EXAM PROCESS CONTROL. 10 DEC 2024. SOLUTION

Problem 1

$$g(s) = 4\,\frac{-0.1s+1}{0.5s+1}$$

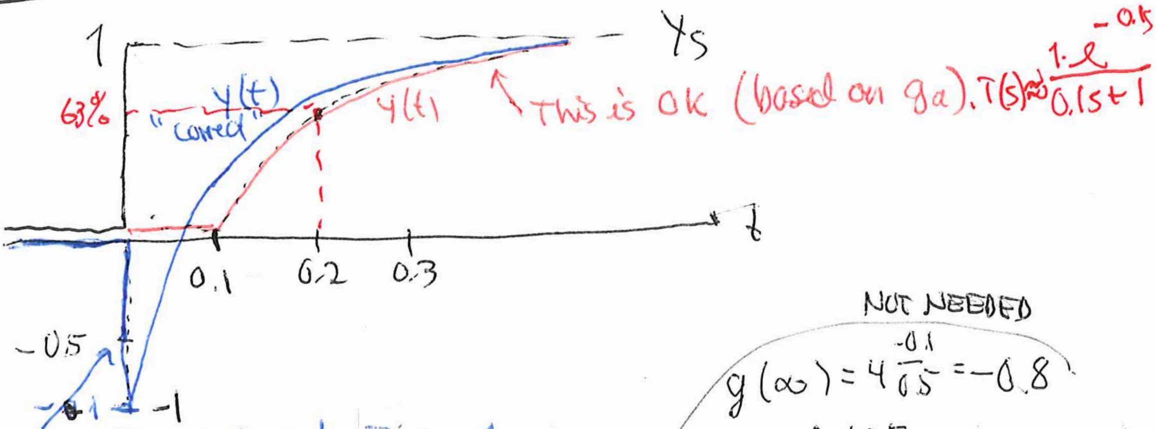
(a) Approximate as $g_a(s) = 4\,\frac{e^{-0.1s+1}}{0.5s+1}$ $\quad k=4,\ \theta=0.1,\ \tau=0.5$

SIMC-rule with $\tau_c=\theta$ ("tight" tunings)

$$K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{4}\frac{0.5}{0.1+0.1} = 0.625$$

$$\tau_I = \min(\tau, 4(\tau_c+\theta)) = \min(0.5, 0.8) = 0.5$$

Sketch



Correct: Jumps down to $T(\infty) = -1$ initially

NOT NEEDED

$g(\infty) = 4\frac{-0.1}{0.5} = -0.8$

$K_c = 0.625$

$\Rightarrow K_c g(\infty) = -0.5$

Jumps down initially to:

$T(\infty) = \frac{K_c g(\infty)}{1+K_c g(\infty)} = \frac{-0.5}{1-0.5} = -1$

CORRECT: (NOT NEEDED)

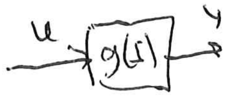
"Correct": $L(s) = g\cdot c = 4\frac{-0.1s+1}{0.5s+1}\cdot 0.625\frac{0.5s+1}{0.5s} = \frac{5}{}\cdot\frac{-0.1s+1}{s}$

$T(s) = \frac{5(-0.1s+1)}{s+5(-0.1s+1)} = \frac{-0.1s+1}{0.1s+1}$ (blue plot)

NOT NEEDED (blue plot)

→ Blue plot (~~so~~ BUT the red plot is ~~not~~ OK)

$$g(s) = 4 \cdot \frac{-0.1s+1}{0.5s+1}$$



(b) Response to step in $u(t)$.

There are many possible ways to find this.

