NTNU – Trondheim
Norwegian University of
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Department of Chemical Engineering

Exam paper for TKP4140 – Process Control

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Examination date: 09 December 2022
Examination time (from-to): 09:00 – 13:00
Permitted examination support material: One (1) A4 double-sided piece of paper with your handwritten notes. Standard calculator.

Other information: State clearly all assumptions you make. You may answer in Norwegian or English

Language: English
Number of pages (front page excluded): 6 (including Bode paper which may be handed in)

Informasjon om trykking av eksamensoppgave
Originalen er:

1-sidig ☐ **2-sidig** ☐

sort/hvit ☐ **farger** ☐

skal ha flervalgskjema ☐

Checked by:

Date Signature

Problem 1 (15%). Approximation of transfer function and tuning

Consider the following process

$$G_0 = \frac{(3s+1)e^{-0.5s}}{(7s+1)\,(2s+1)\,(0.8s+1)}$$

We want to design a PI-controller with τ_c=1.

a) (9%) Approximate the process as a first-order plus delay process.
   - Start by approximating the zero using the rules given below. Select τ_0 so that the ratio T_0/τ_0 is closest to 1.

b) (4%) Derive PI-settings using the SIMC rules.

c) (3%) Would you propose using PID control for this process?

$$\frac{T_0 s+1}{\tau_0 s+1} \approx \begin{cases} T_0/\tau_0 & \text{for } T_0 \geq \tau_0 \geq \cancel{\theta}\,\tau_c & \text{(Rule T1)} \\ T_0/\cancel{\theta}\,\tau_c & \text{for } T_0 \geq \cancel{\theta}\,\tau_c \geq \tau_0 & \text{(Rule T1a)} \\ 1 & \text{for } \cancel{\theta}\,\tau_c \geq T_0 \geq \tau_0 & \text{(Rule T1b)} \\ T_0/\tau_0 & \text{for } \tau_0 \geq T_0 \geq 5\cancel{\theta}\,\tau_c & \text{(Rule T2)} \\ \frac{(\tilde{\tau}_0/\tau_0)}{(\tilde{\tau}_0-T_0)s+1} & \text{for } \tilde{\tau}_0 \overset{\text{def}}{=} \min(\tau_0, 5\cancel{\theta}\,\tau_c) \geq T_0 & \text{(Rule T3)} \end{cases}$$

Problem 2 (20%). Sinusoidal disturbance

Consider the following process (in deviation Laplace variables)

$$y = G\,u + G_d\,d$$

$$G(s) = \frac{1.5}{2s+1}, G_d(s) = \frac{3e^{-2s}}{12s+1}$$

The disturbance d is a sinusoid (of any frequency) with amplitude d_{max}=2, the maximum largest input change is u_{max}=10, and the largest allowed output change is y_{max}=0.5. Thus, in the following consider a persistent disturbance d(t)=2sin(ωt) (where ω may vary).

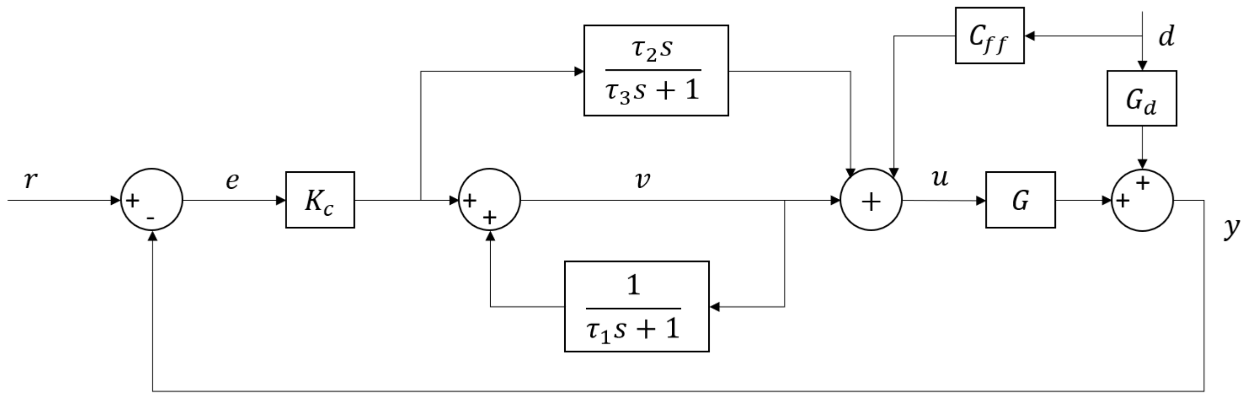
(a) (7%) Consider first no control (u=0). What is the magnitude of the output y as function of ω (give both the analytical expression and make a plot (sketch)).

