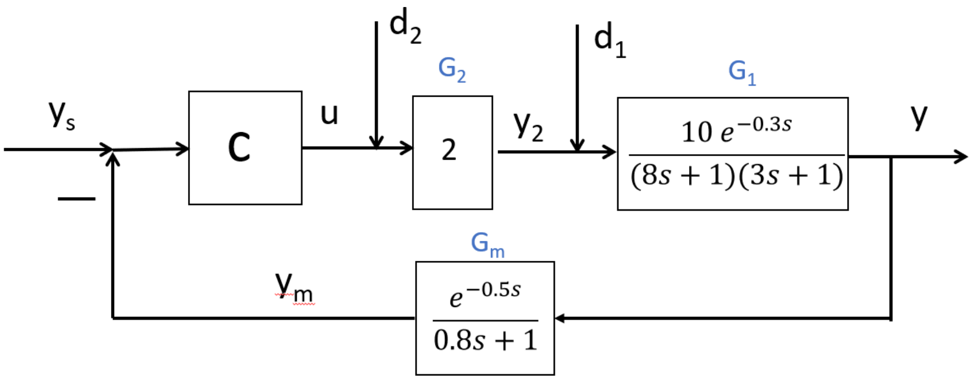
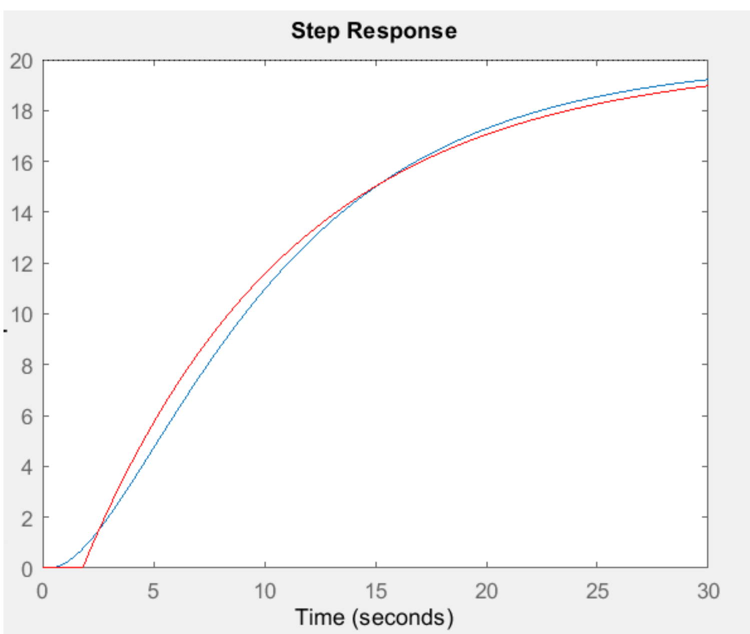
Solution for exam TKP4140 Process Control, 11 Dec. 2023.

The parts in red and most of the plots are not needed for a correct solution.