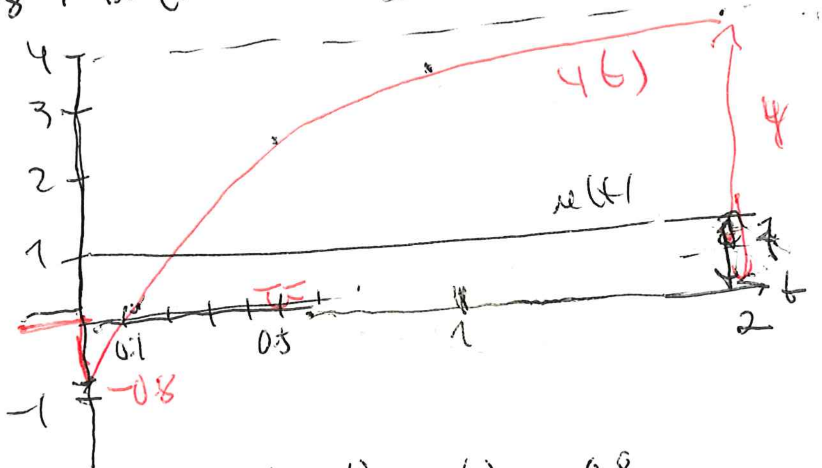
One approach

$$g(s) = -0.8 + \frac{4.8}{0.5s+1}$$

Get for $u(s) = \frac{1}{s}$ (step response):

$$y(t) = -0.8 + 4.8\left(1 - e^{-\frac{t}{0.5}}\right) = 4 - 4.8e^{-2t}$$

t	y(t)
0	-0.8
0.1	0.072
0.5	2.236
1	3.352
2	3.912
∞	4



Note: $y(0^+) = \lim_{s\to\infty} g(s) = -0.8$

$y(\infty) = \lim_{s\to 0} g(s) = 4$

(c) $g = 4\frac{-0.1s+1}{0.5s+1}$ $c = k_c$

Poles: $1+L(s)=0$

$1+4k_c\frac{-0.1s+1}{0.5s+1}=0$

$0.5s+1-k_c 0.4s+4k_c=0$

$\underbrace{(0.5-k_c 0.4)s+(1+4k_c)}=0$

Pole polynomial: $a_1 s + a_0$

Stability $\Leftrightarrow$ $a_1>0$ and $a_0>0$

$\Updownarrow$ $\Updownarrow$

$0.5-k_c \cdot 0.4>0$ $1+4k_c>0$

$k_c<\frac{0.5}{0.4}=1.25$ $k_c>-0.25$

Conclusion: Stable for $k_c \in \langle -0.25, 1.25 \rangle$

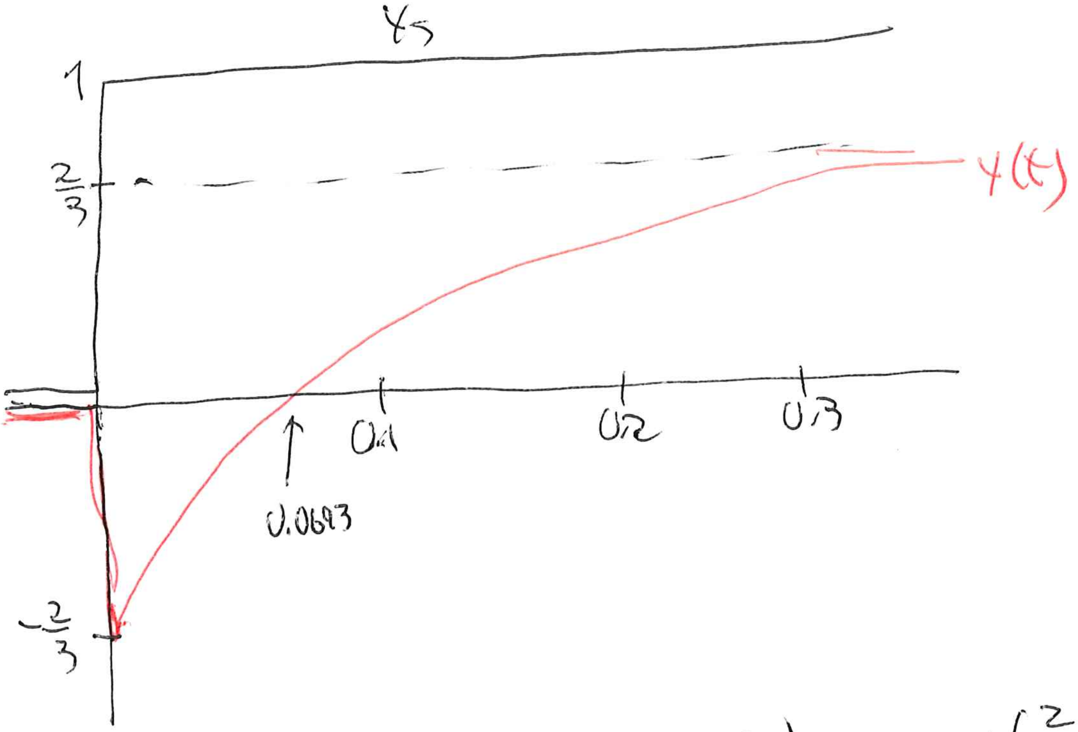
(Can alternatively compute pole, $s=\frac{1+4k_c}{k_c 0.4-0.5}$; gives same solution)