(b) (7%) Consider next feedback control. What PI-tunings do you suggest if we want to use "smooth control" (that is, we want to select the largest possible τ_c that keeps the amplitude $|y|<|y_{max}|$ for any frequency ω).

(c) (6%) Consider finally feedforward control (without feedback). Propose a design for c_{FF}. What is y(t) in this case?

Problem 3 (20%). Block diagram

Consider the system represented by the following block diagram:



a) (10%) Find the transfer function $C(s)$ from e to u, such that $u = C\ e + C_{ff}\ d$. What do the three time constants in C represent?

b) (7%) Using $u = C(s)\ e + C_{ff}(s)\ d$, find the transfer functions $T(s)$ and $T_d(s)$ such that:

$$y = T(s)\ r + T_d(s)\ d$$

c) (3%) (You may answer this without solving part b). In the context of control systems, r represents the reference signal (setpoint), d is a disturbance and y in the controlled variable. If steady-state offset is desired to be zero, what property should we require from $T(s)$ and $T_d(s)$?

Problem 4 (20%). Transfer function responses

Consider the transfer functions:

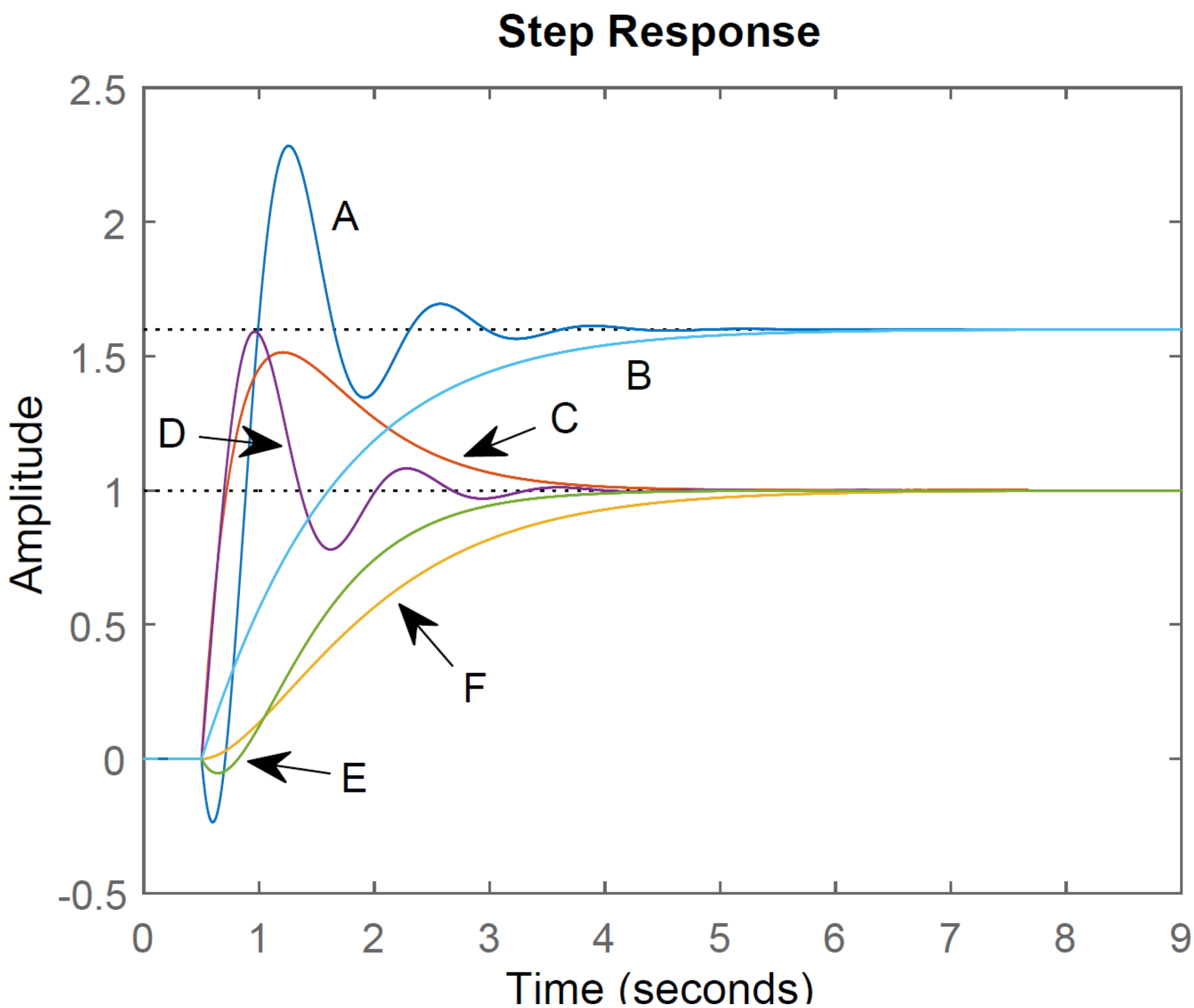
$$G_1(s) = \frac{0.2\,s + 1}{0.04\,s^2 + 0.12\,s + 1} \qquad G_4(s) = \frac{-0.2\,s + 1.6}{0.04\,s^2 + 0.12\,s + 1}$$

