



Solution TKP4140 – Process Control

Examination date: 14 August 2025

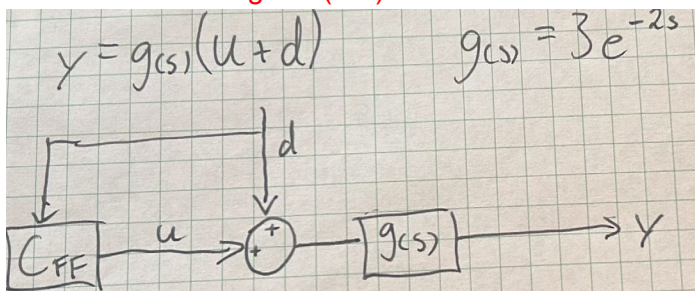
Problem 1 (60%) – Mixed process control questions

- 1) (5%) Given an unknown process, suggest a test (**experiment**) you can perform to check if it is linear. **Solution:** Make two steps of different magnitudes. If linear then the response is identical when scaled. It could be a positive step and negative step of equal size. If the system is linear, the output responses will be symmetric,
- 2) (5%) PID controller. What is the effect on performance (faster/slower) and robustness (more/less) of:
 - (i) decrease in controller gain
 - (ii) increase in integral time
 - (iii) decrease in derivative time.

Solution to all: Slower response and more robust

- 3) (10%) Show a block diagram for feedforward control for a measured input disturbance d , that is, $y = g(s)(u+d)$. What feedforward controller would you suggest for the process $g(s)=3 e^{-2s}$. Plot the expected response to a disturbance in d .

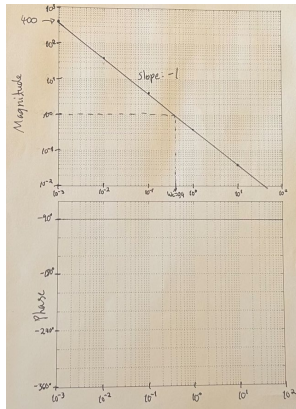
Solution: Block diagram (3%)



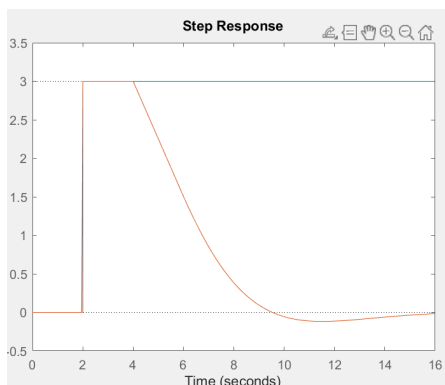
FF(5%): We have $gd=g$ for input disturbance. Ideal: $cFF = -gd/g = -1$.

Response with control (2%). It is perfect (so y stays at 0 all the time, if we use deviation variables).

- 4) (5%) Consider a system with loop transfer function $L(s) = 0.4/s$. Sketch the Bode plot (2%)



- (i) What is the gain and phase at low frequency (as $\omega \rightarrow 0$) ? (1%) Infinite gain, phase=-90 degrees.
 - (ii) What is the gain and phase margin? (1%) Gain margin is infinite since phase never reaches -180 degrees. Phase margin is 90 degrees.
 - (iii) What is the closed-loop time constant of the system? (1%) It is $1/0.4=2.5s$
- 5) (5%) Why is it not recommended to pair on negative steady-state RGA elements?
Because it means that the gain from the paired input to output may change sign if we open or close other loops. This may lead to instability, in particular, if the controller has integral action.
- 6) (5%) Consider a system described by the differential equations $\dot{x}/dt=Ax+Bu$, $y=Cx+Du$. What is the steady-state gain from u to y ? Yes, it is linear. $G(s) = C(sI-A)^{-1}B + D$, so $G(0) = -C A^{-1} B + D$
- 7) (10%) Consider the process $g(s)=3 e^{-2s}$, with $y = g(s) (u+d)$. Plot the output $y(t)$ in response to a unit step change in u (without control) (or it could be d , the response is the same). Suggest PI controller tunings for this process. Plot the expected response $y(t)$ to a unit input disturbance d (with control).