(d) $g = 4\dfrac{-0.1s+1}{0.5s+1}$, $c = k_c = 0.5$

Closed-loop response $T = \dfrac{gc}{1+gc} = \dfrac{2(-0.1s+1)}{0.5s+1+2(-0.1s+1)}$

$$\Rightarrow T = \frac{2}{3}\,\frac{-0.1s+1}{0.1s+1}$$

$$T(0) = \frac{2}{3}, \quad T(\infty) = -\frac{2}{3}$$



- Should have correct for start $(-\frac{2}{3})$ and end $(\frac{2}{3})$
- The crossing should be somewhere around $t = 0.1$

Not required:

- The exact plot is: $y(t) = \frac{2}{3} - \frac{4}{3}e^{-10t}$

(so it will cross 0 at $\frac{4}{3}e^{-10t} = \frac{2}{3} \Rightarrow e^{-10t} = 0.5 \Rightarrow -10t = \ln 0.5$

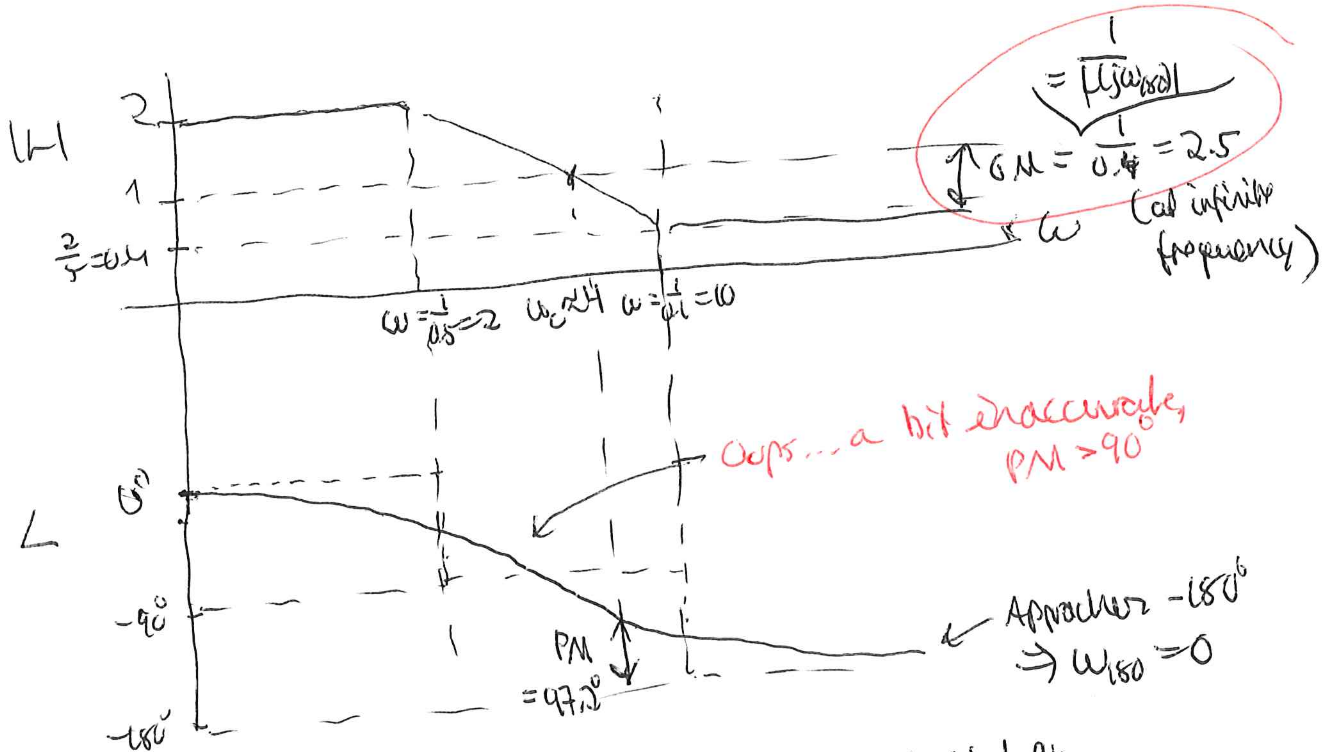
$\Rightarrow t = -\dfrac{\ln 0.5}{10} = \underline{0.0693\,s}$

