PID tuning using the SIMC rule

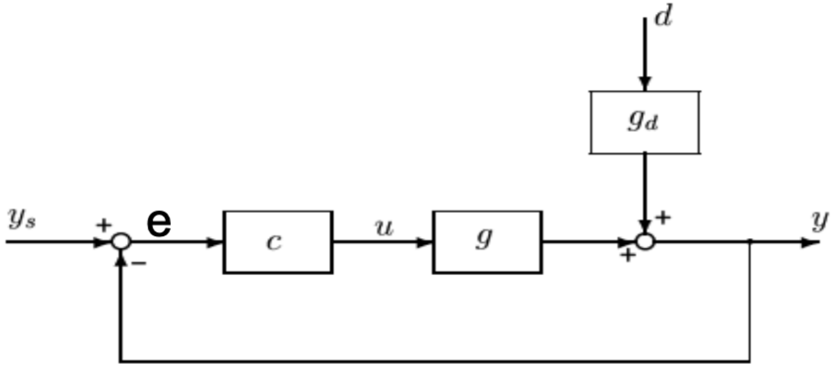
Sigurd Skogestad

Updated 20 Oct. 2025

Lecture outline

- SIMC rule for first order systems
- Choice of tuning constant τ_c
- Some cases

PID controller



- Time domain ("ideal" PID)

$$u(t) = u_0 + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(\xi)\,d\xi + \tau_D\frac{de}{dt}\right)$$

- Laplace domain ("ideal"/"parallel" form)

$$c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

- Usually $\tau_D = 0$. Only two parameters left (K_C and τ_I)…
- How difficult can it be?
  - Surprisingly difficult without systematic approach!

Excellent work – especially considering that it was published only 3 years after the PID controller came on the market (Taylor Model 100 Fullscope, 1939)

Trans. ASME, 64, 759-768 (Nov. 1942).

Optimum Settings for Automatic Controllers

By J.G. ZIEGLER[1] and N. B. NICHOLS[2] • ROCHESTER, N. Y.

In this paper, the three principle control effects found in present controllers are examined and practical names and units of measurement are proposed for each effect.

varying its output air pressure, repositions a diaphragm-operated valve. The controller may be measuring temperature, pressure, level, or any other variable, but we will completely divorce the

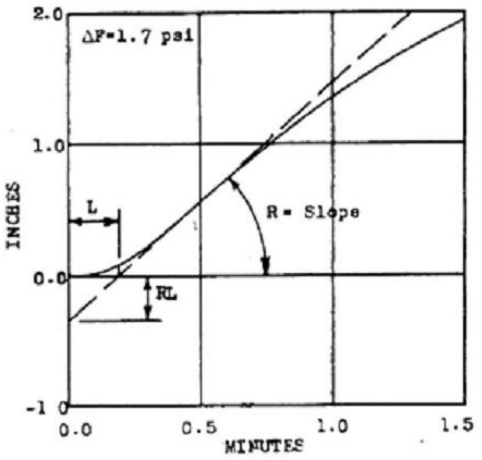


FIG. 8 REACTION CURVE

Reset-Rate Determination From Reaction Curve. Since the period of oscillation at the ultimate sensitivity proves to be 4 times the lag. A substitution of 4 L for P_u in previous equations for optimum reset rate gives an equation expressing this reset rate in terms of lag. For a controller with proportional and auto-matic-reset responses, the optimum settings become

$$Sensitivity = \frac{0.9}{R_1 L}\ psi\ per\ in.$$

$$\text{Reset Rate} = \frac{0.3}{L}\ \text{per min}$$

At these settings the period will be about 5.7L, having been increased, by both the lowering of sensitivity and the addition of automatic reset.

