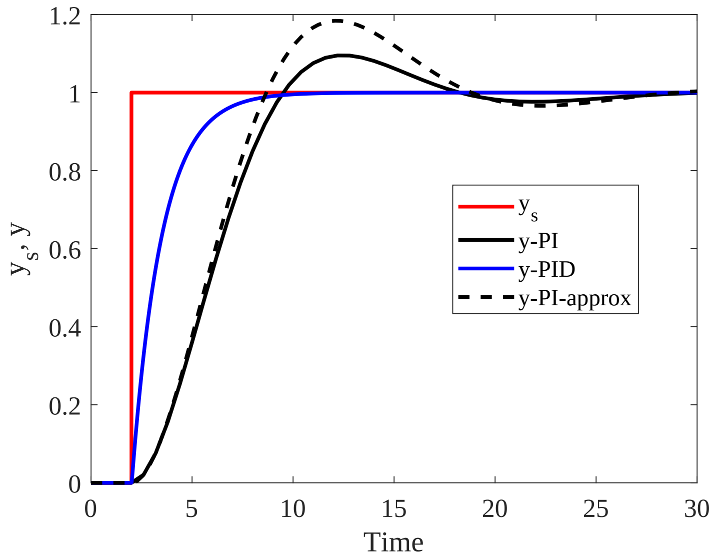

Figure 3: Closed-loop responses for setpoint changes for PI and PID

for PID we have a first order response.

(e) For the PID controller, find also the tunings for the ideal form.

$$
\begin{aligned}
K_c' &= K_c\big(1+\frac{\tau_D}{\tau_I}\big) = 1.67\left(1+\frac{3}{5}\right) = 2.67 \\
\tau_I' &= \tau_I\big(1+\frac{\tau_D}{\tau_I}\big) = 8 \\
\tau_D' &= \frac{\tau_D}{1+\frac{\tau_D}{\tau_I}} = \frac{15}{8} = 1.875
\end{aligned}
$$

(f) Consider a disturbance $d(s)$ at the plant output (i.e.$g_d = 1$). Find the closed-loop transfer function $T_d(s)$ from $d(s)$ to $y(s)$ with the obtained PID tuning parameters.

$$\begin{aligned}
T_d(s) &= \frac{1}{g(s)c(s)} \\
\text{with,} \quad g(s) &= \frac{2}{(5s+1)(3s+1)} \\
\text{and,} \quad c(s) &= K_c\big(1 + \frac{1}{\tau_I s}\big)\big(\tau_D s + 1\big) \\
&= 1.67\big(1 + \frac{1}{5s}\big)\big(3s + 1\big) \\
\text{gives,} \quad T_d(s) &= \frac{1.5s}{1.5s+1}
\end{aligned}$$

Problem 6 (10 points)

Consider a disturbance d at the plant output, with the following closed-loop transfer function from $d(s)$ to $y(s)$: $T_d(s) = \frac{1.5s}{1.5s+1}$

Sketch in Fig. 4 the closed-loop response in y for a unit step change in the disturbance d at time $t = 0$. Also show in Fig. 4 the response in y for a unit step change in the disturbance d without control.

$y(s)$ is given by

$$\begin{aligned} y(s) &= T_d(s)d(s) \\ &= \frac{1.5s}{1.5s+1}\frac{1}{s} \\ &= \frac{1.5}{1.5s+1} \end{aligned}$$

