Transformed inputs for linearization, decoupling and feedforward disturbance rejection

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Zotica, C., Alsop, N., & Skogestad, S. (2020/2021). «Transformed Manipulated Variables for Linearization, Decoupling and Perfect Disturbance Rejection». IFAC World Congress, Berlin.
Sigurd Skogestad, Cristina Zotica, Nicholas Alsop. "Transformed inputs for linearization, decoupling and feedforward disturbance rejection". 2022 (not yet published)

Lecture 7. Transformed inputs

- Motivation: Feedforward ratio control
- Motivation: Decoupling of mixing process
- Inversion of input transformation
  - A. Exact inverse of model
  - B. Approximate inverse using feedback (cascade control)
- Derivation of ideal transformed inputs (for static and dynamic model)
  - Linearization
  - Decoupling
  - Feeforward disturbance rejection

Lecture 8: Examples and discussion

- Examples (many)
- Discussion
  - Chain of transformations (Exact inverse for systems of higher order)
  - Potential internal instability with exact inverse
    - Linear analysis,
    - Bode stability condition
  - Comparison with «feedback linearization»
- Output transformation

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Tuning rules for feedforward control from measurable disturbances combined with PID control: a review

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5.1. Classical solutions

5.1.1. Static feedforward compensator

A static feedforward compensator is a solution widely used in industry, which is given by:

$$C_{ff} = \frac{K_d}{K_v} \tag{15}$$

The reason to use this simple solution is that drastic improvements can be obtained compared with pure feedback control by using just this simple compensator. Moreover, it can be used to account for any non-realisable problem. However, the resulting performance is also limited because of its simplicity.

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Feedforward control

- Feedforward control relies on model
  - as opposed to feedback which relies mostly on data

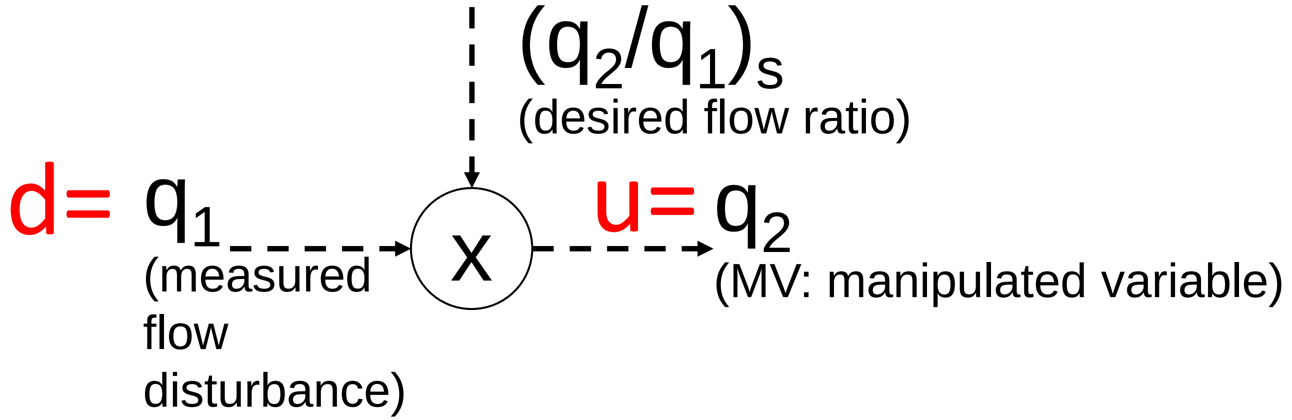
- Feedback control: Linear model is often OK

- Feedforward control: Much less likely that linear model is OK
  - Process changes and disturbances
  - This presentation: Use nonlinear static model

Ratio control
(most common case of nonlinear feedforward)

Example: Process with two feeds q_1(d) and q_2 (u), where ratio should be constant.

Use multiplication block (x):



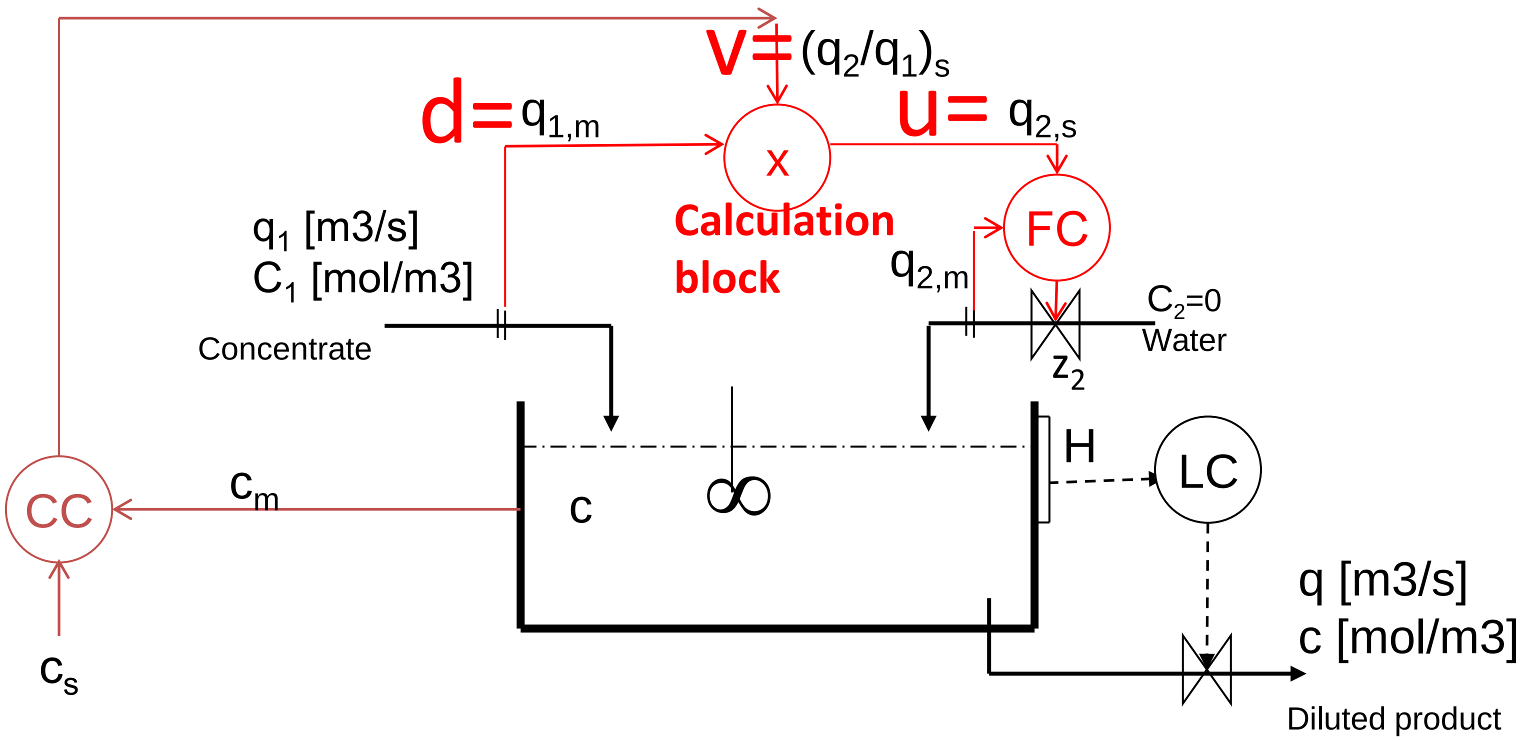
"Measure disturbance ($d=q_1$) and adjust input ($u=q_2$) such that ratio is at given value $(q_2/q_1)_s$"

Usually: Combine ratio (feedforward) with feedback

- Adjust $(q_1/q_2)_s$ based on feedback from process, for example, composition controller.
  - ***Example cake baking (mixing)****: Use recipe (ratio control = feedforward), but adjust ratio if result is not as desired (feedback)*

EXAMPLE: MIXING PROCESS

RATIO CONTROL with outer feedback to adjust ratio setpoint



u = q_2 (or actually valve position z_2) = physical input (MV)

v = q_2/q_1 = u/d = transformed input as seen from feedback controller CC

Feedforward ratio control: Transformed input $v = g(u,d) = u/d$

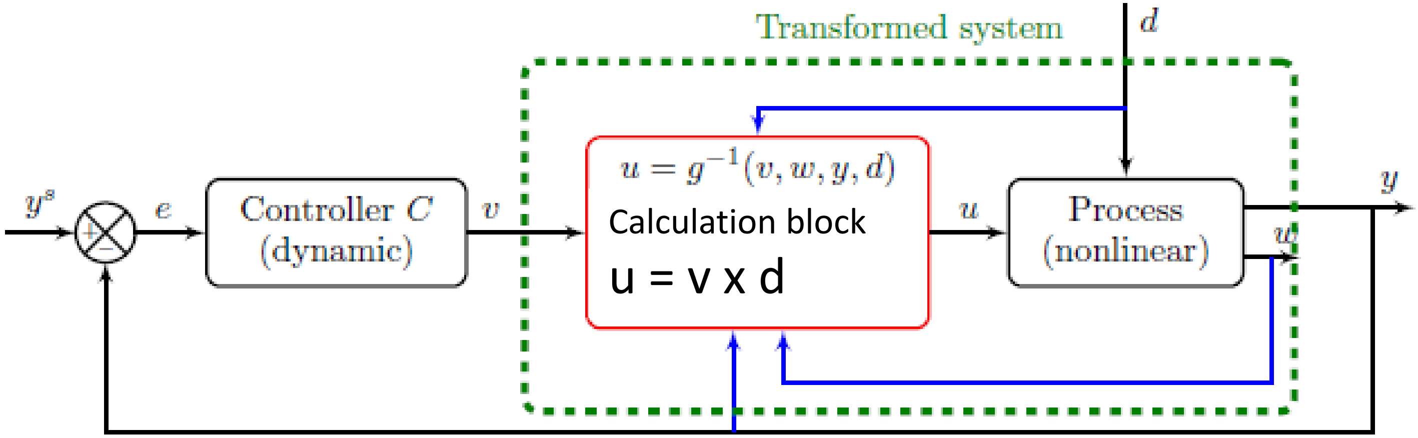


Figure 1: Use of transformed inputs v. For example, the transformed input could be the ratio $v = g(u,d) = \frac{u}{d}$, and the "inverse input transformation" block that inverts this relationship would then be $u = g^{-1}(v,d) = vd$.

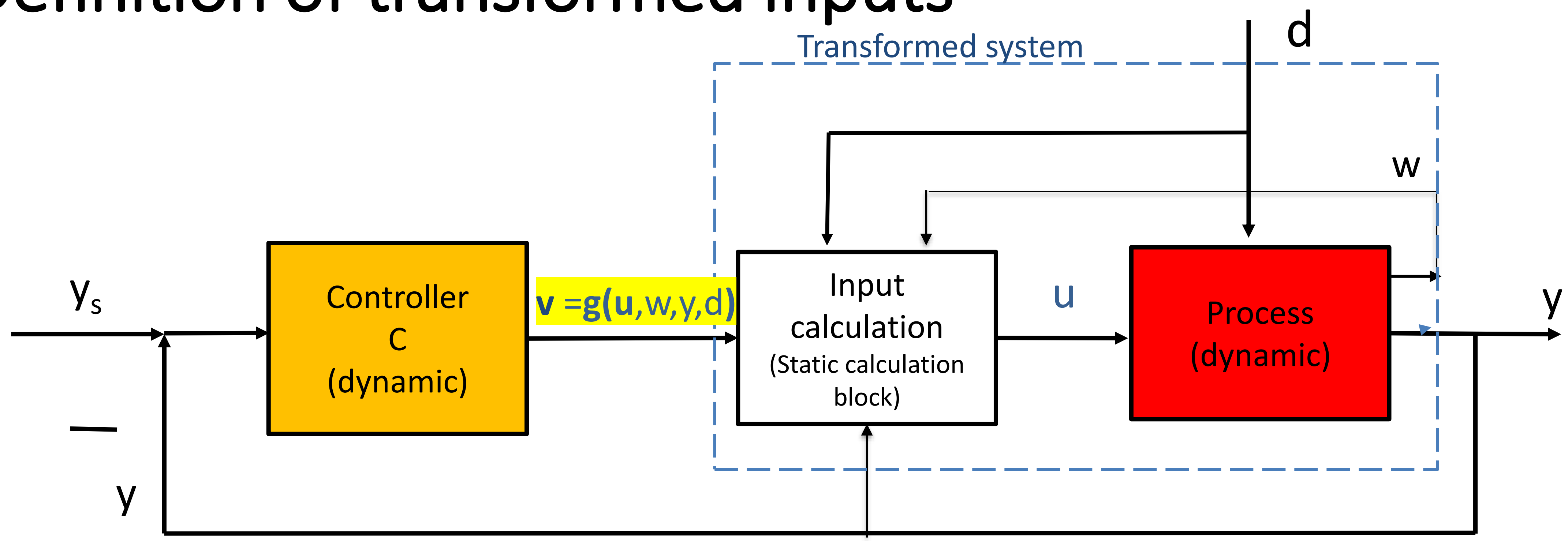
Further motivation transformed inputs

- Industry frequently uses static model-based calculations blocks
  - For feedforward, decoupling, linearization
- … and sometimes combined with cascade control
- Idea: Use physical insight or model equations to derive control strategy
- But little theory for when and how to use

- Shinskey (1981): *"There is no need to be limited to single measurable (y) or manipulable variables (u). If a more meaningful variable happens to be a mathematical combination of two or more measurable or manipulable variables, there is no reason why it cannot be used."*

- Transformed input v: Replaces physical input u as manipulated variable for control

Definition of transformed inputs

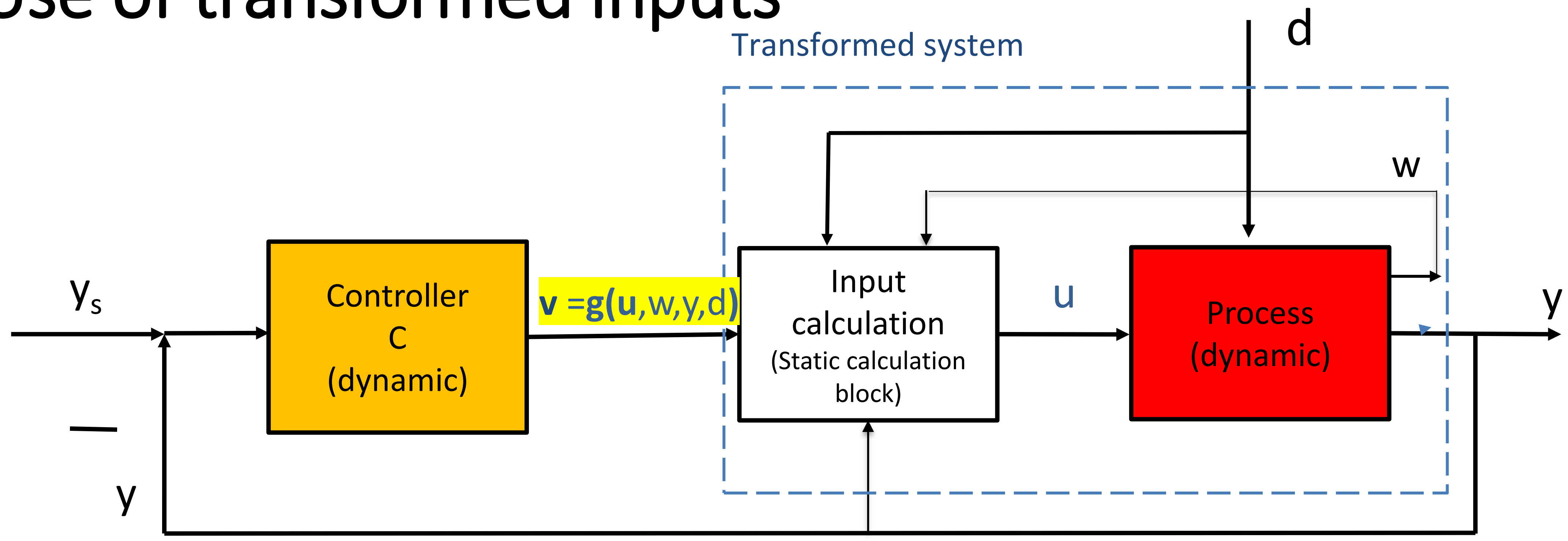


u = physical input
v = transformed input
y = controlled output
w = other measured output (state)
d = disturbance

Transformed input: **v=g(u,w,y,d)** (static function)

- v replaces u as manipulated variable for control (controller output)
- d and w are assumed measured

Use of transformed inputs



Transformed input v =g(u,w,y,d)

- Replaces the physical input u for control of y.
- Aim: **Transformed system** is easier to control
  - May include:
    - Decoupling
    - Linearization
    - Feedforward