Solution: (3%) Open-loop response (blue) has a delay of 2 [seconds] and then jumps up to 3. PI-tunings (4%): Note that $\tau=0$. So SIMC gives pure I-controller $c = KI/s$ with $KI = (1/k)(1/\tau_{\text{auc}}+\theta)$. With $\tau_{\text{auc}}=\theta=2$ we get $KI = (1/(3*4)) = 1/12 = 0.083$.

Closed-loop response (3%) is shown in red; initially (up to $t=4$) it is identical to the blue open-loop response.

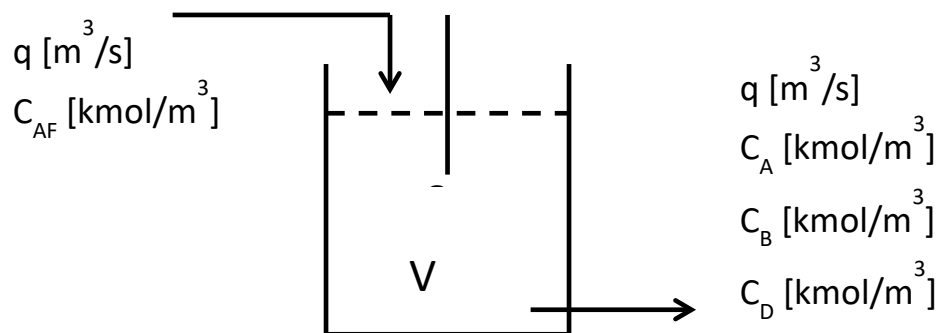
8) (5%) What is anti-windup?

Solution: It is to somehow stop the integral action in periods when the input u has no effect on the output y , for example, due to saturation of u .

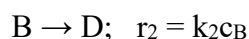
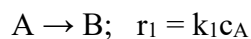
9) (5%) Model predictive controllers (MPC) are widely studied but less used in practice that was expected when they came into use about 40 years ago. What do you think is the reason? Solution; One reason is that it requires quite a lot of effort, especially in terms of getting the model, and the benefit is often small compared to PI control.

10) (5%) When do we use split range control? Give an example. Solution: It is used for MV-MV switching when more than MV is needed for control. Example1: Gas pedal and break in car. Example2: Heating and cooling for car.

Problem 2 (40%) - Reactor dynamics



The following reactions take place in an isothermal mixing tank reactor (CSTR).



where r_j [kmol /m³,s] is the rate of reaction j . The feed only contains component A. The reactor volume V [m³] is assumed constant (Temperature and level controllers needed to keep temperature and volume constant are not shown on the figure).

We are considering using the feedrate (q) to control the concentration of B, so we want to derive a dynamic model for the effect of q on c_B .

- (a) (12%) Derive a dynamic model for the system using balances for components A and B (Note that you need not use the energy balance because the temperature is assumed constant, and you need not use the total balance because the volume is assumed constant).

Solution: General component balance for component A

$$d(\text{holdup A})/dt = \text{inflow A} - \text{outflow A} + \text{generated A in reactions} [\text{molA/s}]$$

where

$$\text{Holdup A} = c_A V, \text{inflow A} = q c_{AF}, \text{outflow A} = q c_A, \text{Gen A} = -r_1 V = -k_1 c_A V.$$

This gives:

$$(\$) \quad d/dt(c_A V) = q (c_{AF} - c_A) - k_1 c_A V$$

Similar for component B:

$$(\$ \$) \quad d/dt(c_B V) = -q c_B + k_1 c_A V - k_2 c_B V$$

- (b) (8%) At steady state we have :

$$V=8\text{m}^3, q=0.4 \text{ m}^3/\text{s}, C_{AF} = 7 \text{ kmol/m}^3, C_A=1 \text{ kmol/m}^3, C_B = 5.5 \text{ kmol/m}^3, C_D=0.5 \text{ kmol/m}^3.$$

What are the values of k_1 [s^{-1}] and k_2 [s^{-1}]?

Steady-state mass balances

$$0 = q (C_{AF} - C_A) - k_1 C_A V$$

$$\rightarrow k_1 = q (C_{AF} - C_A) / C_A V = 0.4 * 6 / (1 * 8) = 0.3 \text{ s}^{-1}$$

$$0 = -q c_B + k_1 C_A V - k_2 C_B V$$

$$\rightarrow k_2 = (-q c_B + k_1 C_A V) / (C_B V) = -0.4 * 5.5 + 0.3 * 1 * 8 / (5.5 * 8) = 0.0045 \text{ s}^{-1}$$

- (c) (10%) Linearize the model. Show that the model can be written as shown in the figure below. Determine the constants k_A , k_{B1} , k_{B2} , τ_A , τ_B . (Note that the sign of k_{B2} is negative whereas the others are positive).