The response can be sketched considering:
at

$$t = 0, y(t) = \lim_{s\to\infty} T_d(s) = 1$$

at

$$t \to \infty, y(t) = \lim_{s\to 0} T_d(s) = 0$$

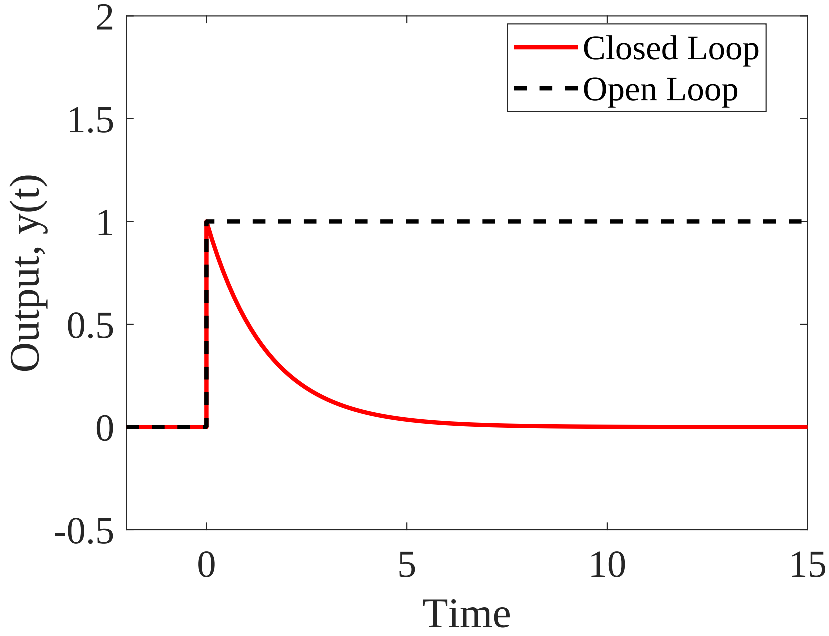

Figure 4: Closed-loop response for a unit change in disturbances for PI controller

Problem 7 (5 points)

Given a system with the transfer function in Eq. 9, obtain the controller setting for a PI controller using the SIMC tuning rule.

$$g(s) = \frac{3e^{-2s}}{s}. \tag{9}$$

The PI controller tunings for an integrating process with the SIMC rules, and with $\tau_C = \theta$ are:

$$\begin{aligned} K_c &= \frac{1}{k'}\frac{1}{\tau_c + \theta} \\ &= \frac{1}{3}\frac{1}{2+2} \\ &= \frac{1}{12} \\ \tau_I &= 4(\tau_C + \theta) \\ &= 4(2+2) \\ &= 16 \end{aligned}$$

Problem 8 (10 points)

Laplace Transforms

(a) Prove that $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$.

$$\begin{aligned}\mathcal{L}\{e^{-at}\} &= \int_0^\infty e^{-at}e^{-st}dt \\ &= \int_0^\infty e^{-(a+s)t}dt \\ &= -\frac{1}{a+s}e^{-(a+s)t}\Big|_0^\infty \\ &= -\frac{1}{a+s}(0-1) \\ &= \frac{1}{a+s}\end{aligned}$$

(b) Given $g(s) = \frac{1}{5s+1}$, derive $y(t)$ (time domain) for:

i. $u(s) = 1$

$$\begin{aligned}y(s) &= g(s)u(s) \\ &= \frac{1}{5s+1}\cdot 1 = \frac{0.2}{s+0.2}\end{aligned}$$

Using the formula proven in *(a)*, we get

$$y(t) = \mathcal{L}^{-1}\left\{\frac{0.2}{s+0.2}\right\} = 0.2e^{-0.2t}$$

.

ii. $u(s) = \frac{1}{s}$

Similarly,

$$\begin{aligned}y(s) &= g(s)u(s) \\ &= \frac{1}{5s+1}\cdot\frac{1}{s} = \frac{0.2}{s(s+0.2)}\end{aligned}$$

We decompose $y(s)$ into 2 fractions according to:

$$\begin{aligned}y(s) &= \frac{0.2}{s(s+0.2)} \\ &= \frac{1}{s} - \frac{1}{s+0.2}\end{aligned}$$

We note that the first fraction is a step response (i.e. $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$), and the Laplace inverse of second fraction is given by the formula in (a) , resulting in

$$\begin{aligned}y(t) &= \mathcal{L}^{-1}\left\{\frac{0.2}{s(s+0.2)}\right\} \\ &= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+0.2}\right\} \\ &= 1 - e^{-0.2t}\end{aligned}$$

Problem 9 (10 points)
Sketch the step response for

(a) $g_1(s) = 2e^{-3s} + \frac{3}{2s+1}$ g_1 is a sum of two responses, $g_{1,a} = 2e^{-3s}$ and $g_{1,b} = \frac{3}{2s+1}$. The individual step responses are shown in Figure 5, together with g_1.

