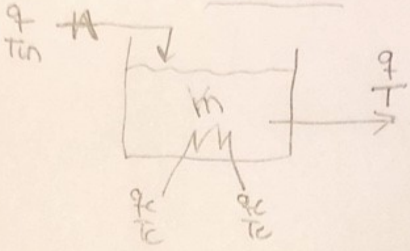
Exam 2020

Solution. Problem 1

19/11-2020

4/12-2020



(a) Energy balance, constant mass and c_p, $m = \rho V$

$$m c_p \frac{dT}{dt} = q c_p (T_{in} - T) + UA (T_C - T)$$

Note that since T_c is assumed constant we do not need the energy balance on the cold side.

(b) Steady state. Set $\frac{dT}{dt} = 0$. Get:

$$T = \frac{q c_p T_{in} + UA T_C}{q c_p + UA} = \frac{10 \cdot 4.2 \cdot 50 + 42 \cdot 10}{10 \cdot 4.2 + 42}$$

$$= \frac{50 + 10}{1 + 1} = \frac{60}{2} = \underline{\underline{30^\circ C}}$$

(c) Linearize

$$m c_p \frac{d\Delta T}{dt} = c_p (T_{in}^* - T^*) \Delta q + q^* c_p (\Delta T_{in} - \Delta T) + UA (\Delta T_C - \Delta T)$$

$$= c_p (T_{in}^* - T^*) \Delta q + q^* c_p \Delta T_{in} + UA \cdot \Delta T_C - (q^* c_p + UA) \Delta T$$

$$\tau \frac{d\Delta T}{dt} = -\Delta T + k_3 \Delta q + k_1 \Delta T_{in} + k_2 \Delta T_C$$

where $\tau = \frac{m c_p}{q c_p + UA} = \frac{5000 \cdot 4.2}{10 \cdot 4.2 + 42} = \frac{500}{2} = 250 s$

$$k_1 = \frac{q^* c_p}{q^* c_p + UA} = \frac{10 \cdot 4.2}{10 \cdot 4.2 + 42} = 0.5$$

$$k_2 = \frac{UA}{q c_p + UA} = 0.5$$

$$k_3 = \frac{c_p (T_{in}^* - T^*)}{q c_p + UA} = \frac{4.2 \cdot (50 - 30)}{10 \cdot 4.2 + 42} = \frac{50 - 30}{20} = 1 \frac{{}^\circ C}{kg/s}$$

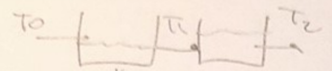
Conclusion

$$g_1 = \frac{0.5}{\tau s + 1}, \quad g_2 = \frac{0.5}{\tau s + 1}, \quad g_3 = \frac{1}{\tau s + 1}, \quad g_4 = 0$$

Note that q_c has no effect on T since T_c is assumed constant.

Comment: It may seem a bit strange that qc has no effect. This will be the case if the boiling fluid which is used for cooling is only partially evaporated. Then increasing qc will not increase the amount which is evaporated (and thus not change Q); it will just lead to more to more excess liquid in the cold outstream. In practice, an increase in qc may increase the U-value for heat transfer, but this neglected.

Problem 2



$\tau_1 = \frac{V_1}{q} = 5s \qquad \tau_2 = \frac{V_2}{q} = 20s$

(a) $T_1 = \underset{g_1}{\frac{1}{5s+1}} T_0 \;, \quad T_2 = \underset{g_2}{\frac{1}{20s+1}} T_1$

$\Rightarrow \quad T_2 = \frac{1}{(20s+1)(5s+1)} T_0$

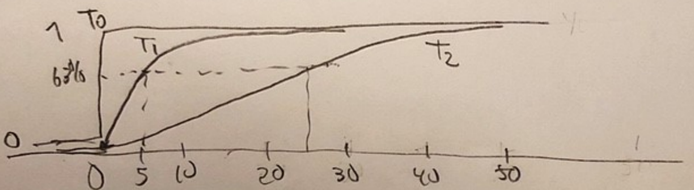
(b) Assume initially at steady state.
Step response

$T_0 = 1 \qquad (t \geq 0)$

$T_1 = (1 - e^{-t/\tau_1}) = 1 - e^{-t/5} \qquad (t \geq 0)$

$T_2 = 1 + \frac{1}{\tau_2 - \tau_1}\left(\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}\right)$

$\quad = 1 + \frac{1}{15}\left(5e^{-t/5} - 20 e^{-t/20}\right)$



(c) Sinusoidal response

$T_0(t) = \sin(\omega t)$ where $\omega = 0.2$ rad/s $\qquad T = \frac{2\pi}{\omega} = 31.4$

$T_1(t) = k_1 (\sin \omega t + \varphi_1) = k_1 \sin(\omega(t - \Delta t_1))$

$T_2(t) = k_2 \sin(\omega(t - \Delta t_2)).$

$k_1 = |g_1(j\omega)| = \frac{1}{\sqrt{(\omega\tau_1)^2 + 1}} = \sqrt{\frac{1}{2}} = 0.7$