$$G_2(s) = \frac{1.6\,s + 1}{0.24\,s^2 + s + 1} \qquad G_5(s) = \frac{-0.2\,s + 1}{0.24\,s^2 + s + 1}$$

$$G_3(s) = \frac{1}{0.6\,s^2 + 1.6\,s + 1} \qquad G_6(s) = \frac{1.6\,s + 3.2}{1.2\,s^2 + 3.2\,s + 2}$$

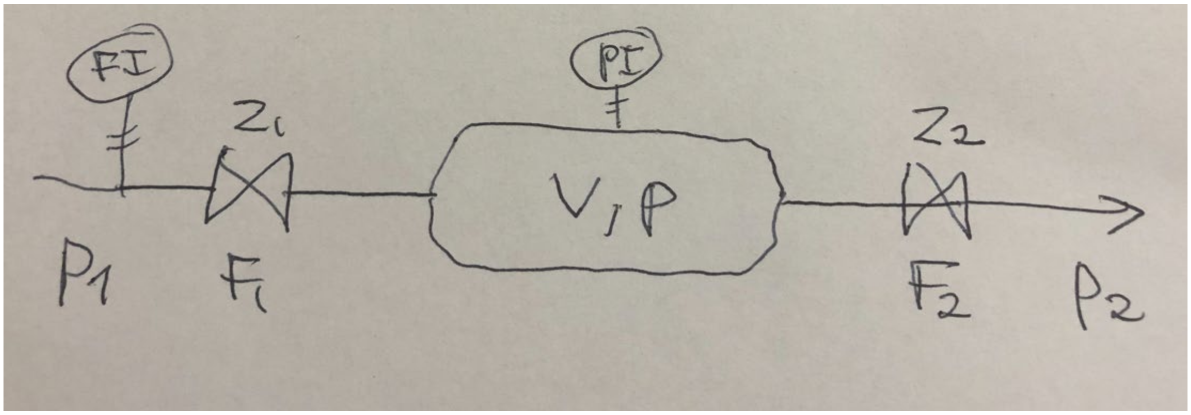
Fill in the missing values in the table below. As conclusion, identify the corresponding responses to a unitary step at $t_0 = 0.5$ provided in the following figure.

TF	Poles	Zeros	Steady-state gain	Initial slope	Conclusion
$G_1(s)$					
$G_2(s)$					
$G_3(s)$					
$G_4(s)$					
$G_5(s)$					
$G_6(s)$					



Step Response
Amplitude
Time (seconds)
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
6
7
8
9
A
B
C
D
E
F

Problem 5 (25%). Modelling and control of flow and pressure



Consider a gas pipeline with two valves. We have measurements of the inflow F_1 and the intermediate pressure p and these should be controlled. The volume of the pipeline can be represented as a tank with volume V as shown in the figure above.

Steady-state data: F_1=1 kg/s, z_1=z_2=0.5, p_1=2 bar, p=1.88 bar, p_2=1.8 bar, V=130 m^3, T=300 K, Parameters: R=8.31 J/K.mol, M_W=18e-3 kg/mol (so the gas is steam).

The following model equations are suggested to describe the system.

(1) $dm/dt = F_1-F_2$
(2) $m = k_p\, p$ where $k_p=VM_w/(RT)$
(3) $F_1 = C_1 z_1 \sqrt{p_1 - p}$
(4) $F_2 = C_2 z_2 \sqrt{p - p_2}$

(a) (3%) Explain what the variables and equations represent. What assumptions have been made?
(b) (3%) Determine the parameters in the model (C_1, C_2, k_p). What is the steady-state value of m? What is the residence time of the gas, m/F_1?
(c) (12%) Linearize the model and find the 2x2 transfer function model from z_1 and z_2 (inputs) to F_1 and p (controlled variables). (Note: To simplify, you can assume p_1 and p_2 are constant)
(d) (4%) What pairings do you suggest for single-loop control (with u = [z_1 z_2], y= [F_1 p])? How could control be improved?
(e) (3%) (This can be answered without solving parts a-d). What control structure would you propose if we instead of p want to control the downstream pressure p_2? Thus, we have u=[z_1 z_2] and y = [F_1 p_2] .

Bode paper: