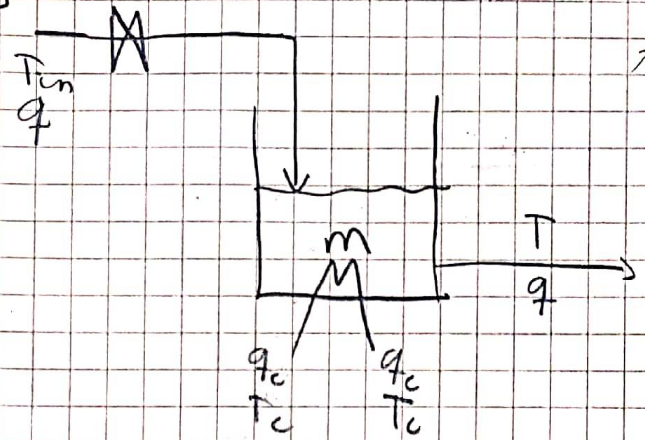
TKP4140 Exam 2020

$\boxed{1}$



Assume $C_p \approx C_v$ and constant C_p

a) Energy balance:

$$\frac{\partial (H \cdot m)}{\partial t} = H_{in} \cdot \dot{m}_{in} - H_{out} \cdot \dot{m}_{out} + \underbrace{Q}_{= UA(T_c - T)}$$

Assume $T_{ref} = 0$ and well mixed tank ($T_{tank} = T_{out} = T$)

$$\Rightarrow \quad H = C_p (T - T_{ref}) = C_p T$$

$$H_{in} = q \cdot T_{in} \cdot C_p$$

$$H_{out} = q \cdot T \cdot C_p$$

Constant m and C_p:

$$\frac{\partial (H \cdot m)}{\partial t} = \frac{\partial}{\partial t} (C_p \cdot T \cdot m) = m \cdot C_p \frac{\partial T}{\partial t}$$

$$\Rightarrow \quad \underline{\underline{m C_p \frac{\partial T}{\partial t} = q \cdot C_p \cdot T_{in} - q \cdot C_p \cdot T + UA(T_c - T)}}$$

b) At steady state:

$$\frac{\partial T}{\partial t} = 0$$

$q = q^* = 10\ kg/s$

$m = 5000\ kg$

$c_p = 4{,}2\ kJ/K\cdot kg$

$q_c = q_c^* = 1\ kg/s$

$T_{in} = T_{in}^* = 50°C$

$T_c = T_c^* = 10°C$

$UA = 42\ kW/K$

$$m \cdot c_p \frac{\partial T}{\partial t} = q \cdot c_p T_{in} - q \cdot c_p \cdot T + UA\,(T_c - T)$$

$$0 = q^* c_p T_{in}^* - T^*(q^* c_p + UA) + UA \cdot T_c^*$$

$$\Rightarrow T^* = \frac{q^* \cdot c_p \cdot T_{in}^* + UA \cdot T_c^*}{q^* c_p + UA}$$

$$= \frac{10\,kg/s \cdot 4{,}2\,kJ/K\,kg \cdot 50°C + 42\,kW/K \cdot 10°C}{10\,kg/s \cdot 4{,}2\,kJ/K\,kg + 42\,kW/K}$$

$$= \underline{\underline{30°C}}$$

c) Original equation:

$$mC_p \frac{\partial T}{\partial t} = qC_p T_{in} - qC_p T + UA(T_c - T) = K$$

Using a first order Taylor expansion:

$$mC_p \frac{\partial \Delta T}{\partial t} = \frac{\partial K}{\partial T_{in}}\Big|_* \Delta T_{in} + \frac{\partial K}{\partial T_c}\Big|_* \Delta T_c + \frac{\partial K}{\partial q}\Big|_* \Delta q + \frac{\partial K}{\partial q_c}\Big|_* \Delta q_c$$
$$+ \frac{\partial K}{\partial T}\Big|_* \Delta T$$

ΔT, ΔT_{in}, ΔT_c, Δq and Δq_c are deviation variables.

$$\frac{\partial K}{\partial T_{in}}\Big|_* = q^* C_p$$

$$\frac{\partial K}{\partial T_c}\Big|_* = UA$$

$$\frac{\partial K}{\partial q}\Big|_* = C_p T_{in}^* - C_p T^* = C_p(T_{in}^* - T^*)$$

$$\frac{\partial K}{\partial q_c}\Big|_* = 0$$

$$\frac{\partial K}{\partial T}\Big|_* = -q^* C_p - UA$$

This yields a linearized model equal to:

$$mC_p \frac{\partial \Delta T}{\partial t} = q^* C_p \Delta T_{in} + UA \Delta T_c + C_p(T_{in}^* - T^*)\Delta q$$
$$+ 0 \cdot \Delta q_c - (q^* C_p + UA)\Delta T$$