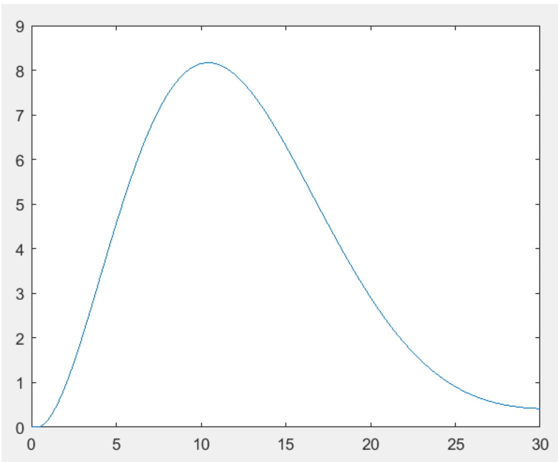
Problem 1 (35%). Controller tuning

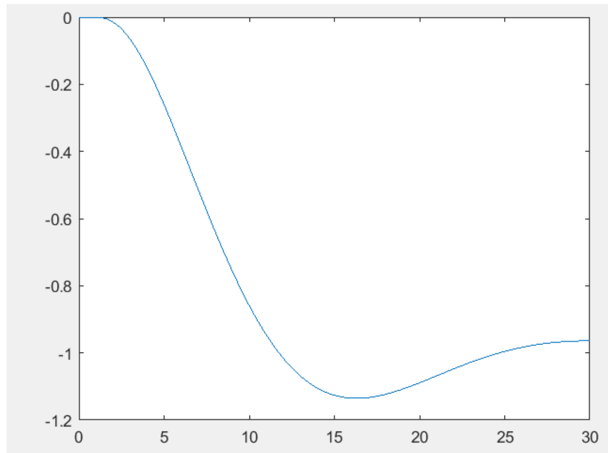


(a) **Response from d2 to y: Gd(s) = G1*G2=20 e-0.3s/(3s+1)(8s+1)**. The step response is second-order with a steady-state gain of 20>1 (see blue curve). So we definitely need control (that is, we need to use u). To sketch it, one may apply the half rule as a starting point; get Gd1=20 e-1.8s/(9.5s+1); it reaches 63% at t=1.8+9.5=11.3s (see red curve; we note that if we make the red curve a bit more S-shaped then we will get close to the correct blue curve.).

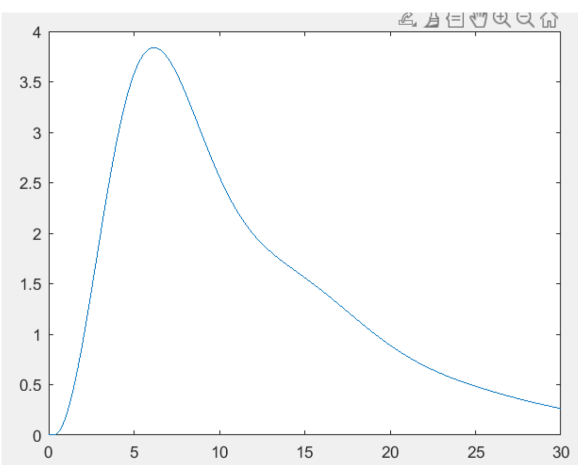


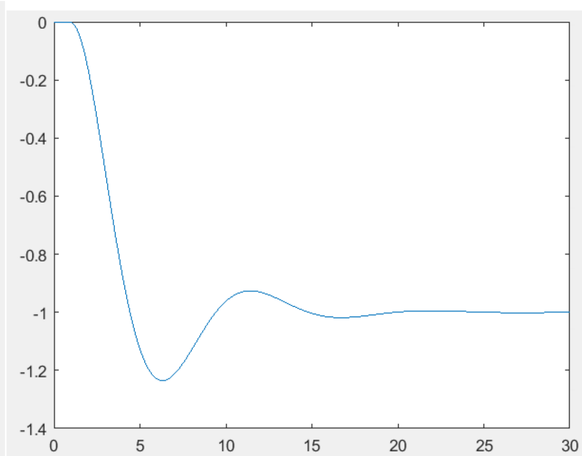
(b) The transfer function for designing c is G1*G2*Gm= **20 e-(0.3+0.5)s/(8s+1)(3s+1)(0.8s+1).** Use half rule to find first-order process, k=20, tau=8+3/2=9.5, theta=0.3+0.5+0.8+3/2=3.1. **SIMC-PI with tight control.** Choose tauc=theta=3.1. Get Kc=(1/k)*tau/(tauc+theta) = 0.077, taui=min(tau,4*(tauc+theta))=min(9.5,24.8) = 9.5 .
Comment: I show below closed-loop simulations for a unit step disturbance in d2. Left: y(t). Right: u(t). This is of course not expected at the exam.