(e) $g = 4\,\frac{-0.1s+1}{0.5s+1}$ $K_c = 0.5$

- In part (c) we found that the system becomes unstable for $K_c = 1.25$, so the gain margin is $GM = \frac{1.25}{0.5} = 2.5$

- We can also find this from the Bode plot.

$$L = 2\,\frac{-0.1s+1}{0.5s+1}$$



$$|L| = 2\,\frac{\sqrt{(0.1\omega)^2+1}}{\sqrt{(0.5\omega)^2+1}} \approx 2\,\frac{1}{0.5\omega} = 1 \quad \text{(to find } \omega_c\text{)}$$

↑ Between $\omega = 2$ and $\omega = 10$ (asymptote)

$\Rightarrow \omega_c \approx 4$ rad/s (from asymptote)

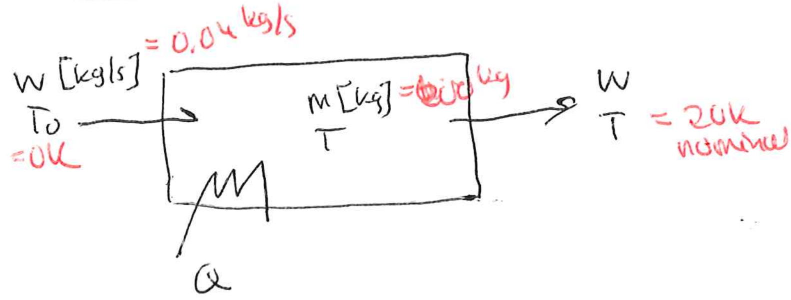
(Exact: $\omega_c = \sqrt{\frac{3}{0.24}} = 3.78$ rad/s)

$$\angle L = -\arctan(\underset{\omega_c}{3.78}\cdot\underset{T_1}{0.1}) - \arctan(\underset{\omega_c}{3.78}\cdot\underset{T}{0.5}) = -20.7^\circ - 62.1^\circ = -82.8^\circ$$

$PM = 180^\circ - 82.8^\circ = 97.2^\circ = 1.696$ rad

$$DM = \Delta\theta_{max} = \frac{PM\,[\text{rad}]}{\omega_c\,[\text{rad/s}]} = \frac{1.696}{3.78} = \underline{0.449\,s}$$

Problem 2



(a)

(i) Energy balance (assume constant c_p for air)

$$\frac{d}{dt}(M c_p T) = Q + W c_p (T_0 - T)$$

(ii) Nominal Q. Use steady-state balance

$$Q^* = W^* c_p (T^* - T_0) = 0.04 \frac{kg}{s} \cdot 1 \frac{kJ}{kg \cdot K} \cdot (20-0)K$$

$$= 0.8\ kW \quad [= kJ/s]$$

(iii) Linearize.

$$M c_p \frac{d\Delta T}{dt} = \Delta Q + W^* c_p (\Delta T_0 - \Delta T) + c_p (T_0^* - T^*) \Delta W$$

Laplace

$$M c_p s T(s) = Q(s) + W^* c_p (T_0(s) - T(s)) + c_p (T_0^* - T^*) W(s)$$

$$\left(\frac{M c_p}{W^* c_p} s + 1\right) T(s) = \frac{1}{W^* c_p} Q(s) + 1 \cdot T_0(s) + \frac{T_0^* - T^*}{W^*} W(s)$$

$\tau_a = \frac{200\ kg}{0.04\ kg/s} = 5000 s$

$k = 25 \frac{K}{kW}$

$k_{d} = 1$

$k_{d2} = 500 \frac{K}{kg/s}$

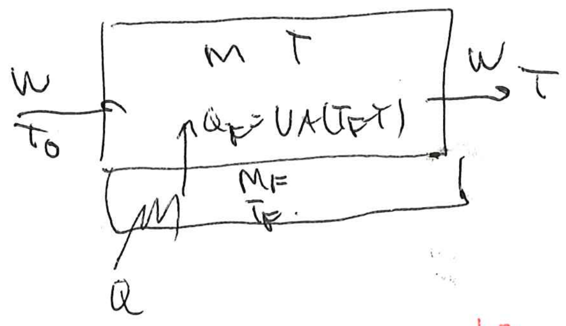
(iv)