My notation:
Model: $R = k', L = \theta$
PI-settings:
$K_c = \frac{0.9}{k'}\frac{1}{\theta}$
$\tau_I = 3.3\theta$

Comment: Very similar to SIMC for integrating process with τ_c=0 (aggressive!):

$K_c = 1/k'\ 1/\theta$
$\tau_I = 4\ \theta$

Disadvantages Ziegler-Nichols:
1. Aggressive settings
2. No tuning parameter
3. Poor for processes with large time delay (θ)

AMERICAN CONTROL CONFERENCE
San Diego, California
June 6-8, 1984

IMPLICATIONS OF INTERNAL MODEL CONTROL FOR PID CONTROLLERS

Manfred Morari
Sigurd Skogestad

California Institute of Technology
Department of Chemical Engineering
Pasadena, California 91125

Daniel F. Rivera

University of Wisconsin
Department of Chemical Engineering
Madison, Wisconsin 53706

252 Ind. Eng. Chem. Process Des. Dev. **1986**, *25*, 252–265

Internal Model Control. 4. PID Controller Design

Daniel E. Rivera, Manfred Morari,* and Sigurd Skogestad

Chemical Engineering, 206-41, California Institute of Technology, Pasadena, California 91125

For a large number of single input–single output (SISO) models typically used in the process industries, the Internal Model Control (IMC) design procedure is shown to lead to PID controllers, occasionally augmented with a first-order lag. These PID controllers have as their only tuning parameter the closed-loop time constant or, equivalently, the closed-loop bandwidth. On-line adjustments are therefore much simpler than for general PID controllers. As a special case, PI- and PID-tuning rules for systems modeled by a first-order lag with dead time are derived analytically. The superiority of these rules in terms of both closed-loop performance and robustness is demonstrated.

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dead time are derived
ustness is demonstrated.

Table I. IMC-Based PID Controller Parameters[a]

	model	$y/y_s = \tilde{g}_+ f$	controller	$k_c k$	τ_I	τ_D	τ_F	comments
A	$\frac{k}{\tau s + 1}$	$\frac{1}{\epsilon s + 1}$	$\frac{1}{k}\frac{\tau s + 1}{\epsilon s}$	$\frac{\tau}{\epsilon}$	τ	–	–	–
B	$\frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{1}{\epsilon s + 1}$	$\frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k\epsilon s}$	$\frac{\tau_1 + \tau_2}{\epsilon}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$	–	–
C	$\frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{1}{\epsilon s + 1}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k\epsilon s}$	$\frac{2\zeta\tau}{\epsilon}$				
D	$k\frac{-\beta s + 1}{\tau s + 1}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{\tau s + 1}{k(\beta + \epsilon)s}$	$\frac{\tau}{\beta + \epsilon}$				
E	$k\frac{-\beta s + 1}{\tau s + 1}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{\tau s + 1}{ks(\beta\epsilon s + 2\beta + \epsilon)}$	$\frac{\tau}{2\beta + \epsilon}$				
F	$k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau + 1}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta + \epsilon)s}$	$\frac{2\zeta\tau}{\beta + \epsilon}$				
G	$k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta\epsilon s + 2\beta + \epsilon)s}$	$\frac{2\zeta\tau}{2\beta + \epsilon}$				
H	$\frac{k}{s}$	$\frac{1}{\epsilon s + 1}$	$\frac{1}{k\epsilon}$	$\frac{1}{\epsilon}$				
I	$\frac{k}{s}$	$\frac{2\epsilon + 1}{(\epsilon s + 1)^2}$	$\frac{2\epsilon s + 1}{k\epsilon^2 s}$	$\frac{2}{\epsilon}$				
J	$\frac{k}{s(\tau s + 1)}$	$\frac{1}{\epsilon s + 1}$	$\frac{\tau s + 1}{k\epsilon}$	$\frac{1}{\epsilon}$	–	τ	–	–
K	k	$2\epsilon s + 1$	$(\tau s + 1)(2\epsilon s + 1)$	$2\epsilon + \tau$	$2\epsilon + \tau$	$2\epsilon\tau$	–	(6)