Taking the Laplace transform of this equation yields:

$$m \cdot c_p \cdot s \cdot T(s) = q^* c_p T_{in}(s) + UA T_c(s) + c_p(T_{in}^* - T^*) q(s) - (q^* c_p + UA) T(s)$$

$$T(s)\left(m \cdot c_p \cdot s + (q^* c_p + UA)\right) = q^* c_p T_{in}(s) + UA T_c(s) + c_p(T_{in}^* - T^*) q(s)$$

$$T(s) = \frac{q^* c_p}{m c_p s + (q^* c_p + UA)} T_{in}(s) + \frac{UA}{m c_p \cdot s + (q^* c_p + UA)} T_c(s) + \frac{c_p(T_{in}^* - T^*)}{m c_p \cdot s + (q^* c_p + UA)} q(s)$$

$$= \frac{\frac{q^* c_p}{q^* c_p + UA}}{\frac{m c_p}{q^* c_p + UA} \cdot s + 1} T_{in}(s) + \frac{\frac{UA}{q^* c_p + UA}}{\frac{m c_p}{q^* c_p + UA} \cdot s + 1} T_c(s) + \frac{\frac{c_p(T_{in}^* - T^*)}{q^* c_p + UA}}{\frac{m c_p}{q^* c_p + UA} \cdot s + 1} q(s)$$

$$\Rightarrow g_1 = \frac{\frac{q^* c_p}{q^* c_p + UA}}{\frac{m c_p}{q^* c_p + UA} \cdot s + 1}, \quad g_2 = \frac{\frac{UA}{q^* c_p + UA}}{\frac{m c_p}{q^* c_p + UA} \cdot s + 1}$$

$$g_3 = \frac{\frac{c_p(T_{in}^* - T^*)}{q^* c_p + UA}}{\frac{m c_p}{q^* c_p + UA} \cdot s + 1}, \quad g_4 = 0$$

Inserting numbers, this makes the transfer functions equal to

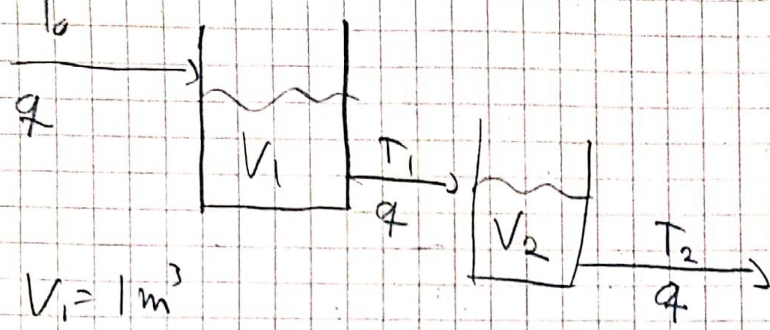
$$g_1 = \frac{0,5}{250 \cdot s + 1}$$

$$g_2 = \frac{0,5}{250 \cdot s + 1}$$

$$g_3 = \frac{1}{250 \cdot s + 1}$$

$$g_4 = 0$$

2)



$V_1 = 1m^3$

$V_2 = 4m^3$

$q = 0{,}2 m^3/s$

a) $T_2(s) = g_2(s) T_1(s)$

$T_1(s) = g_1(s) T_0(s)$

The tanks are well mixed, so g_1 is given as

$$g_1(s) = \frac{k}{\tau_1 s + 1}$$

τ_1 is equal to the residence time in tank 1:

$$\tau_1 = \frac{V_1}{q} = \frac{1m^3}{0{,}2m^3/s} = 5\ s$$

k is the gain, which is equal to 1, since T_1 will equal T_0 at steady state

$$\Rightarrow g_1(s) \frac{1}{5 \cdot s + 1}$$

The same reasoning can be used for g_2:

$$g_2(s) = \frac{k}{\frac{V_2}{q} \cdot s + 1} = \frac{1}{\frac{4}{0{,}2} \cdot s + 1}$$

$$g_2(s) = \frac{1}{20 \cdot s + 1}$$

$T_0(s) = 1/s$

$$T_1(s) = g_1(s)\,T_0(s) = \frac{1}{(5\cdot s+1)\cdot s}$$

$$T_2(s) = g_2(s)\cdot T_1(s) = \frac{1}{(20\cdot s+1)(5\cdot s+1)\cdot s}$$

Transforming to time domain:

$$T_0(t) = S(t) \text{ (unit step at } t=0)$$

$$T_1(t) = 1 - e^{-t/5} = 1 - e^{-0{,}2\cdot t}$$

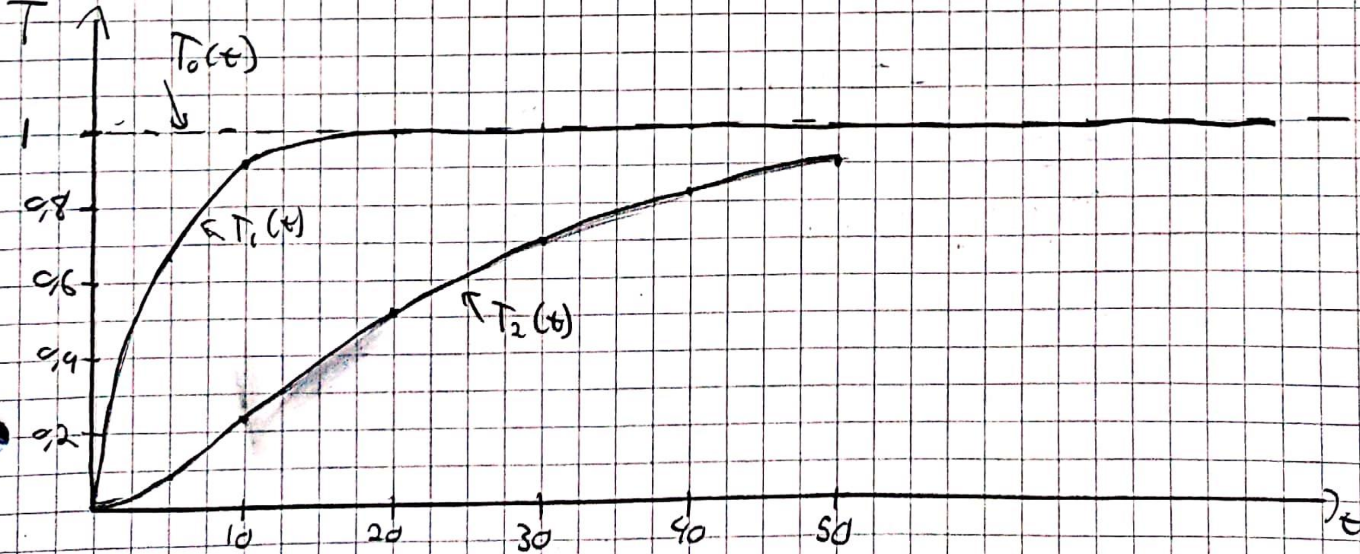
$$T_2(t) = 1 + \frac{1}{5-20}\left(20\,e^{-t/20} - 5\cdot e^{-t/5}\right)$$

$$= 1 - \frac{1}{15}\cdot 5\left(4\,e^{-0{,}05\cdot t} - e^{-0{,}2\cdot t}\right)$$

$$= 1 - \frac{1}{3}\left(4\,e^{-0{,}05\cdot t} - e^{-0{,}2\cdot t}\right)$$

The transformations were done using formulas for Laplace transformations in the lecture notes.

t	0	10	20	30	40	50
T_1	0	0,86	0,98	0,99	1	1
T_2	0	0,24	0,52	0,7	0,82	0,89



c) $T_0 = \sin(0{,}2 \cdot t) \Rightarrow \omega = 0{,}2$

Can find $T_1(t)$ by calculating amplitude ratio AR and phase shift φ:

$$AR = |g_1(\omega \cdot j)| = |g_1(0{,}2 \cdot j)|$$
$$= \frac{1}{\sqrt{(5 \cdot 0{,}2)^2 + 1}} = \frac{1}{\sqrt{2}} \approx 0{,}71$$

$$\varphi = \angle g_1(\omega j) = \angle g_1(0{,}2 \cdot j)$$
$$= -\arctan(5 \cdot 0{,}2) = -\frac{\pi}{4} \approx -0{,}785 \text{ rad}$$

$$\Rightarrow T_1(s) = \underline{\underline{\frac{1}{\sqrt{2}} \sin\left(0{,}2 \cdot t - \frac{\pi}{4}\right)}}$$