$$G(s) = \frac{k}{\tau_a s + 1} = \frac{25}{5000 s + 1}$$

SIMC-rules: $K_c = \frac{1}{k} \frac{\tau_a}{\tau_c + \theta} = 1.5 \Rightarrow \underline{\tau_c = 133.3 s}$ ($k = 25$, $\tau_a = 5000$, $\theta = 0$)

$$\tau_I = \min(\tau_a, 4(\tau_c + \theta)) = \min(5000, 533) = \underline{533 s}$$

(1)



New data: $M_F C_{pF} = 5000 \frac{kJ}{K}$, $UA = 0.5 \frac{kW}{K}$

(i) Balances for energy

Room: $\frac{d}{dt}(M C_p T) = Q_F + w C_p (T_0 - T)$ (1)

Floor: $\frac{d}{dt}(M_F C_{pF} T_F) = Q - Q_F$ (2)

where $Q_F = UA(T_F - T)$ (3)

(ii) Steady-state

$Q = Q_F$

$Q^* = Q_F = w C_p (T_0 - T) = 0.04 \cdot 1 \cdot (20-0) = 0.8 \text{ kW}$ (as before)

From (3)

$T_F^* = \frac{Q}{UA} + T = \frac{0.8}{0.5} + 20 = \underline{\underline{21.6^\circ C}}$

(iii) With w = constant the equations are linear, Laplace give

$(1) \Rightarrow M C_p s T(s) = \overbrace{Q_F(s)}^{UA(T_F(s) - T(s))} + w C_p (T_0(s) - T(s))$

$\Rightarrow \underbrace{\left(\frac{M C_p}{UA + w C_p}\right.}_{\tau_a} s + 1\Big) T(s) = \underbrace{\frac{UA}{UA + w C_p}}_{} T_F(s) + \frac{w C_p}{UA + w C_p} T_0(s)$ (4)

$\tau_a = \frac{200}{0.5 + 0.04} = 370.45$ (with $0.5 + 0.04 = 0.54$)

$k_F = \frac{0.5}{0.54} = 0.926$

$k_d = \frac{0.04}{0.54} = 0.0741$

$(2) \Rightarrow M_F C_{pF} s T_F(s) = \overbrace{-UA(T_F(s) - T(s))}^{-Q_F} + Q(s)$ (5)

$\underbrace{\left(\frac{M_F C_{pF}}{UA}\right.}_{} s + 1\Big) T_F(s) = \frac{UA}{UA} T(s) + \frac{1}{UA} Q(s)$

$\tau_F = \frac{5000}{0.5} = 10000 \text{ s}$

$k_T = 1$

$k_Q = \frac{1}{0.5} = 2$

(iv)

From (5): $T_F(s) = \frac{1}{\tau_F s+1}(T + k_Q Q)$

Put this into (4)

$$(\tau_a s+1)T(s) = \frac{k_F}{\tau_F s+1}(T + k_Q Q) + k_d T_0$$

$$((\tau_F s+1)(\tau_a s+1) - k_F)T(s) = k_F k_Q Q(s) + k_d(\tau_F s+1)T_0(s)$$

($\tau_F = 10^4$, $\tau_a = 374$, $k_F = 0.926$, $k_Q = 2$, $k_d = 0.0741$, $\tau_F = 10^4$)

Numbers

$$(10^4 \cdot 370.4 s^2 + 10374 s + 0.0741)T(s) = 1.852 \cdot Q(s) + 0.0741(10^4 s+1)T_0(s)$$

Divide by 0.0741

$$(50 \cdot 10^6 s^2 + 14 \cdot 10^4 s + 1)T(s) = 25 Q(s) + (10^4 s+1)T_0(s)$$

$$(14 \cdot 10^4 s+1)(358 s+1)$$

Conclusion $T(s) = G_b(s)Q(s) + G_{db}T_0(s)$

$$G_b(s) = \frac{k_b}{(\tau_1 s+1)(\tau_2 s+1)}$$

$k_b = 25$, $\tau_1 = 14 \cdot 10^4$ s, $\tau_2 = 358$ s

$$G_{db} = \frac{k_{db}(\tau_F s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$$