Table II. IMC-Based PID Parameters for $g(s) = ke^{-\theta s}/(\tau s + 1)$ and Practical Recommendations for ϵ/θ

controller	kk_c	τ_I	τ_D	recommended ϵ/θ (> $0.1\tau/\theta$ *always*)
PID	$(2\tau + \theta)/(2\epsilon + \theta)$	$\tau + (\theta/2)$	$\tau\theta/(2\tau + \theta)$	>0.8
PI	$\theta/\tau = 0.1$	1.54		>1.7
improved PI	$(2\tau + \theta)/2\epsilon$	$\tau + (\theta/2)$		>1.7

Disadvantage IMC-PID (=Lambda tuning):

1. Many rules
2. Poor disturbance response for «slow» processes (with large τ_1/θ)

Probably the best **simple** PID tuning rules in the world

Sigurd Skogestad*
Department of Chemical Engineering
Norwegian University of Science and Technology
N–7491 Trondheim Norway

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Abstract

The aim of this paper is to present analytic tuning rules which are as simple as possible and still result in a good closed-loop behavior. The starting point has been the IMC PID tuning rules of Rivera, Morari and Skogestad (1986) which have achieved widespread industrial acceptance. The integral term has been modified to improve disturbance rejection for integrating processes. Furthermore, rather than deriving separate rules for each transfer function model, we start by approximating the process by a first-order plus delay processes (using the "half method"), and then use a single tuning rule. This is much simpler and appears to give controller tunings with comparable performance. All the tunings are derived analytically and are thus very suitable for teaching.

1 Introduction

Hundreds, if not thousands, of papers have been written on tuning of PID controllers, and one must question the need for another one. The first justification is that PID controller is by far the most widely used control algorithm in the process industry, and that improvements in tuning of PID controllers will have a significant practical impact. The second justification is that the simple rules and insights presented in this paper may contribute to a significantly improved understanding into how the controller should be tuned.

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Simple analytic rules for model reduction and PID controller tuning☆

Sigurd Skogestad*

Department of Chemical Engineering, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

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Abstract

The aim of this paper is to present analytic rules for PID controller tuning that are simple and still result in good closed-loop behavior. The starting point has been the IMC-PID tuning rules that have achieved widespread industrial acceptance. The rule for the integral term has been modified to improve disturbance rejection for integrating processes. Furthermore, rather than deriving separate rules for each transfer function model, there is a just a single tuning rule for a first-order or second-order time delay model. Simple analytic rules for model reduction are presented to obtain a model in this form, including the ''half rule'' for obtaining the effective time delay.

Keywords: Process control; Feedback control; IMC; PI-control; Integrating process; Time delay

1. Introduction

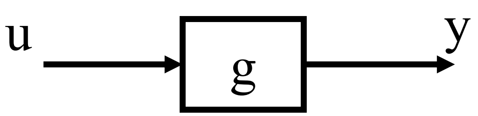
Although the proportional-integral-derivative (PID) controller has only three parameters, it is not easy, without a systematic procedure, to find good values (settings) for them. In fact, a visit to a process plant will usually show that a large number of the PID controllers are poorly tuned. The tuning rules presented in this paper have developed mainly as a result of teaching this material, where there are several objectives:

1. The tuning rules should be well motivated, and preferably model-based and analytically derived.
2. They should be simple and easy to memorize.
3. They should work well on a wide range of processes.