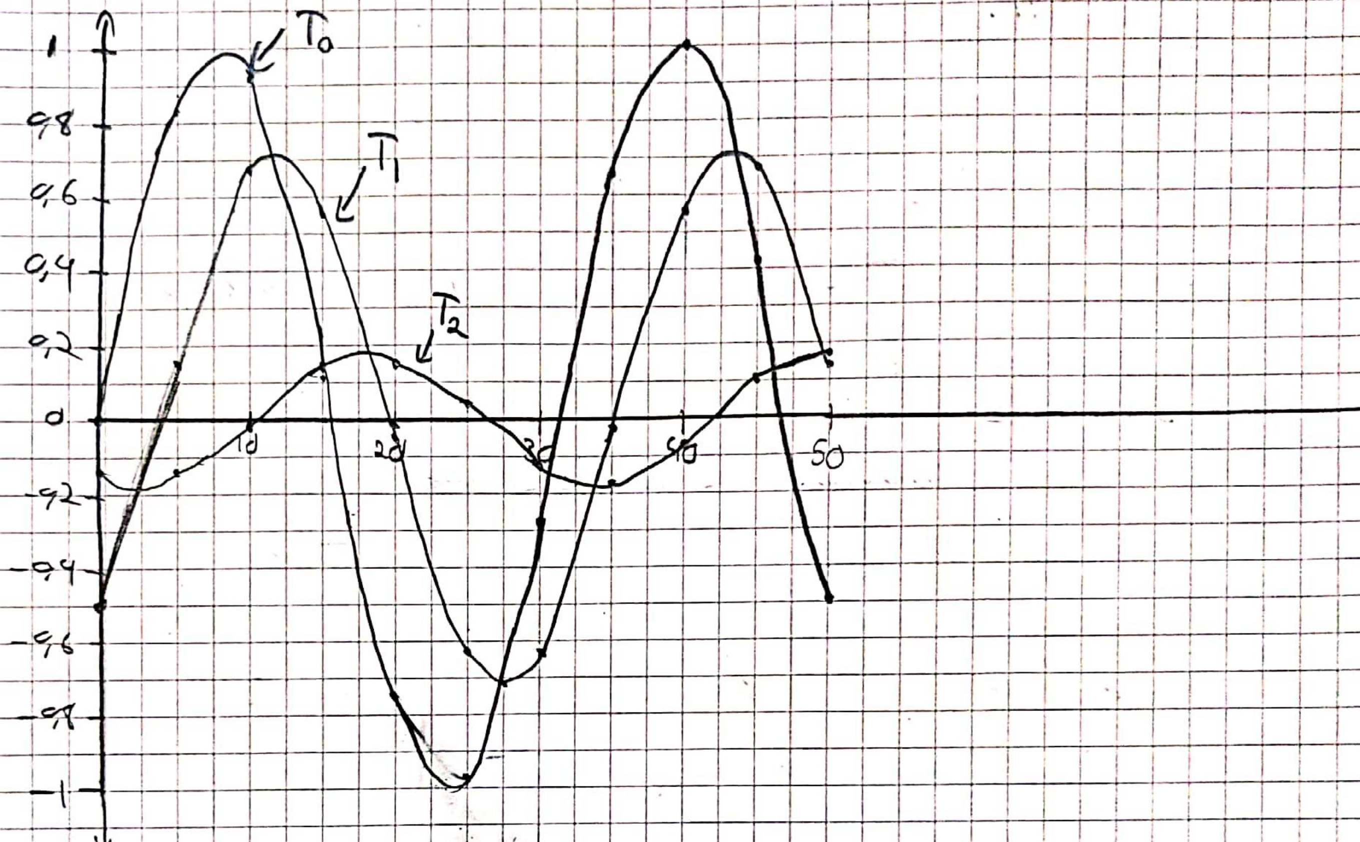
Same procedure can be used for $T_2(t)$:
Transfer function from T_0 to T_2 is equal to $g = g_1 \cdot g_2 = \frac{1}{(20 \cdot s+1)(5 \cdot s+1)}$

$$AR = |g(0{,}2 \cdot j)| = \frac{1}{\sqrt{(20 \cdot 0{,}2)^2+1} \cdot \sqrt{(5 \cdot 0{,}2)^2+1}} = 0{,}17$$

$$\varphi = \angle g(0{,}2 \cdot j) = -\arctan(20 \cdot 0{,}2) - \arctan(5 \cdot 0{,}2)$$
$$= -2{,}11 \text{ rad}$$

$$\Rightarrow T_2(t) = \underline{\underline{0{,}17 \cdot \sin(0{,}2t - 2{,}11)}}$$

t	0	5	10	15	20	25	30	35	40	45	50
T_0	0	0,84	0,91	0,14	-0,76	-0,96	-0,28	0,66	0,99	0,41	-0,54
T_1	-0,5	0,15	0,67	0,57	-0,05	-0,62	-0,62	-0,05	0,57	0,66	0,15
T_2	-0,15	-0,15	-0,02	0,13	0,16	0,04	-0,12	-0,17	-0,07	0,1	0,17



5 a) $u = z$

$y = T$

$y_2 = T_1, F_1, F_2, T_3$

$d = T_2$

b)

Cascade would be good around the heat exchanger, as there are dynamics and nonlinearity which could be handeled by inner loop (*)

=> Can measure T_1 and use it to adjust z_1. Setpoint for T_1 can be provided by master controller for T

Feedforward could be used by measuring T_2 and using this to adjust z. This would make it possible to increase coolant flow quickly if T_2 increases, which would reduce the effect of the disturbance.

Cascade would also be good around the valve Z. Valves usually contain nonlinearity, and a fast slave controller would allow other controllers to adjust F_2 directly, making the response better. The slave controller could be implemented by measuring F_2 and using a flow controller to adjust Z. Setpoint for F_2 would be provided by a master controller.

(*) The cascad around the heat exchanger would also allow for adjustment of T_1 without waiting for measurements of T. This would improve performance, as there is a large delay in measuring T.