$k_{db} = 1$, $\tau_F = 10^4$ s

(v) Using data from exam: $k_b = 30$, $k_{db} = 1$, $\tau_1 = 15 \cdot 10^4$ s, $\tau_2 = 400$ s, $\tau_F = 1.5 \cdot 10^4$ s

Half rule

$$\tilde{G}_b(s) = \frac{30}{(15 \cdot 10^4 s+1)(400 s+1)} \approx \frac{30 e^{-200 s}}{(150200 s+1)} = \tilde{G}_b$$

PI-SIMC

$$K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = 1.5$$

($\tau = 150200$, $k = 30$, $\theta = 200$)

$$\Rightarrow \tau_c = \frac{150200}{30 \cdot 1.5} - 200 = 3337 - 200 = 3137\text{ s}$$

(almost one hour) $= 0.3 \cdot 10^4$ s

$$\tau_I = \min(\tau, 4(\tau_c+\theta)) = 4 \cdot 3337\text{ s} = 13351\text{ s} \quad (3.7\text{ h})$$

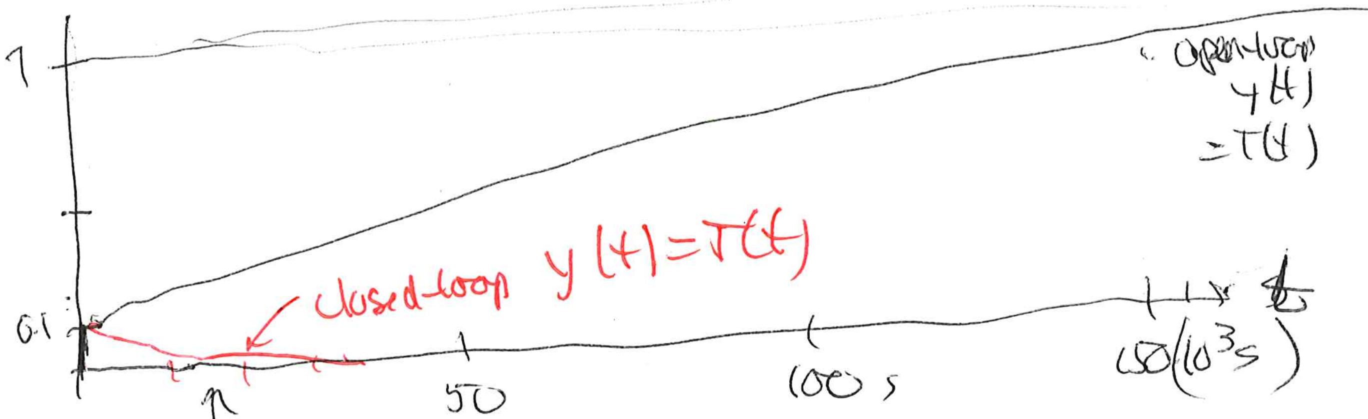
(v) Sketch response

$$G_d = \frac{\cancel{150}(15\cdot10^4 s+1)}{(15\cdot10^4 s+1)\,[0.04\cdot10^5 s+1)} = \frac{15\cdot10^3 s+1}{(150\cdot10^3 s+1)(0.4\cdot10^3 s+1)}$$

Half rule

$$G_d(s) \approx \frac{15\cdot10^3 s+1}{150\cdot10^3 s+1} e^{-0.4\cdot10^3 s}$$

Jumps up to 0,1 initially (after very short delay) and then approaches 1 with time constant $150\cdot10^3$ s

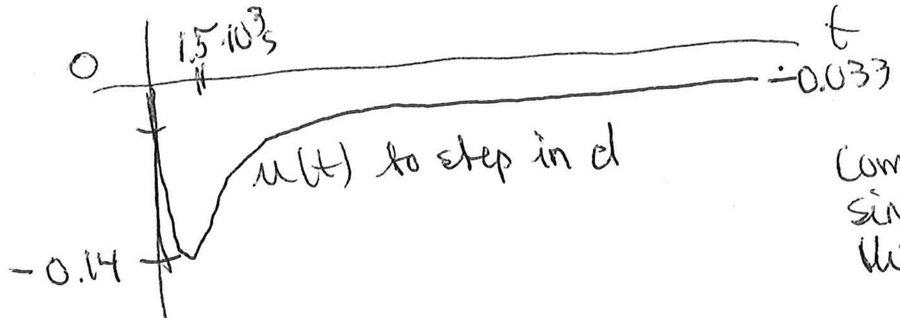


$\tau_c = 13{,}3\cdot10^3$ s so it turns around quickly

Input u(t) (Extra)