General linearization for $V dx/dt = f(x,u)$ is $V * d\Delta x/dt = \Delta f$

where $\Delta x = \Delta C_A(t) = C_A(t) - C_A^*$, etc. and $*$ denotes the steady state value (see part b),

The only nonlinearity for f in (\$) is the bilinear term. For this term the deviation from steady state is

$$\Delta [q (C_{AF} - C_A)] = \Delta q (C_{AF}^* - C_A^*) + q^* (\Delta C_{AF} - \Delta C_A)$$

Taking Laplace we get (where $C_A(s) = \mathcal{L}(\Delta C_A(t))$):

$$s C_A(s) V = q(s) (C_{AF}^* - C_A^*) + q^* (C_{AF}(s) - C_A(s)) - k_1 V C_A(s)$$

Gives $C_A(s)/q(s) = k_A / (\tau_A s + 1)$ where

$$k_A = (C_{AF}^* - C_A^*) (q^* + k_1 V) = 6 / (0.4 + 0.3 * 8) = 2.14$$

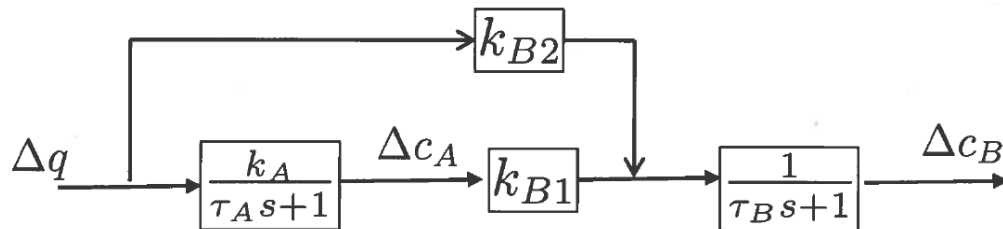
$$\tau_A = V / (q^* + k_1 V) = 8 / 2.8 = 2.86 \text{ s}$$

Same with (\$\$)

$$s C_B(s) V = -q(s) C_B^* - q^* C_B(s) + k_1 V C_A(s) - k_2 V C_B(s)$$

Gives $C_B(s) = (k_{B1} C_A(s) + k_{B2} q(s)) / (\tau_B s + 1)$ where
 $\tau_B = V / (q^* + k_2 V) = 8 / (0.4 + 0.0045 \cdot 8) = 18.3 \text{ s}$
 $k_{B1} = k_1 V / (q + k_2 V) = 0.3 \cdot 8 / ((0.4 + 0.0045 \cdot 8)) = 5.50$
 $k_{B2} = -C_B^* / (q + k_2 V) = -5.5 / (0.435) = -12.61$

- (d) (10%) Determine the transfer function $G(s)$ from $u = \Delta q$ to $y = \Delta c_B$. What are the poles and zeros (you can do this just based on the block diagram below). When you put in numbers, $G(s)$ should have one RHP-zero (please check this if you did part c). Would you recommend using $u = \Delta q$ to control $y = \Delta c_B$?



Solution: Transfer function $c_B(s) = G(s) q(s)$.

$$\begin{aligned} G(s) &= (k_A k_{B1} / (\tau_A s + 1) + k_{B2}) / (\tau_B s + 1) && (4\% \text{ for this, } 4\% \text{ for the rest}) \\ &= (k_A k_{B1} + k_{B2} (\tau_A s + 1)) / ((\tau_A s + 1)(\tau_B s + 1)) \\ &= k (\tau_A s + 1) / ((\tau_A s + 1)(\tau_B s + 1)) \end{aligned}$$

$$\text{Where } k = k_A k_{B1} + k_{B2} = 2.14 \cdot 5.5 - 12.61 = -0.84$$

$$\tau_{Auz} = k_{B2} \tau_A / (k_A k_{B1} + k_{B2}) = k_{B2} \tau_A / k = 12.6 \cdot 2.86 / -0.84 = -42.9 \text{ s}$$

Poles are at $-1/\tau_A, -1/\tau_B$. (LHP-poles, stable)

Zero at $z = -1/\tau_{Auz} = 0.023 \text{ s}^{-1}$ (RHP-zero)

(2%) Have RHP-zero so closed-loop control using $u = q(s)$ to control $y = c_B(s)$ will be poor. Not recommended

Check with ChatGPT (aug. 2024 + July 2025)

- 1) (5%) Given an unknown process, suggest a test you can perform to check if it is linear.