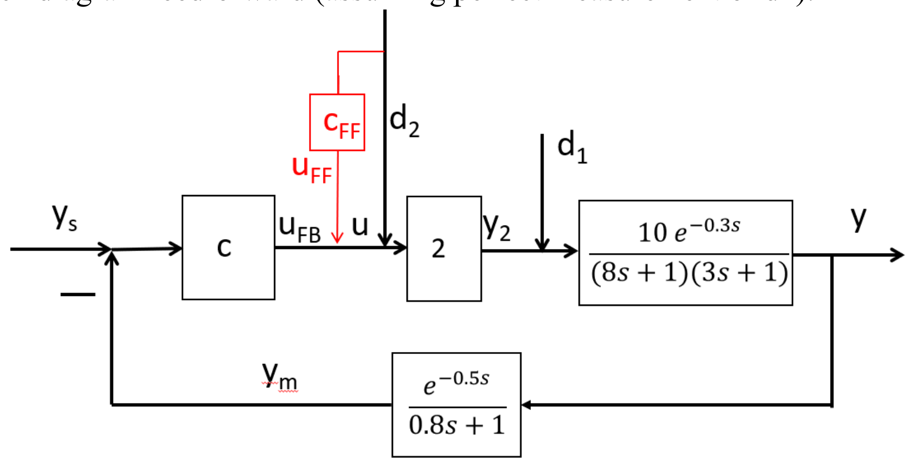
(c) Use half rule to find second-order process, k=20, tau1=8. tau2= 3+0.8/2 = 3.4, theta=0.3+0.5+0.8/2=1.2 **-PID with tight control**. Choose tauc=theta=1.2. PID-control (series form): Kc=(1/k)*tau1/(tauc+theta) = 0.167. taui=min(tau1,4*(tauc+theta) = min(8, 9.6)= 8, taud=tau2=3.4. (In the simulations I added a 1st order filter for the measurement with tauf=0.34). NOTE: We have (1+taud/taui)=1.425 so the tunings for the standard ideal-PID, c(s) = Kc'(1+1/taui'*s + taud'*s), are Kc'=Kc*1.425=0.238, taui'= taui*1.425=11.4, taud'=taud/1.425=2.386.





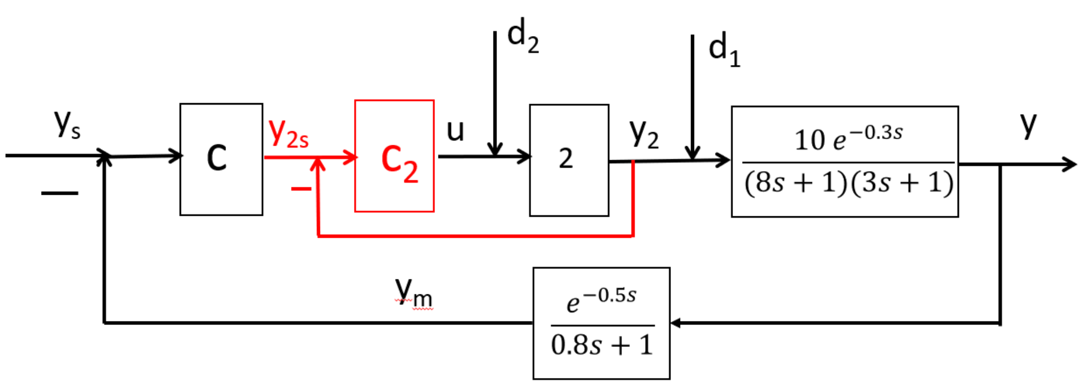
(d) Will it be OK with PI- or PID-control? First, it is clear that PID is beneficial since tau2=3.4>theta=1.2 *(get 1.5 points for saying this)*. However, whether it will be good enough is difficult to tell without doing simulations (as I have done..). A good approach is to look at the frequency wd where |Gd(jwd)|=1. We know that we at least need tauc<1/wd. At high frequencies (w > 1/3=0.33) the gain asymptote becomes: |Gd(jw)|=20/(24w^2). Get |Gd(jwd)|=1 at wd=sqrt(20/24)=0.913 [rad/s]. This means that we need tauc < 1/wd < 1.1. This is certainly not satisfied with PI-control (tauc=3.1), but with PID-control we are almost OK (tauc=1.2). So maybe it is OK? No, the actual requirement is |SGd(jw)|<1 which, since S=1/(L+1), gives the approximation |L|>|Gd| at low frequencies. Since |Gd| has a slop of -2 (while |L| has about -1 at wc=1/tauc) this is difficult to satisfy at w<wc even though the condition tauc<1/wd means that we have satisfied |L|>|Gd| at w=wc.. Indeed, the simulation show that y goes almost up to 4 even with PID control (with PI it goes to 8).

(e) Block diagram feedforward (assuming perfect measurement of d2):



In this case G=G1*G2=Gd (Note that the measurement dynamics for y don't matter when we consider feedforward). Ideal feedforward then gives cff = -Gd/G = -1. Yes, FF is recommended as it ideally gives perfect control for disturbance d2. Here "Ideally" = perfect model and perfect measurement of disturbance. (Comment: FF will also in theory be perfect for d1).