This is not easy. First u(t) will be negative $u = -C\cdot S\cdot G_d\cdot d$ $S = \frac{1}{1+GC}$

It starts out first-order, it has a peak because of the speed-up from $\tau_1 = 150\cdot10^3$ s to $\tau_{c1} = 13.3\cdot10^3$ s (more than 10 times). It ends up at $-\frac{1}{30} = -0.033$ kW. So it looks like this.



Comment: I had to use simulations to find this.

EXTRA

(vi) $\tau_{c2} = 200$ s gives $K_{c2} = \frac{1}{30}\frac{150\,200}{400} = 12.5$

$\tau_{I2} = 4\cdot400\text{ s} = 1600\text{ s} = 1.6\cdot10^3$ s

This also seems reasonable, but for disturbances it tuning 2 is unnecessary fast. ~~Probably~~ For setpoint changes tuning 2 may be OK. What would I choose? Maybe $\tau_c \approx 30 = 600$ s ? (10 min)

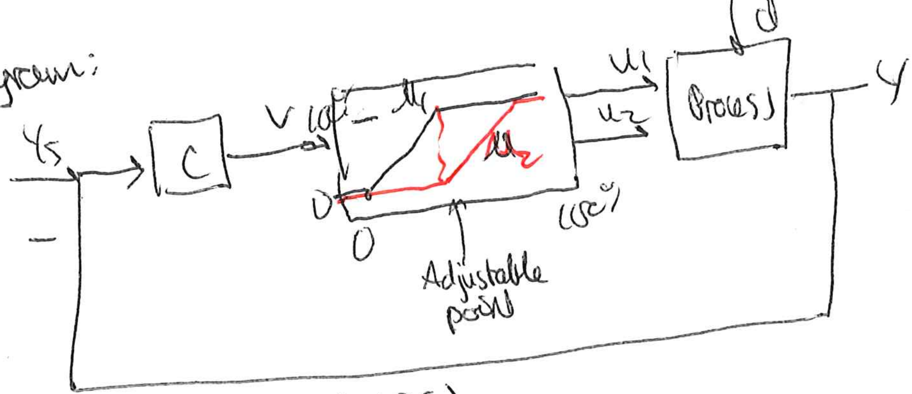
Problem 3. Advanced control

(a) (i) Split range control.

- Use two inputs (MVs) sequentially to cover whole steady-state range

Example: Two heaters. Hot water (u_1) and electricity (u_2)

Block diagram:

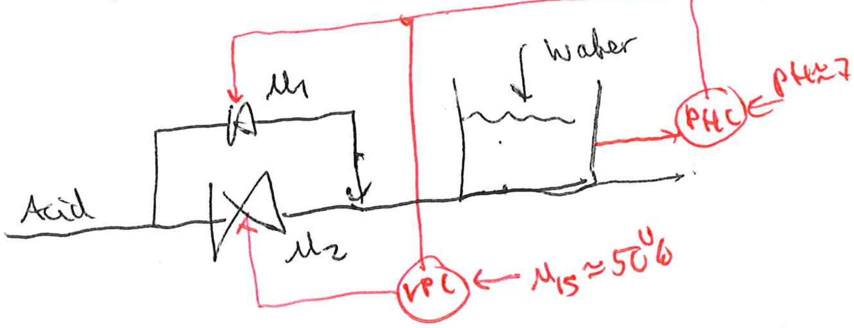


(ii) ~~Selective (override) control~~ Input resetting (VPC).

- In this case we use both inputs at the same time
- One input (u_1) is fast but has small steady-state effect
- Another input (u_2) is slow with large steady-state gain.