ChatGPT: OK, but suggests 3 experiments (u_1 , u_2 , u_1+u_2)

2025: Now it's OK. Suggests only two experiments.:

1) Test for Linearity (5%)

To check if a process is linear, perform a **step test** with two or more different amplitudes. If the output response is proportional (i.e., the shape stays the same and the magnitude scales linearly), the system is likely linear. For example, apply steps of +1 and +2 and compare the output responses.

- 2) (5%) PID controller. What is the effect on performance (faster/slower) and robustness (more/less) of:
- decrease in controller gain
 - increase in integral time
 - decrease in derivative time.

ChatGPT: OK for gain and integral, wrong for derivative (claims response gets faster and less robust with small T_d)

2025: Now it's OK

- 3) (10%) Show a block diagram for feedforward control for a measured input disturbance d , that is, $y = g(s)(u+d)$. What feedforward controller would you suggest for the process $g(s)=3 e^{-2s}$. Plot the expected response to a disturbance in d .

ChatGPT: It's quite impressive what it can do, but it got it wrong to begin with. I had to stress that g is also the transfer function from d to y . Maybe it was my fault because to begin with I didn't define what u is.

2025: Now it's OK

- 4) (5%) Consider a system with loop transfer function $L(s) = 0.4/s$. Sketch the Bode plot.
- What is the gain and phase at low frequency (as $\omega \rightarrow 0$) ?
 - What is the gain and phase margin?
 - What is the closed-loop time constant of the system?

ChatGPT: Wow, it is very impressive, although it got the PM wrong. Maybe this was because it incorrectly claimed that the phase was +90 rather than -90.

Let's analyze the system with the loop transfer function $L(s) = \frac{0.4}{s}$ and address each part of the question step by step.

Bode Plot of $L(s) = \frac{0.4}{s}$

1. Sketching the Bode Plot:

To sketch the Bode plot, we need to determine both the magnitude and phase of the transfer function $L(s)$ as a function of frequency ω .

Magnitude Response:

For $L(s) = \frac{0.4}{s}$, the magnitude $|L(j\omega)|$ is:

$$|L(j\omega)| = \left| \frac{0.4}{j\omega} \right| = \frac{0.4}{\omega}$$



Phase Response:

The phase of $L(j\omega)$ is:

$$\arg\left(\frac{0.4}{j\omega}\right) = \arg(0.4) - \arg(j\omega) = 0^\circ - (-90^\circ) = +90^\circ$$

So, the phase angle is $+90^\circ$ for all frequencies.

Sketch:

- **Magnitude Plot:** Starts at -7.96 dB at $\omega = 1$ rad/s and decreases with a slope of -20 dB/decade.
- **Phase Plot:** Constant at $+90^\circ$ across all frequencies.

a. Gain and Phase at Low Frequency ($\omega \rightarrow 0$)

- **Gain:** As $\omega \rightarrow 0$, $\frac{0.4}{\omega} \rightarrow \infty$. So the gain in dB approaches $+\infty$.
- **Phase:** The phase remains constant at $+90^\circ$.

b. Gain Margin and Phase Margin

To find the gain and phase margins, we first need the open-loop gain crossover frequency (where the magnitude is 0 dB) and the phase crossover frequency (where the phase is -180°).

1. Gain Margin:

- Since $L(j\omega) = \frac{0.4}{j\omega}$, the gain never crosses 0 dB (the magnitude is always negative and decreasing). Therefore, the gain margin is theoretically infinite.