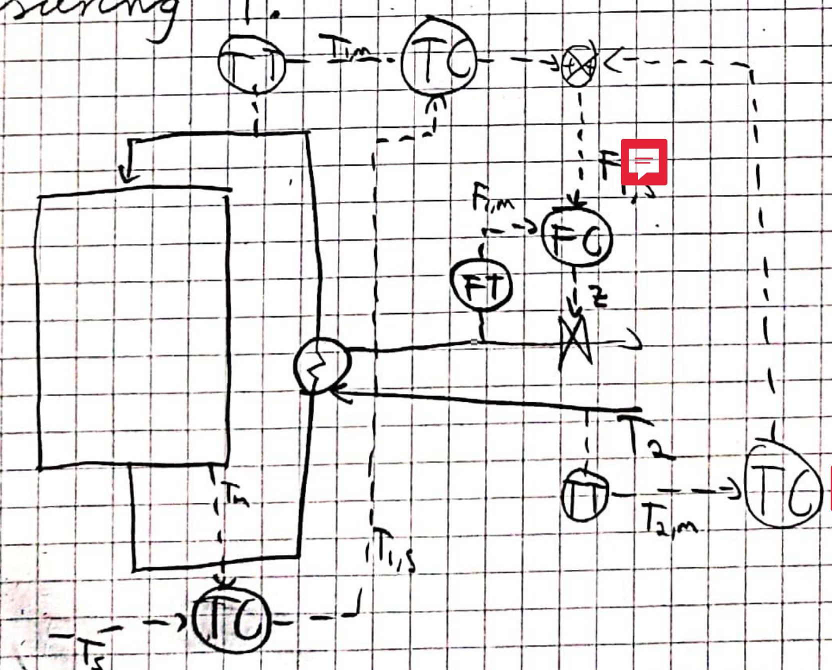


Figure showing the two cascade loops and the feedforward controller.

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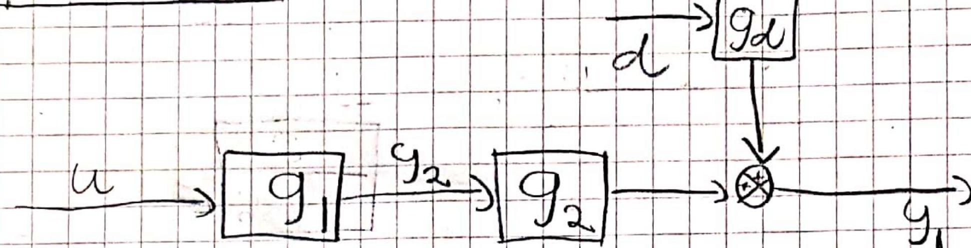
$$y_2 = \frac{3}{2\cdot s+1}\, u = g_1\, u$$

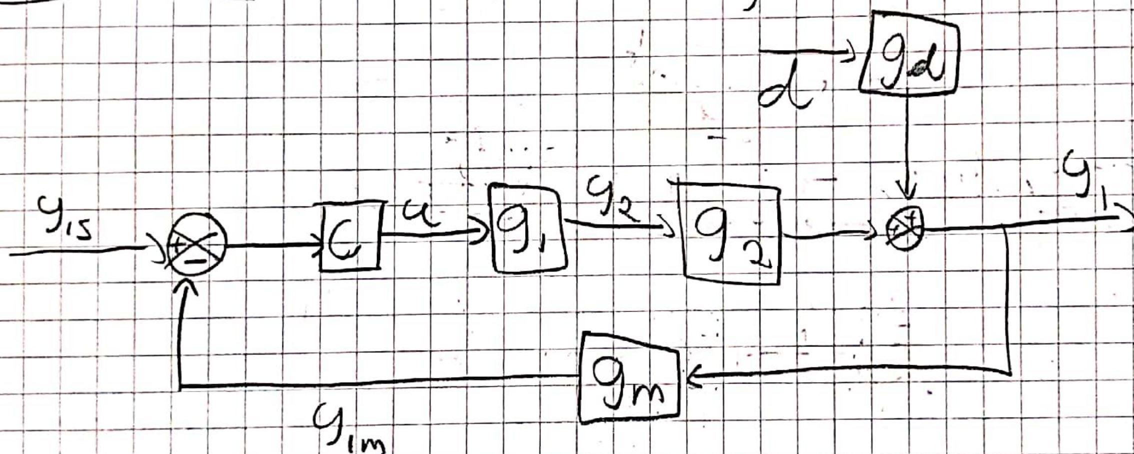
$$y_1 = \frac{y_2 + 2\cdot d}{8\cdot s+1} = \frac{y_2}{8\cdot s+1} + \frac{2}{8\cdot s+1}\, d$$

$$g_m = \frac{1}{0{,}5\cdot s+1} \qquad = g_2\cdot y_2 + g_d\cdot d$$

a) Open loop:



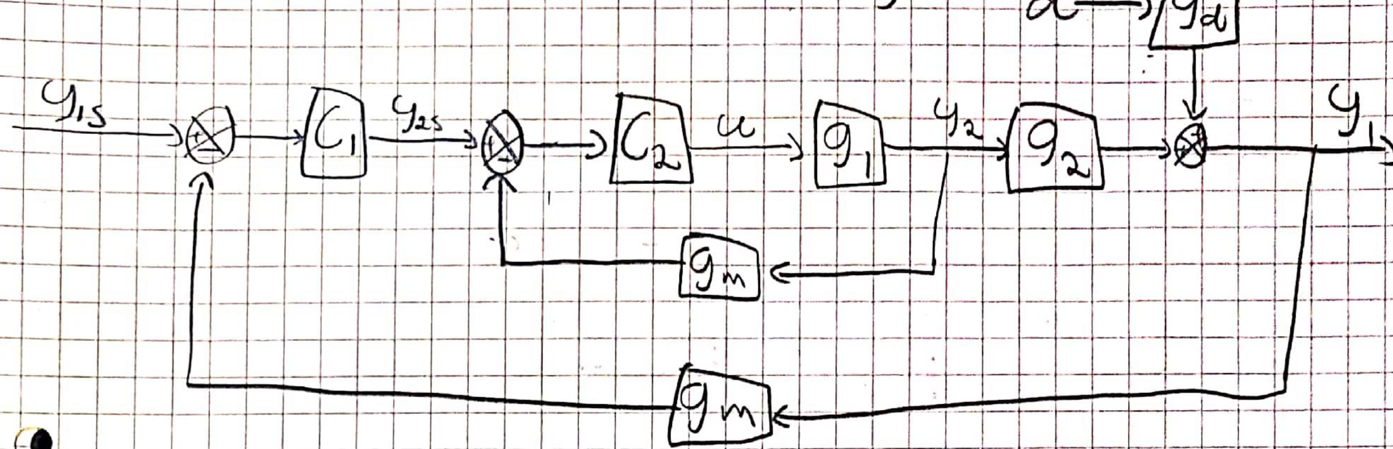
I: feedback: $u = c(s)\cdot(y_{1s} - y_{1m})$



II: Cascade control

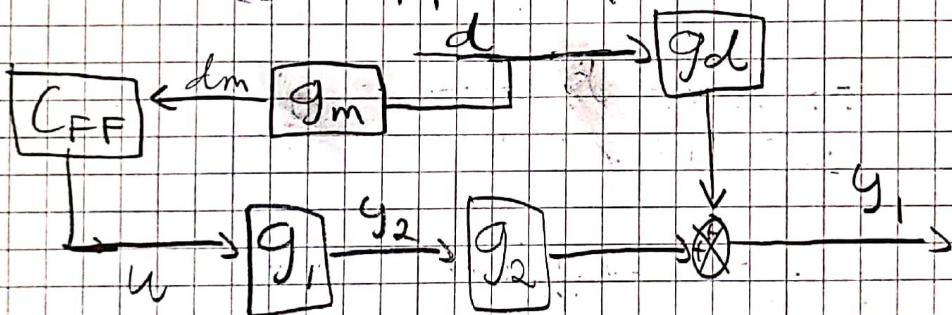
$$u = C_2 \cdot (y_{2s} - y_{2m})$$

$$y_{2s} = C_1 (y_{1s} - y_{1m})$$



III: Feedforward control

$$u = C_{FF} \cdot d_m$$



G) $\underline{\text{I: Tuning } C(s)}$

$$g = g_1 \cdot g_2 \cdot g_m = \frac{3}{2 \cdot s+1} \cdot \frac{1}{8 \cdot s+1} \cdot \frac{1}{0{,}5 \cdot s+1}$$

$$= \frac{3}{(8 \cdot s+1)(2 \cdot s+1)(0{,}5 \cdot s+1)}$$

Half rule:

$$\tilde{\theta} = \theta + \frac{\tau_2}{2} + \tau_3 = 0 + \frac{2}{2} + 0{,}5 = 1{,}5$$

$$\tilde{\tau}_1 = \tau_1 + \frac{\tau_2}{2} = 8 + \frac{2}{2} = 9$$

$$\Rightarrow \hat{g} = \frac{3 \cdot e^{-1{,}5 \cdot s}}{9 \cdot s+1} \Rightarrow \begin{aligned} \theta &= 1{,}5 \\ k &= 3 \\ \tau &= 9 \end{aligned}$$

SIMC rules for PI (with $\tau_c = \theta$ for tight control):

$$K_c = \frac{1}{k} \frac{\tau}{\tau_c + \theta} = \frac{1}{3} \cdot \frac{9}{1{,}5+1{,}5} = 1$$

$$\tau_I = \min(\tau, 4(\tau_c + \theta)) = \min(9, 12) = 9$$

$$\Rightarrow C(s) = 1 \cdot \frac{9 \cdot s+1}{9 \cdot s}$$

II: Tuning C_1 and C_2

Tune slave controller first:

$$g = g_1 \cdot g_m = \frac{3}{2 \cdot s+1} \cdot \frac{1}{0{,}5 \cdot s+1}$$

Half-rule:

$$\tilde{\theta} = \theta + \frac{\tau_2}{2} = 0 + \frac{0{,}5}{2} = 0{,}25$$

$$\tilde{\tau}_1 = \tau_1 + \frac{\tau_2}{2} = 2 + \frac{0{,}5}{2} = 2{,}25$$

$$\Rightarrow \tilde{g} = \frac{3 e^{-0{,}25 \cdot s}}{2{,}25 \cdot s+1} \Rightarrow \begin{array}{l} \theta = 0{,}25 \\ k = 3 \\ \tau = 2{,}25 \end{array}$$