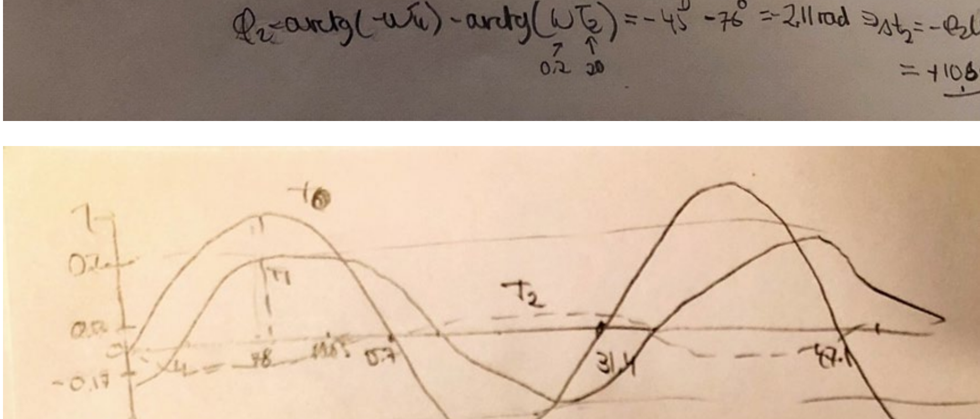
$\varphi_1 = \text{arctg}(-\omega\tau_1) = \text{arctg}(-1) = -45^\circ = -\frac{\pi}{4}$ rad

$\Delta t_1 = -\frac{\varphi_1}{\omega} = \frac{\pi/4}{0.2} = \frac{5}{4}\pi = \underline{3.92s}$

$k_2 = |g_1 g_2(j\omega)| = \frac{1}{\sqrt{(\omega\tau_1)^2+1}\sqrt{(\omega\tau_2)^2+1}} = \sqrt{\frac{1}{2} \cdot \frac{1}{(0.2 \cdot 20)^2 + 1}} = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{17}} = 0.17$

$\varphi_2 = \text{arctg}(-\omega\tau_1) - \text{arctg}(\underset{0.2}{\omega}\,\underset{20}{\tau_2}) = -45^\circ - 76^\circ = -2.11 \text{ rad} \Rightarrow \Delta t_2 = -\varphi_2/\omega$

$= \underline{+10.5s}$



Problem 3 (15%).

Solution:

(a) u = z,
y = T,
d=(F1, T2),
y2 = (T1, T3, F2).

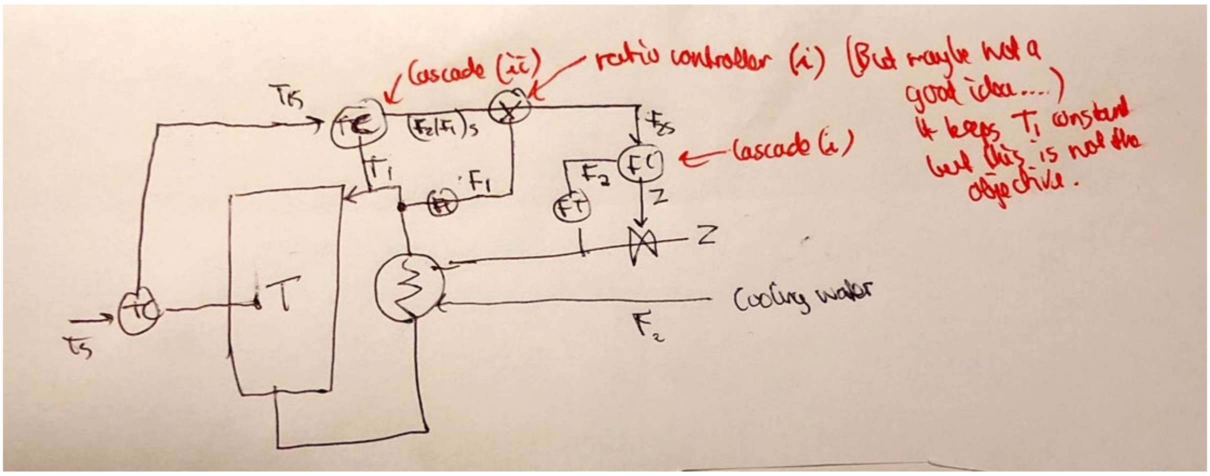
Note that d is independent if u, whereas y2 depends on u. We may use y2 for cascade control and d for feedforward control.

(b) Cascade: (i) Flow slave controller based on y2=F2. Helps for pressure disturbances and linearizes valve.
(ii) Temperature slave controller based on T1 or T3. T1 seems best since it is closer to T. Helps because of delay for T.

Feedforward:
(i) Ratio control from F1 to F2 (as a setpoint to the flow controller for F2).

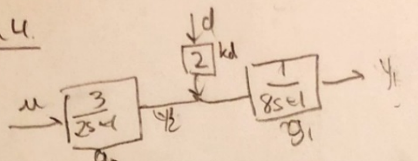
(BUT: This ratio control is probably not a good idea. It may help to keep T1 constant, but what we want to keep constant is T. From the energy balance, the cooling of the tank is given by Q=F1*cp1*(T-T1) , so to keep Q (and thus T) constant we want T1 to increase when F1 increases, so T1 should not remain constant for this disturbance.