2. Phase Margin:

- The phase is always $+90^\circ$. The phase crossover frequency is where the phase is -180° . Since $L(s)$ never reaches -180° , the phase margin is undefined or effectively infinite in this context.

c. Closed-Loop Time Constant

For a system with a loop transfer function $L(s)$, the closed-loop transfer function $T(s)$ is given by:

$$T(s) = \frac{L(s)}{1+L(s)}$$

The time constant of the closed-loop system can be approximated from the dominant pole of $T(s)$. For $L(s) = \frac{0.4}{s}$,



$$T(s) = \frac{\frac{0.4}{s}}{1 + \frac{0.4}{s}} = \frac{0.4}{s+0.4}$$

So, the closed-loop transfer function has a pole at $s = -0.4$.

The time constant τ is the inverse of the pole:

$$\tau = \frac{1}{0.4} = 2.5 \text{ seconds}$$

Summary

1. Gain and Phase at Low Frequency:

- Gain: $+\infty$ dB as $\omega \rightarrow 0$
- Phase: $+90^\circ$

2. Gain Margin and Phase Margin:

- Gain Margin: Infinite
- Phase Margin: Undefined or infinite

3. Closed-Loop Time Constant:

- $\tau = 2.5$ seconds

2025: Now it's correct

5) (5%) Why is it not recommended to pair on negative steady-state RGA elements?

2024: I had to stop here because ChatGPT claims that I have not paid.

2025: OK:

5) Pairing on negative RGA elements (5%)

Pairing on negative steady-state RGA (Relative Gain Array) elements is discouraged because it implies **loop interaction** in the **wrong direction**. Control action in one loop may worsen the performance of the other, leading to instability or poor control.

6) (5%) Consider a system described by the differential equations $\frac{dx}{dt} = Ax + Bu$, $y = Cx + Du$. What is the steady-state gain from u to y ?

2025: ChatGPT correct

7) (10%) Consider the process $g(s) = 3e^{-2s}$, with $y = g(s)(u+d)$. Plot the output $y(t)$ in response to a unit step change in u (without control). Suggest PI controller tunings for this process. Plot the expected response $y(t)$ to a unit input disturbance d (with control).

2025:

The answer on Problem 7) is not correct. The SIMC-rule gives a pure integral controller $c(s) = K_i/s$ for a delay process (where $K_i = K_c/\tau_{ui}$, which would correspond to $K_c=0$ and $\tau_{ui}=0$ for a PI-controller). So the gain K you give is K_i for an I-controller. There should be no integral time.

After correction it writes:

SIMC tuning:

This is a **pure time delay process**, so SIMC recommends a **pure integral controller**:

$$c(s) = \frac{K_I}{s}, \quad \text{where } K_I = \frac{1}{k(\tau_c + \theta)}$$

Given:

- $k = 3, \theta = 2$, and choosing $\tau_c = 2$, we get:

$$K_I = \frac{1}{3(2 + 2)} = \frac{1}{12}$$

So the controller is:

$$c(s) = \frac{1}{12s}$$

8) (5%) What is anti-windup?

2025: ChatGPT is good

8) Anti-windup (5%)

Anti-windup prevents integrator buildup in a PI or PID controller when the actuator is saturated (i.e., output limited). Without it, the integrator continues accumulating error, causing large overshoot when the actuator comes out of saturation.

9) (5%) Model predictive controllers (MPC) are widely studied but less used in practice that was expected when they came into use about 40 years ago. What do you think is the reason?

2025; ChatGPT is good

9) MPC adoption challenges (5%)

Despite its theoretical appeal, MPC is less used due to:

- **Implementation complexity** (requires model, solver)
- **Computational cost**, especially for fast systems
- **Operator unfamiliarity**
- **Sufficient performance from PID** for many processes
- **Maintenance**: models may become invalid

10) (5%) When do we use split range control? Give an example.

2025: ChatGPT is OK

Split range is used when one controller output drives **two or more actuators** in different ranges. Common example:

- Temperature control using both **cooling water** (when output 0–50%) and **steam heating** (50–100%).

I said that the 50% is a tuning parameter and it gave me a new answer;

10) Split Range Control (5%)

Split range control is used when one controller output manipulates **two or more actuators** over different segments of its range. A classic example is temperature control using both **cooling** and **heating**.

Example:

- The controller output may control a cooling valve in the range **0–50%**, and a steam (heating) valve in the range **50–100%**.

Note:

The **split point (50%)** is typically the default, but it is actually a **tuning parameter**. Adjusting this point allows you to change the **effective feedback gain** for each actuator. This can improve control performance, especially when one actuator is more effective or should be prioritized.