SIMC with $\tau_c = \theta$:

$$K_c = \frac{1}{k} \frac{\tau}{\tau_c + \theta} = \frac{1}{3} \frac{2{,}25}{0{,}25+0{,}25} = 1{,}5$$

$$\tau_I = \min(\tau, 4(\tau_c+\theta)) = \min(2{,}25, 2) = 2$$

$$\Rightarrow C_2 = 1{,}5 \frac{2 \cdot s+1}{2 \cdot s}$$

Need to estimate closed loop response in inner loop to tune C_1:

$$T_{inner} = \frac{C_2 \cdot g_1 \cdot g_m}{1 + C_2 \cdot g_1 \cdot g_m} \approx \frac{e^{-\theta s}}{\tau_c \cdot s + 1}$$

$$= \frac{e^{-0{,}25 \cdot s}}{0{,}25 \cdot s + 1}$$

$$g_{outer} = T_{inner} \cdot g_2 \cdot g_m = \frac{e^{-0{,}25 \cdot s}}{0{,}25 \cdot s + 1} \cdot \frac{1}{8 \cdot s + 1} \cdot \frac{1}{0{,}5 \cdot s + 1}$$

Half rule:

$$\tilde{\theta} = \theta + \tau_2/2 + \tau_3 = 0{,}25 + \frac{0{,}5}{2} + 0{,}25 = 0{,}75$$

$$\tilde{\tau}_1 = \tau_1 + \tau_2/2 = 8 + \frac{0{,}5}{2} = 8{,}25$$

$$\Rightarrow \tilde{g}_{outer} = \frac{e^{-0{,}75 \cdot s}}{8{,}25 \cdot s + 1} \Rightarrow \begin{array}{l} \theta = 0{,}75 \\ k = 1 \\ \tau = 8{,}25 \end{array}$$

SIMC with $\tau_c = \theta$:

$$K_c = \frac{1}{k} \frac{\tau}{\tau_c + \theta} = \frac{1}{1} \cdot \frac{8{,}25}{0{,}75 + 0{,}75} = 5{,}5$$

$$\tau_I = \min(\tau_1, 4(\tau_c + \theta)) = \min(8.25, 6) = 6$$

$$\Rightarrow \underline{\underline{C_1(s) = 5{,}5 \cdot \frac{6 \cdot s + 1}{6 \cdot s}}}$$

III: Tuning C_{FF}

- Ideal controller is given as

$$C_{FF}^{ideal} = -\frac{g_d}{g_m \cdot g_1 \cdot g_2}$$

$$= -\frac{\frac{2}{8\cdot s+1}}{\frac{1}{9{,}5\cdot s+1}\cdot\frac{3}{2\cdot s+1}\cdot\frac{1}{8\cdot s+1}} = -\frac{2}{3}(2\cdot s+1)(9{,}5\cdot s+1)$$

This is not realizable, as it has more zeros than poles. It can be made realizable by removing one of the zeros and adding a tunable pole:

$$\underline{\underline{C_{FF} = -\frac{2}{3}\,\frac{2\cdot s+1}{\tau\cdot s+1}}}$$

, τ adjusts how agressive the controller is. Small τ gives fast response. Could for example choose $\tau = 0{,}1$

c) Closed loop response for feedback is approximately:

$$T_F \approx \frac{e^{-\theta \cdot s}}{\tau_c \cdot s+1} = \frac{e^{-1,6 \cdot s}}{1,6 \cdot s+1}$$

For cascade this is

$$T_{outer} \approx \frac{e^{-0,75 \cdot s}}{0,75 \cdot s+1}$$

Cascade control therefore gives faster control than feedback.
Would not prefere feedforward, as the ideal controller is not realizable due to $g_m \cdot g_1 \cdot g_2$ being slower than g_d

=> Cascade control is therefore preferable

For C and C_1, the closed loop is dominated by $\tau_1 = 8$, which is much larger than the other time constants. As τ_2 therefore has little effect, I would not suggest using PID here.

For C_2, $\overset{(=2)}{\tau_1}$ and $\overset{(=0,5)}{\tau_2}$ are a bit closer, and PID could be used as τ_2 is also larger than the time delay, which is zero. This could therefore improve the performance of the slave controller.

5 a) Bode stability condition:

$$|L(\omega_{180^\circ} j)| < 1$$

As shown in the figure,

$$|L(\omega_{180^\circ} j)| \approx 0{,}3$$

for this system. It is therefore stable.

b) $GM = \dfrac{1}{|L(\omega_{180^\circ} j)|} \approx \dfrac{1}{0{,}3} \approx \underline{\underline{3{,}3}}$

$PM = \angle L(\omega_c j) + 180^\circ \approx -135^\circ + 180^\circ$

$= \underline{\underline{45^\circ}} = \pi/4 \text{ rad}$

Time delay margin $= DM = \dfrac{PM\,[\text{rad}]}{\omega_c}$

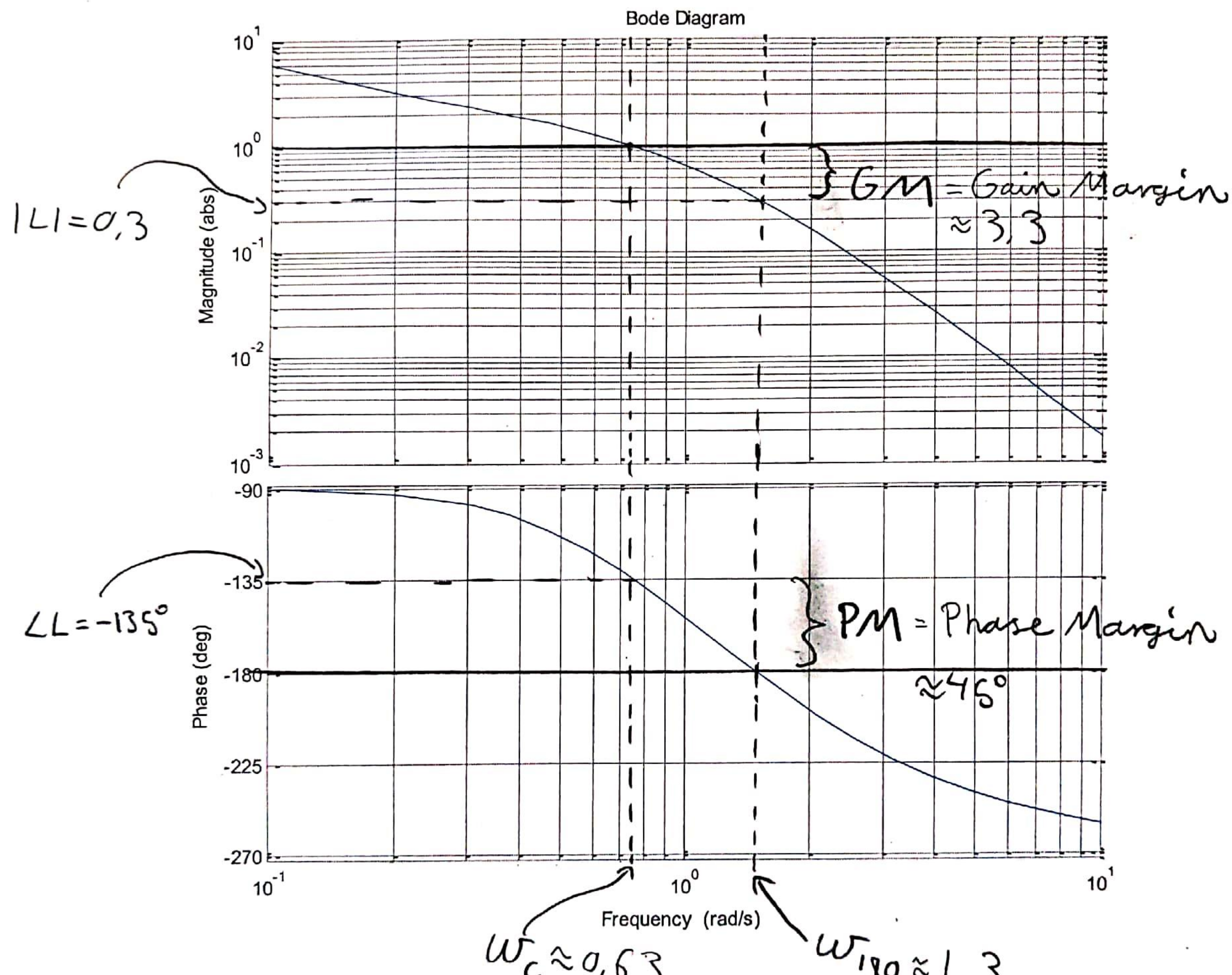
$= \dfrac{\pi/4 \text{ rad}}{0{,}63 \text{ rad/s}} = \underline{\underline{1{,}25 \text{ s}}}$

The figure is shown on the next page.

Problem 5 (10 %)

The frequency response of a loop transfer function $L(s) = g(s)c(s)g_m(s)$ is shown in the Bode diagram below.

(a) (2%) Formulate the Bode stability condition. Is the system stable?
(b) (8%) What is the gain margin, phase margin (show on the figure) and what is the allowed extra time delay in the loop to remain stable?



Comment: *You may write on this paper and use it as your solution.*