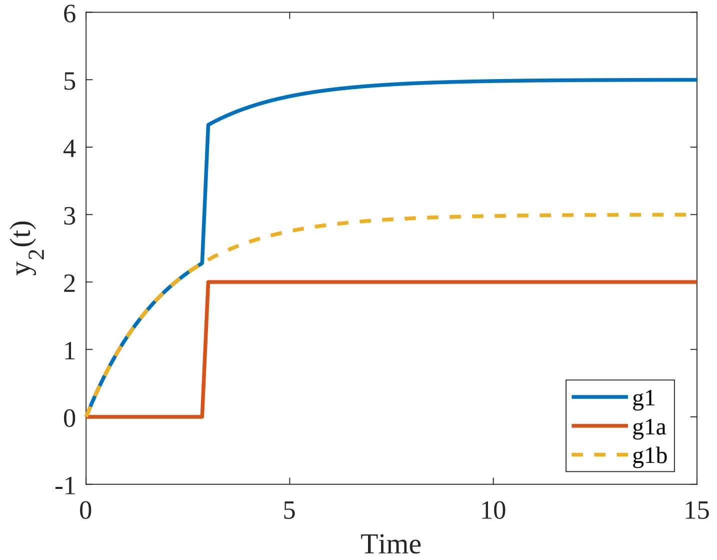

Figure 5: Step Response g_1

(b) $g_2(s) = \frac{2s-1}{(4s+1)(s+1)}$ The system has a RHP zero at $z = 0.5$, i.e. the system has an inverse response.

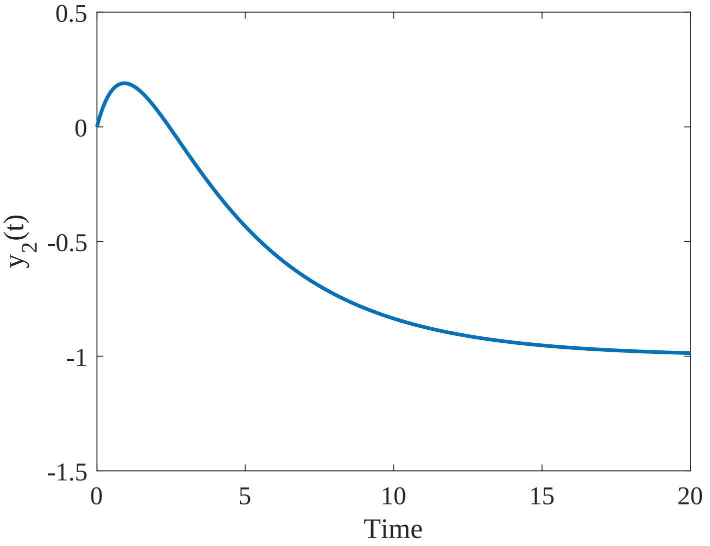

Figure 6: Step Response g_2

Problem 10 (5 points)
Given the plot in Fig. 7 for a second order transfer function and the formulas in Eq. 10:

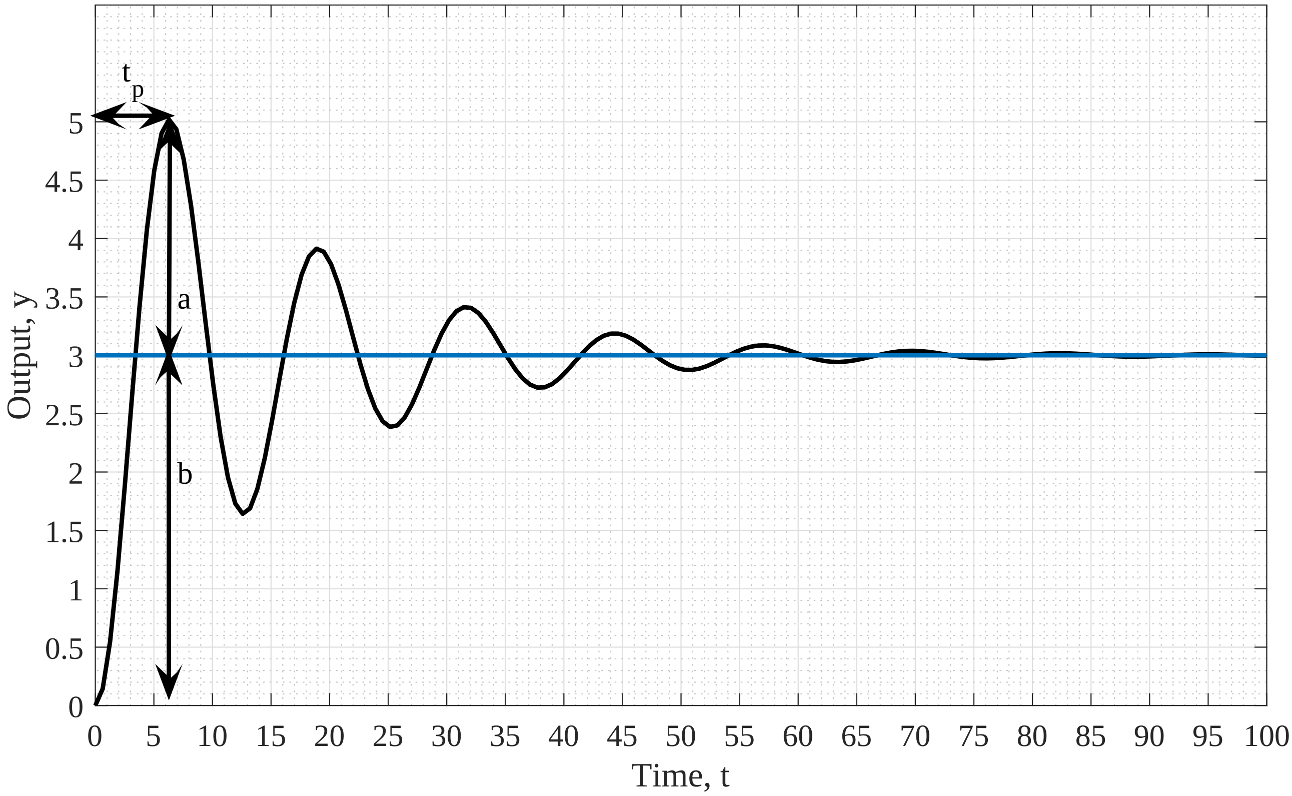

Figure 7: Step response for $g(s)$

$$t_p = \frac{\pi\tau}{\sqrt{1-\zeta^2}} \approx \pi\tau; \qquad \frac{a}{b} = exp\Big(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\Big) \approx exp(-\pi\zeta) \tag{10}$$

where, t_p is the time to first peak and $\frac{a}{b}$ is the overshoot. These approximations hold for a small ζ, and can be used in this case.

(a) Show a, b in Fig.7. See Figure 7

(b) Find the transfer function $g(s)$.
The general equation for a second order process is

$$g(s) = \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1} \tag{11}$$

From figure 7, we read:

$$t_p \approx 6.3$$
$$a \approx 2$$
$$b \approx 3$$
$$K = 3$$

Using the given formulas,

$$\begin{aligned}\tau &= \frac{t_p}{\pi} \\ &= \frac{6.3}{\pi} \\ &= 2 \\ \zeta &= -\ln\left(\frac{a}{b\pi}\right) \\ &= -\ln\left(\frac{2}{3\pi}\right) \\ &= -0.129\end{aligned}$$

Substituting into Eq.11, gives:

$$\begin{aligned}g(s) &\approx \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1} \\ &\approx \frac{3}{2^2 s^2 + 2 \cdot (2 \cdot 0.129)s + 1} \\ &\approx \frac{3}{4s^2 + 0.516s + 1}\end{aligned}$$

This is close to the plotted function, given by

$$g(s) = \frac{3}{4s^2 + 0.5s + 1}$$

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