Examples

- $v = u/d$
- $v = u_1/u_2$
- $v = u_1 + u_2$
- $v = w(u)$ -> Cascade control

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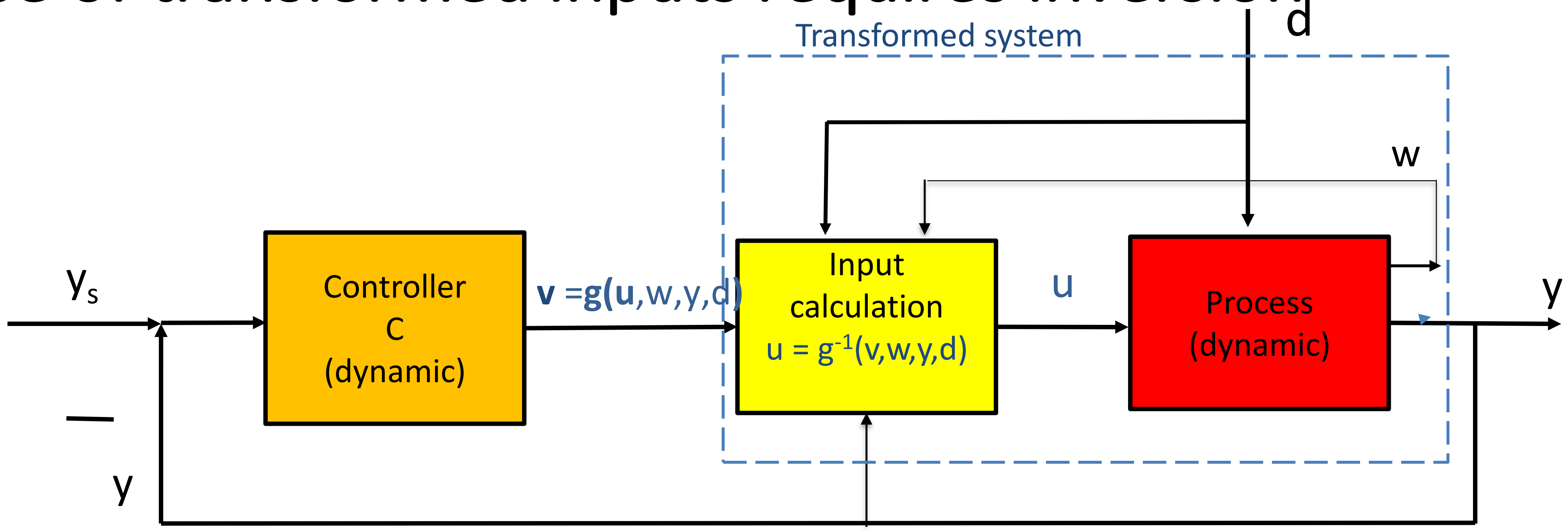
Use of transformed inputs requires inversion



Input calculation block: Need to "invert / reverse" the transformation:

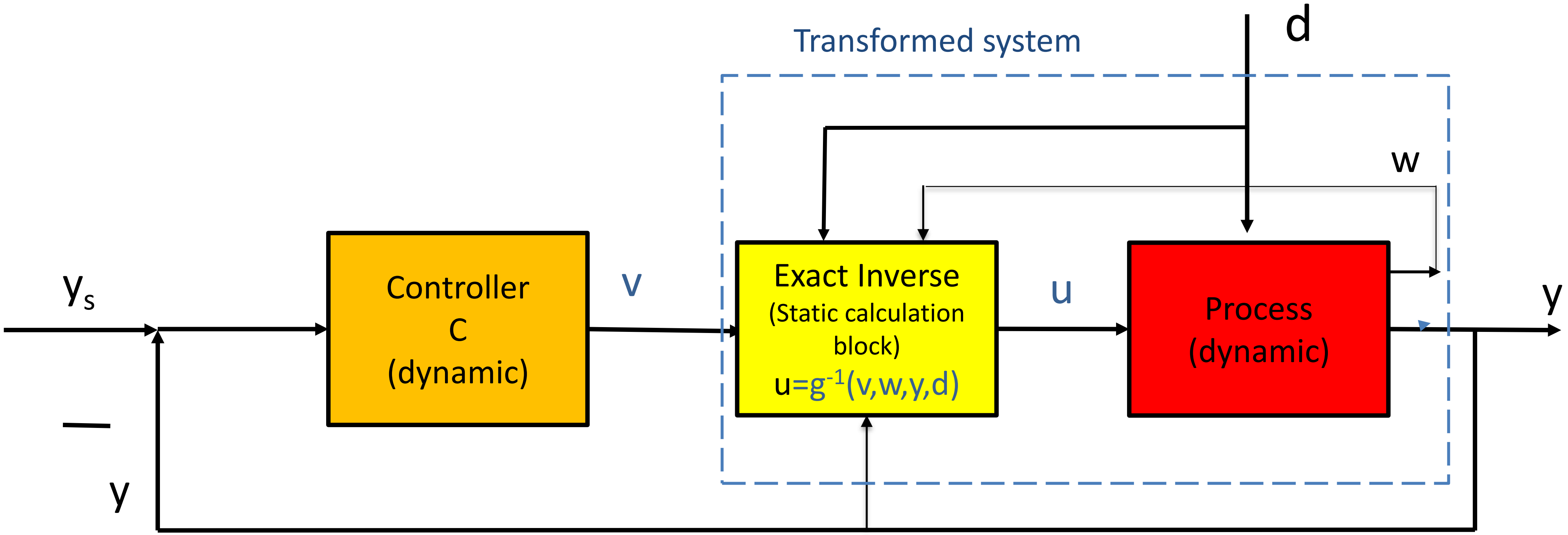
$u = g^{-1}(v,w,y,d)$

- Two main options:

  A: Exact inverse

  B. Approximate inverse by feedback
- Can also use combination (C)

A. EXACT INVERSE

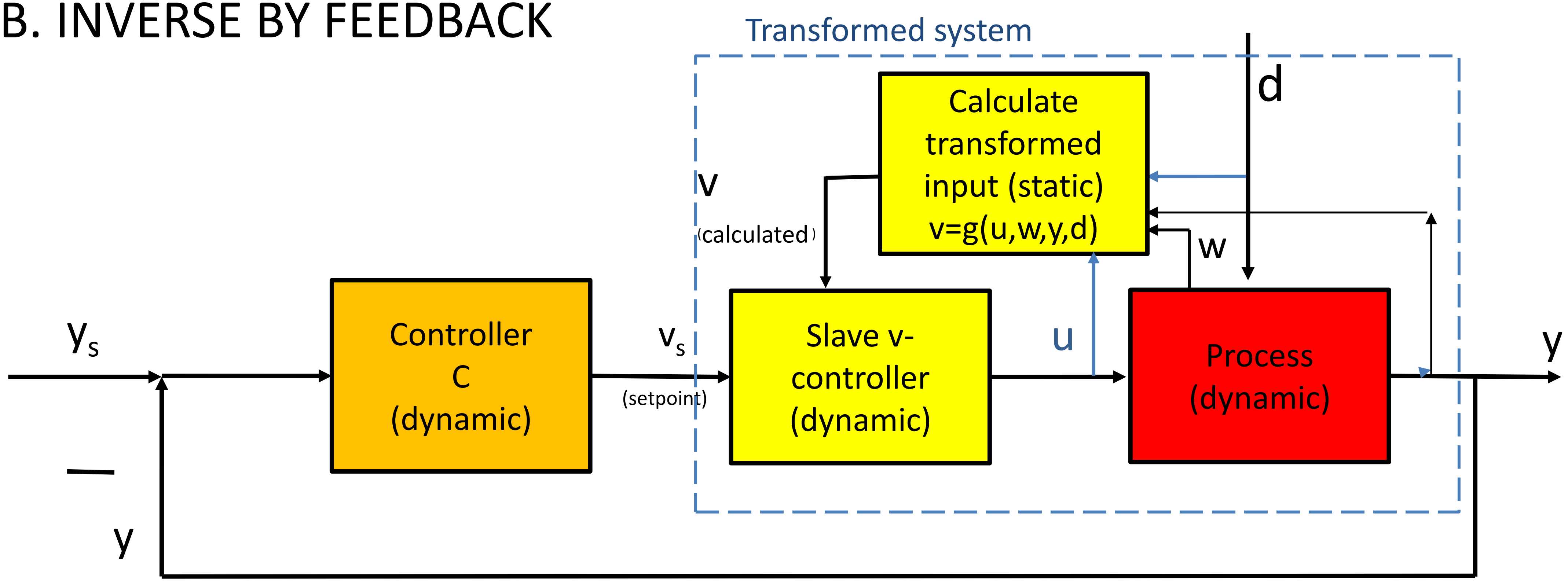


Exact inverse requires that v=g(u,w,y,d) depends explicitly on u

Potential problems:

- Inverse may be complicated, easy to do mistakes
- May get internal instability (because of feedback from w and y)

B. INVERSE BY FEEDBACK



Only option when v=g(w(u),y,d) does not depend explicitly on u.

Example: v = w (flow controller where w=F)

Other advantages:

- Avoid internal instability with exact inverse
- Avoid complicated inverse (and reduce errors!)

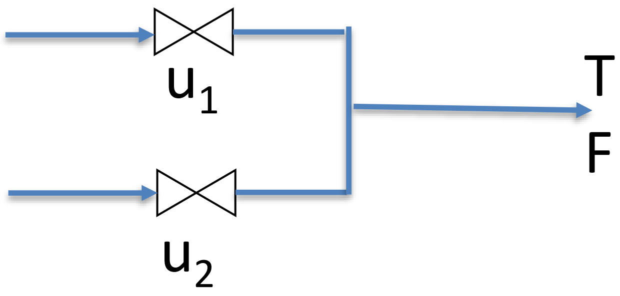
Disadvantages:

- Inverse not perfect dynamically (need fast slave controller)

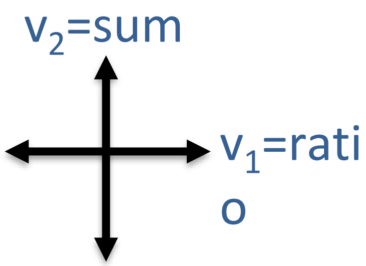
Examples of transformed inputs

- Feedforward ratio control, v = g(u,d)=u/d
- Example 1: Decoupling for mixing process (exact inverse)
- Example 2 (Industrial): Feedforward for control of reactor temperature (inverse by feedback)

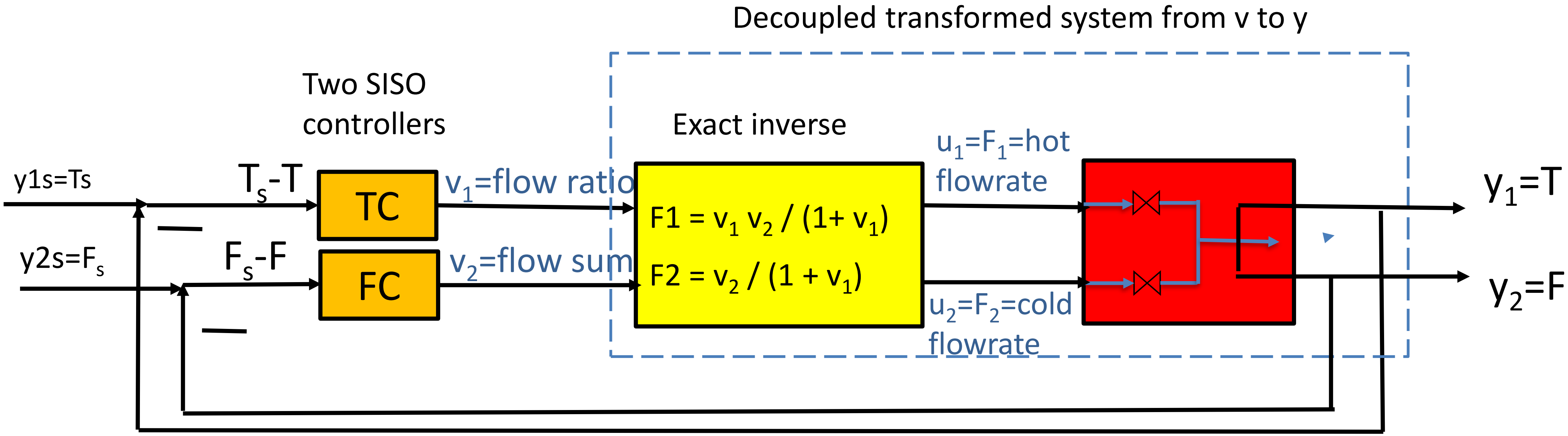
Example 1: Mixing of hot and cold water



- Want to control
  - y_1 = Temperature T
  - y_2 = total flow F
- Inputs (Manipulated variables)
  - $u_1 = F_1$ = hot water flowrate
  - $u_2 = F_2$ = cold water flowrate
- Want to use two SISO PI-controllers (TC, FC)
- But the process is very coupled
- **Get decoupling with flow ratio and flow sum as transformed inputs**
  - TC sets **flow ratio**, $v_1 = u_1/u_2$
  - FC sets **flow sum**, $v_2 = u_1 + u_2$
- Exact inverse («static calculation block»):
  - $u_1 = v_1 \, v_2 / (1+ v_1)$
  - $u_2 = v_2 / (1 + v_1)$



A. EXACT INVERSE



TC = temperature controller,
FC = flow controller
Pairings:

- T – v_1
- F – v_2

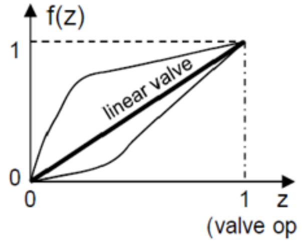
No interactions for setpoint change

BUT: Flows (F_1, F_2) cannot be implemented directly.
The real physical inputs are the valve positions (z_1, z_2)

BUT

- Flows F cannot usually be implemented directly
- Phycial input (u) = valve position (z)
- Valve equation

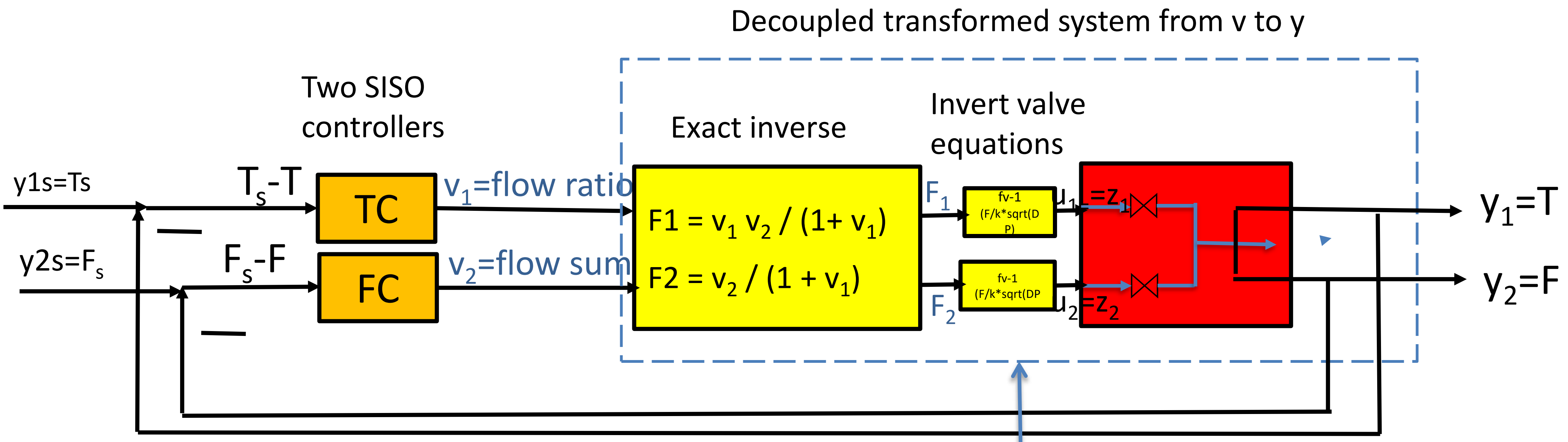
$$F = k f_v(z)\sqrt{\Delta P}$$



- The valve equation needs to be inverted

$$z = f_v^{-1}\left(\frac{F}{k\sqrt{\Delta P}}\right)$$

A. EXACT INVERSE with inversion of valve equation



u1 and u2: physical inputs = valve positions

$$z = f_v^{-1}\left(\frac{F}{k\sqrt{\Delta P}}\right)$$

Assumes:

- Know valve equation and valve parameters
- Measure pressure drop d=ΔP

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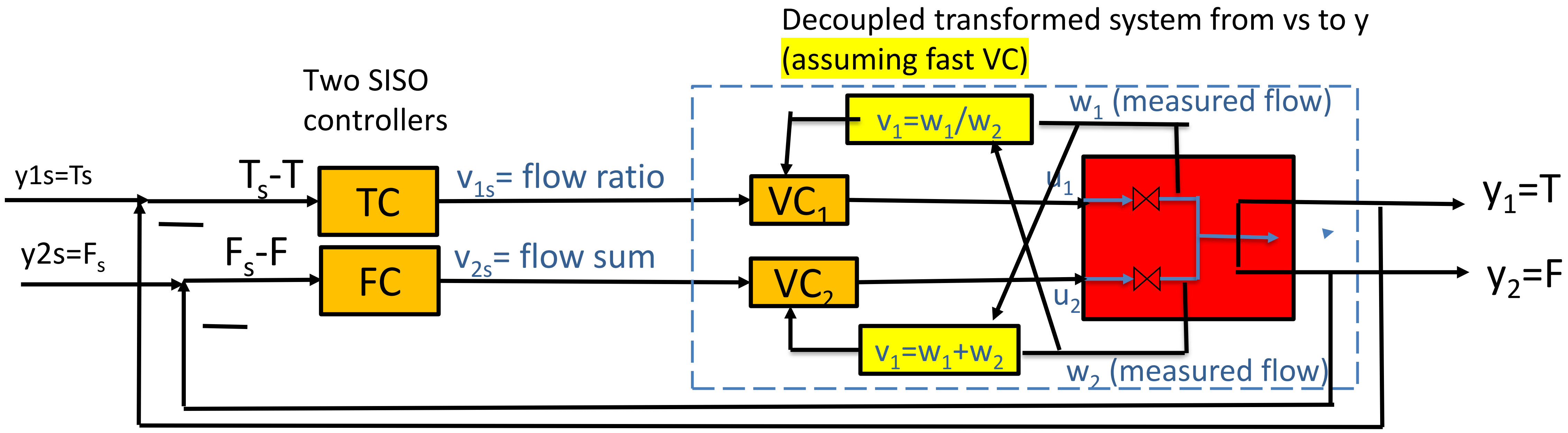
B. ALTERNATIVE APPROACH: Inverse by feedback (v-controllers)



VC = v-controller

Requires: Measurement of individual flows ($w=F_1$, F_2)

Problem: Strong coupling between v-controllers (VC1 and VC2)

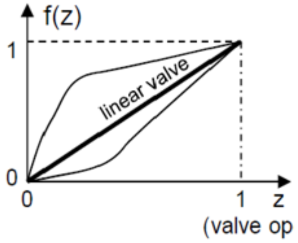
- May take some iterations (time) to converge to the correct inverse

Any better solution?

