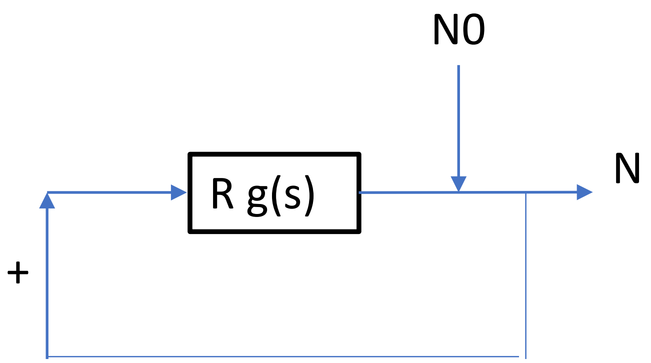
Simple positive feedback COVID model

Positive feedback model with $L = R\,g(s)$

November 2020

21 Dec. 2020

POSITIVE FEEDBACK MODEL



N0 = number of initial cases (persons)
N = number of total cases
R = no. infected from one person
g(s) : dynamics for each person, assumed equal

Response: N = S(s) N0 where **S(s) = 1/(1-L)** and L=R g(s)

For first-order dynamics, g(s) = 1/(tau1*s+1),
S(s) = 1-k/(tau*s-1), k=R/(R-1), tau=tau1/(R-1).
Resulting cases N for step in N0:
N = (N0/(R-1))*(R*exp(rt)-1), where **r = (R-1)/tau1**
Note that this not quite on the form N = k*exp(rt), except when rt is large.

Key parameter:

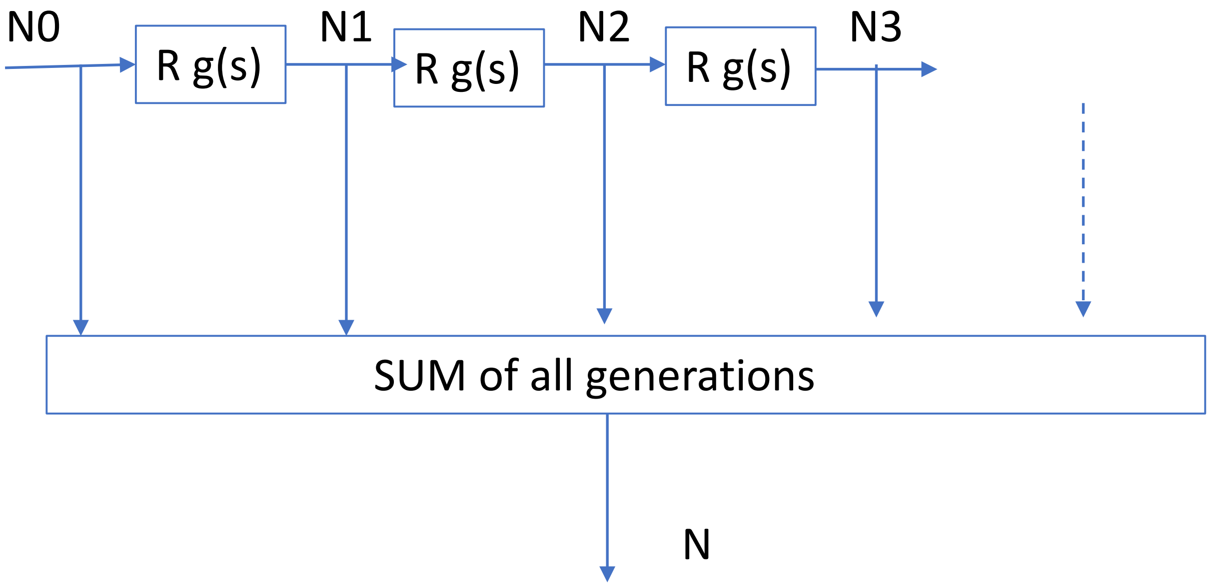
- R – reproductive number
- With herd immunity
  - $R(N) = R_0 (1-N/N_{pop})$
- R should be less than 1 for stability

Is this simple positive feedback model really correct?

Example of dynamics

- $g(s) = 1/(\tau_1 s+1)$
- $g(s) = 1/(\tau_1 s+1)(\tau_2 s+1)$
- $g(s) = \exp(-\theta s)/(\tau_1 s+1)$

Sum Generations model



N0 = number of initial cases (persons)
N = number of total cases
R = no. infected from one person
g(s) : dynamics for each person, assumed equal

STEP RESPONSE for N0

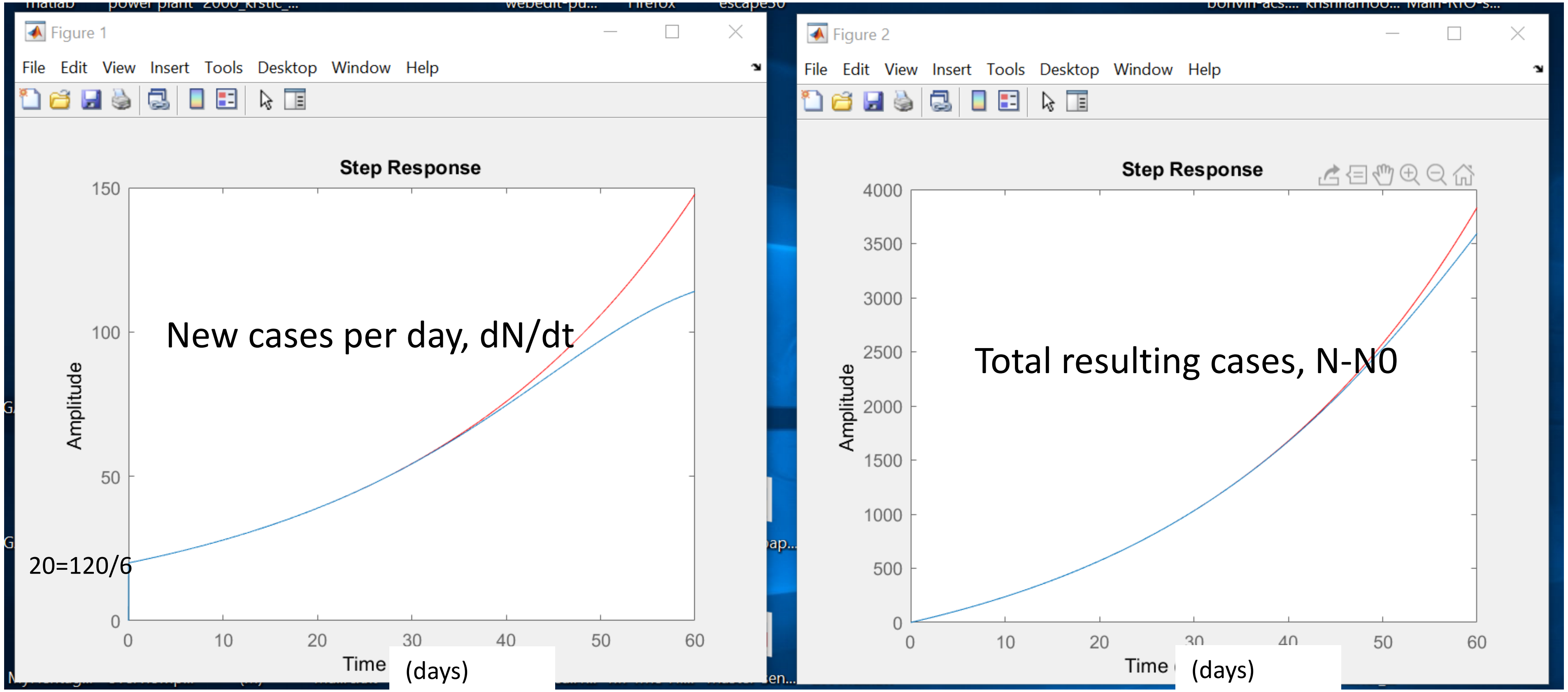
«Numerical proof» of Positive feedback model.

Compare with sum of generations model

N0=100

R=1.2

τ_1=6 days




```
R=1.2; T0=100; G = 1/(6*s+1);L = R*G;
% positive feedback model:
T=L*T0/(1-L); Tnew=s*T/(0.001*s+1); tt=60; figure(1); step(Tnew,tt,'red'); figure(2); step(T,tt,'red')
% sum generations modcel:
T=0;
for n=1:15          % could only go to n=15 for numerical reasons
TT(n) = L^n*T0;     % n = generation number
T =  TT(n)+T;
end
Tnew=s*T/(0.001*s+1); figure(1); step(Tnew,tt); figure(2); step(T,tt)
```

Mathematical proof.

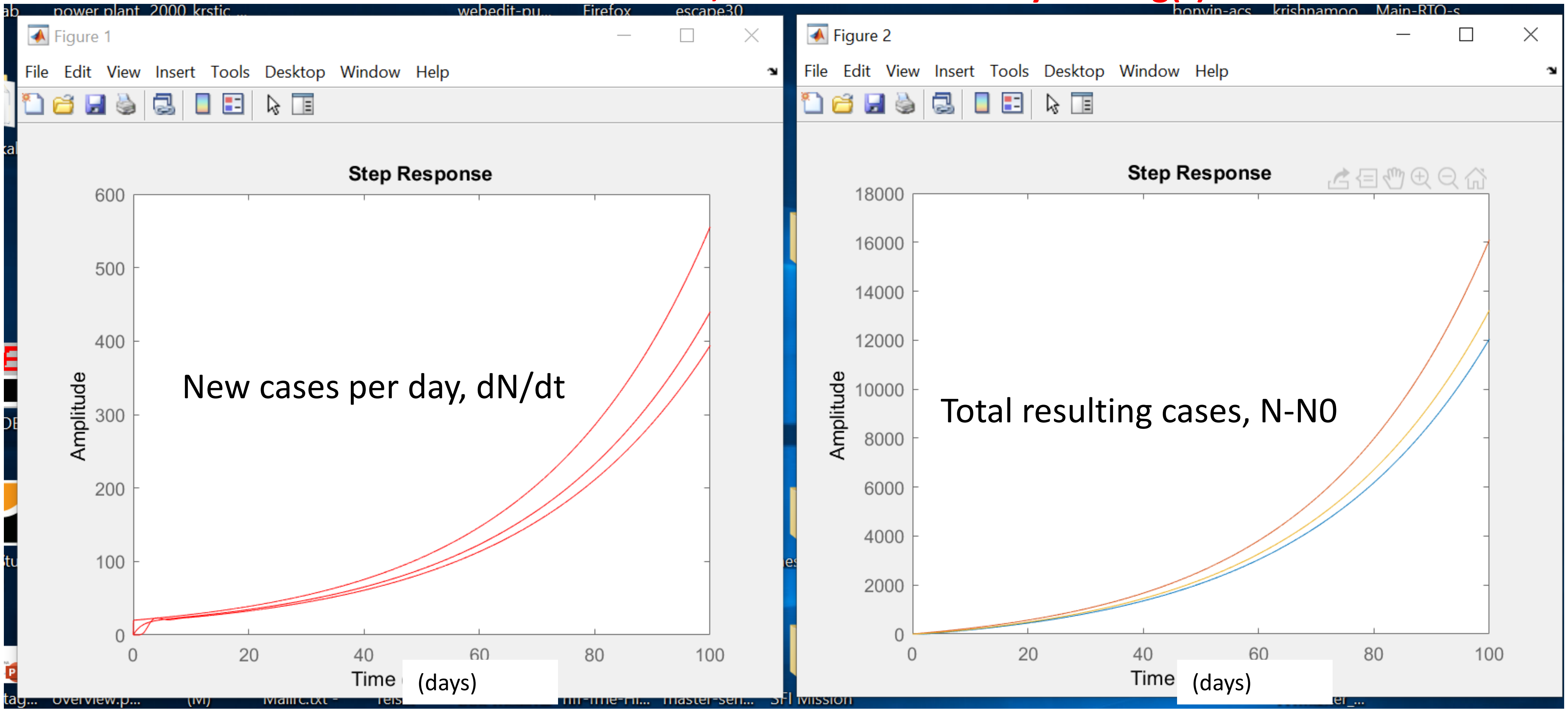
Converging geometric series:

$$1+L+L^2+L^3+\ldots\ldots = 1/(1-L)$$

The positive feedback model is clearly a simplification, since each person has their own dynamics g(s). So the fulll model is high-order, but the response is equal to the low-order positive feedback model

Positive feedback model, effect of different dynamics g(s)

$N0=100$
$R=1.2$
$\tau_1=6$ days



R=1.2, sum_i tau_i = 6, N0=100
Upper: First-order, tau=6
Middle: Second-order, tau1=tau2=6
Lower: With delay, theta=3, tau=3

Conclusion

- Very simple positive feedback model
- Can easily include any dynamics, g(s)
- I looked around, but I did not easily find such a simple model in the literature
- Feedback control by government: Adjust regulations to keep R below 1, maybe setpoint around 0.7 to avoid too much restrictions