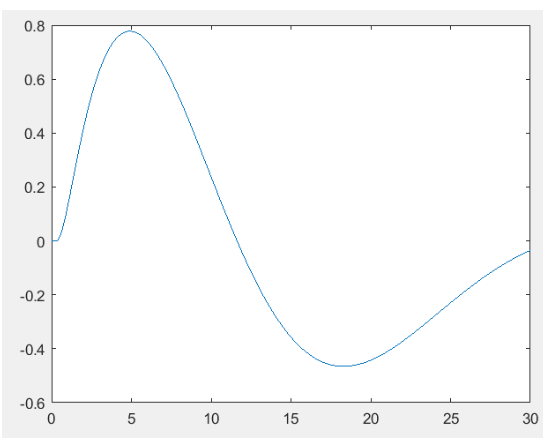
(f) Block diagram cascade (assuming here perfect measurement of y2, that is, $g_{2m}(s)=1$):

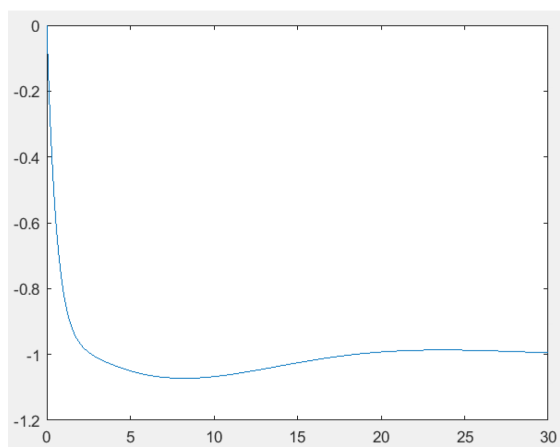


C2 is designed based on G2=2 (so tau2=0, theta2=0). We use tauc2=0.6. This value is reasonable if the effective delay in G2 is less than about 0.6/2=0.3. Get with SIMC: Kc2=0. taui=min(0,4*0.6) =0 (!). The integral time is zero so this is actually an I-controller with KI = (1/2)*1/(tauc2+theta2) = 1//(2*0.6) = 0.833. So c2 = 0.833/s. (if you want to approximate it s a PI-controller then select Kc small and use taui=Kc/0.833). Yes, cascade control will be helpful for d2 because of the dynamics and delay in both G and the measurement (it will be helpful even with feedforward because there are always nonlinearity in G2 and error in the measurement of d2). Yes, we need to retune c, in particular, because the gain in T2 is 1 and in G2 it is 2.

(g) New tunings for c with cascade. Replace G2 by T2, where T2 = 1/(tauc2*s+1) = 1/(0.6s+1) (in this case this is exact; not an approximation). The half rule now gives the following 1^{st} order with delay model for design of c: k=10, tau=8+3/2=9.5, theta=0.3+0.5 + 3/2+ 0.8 + 0.6 = 3.7. SIMC-PI with tight control gives tauc=theta=3.7. New PI-controller c becomes: Kc=(1/k)*tau/(tauc+theta) = 0.13, taui=min(tau,4*(tauc+theta) = min(9.5, 29.6)= 9.5.

Simulations with cascade (below, y is left, u is right) show that we (as expected) get good response for disturbance d2 with |y|<0.8<1. We see that slave controller c2 makes the input u(t) drop quickly down to the "ideal" value of -1.





Problem 2 (35%). Closed-loop stability

Solution

C(s) = Kc(tauis+1)/tauis. With taui=20 and 4/20=0.2 we get $L(s) = G(s)C(s) = \frac{0.2\,K_c\,(-2s+1)}{s(6s+1)}$