REACALL

- Flows F cannot usually be implemented directly
- Phycial input (u) = valve position (z)
- Valve equation

$$F = k f_v(z)\sqrt{\Delta P}$$



- The valve equation needs to be inverted
- Two solutions
  - Ideal inverse (model-based), $z = f_v^{-1}\left(\frac{F}{k\sqrt{\Delta P}}\right)$
  - By feedback: Slave flow controller (w-controller)

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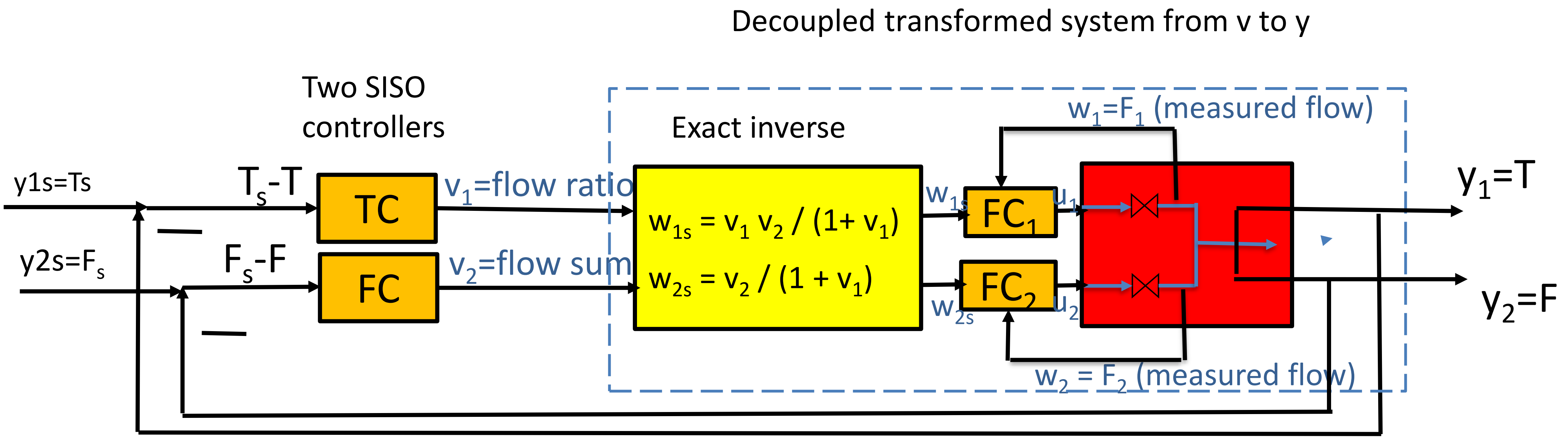
C. COMBINE Exact Inverse with feedback inverse
Use slave flow controllers (w-controllers)

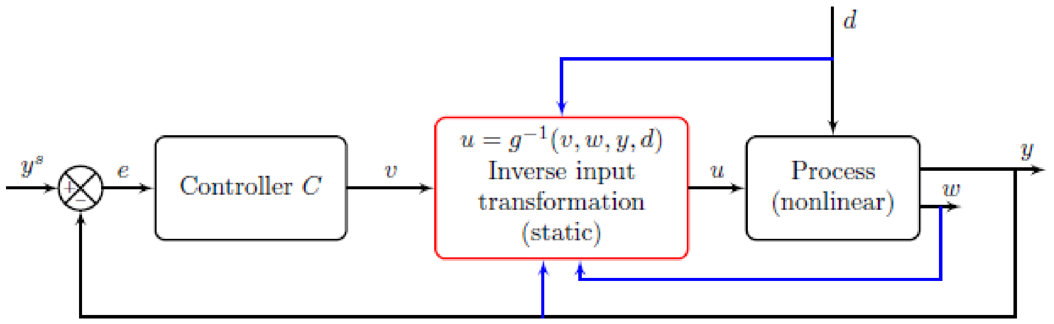


Requires: Measurement of individual flows (w), F_1 and F_2

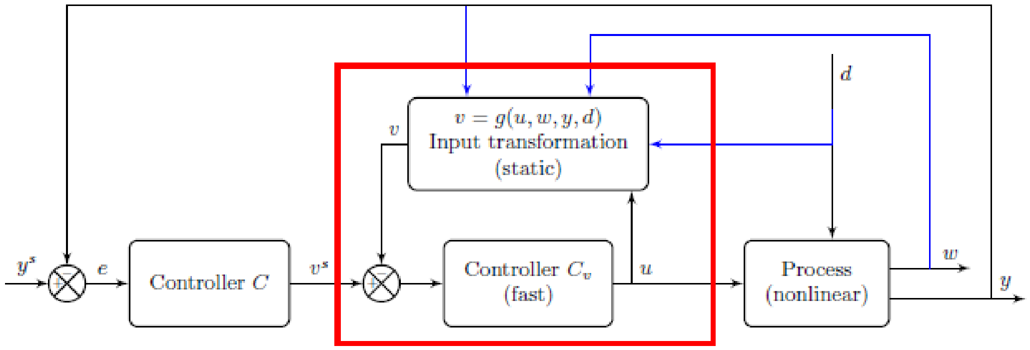
Advantage: No interaction between flow(w)–controllers (FC1 and FC2) .

- Inversion fast if we have fast flow control

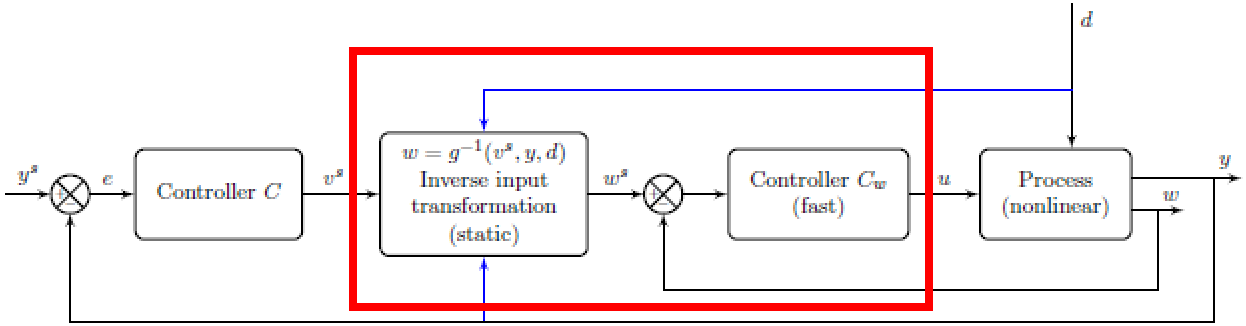
Summary: Alternatives for inverting input transformation



(a) Model-based implementation A of transformed input $v = g(u, w, y, d)$. The



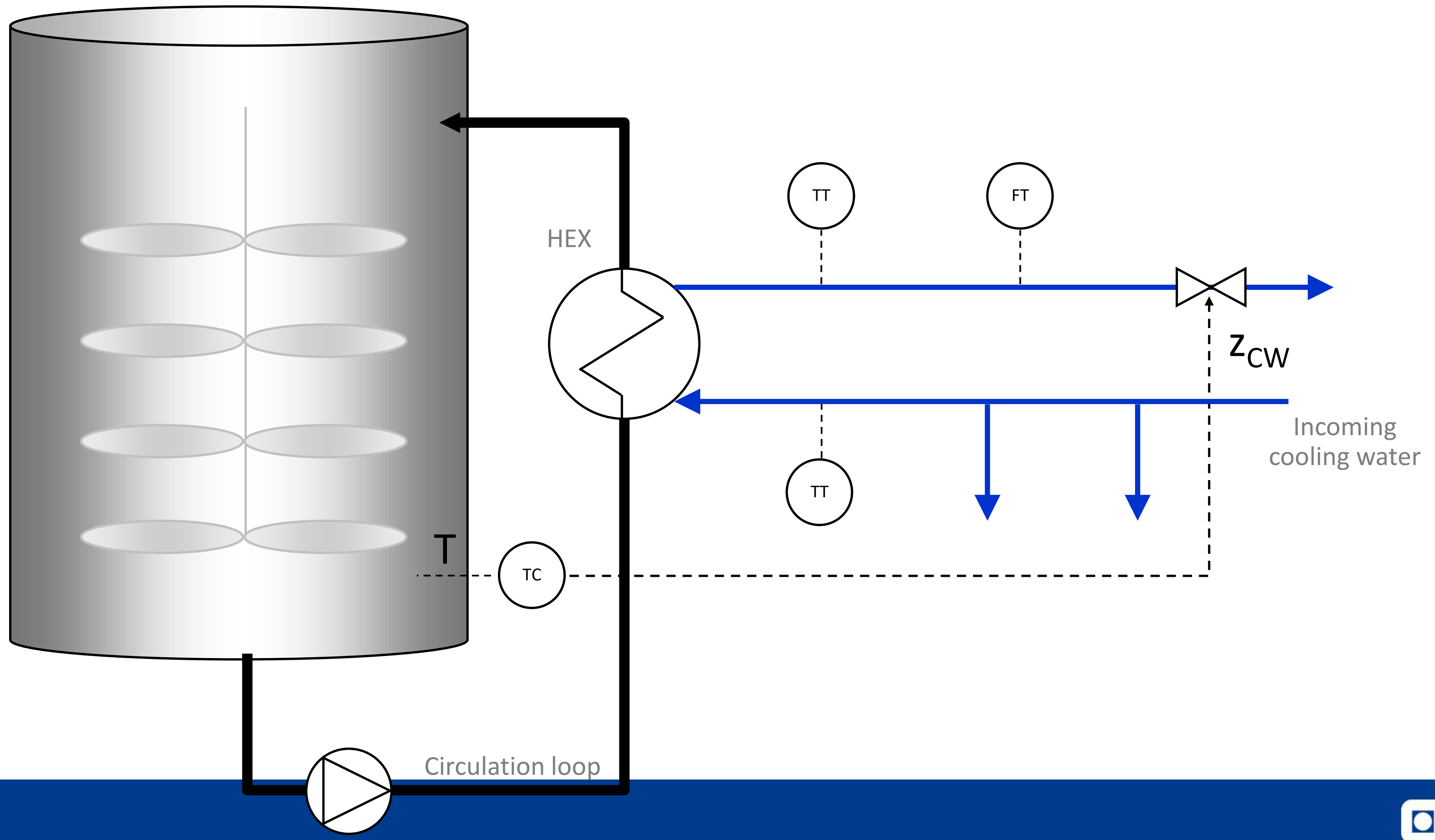
(b) Feedback implementation B of transformed input $v = g(u, w, y, d)$ using



(c) Combined model-based and feedback implementation C of transformed input

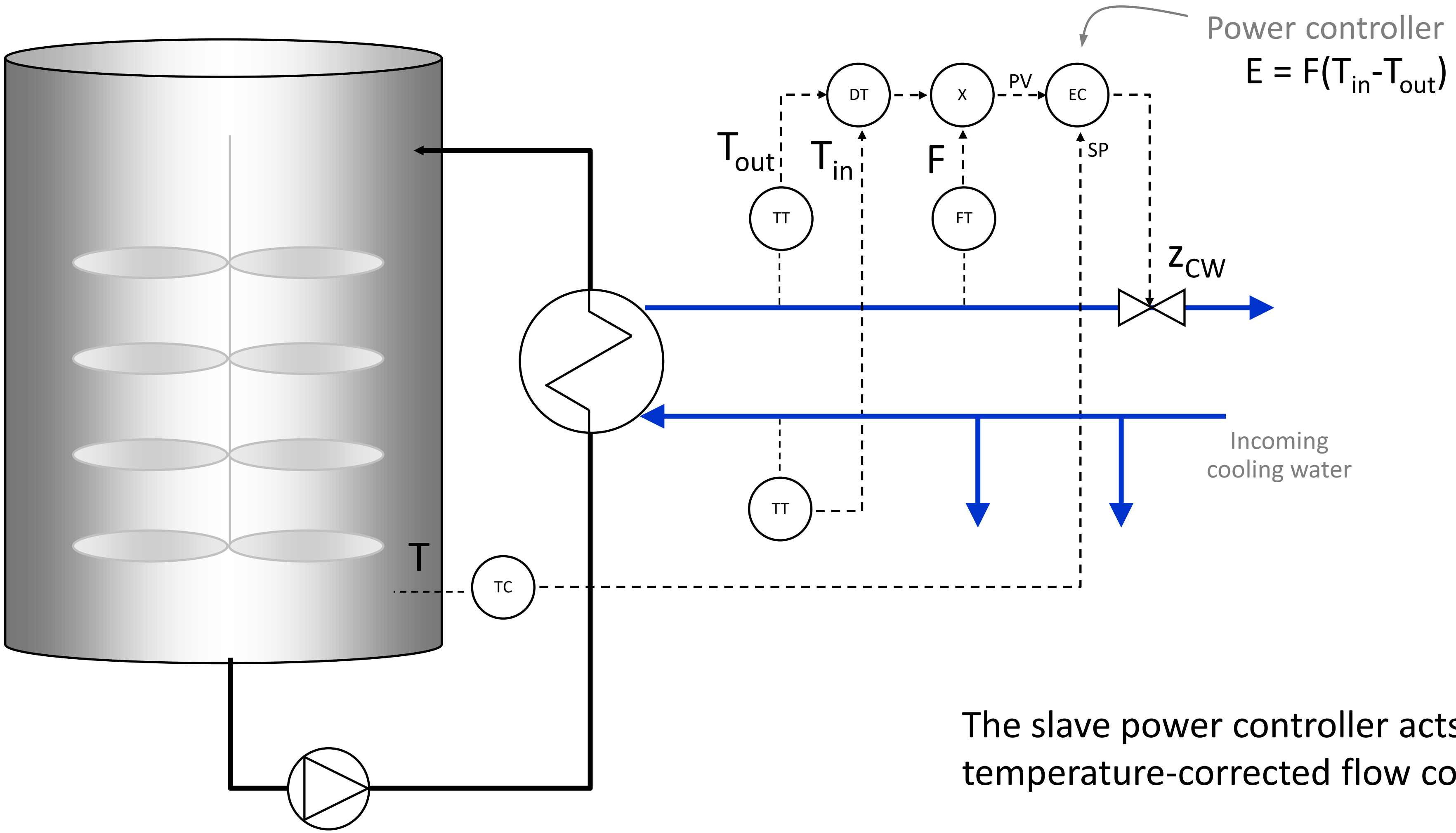
Example 2: Reactor temperature control (batch process)

The reactor solution is circulated through a heat exchanger (cooler).
The reaction is very exothermic: it is important to control the temperature.
Typical variations/disturbances: Cooling water header pressure, CW temperature



$y = T$
$u = z_{CW}$

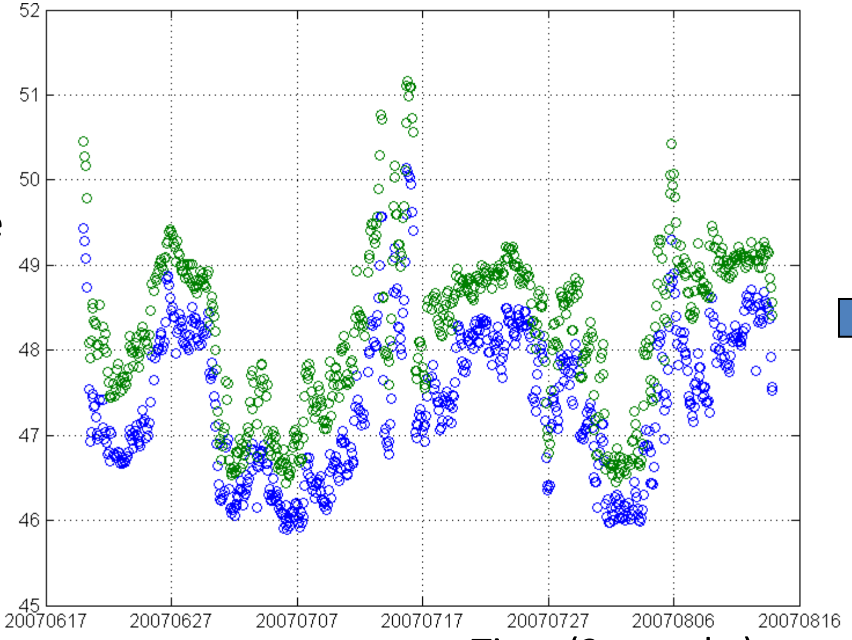
New control structure: Power (E) control



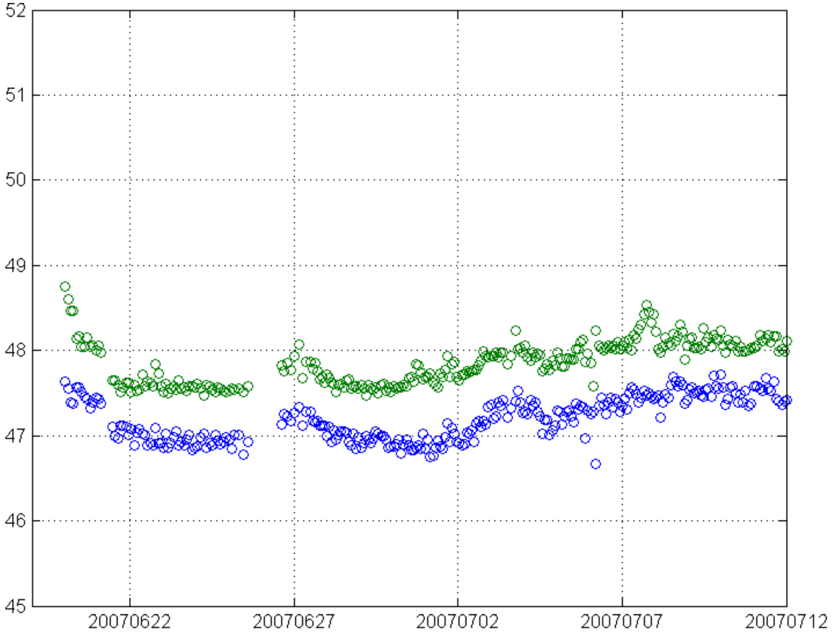
The slave power controller acts as a temperature-corrected flow controller

HEX power control reduces variations between batches

Temperature



Time (2 months)

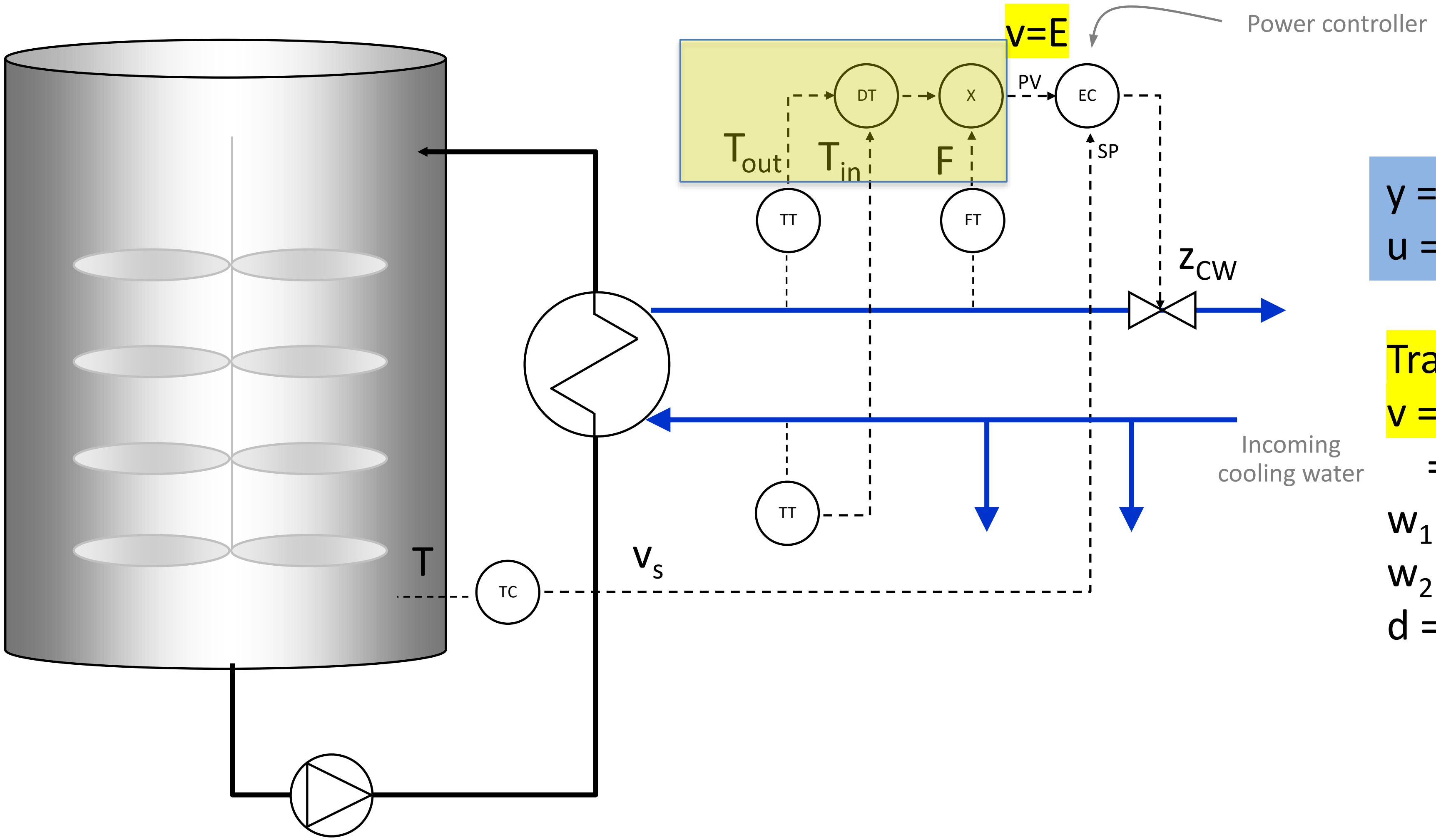


Before

With slave power controller

Thanks to: Krister Forsman

New control structure: Power (E) control



$y = T$

$u = z_{CW}$

Transformed input

$v = g(w,d) = E$

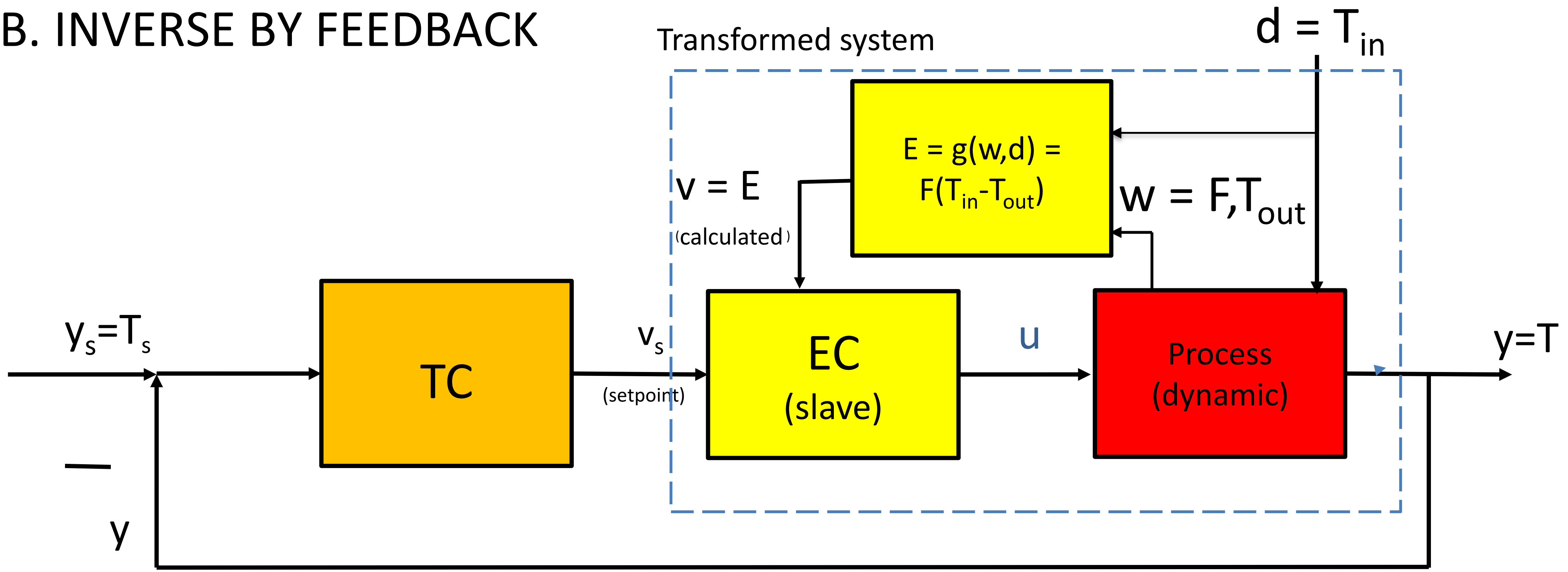
$= F(T_{in}-T_{out})$

$w_1 = F$

$w_2 = T_{out}$

$d = T_{in}$

B. INVERSE BY FEEDBACK



Must generate inverse by feedback (slave v-controller EC) since v=E does not depend explicitly on $u=z_{CW}$

Looks good..... Works in practice...But is there any theory?

- Not too much

- Question 1: How to derive input transformations in a systematic matter?

- Question 2: Properties of transformed systems. Stability?
  - Potential internal instability if transformed variable v depends on outputs (w)

Q1. Systematic derivation of input transformations

- From static model
- From dynamic model

Ideal transformed input v_0 from static model

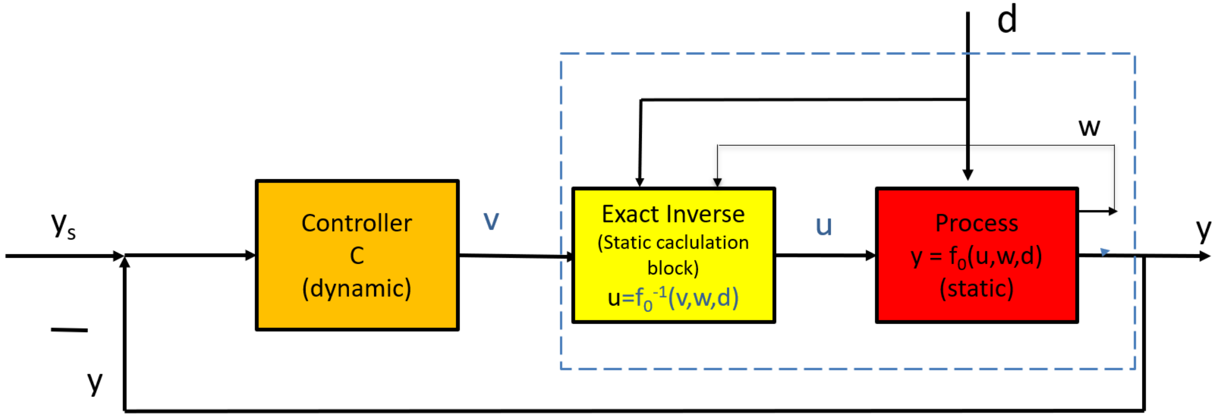
- Write nonlinear process model on form

  $y = f_0(u, w, d)$
- Introduce transformed inputs as RHS

  $v_0 = f_0(u, w, d)$ **(*)**
- Exact inverse: u is solution to (*) for given v_0:

  $u = f_0^{-1}(v_0, w, d)$
- Resulting transformed system (at steady state)

  $y = v_0$
- Decoupled, linear and independent of disturbances



Amazingly simple!
But it works!!!

- **Assumptions**
  - Know model and measure all disturbances (d)
  - The solution to the static inverse problem exists and satisfies certain properties.
- **If f_0 (and v_0) does not depend explicitly on u: Use feedback to generate approximate inverse**

Ideal transformed input v_A from dynamic model

- Nonlinear dynamic process model on form*

$$\frac{dy}{dt} = f(u, w, y, d)$$

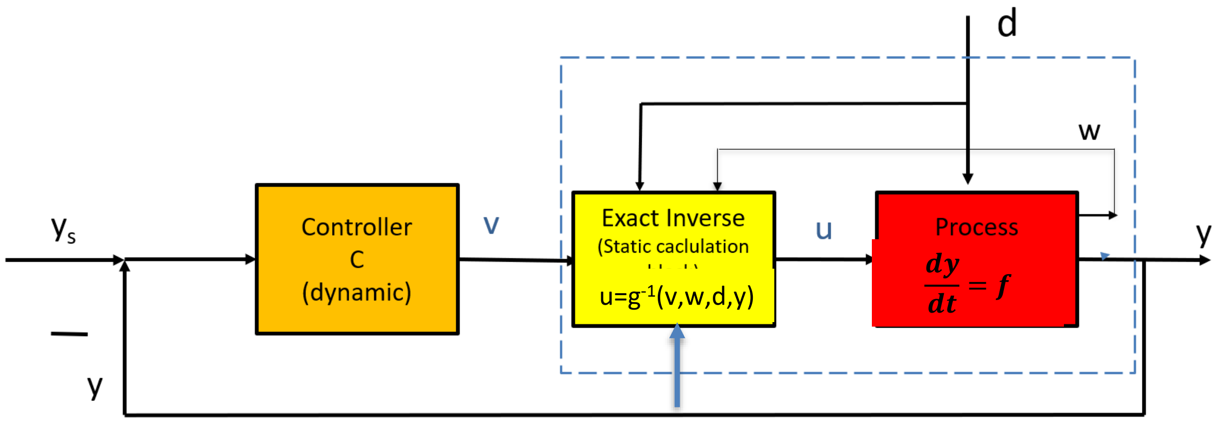
- Introduce transformed inputs from RHS

$$vA = B^{-1}[f(u, w, y, d) - Ay\,] \quad (*)$$

- Tuning parameters (usually diagonal matrices):
- Exact inverse: u is solution to (*) for given v_A
- Resulting transformed dynamic system

$$\frac{dy}{dt} = Ay + Bv_A$$

  - linear, decoupled (with A and B diagonal) and independent of disturbances!
- **Assumptions**
  - Know model and measure all disturbances (d)
  - The solution to the static inverse problem exists and satisfies certain properties
- **If f (and v_A) does not depend explicitly on u: Use feedback to generate approximate inverse**



Also simple!
And it works!

*This model looks fairly general, but it's not. For scalar case, we get first-order system.
However, we may handle higher-order systems by use of measured w-variables.

Ideal transformed input v_A from dynamic model

- Nonlinear dynamic process model on form*

$$\frac{dy}{dt} = f(u, w, y, d)$$

- Introduce transformed inputs from RHS

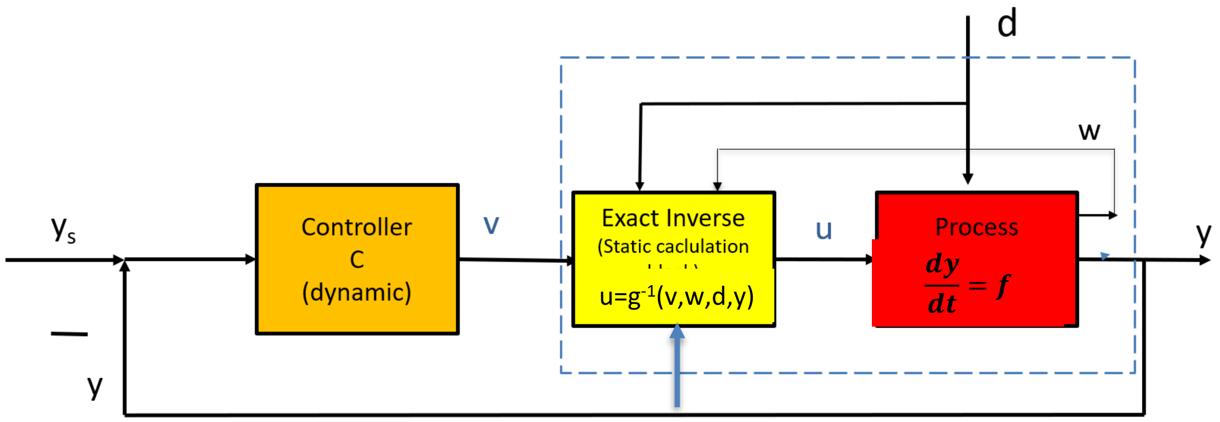
$$vA = B^{-1}[f(u, w, y, d) - Ay\,] \quad (*)$$

- Tuning parameters (usually diagonal matrices):
- Exact inverse: u is solution to (*) for given v_A
- Resulting transformed dynamic system

$$\frac{dy}{dt} = Ay + Bv_A$$

- Choices for B

1. B=I, $\rightarrow$ $\frac{dy}{dt} = Ay + v$
2. B=−A, $\rightarrow$ $\frac{dy}{dt} = A(y - v)$ (y=v at steady-state, which is nice!)



Also simple!
And it works!

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Similar to: Feedback linearization for system of relative order = 1 (Isidori)

- Nonlinear dynamic system (process)

$$\frac{dy}{dt} = f(u, y, d) = f_1(y, d) + f_2(y, d)u$$

- Introduce transformed inputs

$$v = f(u, y, d)$$

- New transformed system is linear, integrating, decoupled and independent of disturbances:

$$\frac{dy}{dt} = v$$

- Corresponds to B=I and **A=0**

Why is A=0 a poor choice?

- Feedback linearization: Transformed linear system is integrating:

$$\frac{dy}{dt} = v$$

- A=0: Transforms stable process into integrator (positive feedback from y)
  - Transformed system cannot be operated alone
  - Unknown disturbances will integrate.
  - Industrial experience: Bad!
- Imagine that we want fast control of a process which is already fast.
  - First make slow (integrating) by using A=0 (positive feedback)
  - Then make fast again using controller C (negative feedback)
    - Does not make much sense!
    - Also: Integrating systems are not easy to control using C
- Fortunately, it is not necessary to make choice A=0 in feedback linearization
    - Theory still holds
    - A=0 was chosen as an example for simplicity (Isidori)
- Feedback linearization theory applies to input transformations

Nonlinear Decoupling via Feedback: A Differential Geometric Approach

ALBERTO ISIDORI, MEMBER, IEEE, ARTHUR J. KRENER, MEMBER, IEEE, CLAUDIO GORI-GIORGI, AND SALVATORE MONACO

Abstract—**The paper deals with the nonlinear decoupling and noninteracting control problems. A complete solution to those problems is made possible via a suitable nonlinear generalization of several powerful geometric concepts already introduced in studying linear multivariable control systems. The paper also includes algorithms concerned with the actual construction of the appropriate control laws.**

Manuscript received June 25, 1979; revised October 19, 1979 and August 25, 1980. Paper recommended by M. Vidyasagar, Past Chairman of the Stability, Nonlinear, and Distributed Systems Committee. This work was supported by the University of Rome under Grant 7.8*P/1978 and by the National Research Council of Italy.

A. Isidori, C. Gori-Giorgi, and S. Monaco are with the Istituto di Automatica, University of Rome, Rome, Italy.

A. J. Krener is with the Department of Mathematics, University of California, Davis, CA 95616.

I. Introduction

CONSIDER a nonlinear system of the form

$$\dot{x}=f(x)+g(x)u \tag{1.1a}$$

$$x(0)=x^0 \tag{1.1b}$$

$$y=h(x) \tag{1.1c}$$

where the input u, the output y, and the state x are l, m, and n dimensional, f and h are vector-valued differentiable functions, and g is a matrix-valued differentiable function, all of the appropriate dimensions. We shall be more precise later on. The input–output behavior of such a system can

0018-9286/81/0400-0331$00.75

From: Alberto Isidori <albisidori@diag.uniroma1.it>
Sent: Sunday, October 4, 2020 5:24 PM
To: Sigurd Skogestad <sigurd.skogestad@ntnu.no>
Subject: Re: Feedback linearization generalization

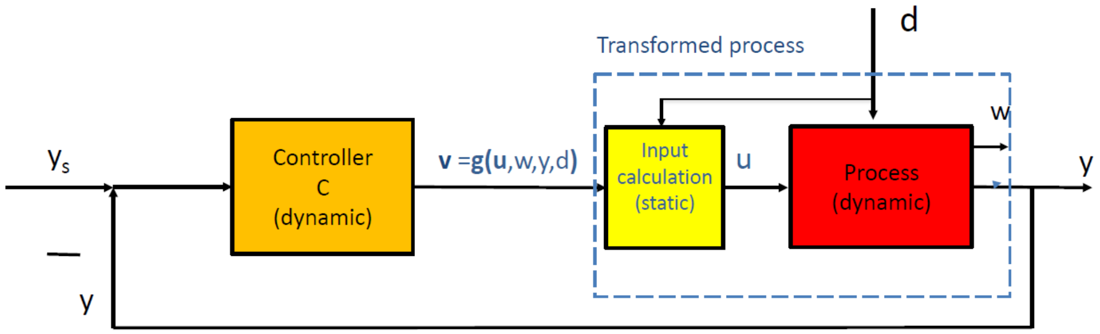
Dear Sigurd
It is nice to hear from you….
Let's move to your questions. I believe that an answer could be as follows.
In feedback linearization, one picks A=0 just as an example. …. The equation
f(y,u) = Ay - v
must be solvable for u. This entails, in the higher-dimensional case, a definition of "relative degree" ….

Best regards
Alberto

Choice of Tuning parameter A

- One idea: Select $A = \frac{df}{dy}$ at nominal operating point
  - Then: No feedback from y into transformation (nominally)
  - Transformed system has the same dynamics as the original system (nominally)
- To get decoupling may choose: $A = diag(\frac{df}{dy})$
  - Will get some feedback from y also nominally.
- May want to «speed up» the response of the transformed systems by selecting a larger A.
  - This involves negative feedback from y, and may as usual give robustness problems if time delay for y
  - «Slowing down» the response (positive feedback from y) does not have robustness problems

Commoin choice: B = -A (gives y=v at steady-state)



- $\frac{dy}{dt} = f(u, w, y, d)$
- Define **T_A = -A^{-1}**
- Get: $v_A = TA\, f(u, w, y, d) + y$

- Get transformed system*: $T_A \frac{dy}{dt} = -y + vA$
- Transformed system has linear «setpoint» response (from v_A to y) with
  - time constant T_A
  - steady-state gain I
- Looks great! May in theory avoid the outer feedback controller C
  - But note that the transformation works by feedforward action
- **The outer controller C is needed to correct for model errors and unknown disturbances**

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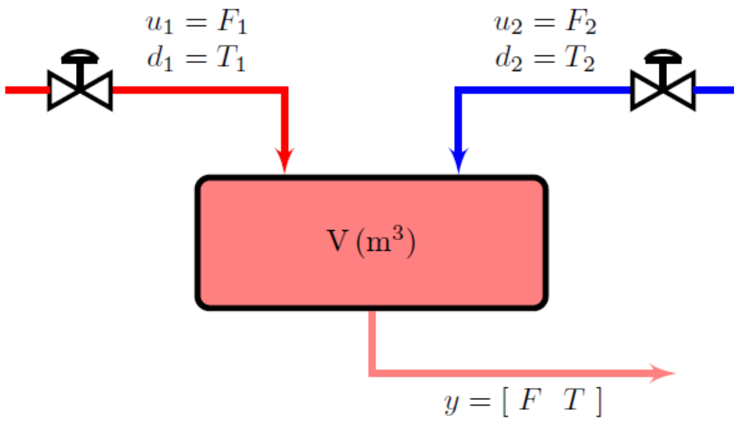
Examples («magic»)

- Example 1 (revisit): Mixing with one static and one dynamic equation
- Example 2 (revisit): Reactor temperature control (dynamic)
- Example 3: heated tank
- Example 4: Level control
- Example 5: Heat exchanger (static applied to dynamic system)

Cascade of transformations:

- Example 6: CSTR (with exact inverse using w)

Example 1. Mix hot (1) and cold (2) water (shower), y=[F T]



Mass balance: $q = F_1 + F_2$ (static equation for y_1=F)

$v_0 = F_1 + F_2$

Energy balance: $\frac{dT}{dt} = \frac{F_1}{V}(T_1 - T) + \frac{F}{V}(T_2 - T)$ (dynamic equation for y_2=T)

$v_A = \frac{F_1}{V}(T_1 - T) + \frac{F_2}{V}(T_2 - T) - AT$ (choosing B=I)

New transformed inputs: v_0 and v_A

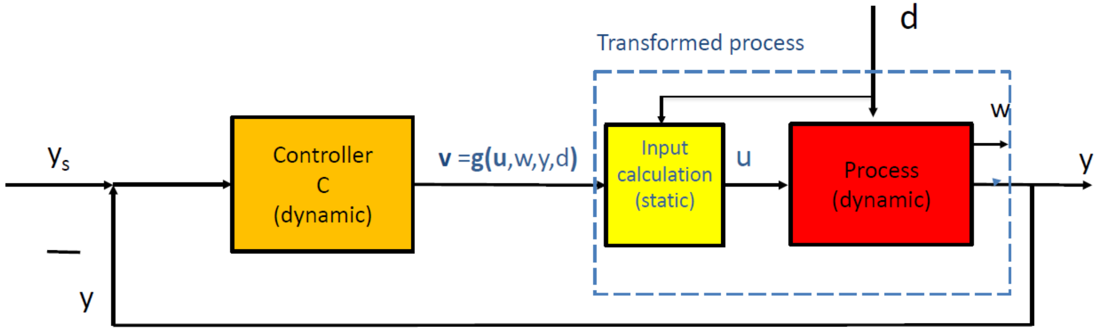
- v_0 = sum of flows (as before)
- v_A: not ratio (but would be similar to ratio if we used static energy balance)

Exact inverse transformation (with u_1=F_1 and u_2=F_2):

$$F_1 = \frac{V\left(v_A + AT\right) + v_0\left(T - T_2\right)}{T_1 - T_2}$$

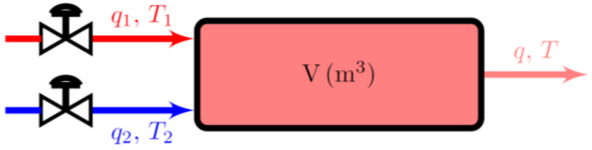
$$F_2 = v_0 - F_1$$

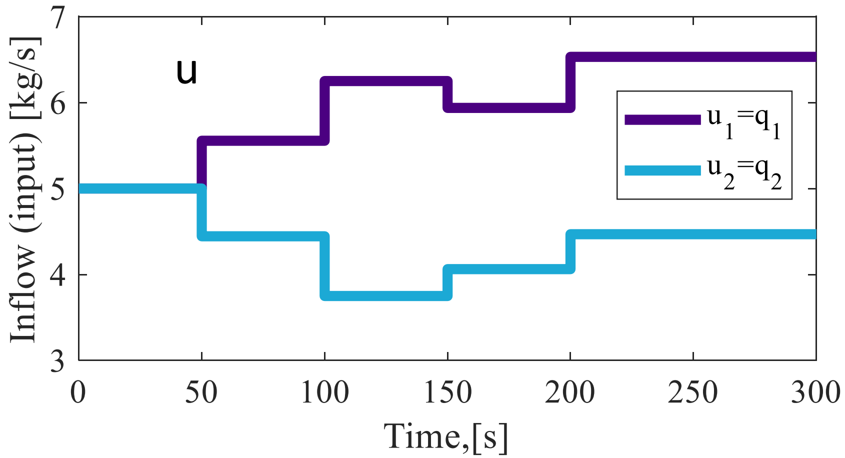
Tuning parameter, A = -(q/V)* (nominal)

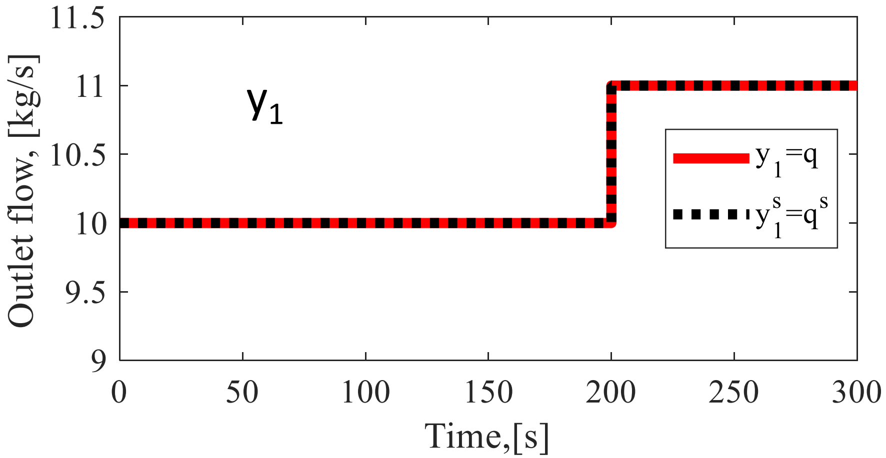


Example 1. Simulation responses with transformation only.

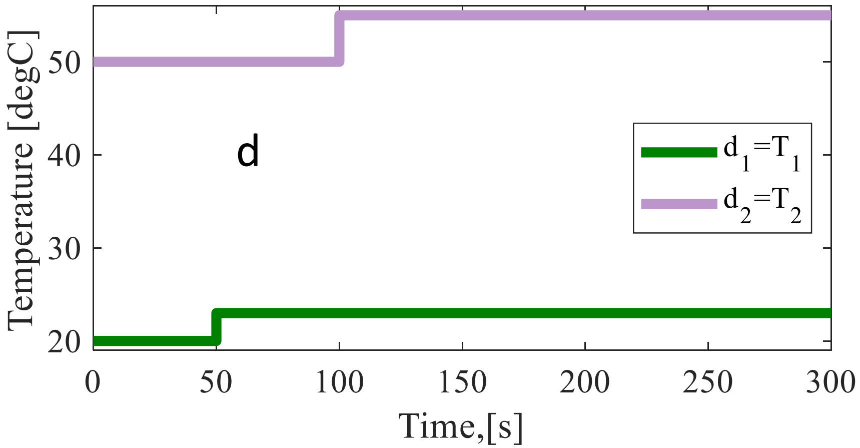
-> Perfect disturbance rejection and decoupling (as expected)

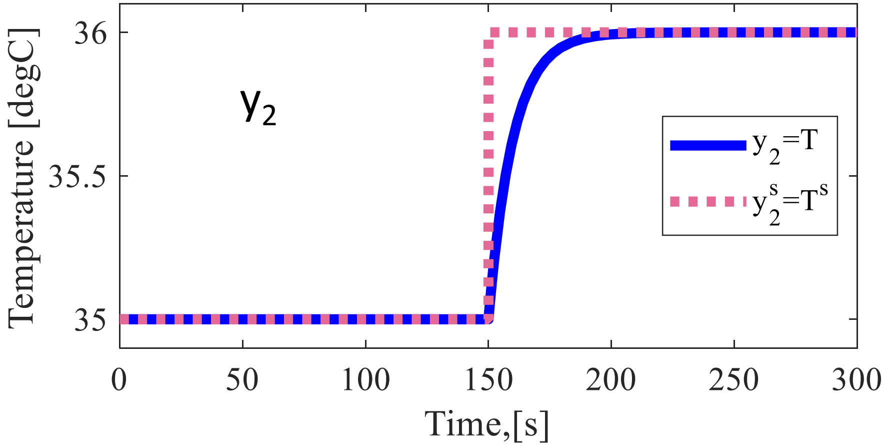






1. $d_1=T_1$: 20 ->22 °C at t = 50 s
2. $d_2=T_2$: 50 ->55 °C at t = 100 s
3. $y_{2s}=T_s$: 35 ->36 °C at t = 150 s
4. $y_{1s}=q_s$: 10 ->11 kg/s at t = 200 s





In practice

- May not measure all disturbances
- Transformation will not longer be «perfect» but still useful

Example 2. Control of reactor temperature,

Energy balance tank:

$$m_1 c_{p1} dT/dt = Q + Q_{rx}$$

Static energy balance for cold side:

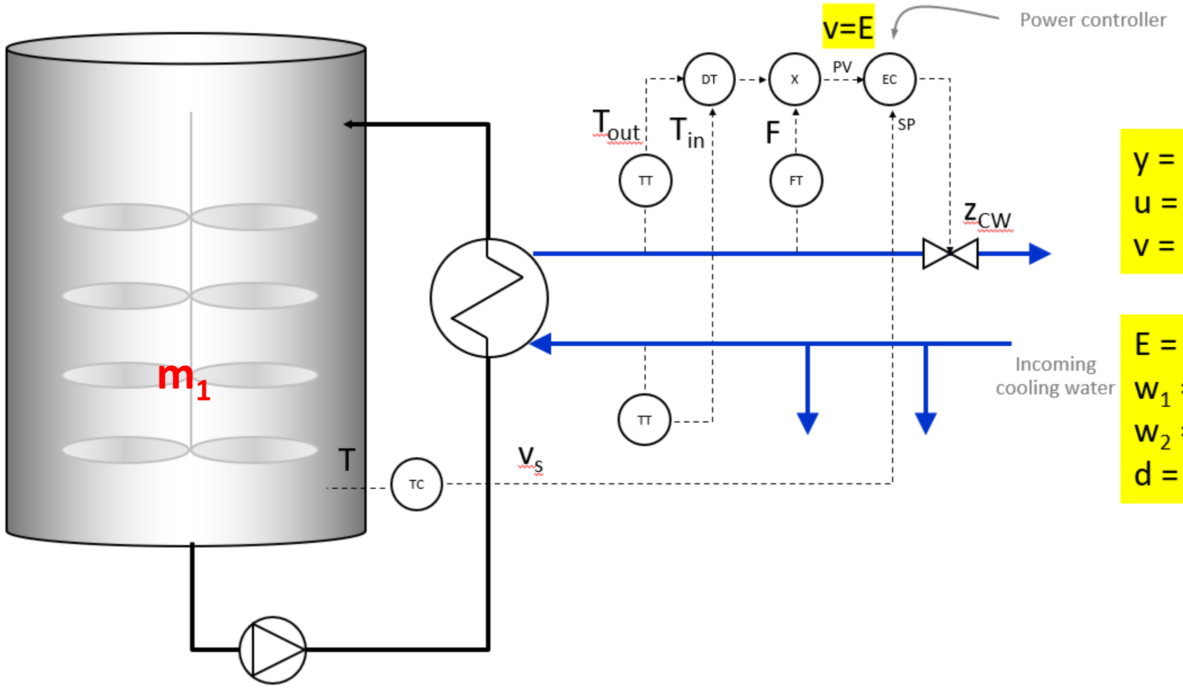
$$Q = F c_p (T_{out} - T_{in}) = E c_p$$

Neglecting heat of reaction Q_{rx}, we get

$$dT/dt = k E, \quad k = c_p/(c_{p1} m_1)$$

Transformed input with systematic approach

$$v = kE - AT$$



$y = T$
$u = z_{CW}$
$v = E$

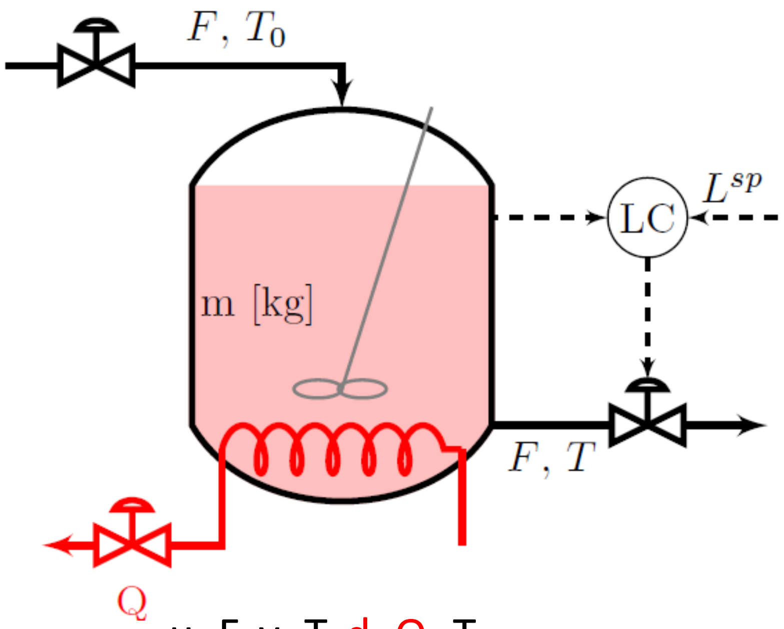
$E = g(w,d) = F(T_{in}-T_{out})$
$w_1 = F$
$w_2 = T_{out}$
$d = T_{in}$

Note: choice $v=E$ corresponds to $A=0$

- Some self-regulation removed by EC
- Maybe not so bad for this process which anyway needs to be stabilized (because of Q_{rx})

$Q_{rx}(T)$ is mainly a function of T. Can be handled by design of TC

Example 3. Heated tank



u=F, y=T, d=Q, T_0

Energy balance:

Static model: $$T = f_0(u,d) = T_0 + \frac{Q}{Fc_P}$$

Dynamic model: $$\frac{dT}{dt} = f(u,y,d) = \frac{1}{mc_P}(Fc_P(T_0 - T) + Q)$$

Comparison of static and dynamic case

Transformed inputs (with B=-A for dynamic case)

$$v_0 = f_0(u,d) = T_0 + \frac{Q}{Fc_P}$$

$$v_A = -A^{-1} f(u,y,d) + y = -A^{-1}\left(\frac{F}{m}(T_0 - T) + \frac{Q}{mc_P}\right) + T$$

Ideal inverse of input transformation

$$u = F = \frac{Q}{c_p(v_0 - T_0)}$$

$$u = F = \frac{Q + Amc_P(v_A - y)}{\rho c_p(y - T_0)}$$

Resulting transformed system (from v to y=T)

$$\frac{dT}{dt} = \frac{F}{m}(v_0 - T)$$

$$\frac{dT}{dt} = -A(v_A - T)$$

Static and Dynamic: Identical with the following choice

$$A = -\left(\frac{\partial f}{\partial T}\right)_* = -\frac{F^*}{m},$$

v_0 simpler: Does not depend on y and no tuning parameter A

Only disadvantage: Transformed system nonlinear (depends on F/m)

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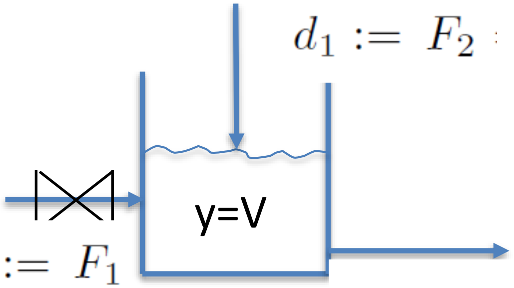
Example 4: Level control

$d_1 := F_2$



$u := F_1$

$F_3 = k\sqrt{k_1 V + p_0}$

$d_2 := p_0 = \text{pressure difference}$

dynamic model

$$\frac{dV}{dt} = F_1 + F_2 - F_3$$

Use scaled variables: $k=k_1=1$

$$\frac{dy}{dt} = f(u,y,d) = u + d_1 - \sqrt{y + d_2}$$

Dynamic case (selecting B=-A)

$$v_A = y - A^{-1} f(u,y,d) = \underbrace{y - A^{-1}(u + d_1 - \sqrt{y + d_2})}_{g(u,y,d)}$$

Ideal inverse: $u = g^{-1}(v_A, y, d) = A(y - v_A) - d_1 + \sqrt{y + d_2}$

Transformed system: $\frac{dy}{dt} = A(y - v_A)$ $\quad A = \left(\frac{\partial f}{\partial y}\right)_* = -\frac{1}{2\sqrt{y^* + d_2^*}}$

Static case

$$y = f_0(u,d) = (u + d_1)^2 - d_2$$

$$v_0 = f_0(u,d) = (u + d_1)^2 - d_2$$

Ideal inverse: $u = f_0^{-1}(v_0, d) = \sqrt{v_0 + d_2} - d_1$

Transformed system when apply v_0 to dynamic model: $\frac{dy}{dt} = \sqrt{v_0 + d_2} - \sqrt{y + d_2}$

Linerarized: $\frac{d\Delta y}{dt} = \frac{1}{2\sqrt{y* + d_2^*}}(\Delta v_0 - \Delta y)$

In practice: v_0 behaves very similar to v_A!

- At steady state $y=v_0$ (perfect control)
- Also perfect control dynamically for disturbances

v_0 simpler: Does not depend on y and no tuning parameter A

Only disadvantage: Transformed system nonlinear

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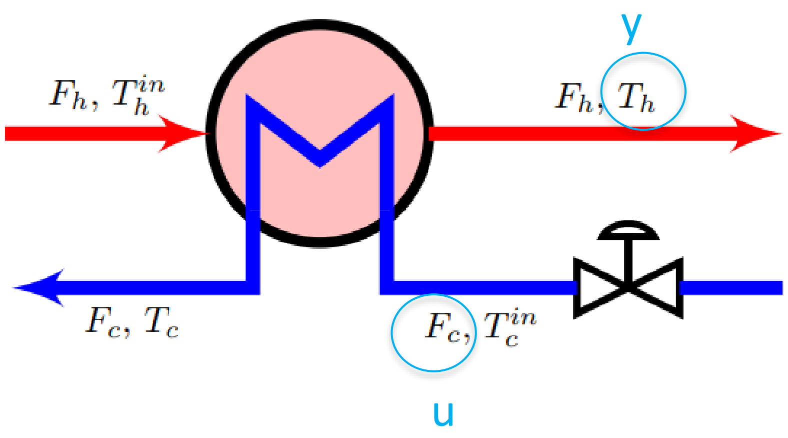
Example 5. Heat exchanger (static)



MVs (original inputs):

$$u = F_c \ [\mathrm{kg/s}]$$

CVs (outputs):

$$y = T_h \ [^\circ\mathrm{C}]$$

DVs (disturbances):

$$d_1 = T_c^{in} \ [^\circ\mathrm{C}]$$
$$d_2 = T_h^{in} \ [^\circ\mathrm{C}]$$
$$d_3 = F_h \ [\mathrm{kg/s}]$$

Energy balance, countercurrent flow, $Q = UA\Delta T_{LM}$

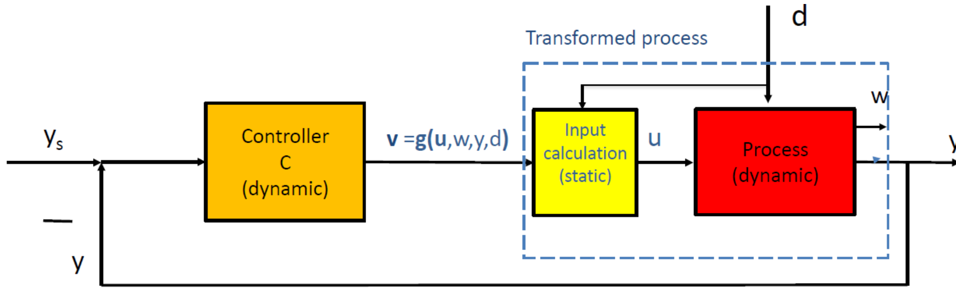
$$T_h = \underbrace{(1-\epsilon_h)T_h^{in} + \epsilon_h T_c^{in}}_{v_0}$$

$$N_{tu} = \frac{UA}{F_c c_{p,c}}$$

$$C = \frac{F_c c_{p,c}}{F_h c_{p,h}}$$

$$\epsilon_c = 1 - \frac{\exp(-N_{tu}(C-1))}{C - \exp(-N_{tu}(C-1))}$$

$$\epsilon_h = \epsilon_c C$$



Input calculation:
Use numerical inverse (to find u for given $T_h = v_0$)

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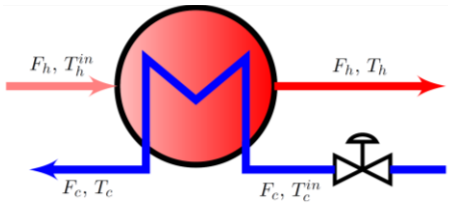
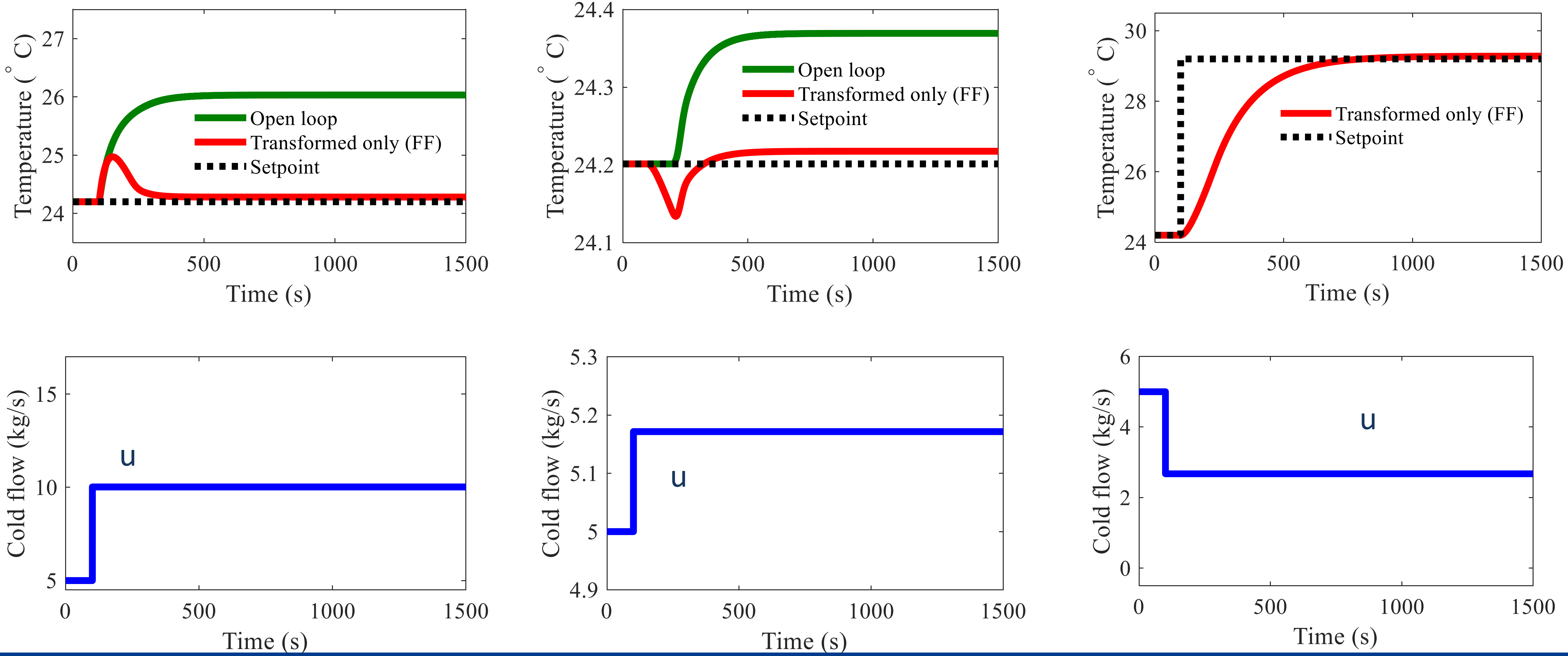
Simulation: Static v_0 with cell dynamic model



$d_1 = \Delta T_c^{in} = +2\ °C$

$d_2 = \Delta T_h^{in} = +2\ °C$

$v_0 = \Delta T_h^s = +5 °C$



Summary so far

- Transformed input v_0 based on static model is usually good enough!
  - Model $y = f_0(u,d,w)$
  - Simple select transformed variable as RHS, $v_0 = f_0(u,d,w)$
    - No tuning, independent of y
- Get transformed system $y=v_0$ at steady state
  - Steady-state: Perfect feedforward control, decoupled, linear
  - Dynamically (for model $dy/dt=f(u,d,w)$): Perfect feedforward control, usually decoupled
- The main advantage with v_A based on dynamic model is that it linearizes the system also dynamically
  - But this is probably not so important in most cases
- If we do not measure all disturbances then transformations will no longer be «perfect» but may still be very useful!

Discussion

Extension: Chain of transformations

- Idea: **Extend exact inverse to systems of higher relative order (when v does not depend explicitly on u)**
- Model for y (static case) or dy/dt (dynamic case)

$$y = f_0(w,d) \quad or \quad \frac{dy}{dt} = f(w,y,d)$$

  - Until now: Cannot use exact inverse
- Alternative 1 (until now): Use feedback control to generate approximate inverse
- Alternative 2 (chain of transformations): Make use of known model for w

$$\frac{dw}{dt} = f_2(u,w,y,d_2)$$

  - Use two exact inverses; find w from f, find u from f_2.
  - May be viewed as alternative to «feedback linearization» for systems with high relative order

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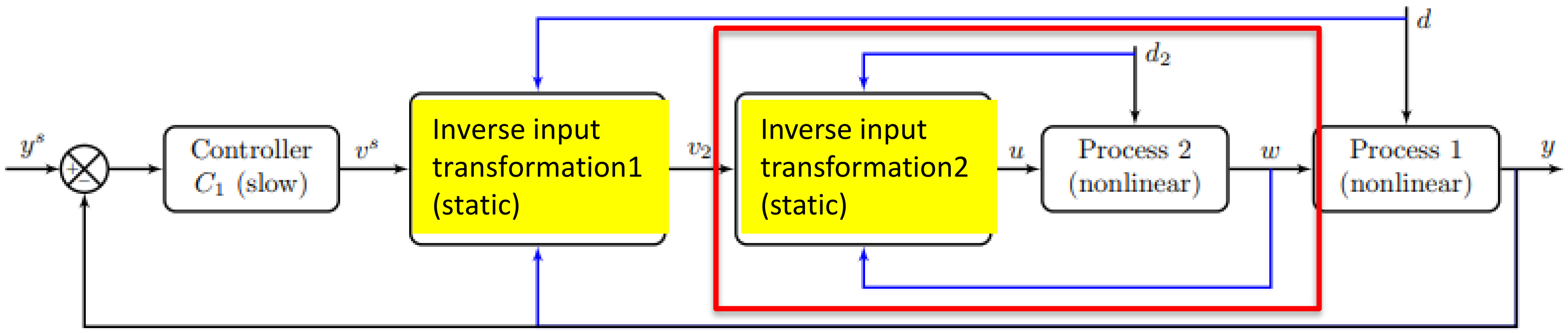
Chain of transformations

$$\boxed{dy/dt = f(w, y, d) \quad = Ay + v}$$ (Inverse 1: Solve for given v to find w^s)

$$\frac{dw}{dt} = \boxed{f_2(u, w, y, d_2) \; = A_2(w - v_2)}$$ (Inverse 2: Solve for given given $w^s = v_2$ to find u)



- Input u has relative order 2 (from u to y)
- Get perfect disturbance rejection for d_2 (enters same place as u)
- But not for d since it must go through subystem 2

$$\boxed{\tau_2 \frac{dw}{dt} = v_2 - w}$$

$\tau_2 = -A_2^{-1}$

$v_2 = w^s$

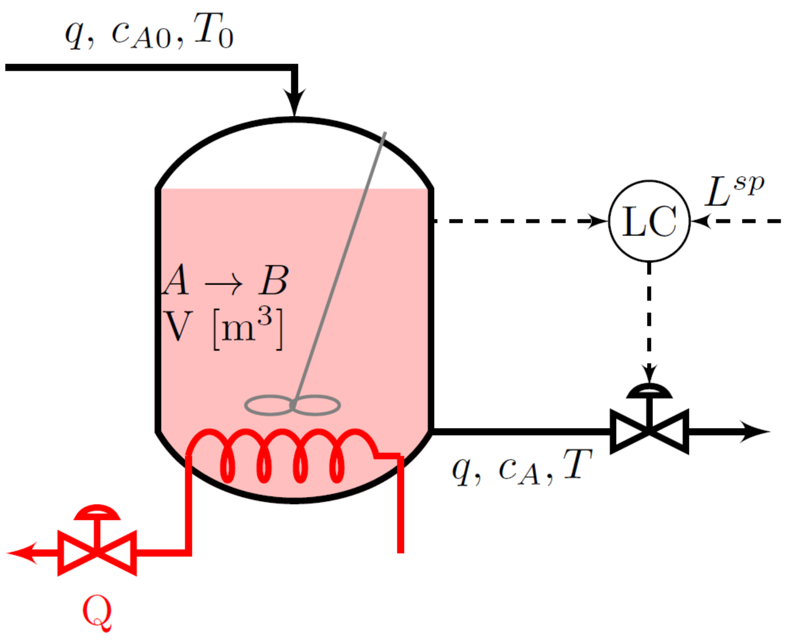
- Note: Choose B=-A in inner transformations to get steady-state gain of I ($v_2 = w^s$)

Example 6. CSTR. $y = c_A$, $u = Q, w = T$

Component balance: $\frac{\mathrm{d}c_A}{\mathrm{d}t} = \frac{\mathrm{q}}{\mathrm{V}}(\mathrm{c}_{A0} - c_A) - k(T)c_A$ $\qquad k = k_0 \exp\left(-\frac{E_A}{R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right)$

$$v_A = \frac{q}{V}(c_{A0} - c_A) - k(T)c_A - Ac_A$$

Energy balance: $\frac{dT}{dt} = \frac{q}{V}(T_0 - T) + \frac{Q}{V\rho c_p} - \frac{\Delta H_{rx} k(T) c_A}{\rho c_p}$



Alternative 1: Cascade control of v_A

Alternative 2: Use also model for w=T (energy balance)

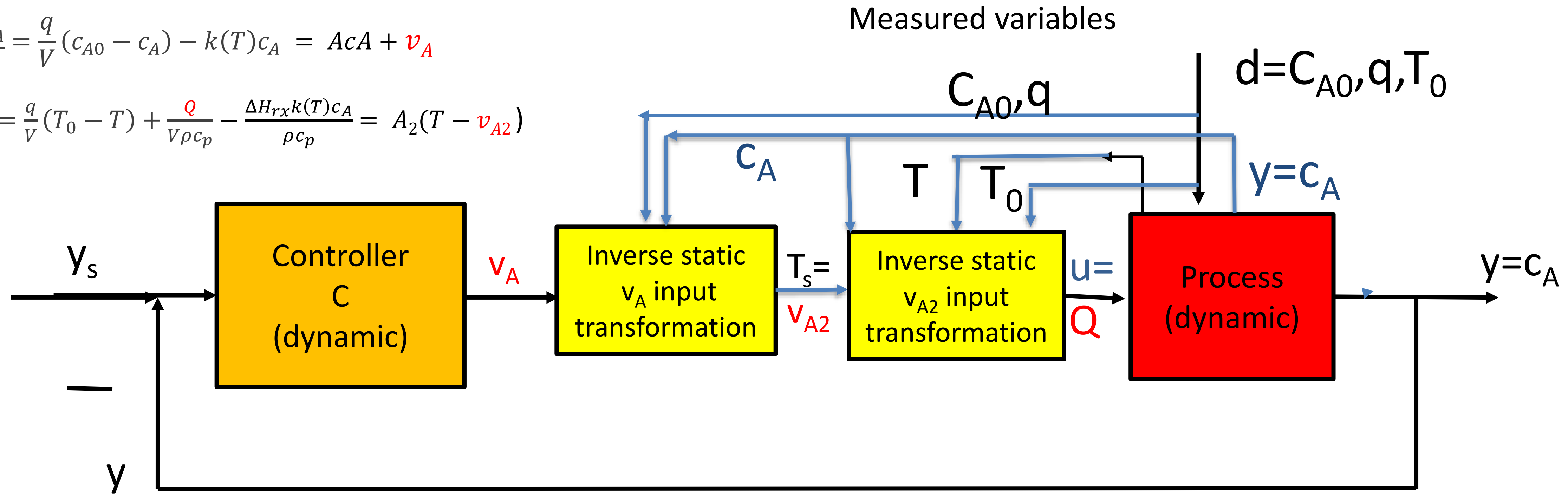
$$\frac{q}{V}(T_0 - T) + \frac{Q}{V\rho c_p} - \frac{\Delta H_{rx} k(T) c_A}{\rho c_p} = A_2\,(T - v_{A2})$$

and use chain of transformations.

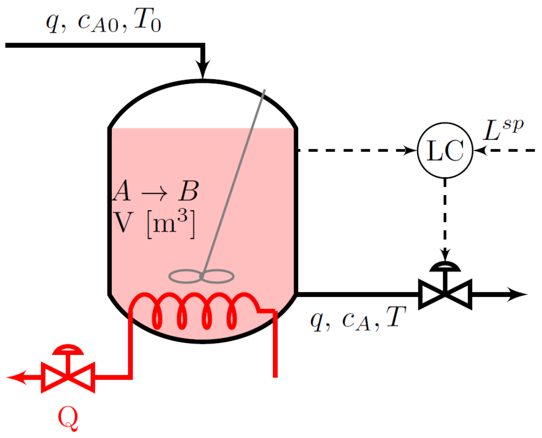
Alt.2 Chain of inverse transformations

$$\frac{dc_A}{dt} = \frac{q}{V}(c_{A0} - c_A) - k(T)c_A \ = \ AcA + v_A$$

$$\frac{dT}{dt} = \frac{q}{V}(T_0 - T) + \frac{Q}{V\rho c_p} - \frac{\Delta H_{rx}k(T)c_A}{\rho c_p} = \ A_2(T - v_{A2})$$



- Make use of model for w=T
- Feedforward control for T_0 will be perfect, but not for C_{A0} and q.
- Need to invert expression for v_A with respect to T -> invert k(T).



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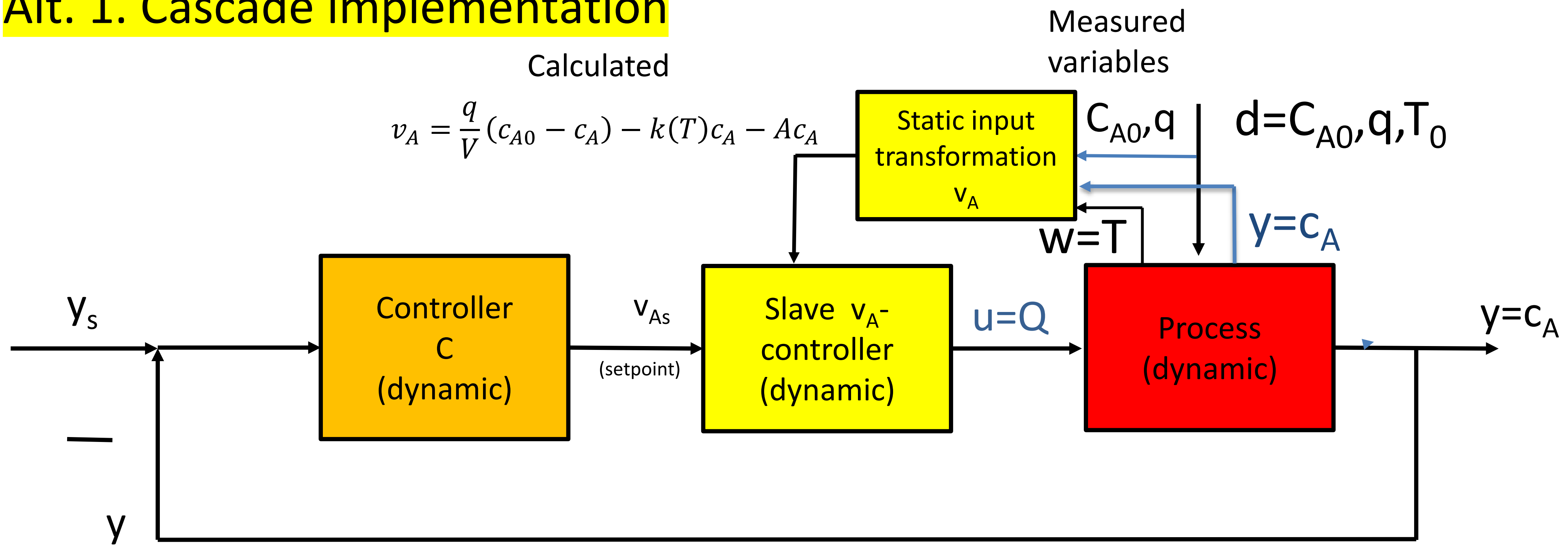
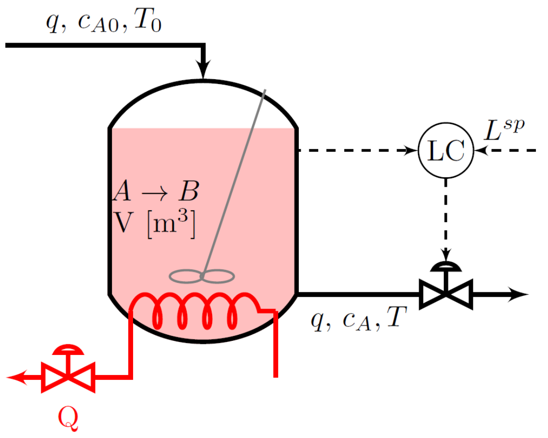
Alt. 1. Cascade implementation



Two reasons to use slave controller

1. u=Q does not appear in v_A
2. Avoid inverting expression for v_A with respect to T

But do not get perfect feedforward control for T_0



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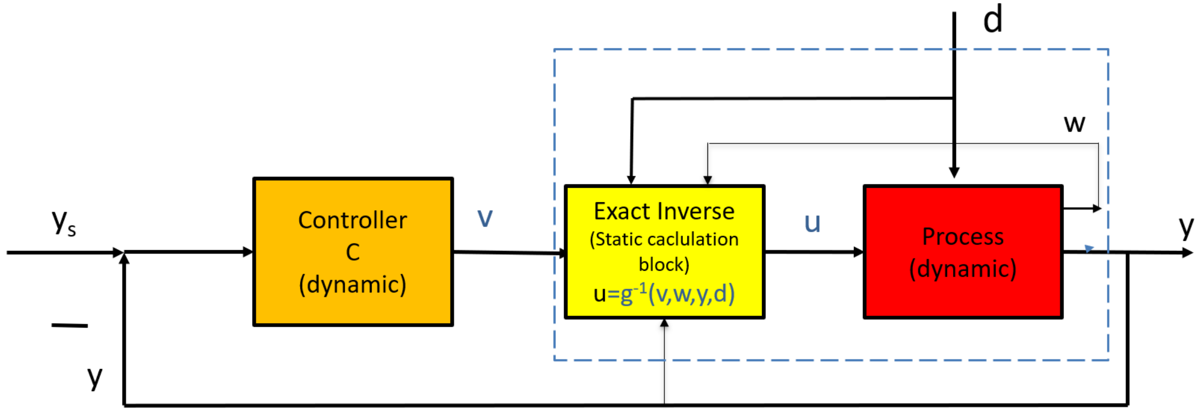
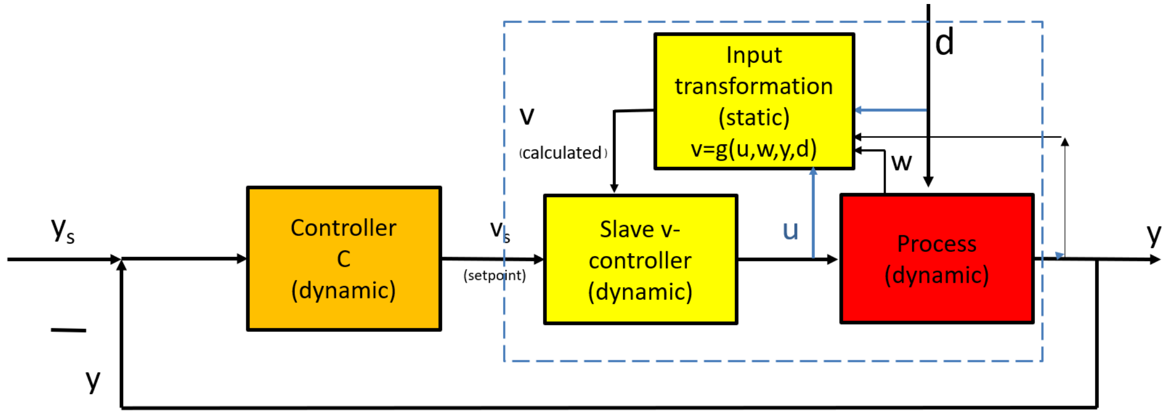
Discussion: Stability problems?

- Consider any transformed input, v = g(u,w,y,d)
- With exact inverse the **transformed system may be internally unstable** because we treat w as disturbance, but actually w depends on u
  - Happens when w causes **unstable zero dynamics** from u to v
- Stability problems can be avoided with feedback (cascade) implementation which gives approximate inverse

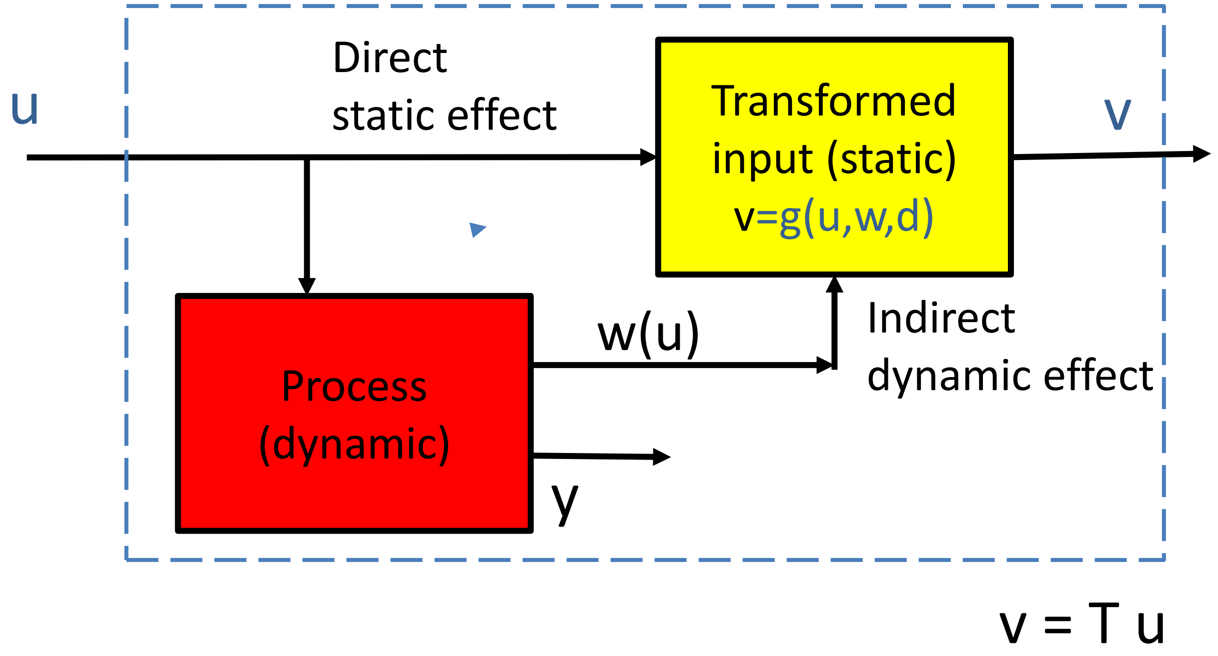




Unstable zero dynamics

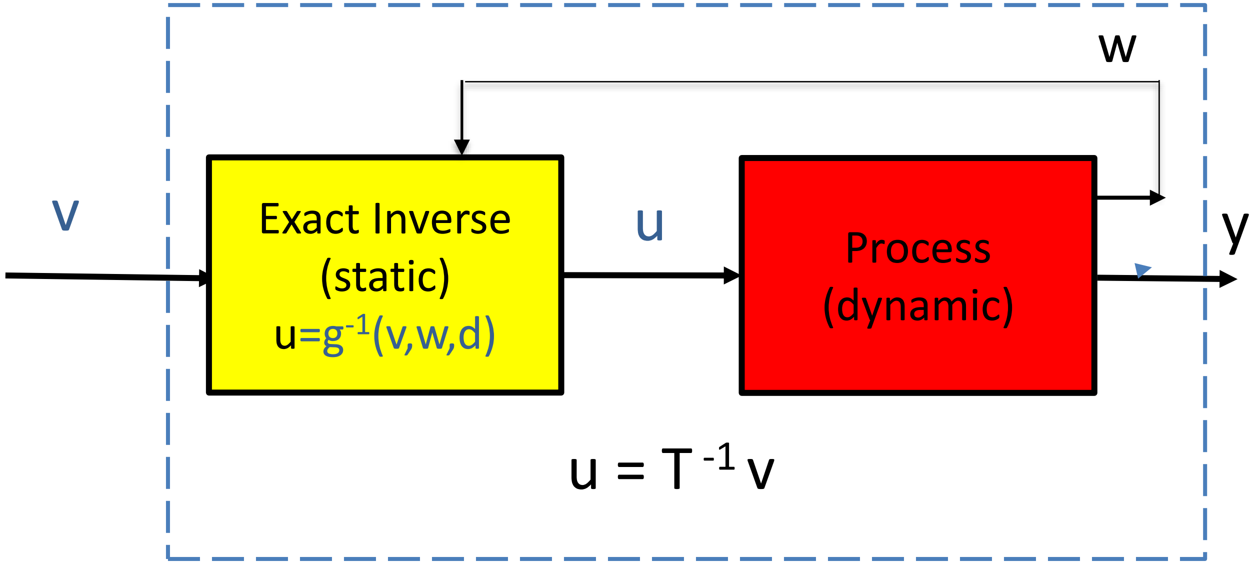
Indirect effect through w may cause

- unstable zero dynamics for T (from u to v)
- = inverse response from u to v (scalar)



Internal instability

Unstable zero dynamics for T give internal unstable tranformed system if we use exact inverse



Internally unstable:

Response from v to y is stable (apparently), but internal signals u and w are unstable

Example 7: Internal instability

Process

$$y = u + w + d$$

$$\boxed{w = \frac{-2u}{4s+1}}$$

Transformed input

$$v_0 = g(u, w, d) = u + w + d$$

$$\boxed{v_0 = \frac{4s-1}{4s+1}u + d}$$

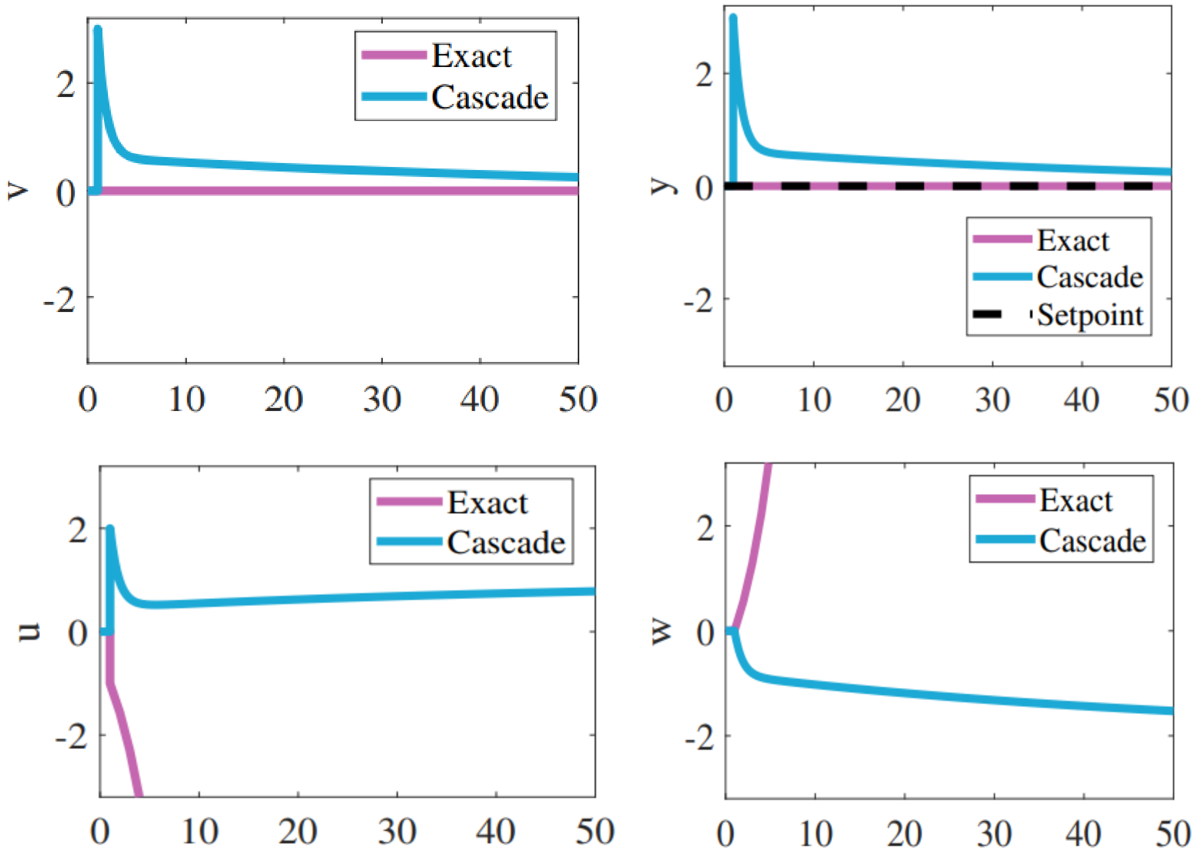
Exact inverse

$$u = v_0 - w - d \qquad \boxed{u = \frac{4s+1}{4s-1}(v_0 - d)}$$

Response of transformed system

$$y = v_0$$

Response to step disturbance (d=1)



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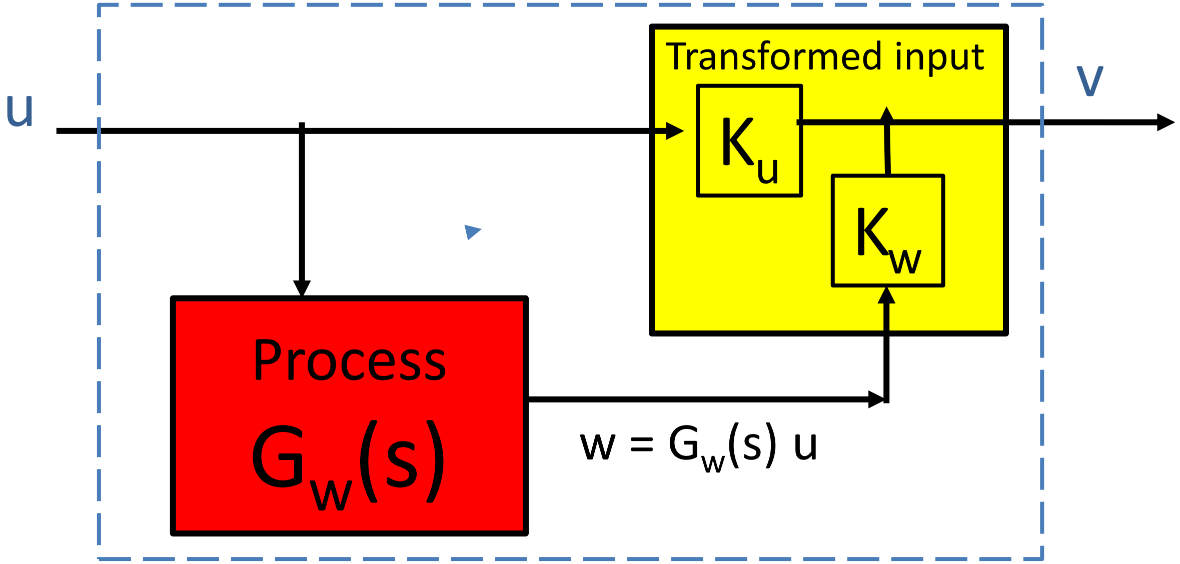
Linear analysis

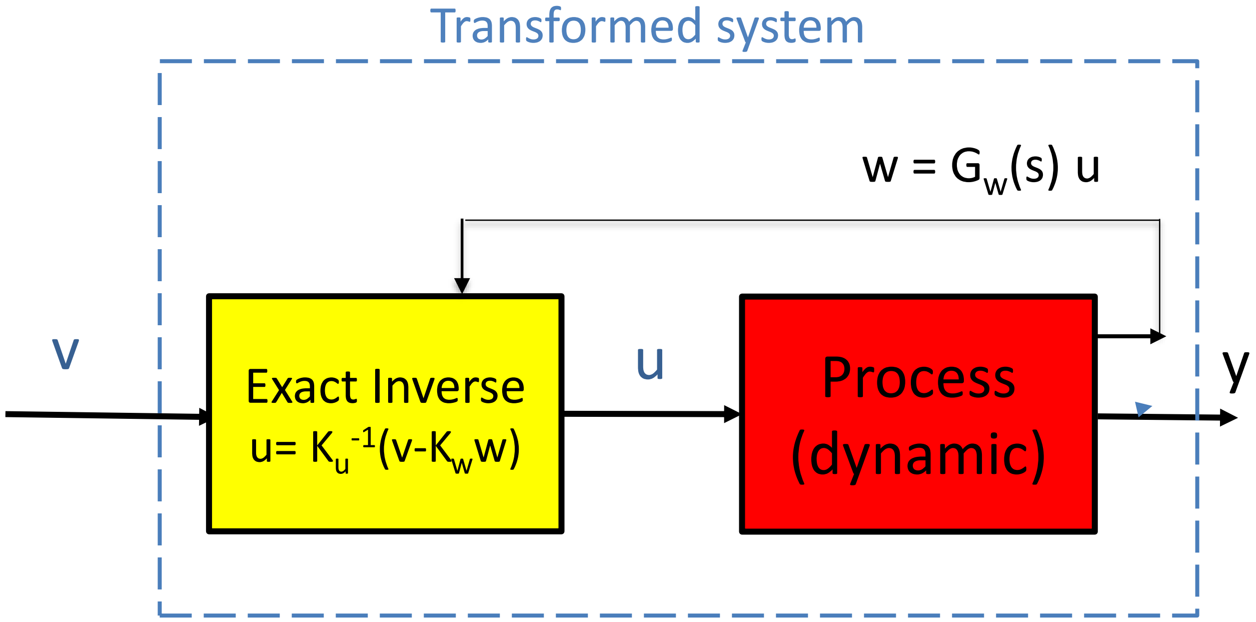


Transformed input

$$v = K_u\, u + K_w\, w$$
$$= (K_u + K_w\, G_w) u = T\, u$$

where

$$T = (K_u + K_w\, G_w) = K_u(I+L_w)$$
$$L_w = K_u^{-1} K_w\, G_w$$



Transformed system with exact inverse

$$u = T^{-1}\, v = (I+L_w)^{-1}\, K_u^{-1}\, v$$

For internal stability of transformed system:

$$T^{-1} = (I+L_w)^{-1}\, K_u^{-1} \text{ must be stable}$$

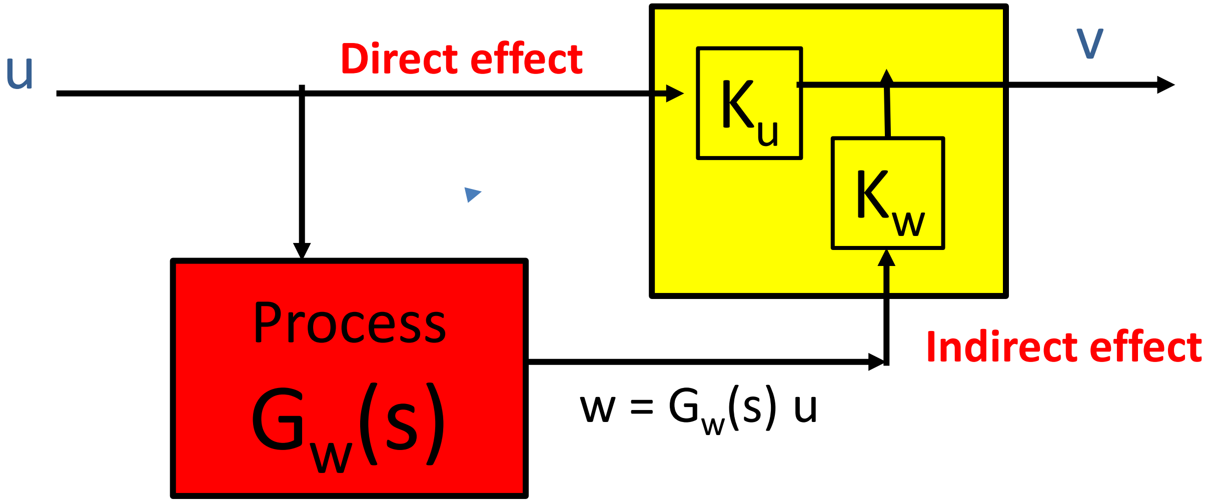
Equivalently: Transfer function

$$T= K_u(I+L_w)$$

from u to v must have stable zero dynamics

Trick: Can use Nyquist/Bode stability condition for L_w

Linear stability theorems



Transformed input

$v = K_u(I+L_w)\ u$

where

$L_w = K_u^{-1} K_w\ G_w$

Stability.
Transformed system is internally stable if and only if

$(I+L_w)^{-1}$ is stable

Bode stability condition: Internally stable

if and only if $|L_w(j\omega_{180})|<1$ (scalar)

Small gain theorem. Stable

if $|L_w(j\omega)|<1$ at all ω

In words: Stable if «indirect effect» K_wG_w (through w)

is smaller than «direct effect» K_u (through u).

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Bode stability condition

$L_w = K_u^{-1} K_w G_w$

Bode (scalar): Internally stable if and only if $|L_w(j\omega_{180})|<1$

Two cases

1. $L_w(0) < 0$: Direct and indirect effect are opposite at steady-state. $\omega_{180}=0$

   Get internal instability iff $|L_w(0)|>1$

   - When Indirect effect is larger and opposite at steady state

   Example 5: $K_u=K_w=1$ and $G_w=-2/(4s+1)$ so $L_w(0)=-2$ <-> internally unstable

   Note: Transfer function from u to v is $T = K_u(I+L_w) = (4s-1)/(4s+1)$.

2. $L_w(0) > 0$: Direct and indirect effect in same direction at steady state.

   - Internal instability is less likely.
   - Requires that indirect effect is large and that G_w has unstable zeros (inverse response) or delay

   Example 6: $K_u=1$, $G_w = (-s+1)/(s+1)$. $\omega_{180}=\infty$. $|L_w|=K_w$ at all ω. Get internal instability iff $K_w>1$.

   Note: With $K_w=2$, transfer function from u to v is $T= K_u(I+L_w) = (3-s)/((s+1)$.

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What if uncertain about internal instability?

- Use feedback (cascade) implementation
- Slave loop involves controlling v = T(s) u.
  - $T(s) = K_u + K_w G_w(s)$
  - Unstable (RHP) zero or time delay in T(s) implies that slave loop cannot be fast
  - Uncertain model: Can tune slave controller based on experimental T.

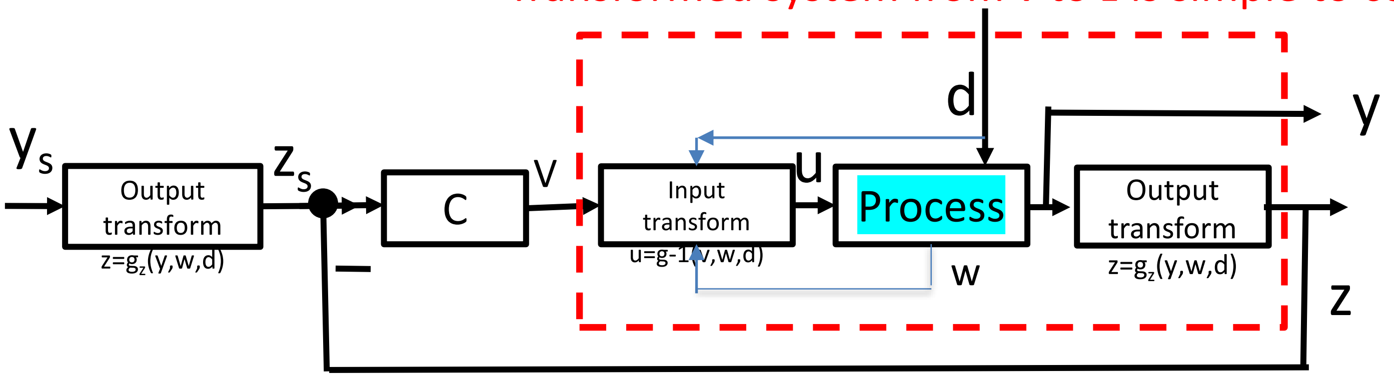
Transformed output

$z = g_z(y, {}_{w,d})$

Main idea: Simpler/more linear model for z than for y

No fundamental advanage.

Transformed system from v to z is simple to control



Since we use the same transformation on both y and y_s, we will at steady state get $y = y_s$.

Example: y=T (temperature), z=H(T,p,x) (enthalpy).

Easy to write energy balance in terms of z=H

Further discussion…

- We have looked at many other examples
- And in particular we have looked at the effect of uncertainty
  - No big surprises
  - It’s fairly robust!
  - Mater theses by Callum Kingstree and Simen Bjorvand

Conclusion

- **Transformed input v = g(u,w,d,y)**
- **Based on simple process models, easy to understand and implement**
- Inversion to generate u from v: Exact inverse or approximate inverse by feedback

- Systematic from Static model:
  - $y=f_0(u,w,d)$.
  - Transformed input: $v_0 = f_0(u, w, d)$
- Systematic from Dynamic model:
  - $\frac{dy}{dt} = f(u, w, y, d)$
  - Transformed input (B=I): $v_A = f(u, w, y, d) - Ay$
- Resulting transformed system from v to y:
  - Linear, independent of disturbances, decoupled
- Potential internal instability with exact inverse
  - No problem if indirect effect on v through w is small
  - Otherwise use cascade implementation

Academic control community fish pond

Complex optimal centralized Solution (**EMPC, FL**)

Simple solutions that work (PID++)