Example: Large and small valves in parallel

Flowsheet



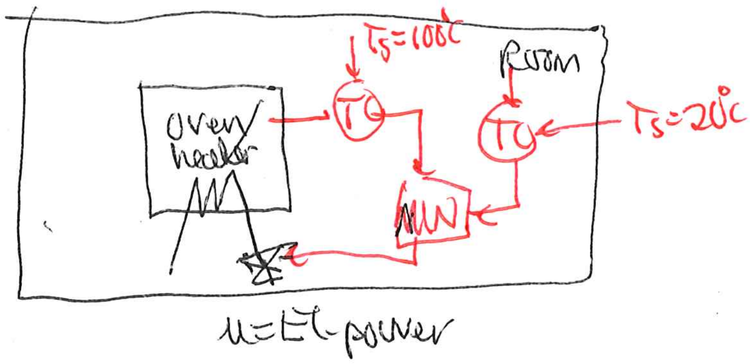
(b) (i) Cascade control: y_2 is an extra measurement but does not have a given setpoint. Example:

y_1 = level
y_2 = flowrate
u = valve position

- Setpoint for y_2 is set by master controller
- Slave controller (fast) controls y_2 and manipulates u.

(ii) Selective (override control). In this case both y_1 and y_2 have setpoint, or at least constraint values. Can only control one at a time, but then the other is "oversatisfied". (constraint)

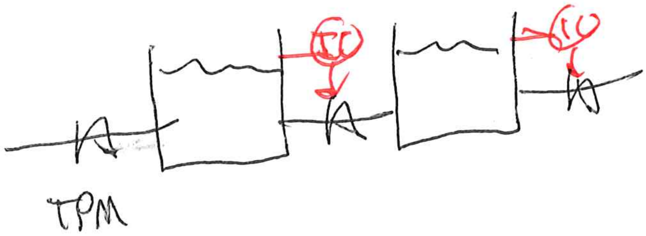
Example: Avoid too high temperature for heater



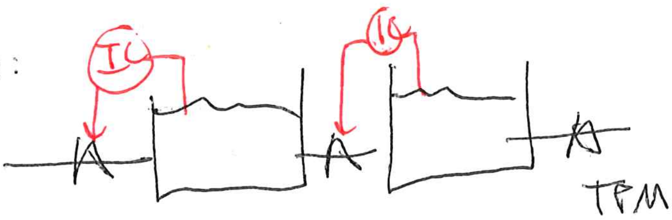
(c)
TPM = throughput manipulator = where production rate is set.
"Radiation rule: Inventory control must be radiation around TPM.

6i

TPM at feed:



TPM at product:



Some remarks regarding how the students performed:

Problem 1

a. Almost everybody got the tuning correct, few people got full marks for the correct sketch (some forgot it all together, most people did not read correctly that the closed-loop was asked for)
b. People either knew how to approach this problem, or drew something that they made up. Others approximated the time-delay.
c. Most people knew how to approach this problem, but only few got full marks. Very few people got both bounds, as most only considered the a1 part of the polynomial
d. This was similarly successful to b).
e. Very few people used the Bode paper, but many people got this one correct. Most people did not recognize that they could get the gain margin from the Kc,max they previously calculated.

Problem 2a

Generally, this was the most successful part for the students and many got full marks.

Problem 2b

i. Most people managed to set up the correct balance, even if they did very poorly in Part 1 (Still good chemical engineers, even if Process Control is not their strong suit 😊). Quite some people were confused however and tried to add some convective term to the floor. Another common mistake was that the cp only appeared on the LHS, and similar mistakes.
ii. This was also easy for most people.
iii. The lineariziation was okay for most people.
iv. Some (few) people managed to get the correct time constants in this task. Many others good close to the correct form and then described that they needed to factorize the polynomial but did not have the time, and they were awarded most of the points also.
v. This was also easy for most people. The most common people that people did was to divide by 2*tau_c in the equation for Kc, even though they had just estimated an effective theta.
vi. Few people got full marks for this. Many based their plots on the wrong transfer function (i.e., from Q to T), which then defied thermodynamics – increasing the inlet temperature by 1 K increased the room temperature by 30K! Also, most people did not bother to use the insights from the time constants into their plots, which also meant that they did not get full marks. Only one person attempted the bonus exercise (this person did well).

a. This was okay, most students recommended to use cascade and gave a reason (even though using cascade is not recommended, we still gave some points to the students who gave some reasonable reason).

Problem 3

This was the “worst” part of the exam, maybe also because it was at the end. Many people made things up very generically for some of the concepts. The people used mostly the same examples (I guess from the lecture), and those that had examples usually also appeared to have understood the respective concept.

A more general gripe that I had was that almost none of the students drew proper plots. Almost nobody had any values on their x or y-axis, and a lot of people even did not have proper labels on their plots. I would expect that to be second nature at this point in their studies.