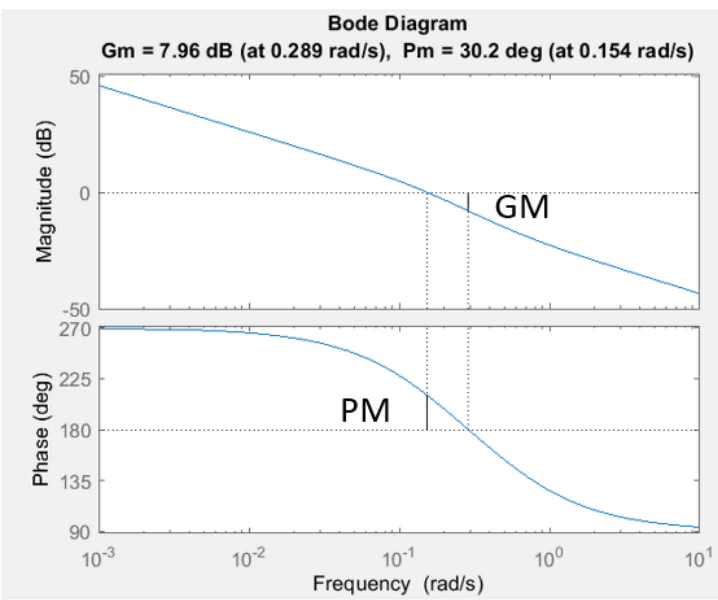
Closed-loop transfer functions have 1+L(s) in denominator. Multiplying to get a polynomial, we find that

d(s) = s(6s+1) + 0.2Kc(-2s+1)= 6s^2 +(1 -0.4 Kc) s + 0.2Kc.

With Kc=1 we get d(s) = 6s^2 + 0.6s + 0.2 (note that we may multiply d(s) by any constant). All coefficients are positive so it's closed-loop stable. Yes, the condition is necessary and sufficient for a 2^{nd} order system.

(b) Gain margin. The second coefficient in d(s) becomes zero for Kc=1/0.4=2.5 (so this is the maximum gain to have stability). So with Kc=1, we have that GM=2.5 (20 log(2.5) = 7.96 dB).

(c) Bode plot with GM and PM. I here used Matlab (the phase plot is a bit strange; it should start from -90 from the integrator, but Matlab adds 360 so it starts from 270; mathematically it's the same because a complex number comes back to the same value of you add any multiple of 360 degrees). For the solution it OK with an approximate plot based on asymptotes. There are three asympotes for L (using Kc=1): low w: L=0.2/s (slope=-1,phase=-90). w>wb1=1/6: L=0.2/s*6s (slop=-2, phase=-180), w>wb2=1/2: L=-0.2*2/6s (slope=-1, phase -270).



GM is evaluated at frequency w180 where phase of L is -180

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s=tf('s')
L=0.2*(-2*s+1)/(s*(6*s+1))
Margin(L)
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(d) DM=PM/wc . PM is evaluated at frequency wc where |L(jwc)|=1. Here PM=30.2 degrees = 30.2 * pi/180 = 0.527 rad. W c=0.154 rad/s (see Bode plot). So DM = 0.527/0.154 = 3.42s (so the system goes unstable if we add a delay theta=3.42 s).

To find PM analytically we first find wc by solving

|L(jwc)|=0.2*sqrt(4*wc^2+1)/(wc*sqrt(36*wc^2+1)) = 1.

This gives wc=0.154. At this frequency the phase is:

Phase L = -pi/2 - atan(2*wc) – atan(6*wc) = -2.6155 rad,

so the phase margin is 3.14 - 2.615 = 0.527 rad. QED,

Similarly, to find GM analytically, we find w180 as the frequency where phase(L) = - pi rad and then we evaluate |L| at this frequency. We iterate on w until the phase is -180. This gives w=0.289 and

|L(jw180)|=0.2*sqrt((2*0.289)^2+1) / 0.289* sqrt((6*0.289)^2+1) = 0.40,

so GM=1/|L(jw180)|=2.5.

(e) P-control. $L(s) = G(s)C(s) = \frac{4\,K_c\,(-2s+1)}{(20s+1)(6s+1)}$.

Pole polynomial: d(s) = (20s+1)(6s+1) + 4 Kc(-2s+1)= 120 s2 + (26-8Kc)s + (1+4Kc).

2nd coeff. Is zero when Ku=26/8 = 3.25.

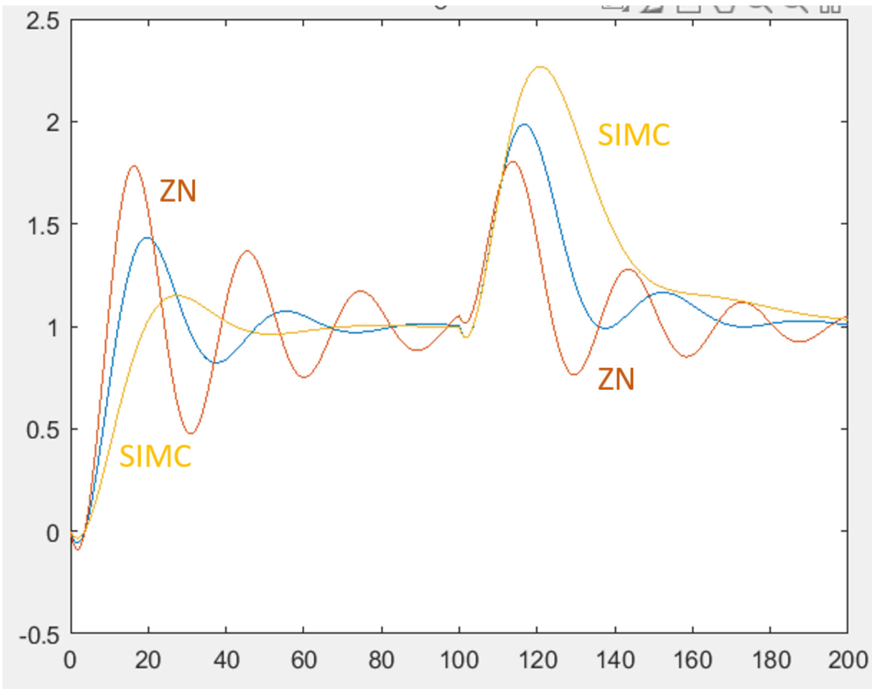
At this point d(s) = 120 s^2 + (1+4Ku) = 120 s^2 + 14

Poles are solutions to d(s)=0: Get $s = \pm j\omega_u$ where ω_u = sqrt(14/120) = 0.3416 rad/s. So Pu = 2pi/wu = 18.4 s.

ZN-tunings: Kc=0.45 Ku= 1.46 and taui=Pu/1.2 = 15.3 s.

(f)SIMC: Half rule gives theta=6/2+2=5, tau=20+6/2 = 23. With tauc=theta we get: Kc=(1/4)*23/(5+5) = 0.575. taui=23 (which is much less aggressive than ZN)

NOT EXPECTED ON EXAM: Below is a comparison of the response y(t) with the three PI-controllers (Blue; Kc=1, taui=20, Brown: ZN, Yellow: SIMC). “As usual”, the response y(t) is for a step setpoint change (at t=0) followed by a step input disturbance (as t=100). I think SIMC is the best although the disturbance response is a bit slow; it has the best setpoint response and is by far the most robust (see box). ZN will easily go unstable since the gain margin is only 1.54.

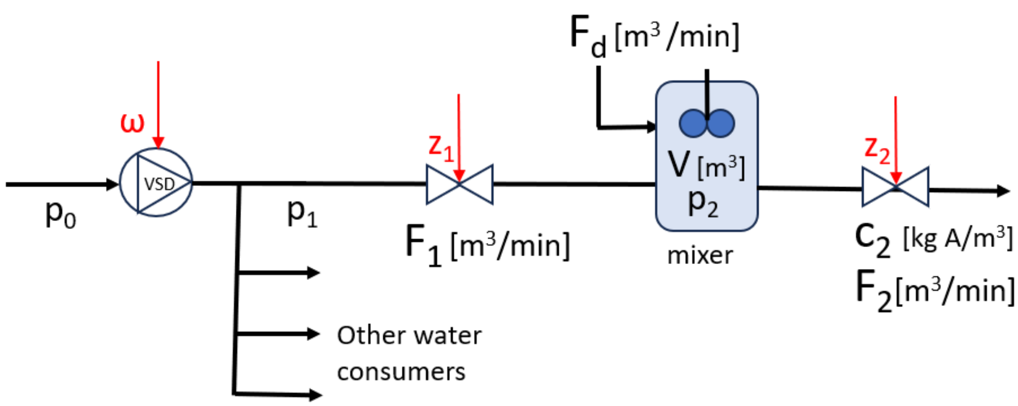


SIMC: GM=4.5, DM=9.11s

ZN: GM=1.54, DM=1.12 s

Blue: GM=2.5, DM= 3.42s.