(ii) Feedforward control from T2 to z (seems a bit difficult).



Problem 4.

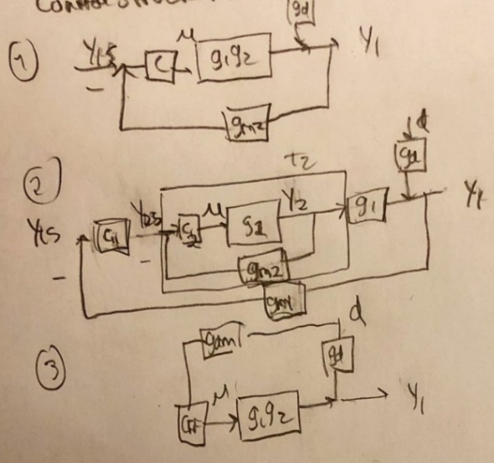
(a) Process



$g_d = \frac{2}{8s+1}$

$g_m = \frac{1}{0.5s+1}$

Control structures



(b) Control settings

① Design C based on $g_1 g_2 g_m = \frac{3}{(8s+1)(2s+1)(0.5s+1)} \approx \frac{3e^{-1.5s}}{(9s+1)}$

$\tau_c=\theta=1.5 \Rightarrow K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\frac{9}{3} = 1$

$\tau_I = \min(\tau_1, 4(\tau_c+\theta)) = \min(9, 12) = 9$

② Design C_2 based on $g_2 g_{2m} = \frac{3}{(2s+1)(0.5s+1)} \approx \frac{3e^{-0.25s}}{2.25s+1}$

$\tau_{c2}=\theta=0.25$ $\quad K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\frac{2.25}{0.5} = 1.5$

$\tau_I = \min(2.25, 2) = 2$

Design C_1 based on $T_2 g_1 g_{m1}$ where $T_2 \approx \frac{1}{0.25s+1}$

So $T_2 g_1 g_{m1} = \frac{1}{8s+1}\,\frac{1}{0.5s+1}\,\frac{1}{0.25s+1} \approx \frac{e^{-0.5s}}{8.25s+1}$

$\tau_{c1}=\theta=0.5 \Rightarrow K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{1}\frac{8}{1} = 8$

$\tau_I = \min(\tau_1, 4(\tau_c+\theta)) = \min(8, 4) = 4$

Comments on cascade. 1) Design of c1: Will also get full score if use T2 =approx. exp(-0.25s)/(0.25s+1). But note that the measurement g2m is not in the direct path from, y2s to y2.

2) The simulations (you can try!) for the cascade show OK responses, but there are some potential problems. First, the inner "slave" loop should not be tuned so tight that it may cause instability, so one should probably select tauc2 larger than theta2=0.25 to get better robustness. Second, I normally recommend a factor of at least 5 between tauc for the slave and master loops (to avoid interactions between the slave and the master loops), and here we only have a factor 2. So, also tauc1 should be larger in practice. Maybe tauc2=0.5 and tauc1=2 would work well, but then there will be little or no benefit compared to structure 1 with no cascade (which has tauc1=1.5).

③ Design C_{FF}.

~~Ideal~~ $y_1 = g_d \cdot d + g_1 g_2 \cdot C_{FF} \cdot g_{dm} \cdot d \overset{0}{}$

Ideal $y_1 = 0 \Rightarrow$

$$C_{FF,ideal} = -\frac{g_d}{g_1 g_2 g_{dm}} = -\frac{2\,\cancel{(8s+1)}(2s+1)}{\cancel{8s+1} \cdot 3} \cdot (0.5s+1)$$

$$= -\frac{2}{3}(2s+1)(0.5s+1)$$

This is not realizable. ~~The~~ The simplest is to choose

$$C_{FF} = -\frac{2}{3}$$

(c) It's not so obvious which is the best because the disturbance d is not inside the slave loop.

- However, we see that we can make $\tau_{c1} = 0.5$ with cascade control, whereas $\tau_c = 1.5$ without, so we can speed up the response with cascade (when using PI).

- An alternative would be to use a PID-controller for C (structure 1). We have

$$g_1 g_2 g_{1m} \approx \frac{3e^{-0.25s}}{(8s+1)(2.25s+1)}$$

So we could then choose $\tau_c = 0.25$ (PID) which would be even faster than cascade control (two PI's).

- Feedforward control (if used) should be combined with feedback

Problem 5

Solution.

Yes, the system is stable since it satisfied the Bode stability criterion: For closed-loop stability the Loop gain |L| must be less than 1 (it is about 0.32 in this case) at the frequency w180 where the phase shift around the loop (in L) is -180 degrees.

At frequency w180: GM=1/0.32 = 3.12,

At frequency wc: PM = -133 + 180 = 47 degrees = 0.82 rad

Wc = 0.73 rad/s, DM = PM/wc = 0.82/0.73 = 1.12 s