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Self-Optimizing Control for Recirculated Gas lifted Subsea Oil Well Production

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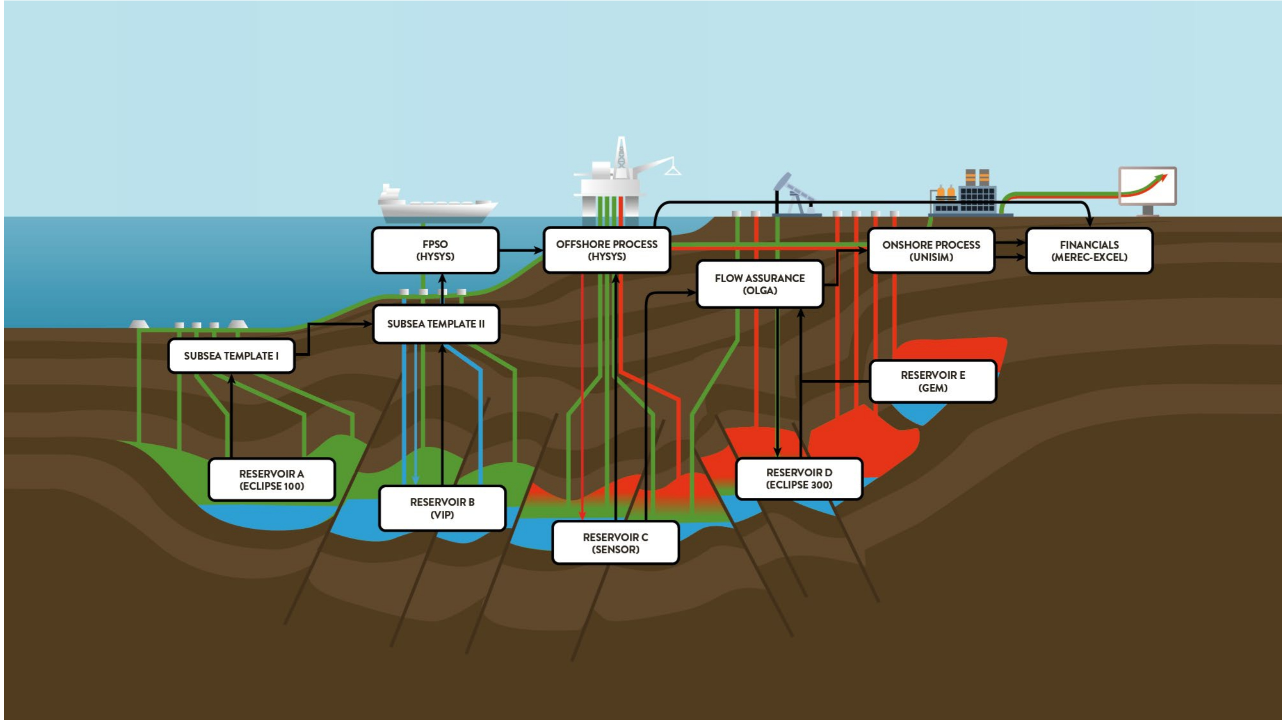
SFI SUBPRO

Optimization in Oil & Gas Industry



Main Research Questions

How to optimize the operation of a

- *complex, large-scale oil and/or gas production system,*
- *varying timescales,*
- *numerous potential constraints,*

Preferably utilizing simple tools like

- *PID controllers,*
- *selectors,*
- *and small-scale solvers (if necessary)?*

Outline

- Conventional RTO

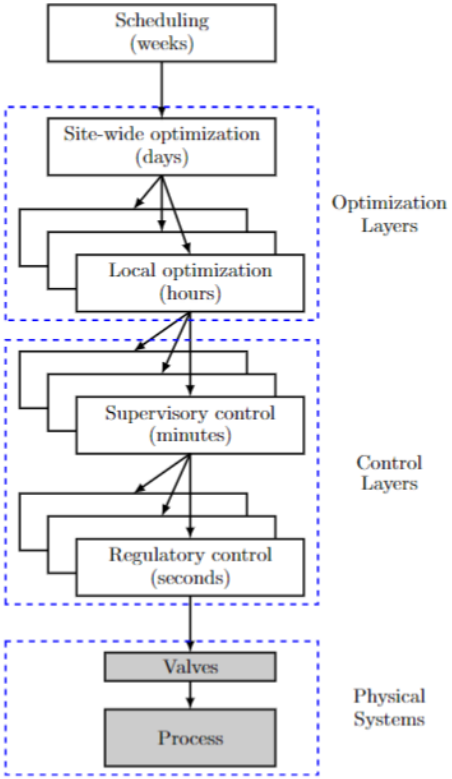
Put optimization into control layer:

- Self-optimizing control (SOC)
  - Marathon runner
- Case study using SOC
- New results on gradient-based control for changing active constraints
  - Primal-dual using Lagrange multipliers
  - Region-based with selectors

Optimal Operation

RTO

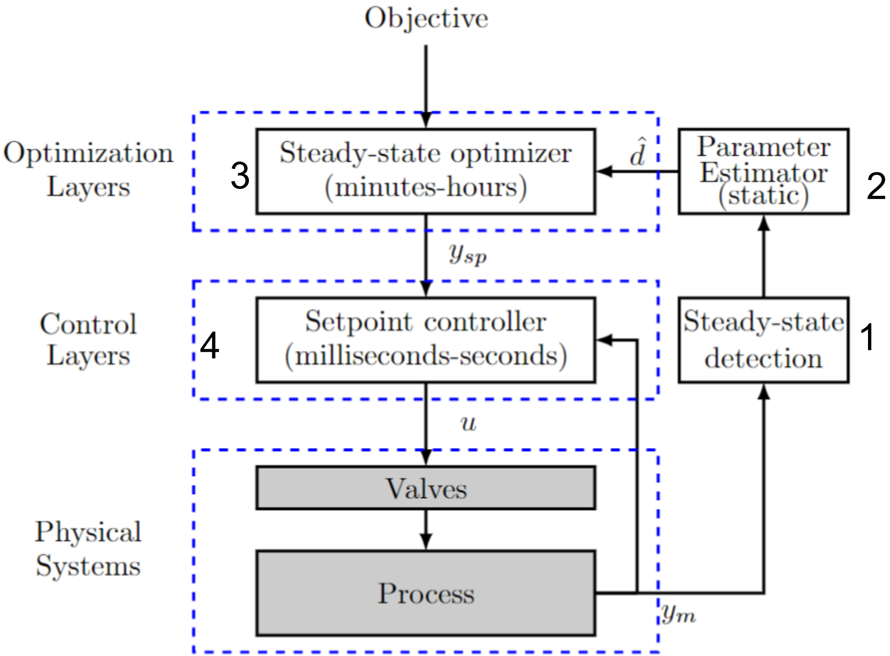
MPC or ARC



RTO = real-time optimization
MPC = model predictive control
ARC = advanced regulatory (PID) control

Optimal Operation

- Traditional RTO



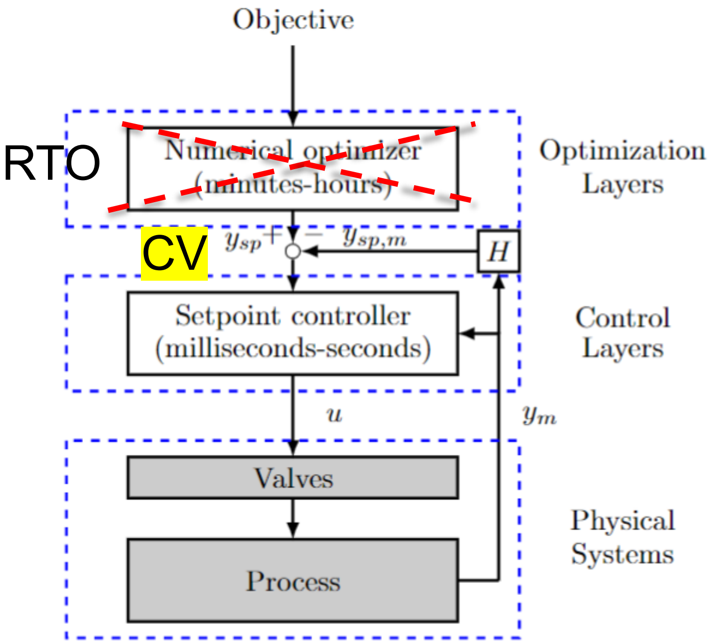
Issue : Steady-state wait time

Issue : Non-transparent constraint control

Issue : Complex, need on-line model

Optimal Operation

- Self-optimizing control: Select good CV



Advantage : Transparent and simple

Advantage : Fast

Issues : Nonlinearity (some loss in optimality)
+ not optimal if constraints change

CV = controlled varable

Example: Optimal operation of runner

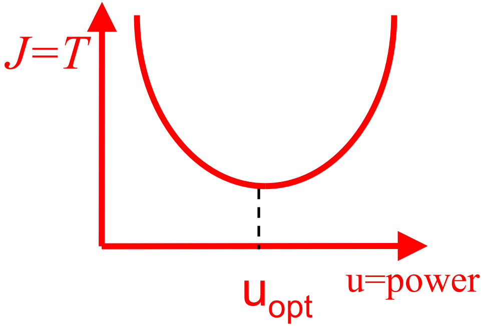
- Cost to be minimized, J=T
- One degree of freedom (u=power)
- What should we control (CV)?

Self-optimizing CV?

- Sprinter (100m):
  - «Run as fast as you can»
  - Active constraint control
  - CV=u (no controller needed), CV_s = max

Example: Optimal operation of runner

– Marathon (40 km)



CV_1 = distance to leader of race
CV_2 = speed
CV_3 = heart rate
CV_4 = level of lactate in muscles

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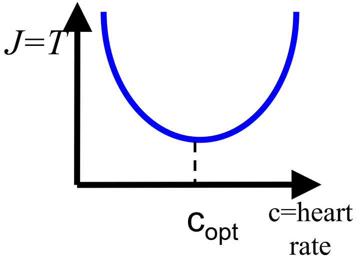
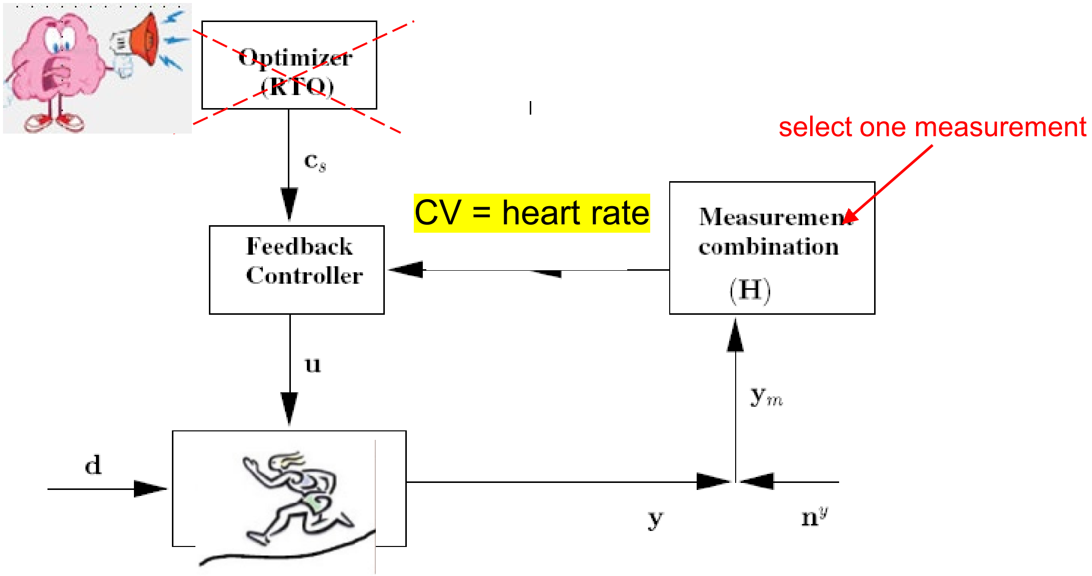
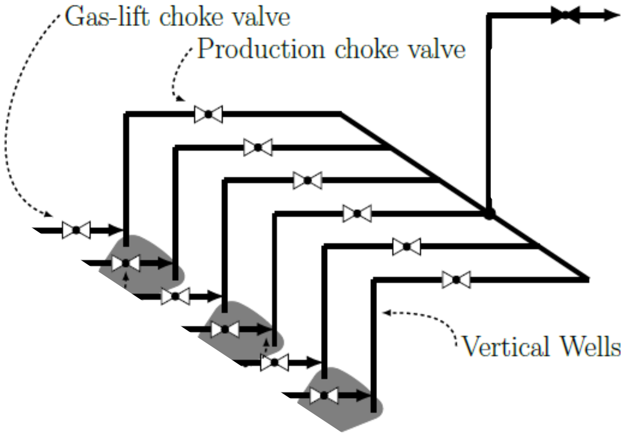
Conclusion Marathon runner



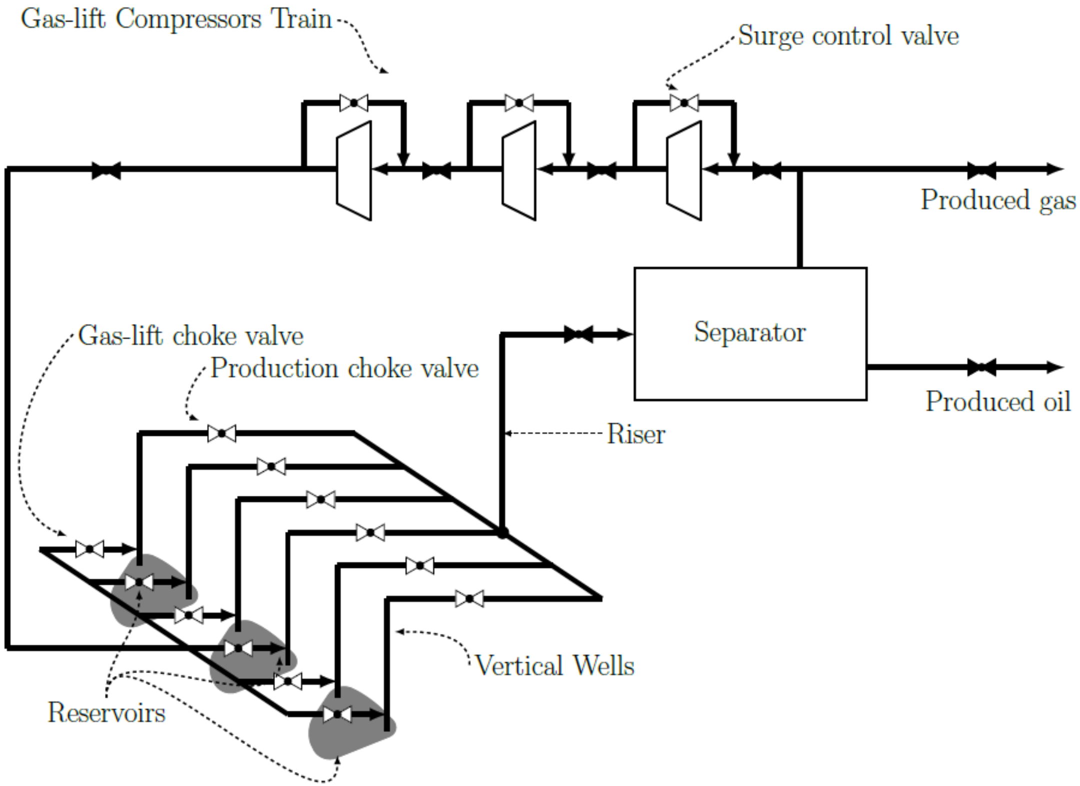


- CV = heart rate is a good "self-optimizing" variable
- Disturbances are indirectly handled by keeping a constant heart rate
- May have infrequent adjustment of setpoint (c_s)

Gas-Lifted Optimization Problem



Recirculated Gas-Lifted



Steady-state optimization problem

$$
\begin{aligned}
\min_{\mathbf{u}} \quad & J(\mathbf{u}, d) = -p_{oil} w_{os} + p_{en} \Phi_{gl} \\
\text{s.t.} \quad & g_{z_{gl,i}}(\mathbf{u}, d) : z_{gl,i} - 1 \leq 0 \quad i = 1, \ldots, 6, \\
& g_{z_{s,i}}(\mathbf{u}, d) : -z_{s,i} + 0 \leq 0 \quad i = 1, \ldots, 3, \\
& g_{s_i}(\mathbf{u}, d) : s_i - \bar{s}_i \leq 0 \quad i = 1, \ldots, 3, \\
& g(\mathbf{u}, d) : w_{gs} - \bar{w}_{gs} \leq 0
\end{aligned}
$$

Maximize oil revenue ($-p_{oil} w_{os}$) Minimize gas lift cost ($+p_{en} \Phi_{gl}$)

GLC has max. opening

SCV has min. opening

Surge constraints

Max export/produced gas constraints

$$\mathbf{y} = \begin{bmatrix} p_{bh,2} & p_{wh,2} & p_{d,3} & p_s \end{bmatrix}^\top$$

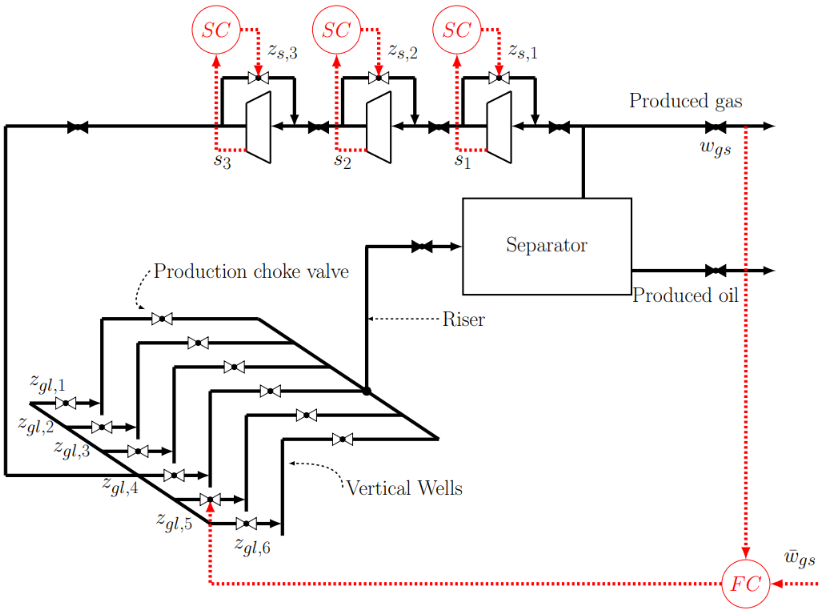
Available measurements

$$d = GOR_2$$

Disturbances

Self-optimizing Control Structures

- Structure 1
  - Keep the valve positions constant $(\mathbf{u} = \mathbf{u}^*)$
- Structure 2
  - Control active constraints
    - $z_{gl,5} \rightarrow g(\mathbf{u}, d)$
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$



Self-optimizing Control Structures

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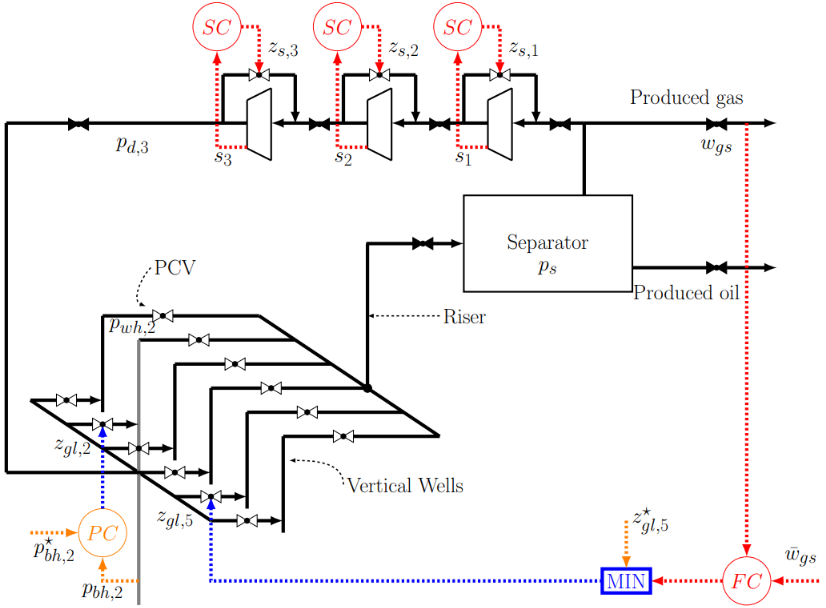
- Structure 3

  Region I
  - Control active constraints
    - $z_{gl,5} \rightarrow g(\mathbf{u}, d)$
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control bottomhole pressure as self-optimizing control variable
    - $z_{gl,2} \rightarrow p_{bh,2}$

  Region II
  - Control active constraint
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control self-optimizing control variables
    - $z_{gl,2} \rightarrow p_{bh,2}$
    - $z_{gl,5} = z_{gl,5}^{*}$



Allowing active constraint switching

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PART III

Self-optimizing Control Structures

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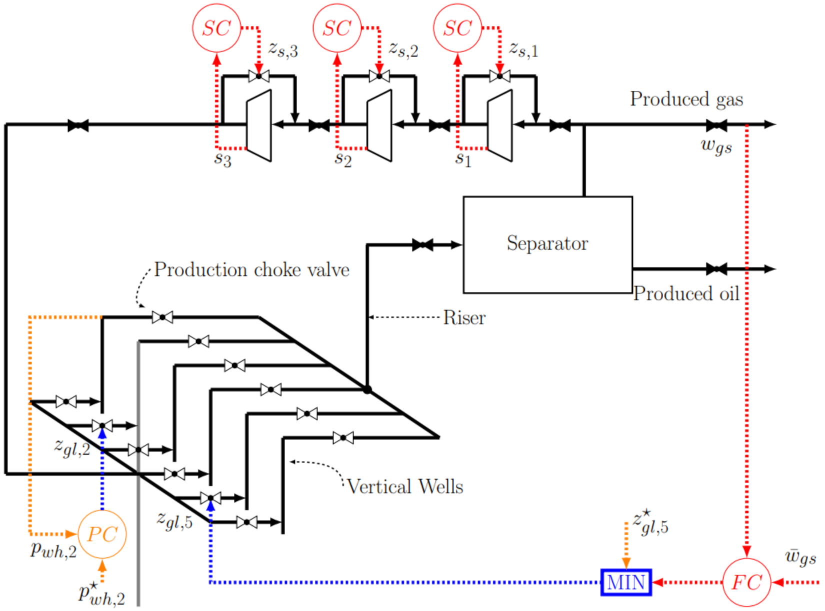
- Structure 4

  Region I

  - Control active constraints
    - $z_{gl,5} \rightarrow g(\mathbf{u}, d)$
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control wellhead pressure as self-optimizing control variable
    - $z_{gl,2} \rightarrow p_{wh,2}$

  Region II

  - Control active constraint
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control self-optimizing control variables
    - $z_{gl,2} \rightarrow p_{wh,2}$
    - $z_{gl,5} = z_{gl,5}^{*}$



Self-optimizing Control Structures

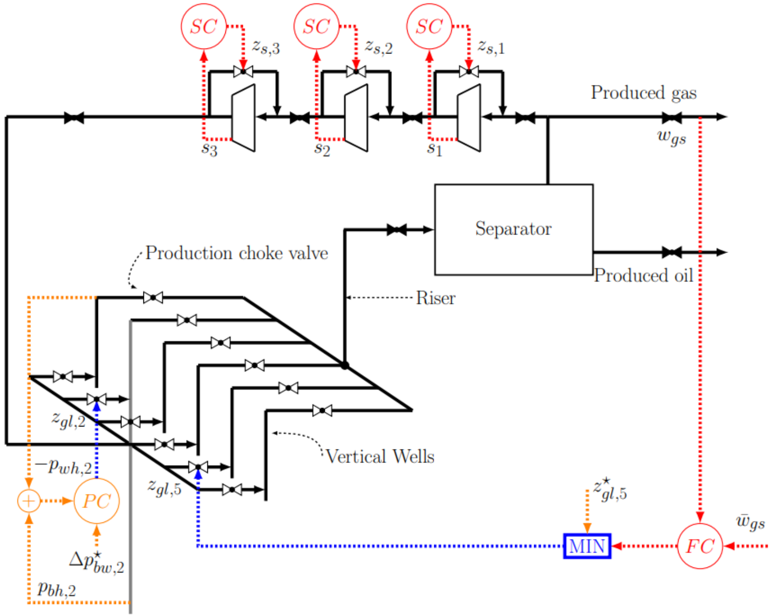
- Structure 5

  Region I

  - Control active constraints
    - $z_{gl,5} \rightarrow g(\mathbf{u}, d)$
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control tubing pressure as self-optimizing control variable
    - $z_{gl,2} \rightarrow \Delta p_{bw,2}$

  Region II

  - Control active constraint
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control self-optimizing control variables
    - $z_{gl,2} \rightarrow \Delta p_{bw,2}$
    - $z_{gl,5} = z^{*}_{gl,5}$



Self-optimizing Control Structures

- Structure 6

  Region I

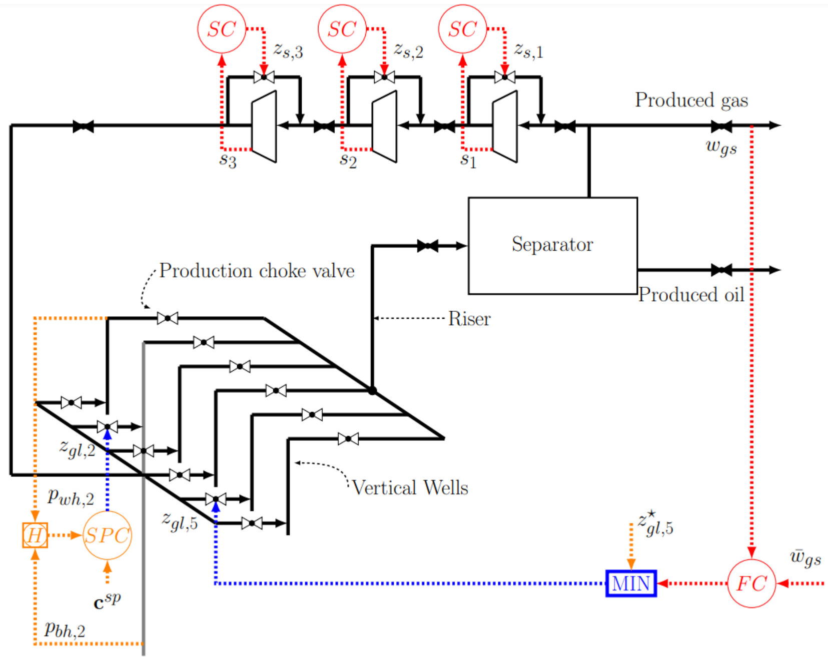
  - Control active constraints
    - $z_{gl,5} \to g(\mathbf{u}, d)$
    - $z_{s,i} \to g_{z_{s,i}}(\mathbf{u}, d)$
  - Control mix of tubing and wellhead pressure as self-optimizing control variable
    - $z_{gl,2} \to c := 0.521 p_{bh,2} + 0{,}854 p_{wh,2}$

  Region II

  - Control active constraint
    - $z_{s,i} \to g_{z_{s,i}}(\mathbf{u}, d)$
  - Control self-optimizing control variables
    - $z_{gl,2} \to c := 0.521 p_{bh,2} + 0{,}854 p_{wh,2}$
    - $z_{gl,5} = z_{gl,5}^{*}$

$$\mathbf{y} = \begin{bmatrix} p_{bh,2} & p_{wh,2} \end{bmatrix}^\top \qquad \mathbf{F} = \frac{\partial \mathbf{y}^\star}{\partial \mathbf{d}} \qquad \mathbf{c} = \mathbf{H}\mathbf{y}$$

Null space method: $\mathbf{HF} = \mathbf{0}$



Self-optimizing Control Structures

- Structure 7

  Region I

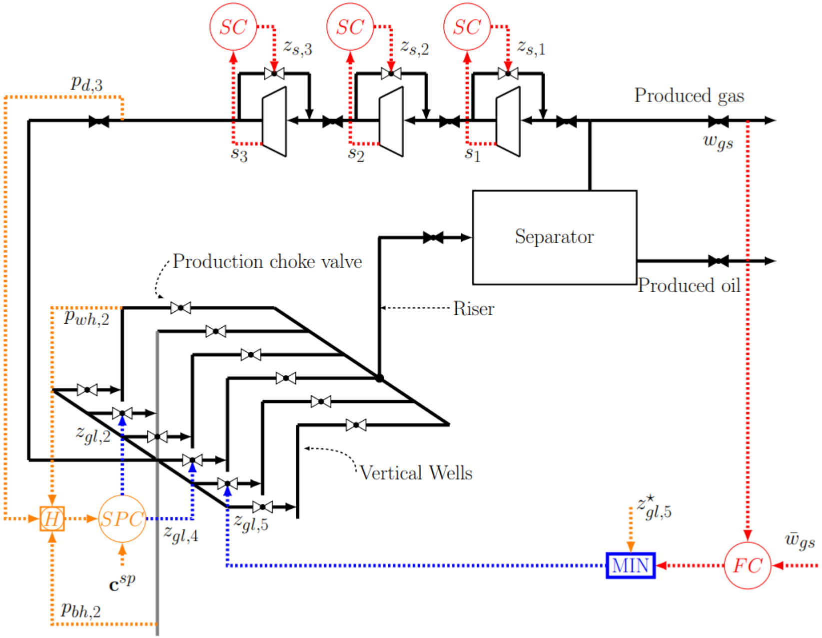
  - Control active constraints
    - $z_{gl,5} \rightarrow g(\mathbf{u}, d)$
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control two optimal self-optimizing control variables
    - $z_{gl,2} \rightarrow \mathbf{c}(1)$
    - $z_{gl,4} \rightarrow \mathbf{c}(2)$

  Region II

  - Control active constraint
    - $z_{s,i} \rightarrow g_{z_{s,i}}(\mathbf{u}, d)$
  - Control self-optimizing control variables
    - $z_{gl,2} \rightarrow \mathbf{c}(1)$
    - $z_{gl,4} \rightarrow \mathbf{c}(2)$
    - $z_{gl,5} = z^{*}_{gl,5}$

$$\mathbf{y} = [p_{bh,2} \quad p_{wh,2} \quad p_{d,3}]^\top \qquad \mathbf{F} = \frac{\partial \mathbf{y}^\star}{\partial \mathbf{d}} \qquad \mathbf{c} = \mathbf{H}\mathbf{y}$$

Null space method: $\mathbf{HF} = \mathbf{0}$

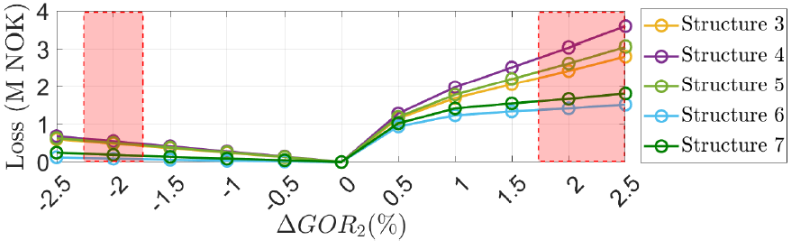


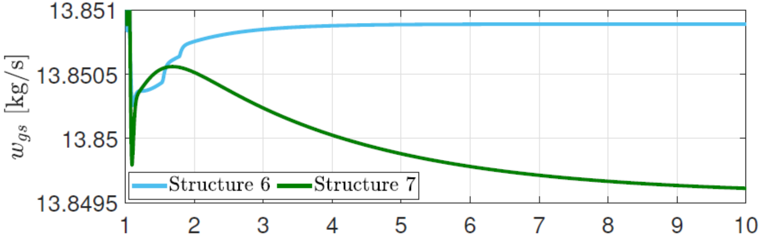
Simulations Results

- Steady-state monthly loss

Table 11.2: Steady-state monthly loss

Control Structure	$-2.5\%\ GOR_2$	$+2.5\%\ GOR_2$ (est.)
1	NOK 59.544	Inf
2	NOK 6.116.745	NOK $\sim$ 3.444.831
3	NOK 604.897	NOK $\sim$ 2.810.376
4	NOK 686.095	NOK $\sim$ 3.595.481
5	NOK 633.027	NOK $\sim$ 3.065.285
6	NOK 124.246	NOK $\sim$ 1.523.036
7	NOK 248.667	NOK $\sim$ 1.817.930





Case study summary

- Extend gas lift model to recirculated gas-lift oil production.
- Reconfirms the SOC can be an alternative for optimization
- Selector allows active constraint region switching.
- Structure 6 is recommended. From nullspace method:

$$\text{CV} = 0.52p_{bh,2} + 0.85p_{wh,2}$$

SOC: changing constraints are not handled optimally

- We have some more recent results based on KKT optimality conditions
- $\lambda =$ Lagrange multiplier
- Cost gradient: $\nabla_u J \equiv J_u$

Theorem 2.3: Karush-Khun-Tucker (KKT) Optimality Conditions

Suppose that the objective function $J(\mathbf{u}, \mathbf{d})$ and constraint $\mathbf{g}(\mathbf{u}, \mathbf{d})$ have subderivatives at point $\mathbf{u}^\star$. If $\mathbf{u}^\star$ is a local optimum and the optimization problem satisfies some regularity or *KKT conditions* (see below), then there exist constants $\boldsymbol{\lambda}$, called *KKT multipliers* or *Lagrange multipliers* or *dual variables* , such that the following conditions hold:

$$\nabla_{\mathbf{u}} \mathcal{L}(\mathbf{u}, \mathbf{d}, \boldsymbol{\lambda}) = 0 \tag{2.9a}$$

$$g_i(\mathbf{u}, \mathbf{d}) \leq 0, \ \forall i = 1, \ldots, n_{\mathbf{g}} \tag{2.9b}$$

$$\lambda_i \geq 0, \ \forall i = 1, \ldots, n_{\mathbf{g}} \tag{2.9c}$$

$$\lambda_i g_i(\mathbf{u}, \mathbf{d}) = 0, \ \forall i = 1, \ldots, n_{\mathbf{g}} \tag{2.9d}$$

where

$$\nabla_{\mathbf{u}} \mathcal{L}(\mathbf{u}, \mathbf{d}, \boldsymbol{\lambda}) = \nabla_{\mathbf{u}} \mathrm{J}(\mathbf{u}, \mathbf{d}) + \nabla_{\mathbf{u}}^\top \mathbf{g}(\mathbf{u}, \mathbf{d}) \boldsymbol{\lambda},$$

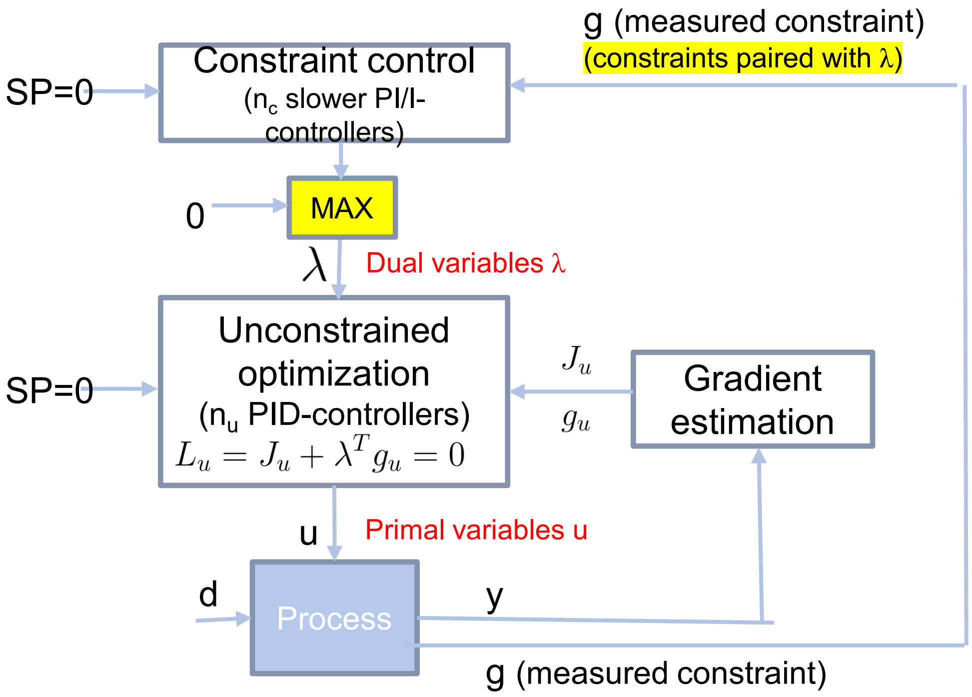
$$\mathbf{g}(\mathbf{u}, \mathbf{d}) = \begin{bmatrix} g_1(\mathbf{u}, \mathbf{d}) & \ldots & g_{n_{\mathbf{g}}}(\mathbf{u}, \mathbf{d}) \end{bmatrix}^\top,$$

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \ldots & \lambda_{n_{\mathbf{g}}} \end{bmatrix}^\top,$$

Eq. (2.9a) is called stationary condition, Eq. (2.9b) is called primal feasibility condition, Eq. (2.9c) is called dual feasibility condition, and Eq. (2.9d) is called complementary slackness condition [36].

I. Primal-dual control based on KKT conditions:

Tracks active constraints by adjusting Lagrange multipliers (= shadow prices = dual variables) λ



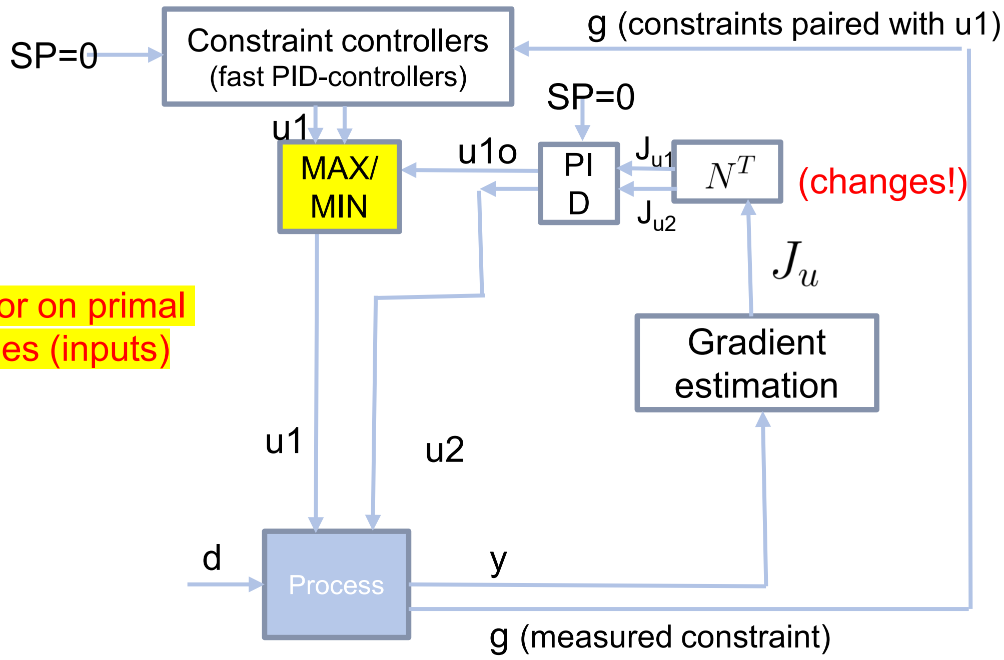
$$L_u = J_u + \lambda^T g_u = 0$$

Inequality constraints: $\lambda \geq 0$

- Problem: Constraint control using dual variables is on slow time scale

- D. Krishnamoorthy, A distributed feedback-based online process optimization framework for optimal resource sharing, J. Process Control 97 (2021) 72–83,
- R. Dirza and S. Skogestad . Primal–dual feedback-optimizing control with override for real-time optimization. J. Process Control, Vol. 138 (2024), 103208.

II. Region-based feedback solution with «direct» constraint control (for case with more inputs than constraints)



- Selector on primal variables (inputs)

KKT: $L_u = J_u + \lambda^T g_u = 0$

Introduce N: $N^T g_u = 0$

Control
1. Reduced gradient $N^T J_u = 0$
   - «self-optimizing variables»)
2. Active constrints $g_A = 0$.

- Jaschke and Skogestad, «Optimal controlled variables for¨polynomial systems». S., J. Process Control, 2012
- D. Krishnamoorthy and S. Skogestad, «Online Process Optimization with Active Constraint Set Changes using Simple Control Structure», I&EC Res., 2019
- Bernardino and Skogestad, Decentralized control using selectors for optimal steady-state operation with changing active constraints, J. Process Control, Vol. 137, 2024

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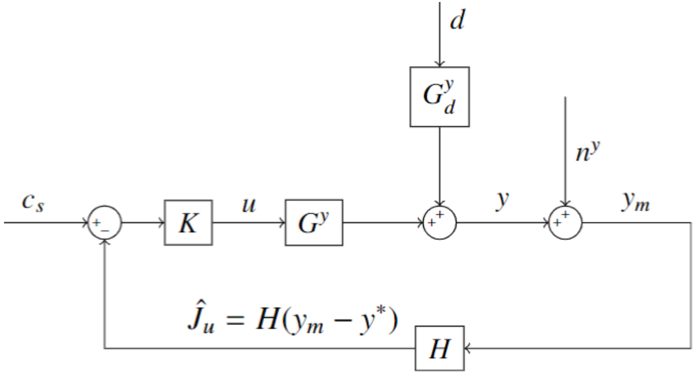
New static gradient estimation based on SOC:
Very simple and works well!



From «exact local method» of self-optimizing control:

$$H^J = J_{uu}\left[G^{yT}\left(\tilde{F}\tilde{F}^T\right)^{-1}G^y\right]^{-1}G^{yT}\left(\tilde{F}\tilde{F}^T\right)^{-1}$$

where $\tilde{F} = [FW_d \quad W_{n^y}]$ and $F = \frac{dy^{opt}}{dd} = G_d^y - G^y J_{uu}^{-1} J_{ud}$.

- Bernardino and Skogestad, Optimal measurement-based cost gradient estimate for real-time optimization, Comp. Chem. Engng., 2024

Conclusion

Move optimization into control layer by selecting good CVs

- CV = Active constraints

Unconstrained degrees of freedom:

- CV = Self-optimizing variables
- CV = Gradients

Reminder: DYCOPS conference in Bratislava (Slovakia) 16-19 June 2025.
I hope to see you there!

CV = controlled varable