Problem 3 (30%). Modelling and control of mixing process



Solution:

(a) Linear valve equation: F1 = Cv*z1*sqrt((p1-p2)/rho). Linearized: $\Delta F_1 = k\ \Delta z_1$ where k= Cv*sqrt((p1-p2)/rho). We note that process gain k varies with the square root of the pressure difference DP=p1-p2. But k is constant if we control p1 and p2.

(b) (i) Total mass balance (assuming constant density):

$$\frac{dV}{dt} = F_1 + F_d - F_2 \qquad (1)$$

Component balance:

$$\frac{d(c_2V)}{dt} = F_1c_1 + F_d\, c_d - F_2\, c_2 \qquad (2)$$

(ii) At steady state, the last balance gives (with c1=0): 0=Fd cd – F2 c2. Thus, Fd=F2 c2/cd = 2*20/700 = 0.0571 m3/min.

(iii) Assuming V constant and using c_1=0, the balances become:

$$F_1 + F_d = F_2$$

$$V\frac{dc_2}{dt} = F_d\, c_d - F_2\, c_2$$

Linearize and deviation variables (assuming F_d and c_d constant):

$$\Delta F_1 = \Delta F_2$$

$$V\frac{d\Delta c_2}{dt} = -\, F_2^*\, \Delta c_2 - c_2^*\, \Delta F_2$$

Combining

(*)
$$V\frac{d\Delta c_2}{dt} = -\, F_2^*\, \Delta c_2 - c_2^*\, \Delta F_1$$

Taking Laplace (in deviation variables):

$$Vsc_2(s) + F_2^* c_2(s) = -c_2^*\, F_1(s)$$

So

$$c_2(s) = -\frac{\frac{c_2^*}{F_2^*}}{\tau s+1} F_1(s) = -\frac{10}{\tau s+1} F_1(s)$$

where $\tau = V/F_2^* = 0.36/2 = 0.18$ min = 11 s.

Comment: It is also possible to linearize everything without making any assumptions. It gives the same result, and we don't need to assume V constant: Combining with (1), the left hand side of the component balance (2) becomes $\frac{d(c_2V)}{dt} = V\frac{dc_2}{dt} + c_2\frac{dV}{dt} = V\frac{dc_2}{dt} + c_2(F_1 + F_d - F_2)$ and we get (exactly!)

$$V\frac{d(c_2)}{dt} = F_1(c_1 - c_2) + F_d\,(c_d - c_2)$$

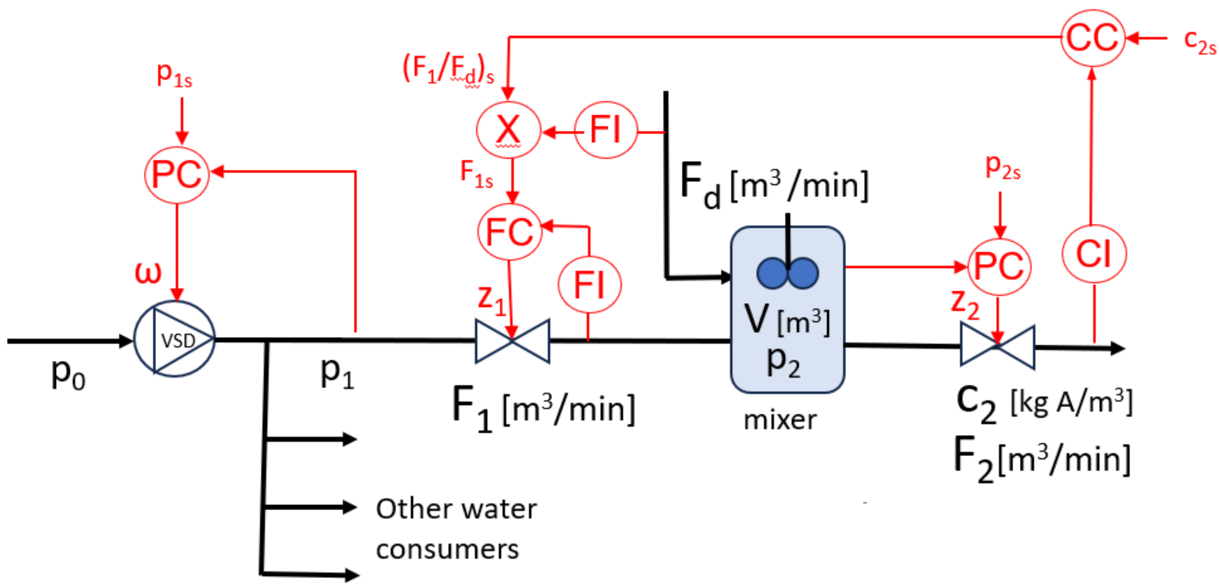
Note that the outflow F_2 drops out of the component balance (as usual). Linearize and deviation variables:

$$V^*\frac{d(\Delta c_2)}{dt} = F_1^*(\Delta c_1 - \Delta c_2) + F_d^*\,(\Delta c_d - \Delta c_2\,) + (c_1^* - c_2^*)\,\Delta F\_1 \quad + (c_d^* - c_2^*)\Delta F\_d$$

Assuming pure water in stream 1 (c_1=0, Δc_1=0), and assuming F_d and c_d constant (ΔF_d =0, Δc_d = 0) and using $F_2^* = F_1^* + F_d^*$ gives as before equation (*):

$$V^*\frac{d\Delta c_2}{dt} = -\, F_2^*\, \Delta c_2 - c_2^*\, \Delta F_1$$

(c)



Comments for control loops:

- Ratio control from Fd to F1 (we almost always have ratio of feeds to a mixer). It would be possible to have ratio control from Fd to F2, but it is most common to use it on the feeds.,
- The inventory control (involving p1 and p2) is radiating around the given flow (Fd, F1).
- The pressure control on p2 in the tank may be replaced by a level controller on V. This would assume that there is gas inside the mixer so that there is a level, and the pressure p2 inside the tank would then need to be set some other way, for example, by a vent.
- The ratio setpoint is set by a slow composition controller (CC)
- The slave flow controller for F1 is needed for implementing ratio control, but it also give disturbance rejection for p1 and p2 (they are withing the slave loop) and it linearizes the nonlinear valve.
- Order of tuning: First implement FC for F12 (fast), then the ratio controller (X), then the two pressure controllers and finally the composition controller.

Comment: The flowsheet in Problem 3 may correspond to parts of the block diagram in Problem 1 (with time in seconds and excluding z2 and ω). We have d2=p1, d1=Fd, u=z1, y=c and y2=F1. G2=2 represents the valve. For the mixer (G), the two time constants of 8s and 3s (rather than a single time constant of 11s) may represent some zones in the mixer (mixing is not immediate).The delay in G of 0.3s may be due to the pipeline out of the mixer. The measurement delay is 0.5s and there is also a time constant of 0.8s for the measurement of y=c2.

COMMENTS MADE AFTER CORRECTING THE EXAMS (from Lucas Cammann)

Problem 1

1) As you already saw in the exams that you corrected, not many people drew the second order response correctly. There were some however, and others drew the response obtained from the half-rule and wrote next to it that it should have an S-shape.
2) Many people forgot to include Gm when doing the tunings in exercise 1 b) and c). I deducted three points in b) for this, and another point in c).
3) I noted that almost all the students seemed to understand cascade well, but were struggling with the concept of feedforward. Here I was also a bit strict with the sketch, most of the students did not manage to put c_ff into the block diagram correctly. But this was only one point anyways.

Problem 2

4) I was surprised that quite some students managed to get more points in the second problem than in the first because this seems to have more "easy points" with the two tuning exercises.
5) The most common mistake I saw was a sign error in the phase calculation, where students neglected the minus of the RHP zero.
6) Another common mistake, sadly, was that some students constructed the Bode plots for G instead of L.

Problem 3

7) Problem 3 gave the least points, relatively speaking, potentially also because there were no bonus points to be obtained.
8) Very little people got rewarded the entire 5 points of a), because almost nobody reported the gain.
9) Problem b) worked relatively well, but some people did very weird things in the linearization. Something a lot of people did was to only set up the steady-state mass and component balances, linearize them and then have an "s" magically appear out of nowhere during the Laplace transform.
10) Exercise c) was hit or miss. There were some weird suggestions, and often times I had the impression that the students remembered something vaguely and then tried to make up the rest to make it fit. But this might also be because this was the last exercise, if the students worked through the exam from start to finish.