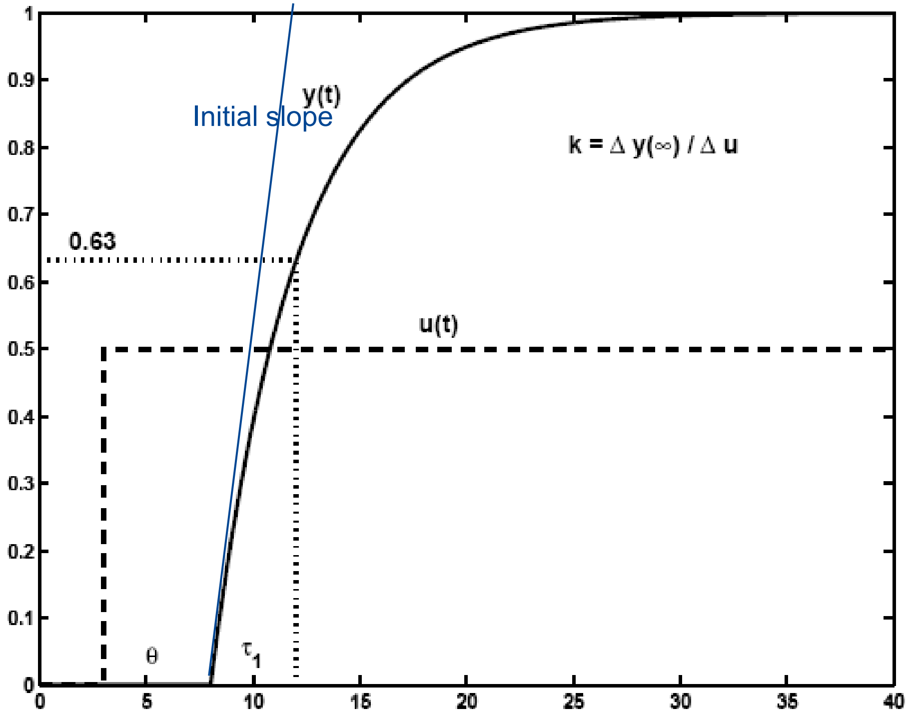
Step 2. Derive model-based controller settings. PI-settings result if we start from a first-order model, whereas PID-settings result from a second-order model.

There has been previous work along these lines, including the classical paper by Ziegler amd Nichols [1], the IMC PID-tuning paper by Rivera et al. [2], and the closely related direct synthesis tuning rules in the book by Smith and Corripio [3]. The Ziegler–Nichols settings result in a very good disturbance response for integrating processes, but are otherwise known to result in rather aggressive settings [4,5], and also give poor performance for processes with a dominant delay. On the other hand, the analytically derived IMC-settings in [2] are known to result in a poor disturbance response for integrating processes (e.g., [6,7]), but are robust and

Process model: First-order with delay



Open-loop step response

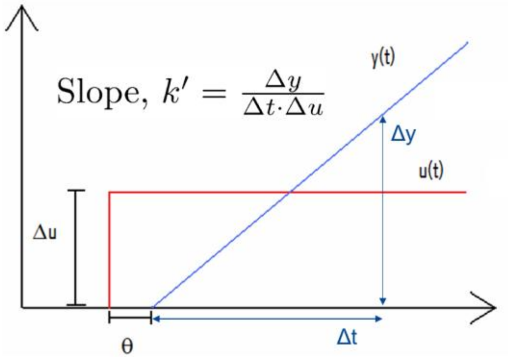


$$g(s) = k\,\frac{e^{-\theta s}}{\tau_1 s + 1}$$

k = gain
θ = time delay
τ = time constant (additional time to reach 63%)

Initial slope:
k' = k/ τ_1

Slow/Integrating process: Don't need τ for tuning – obtain slope k' instead
So if not settling after about 5θ: stop experiment and read off slope k'.



SIMC PI tuning rule

1. Approximate process as first-order with delay
   - k = process gain
   - τ_1 = process time constant
   - θ = process delay

2. Derive SIMC tuning rule:

$$K_c = \frac{1}{k}\frac{\tau_1}{(\tau_c+\theta)} = \frac{1}{k'}\frac{1}{(\tau_c+\theta)}$$

$$\tau_I = \min(\tau_1, 4(\tau_c+\theta))$$

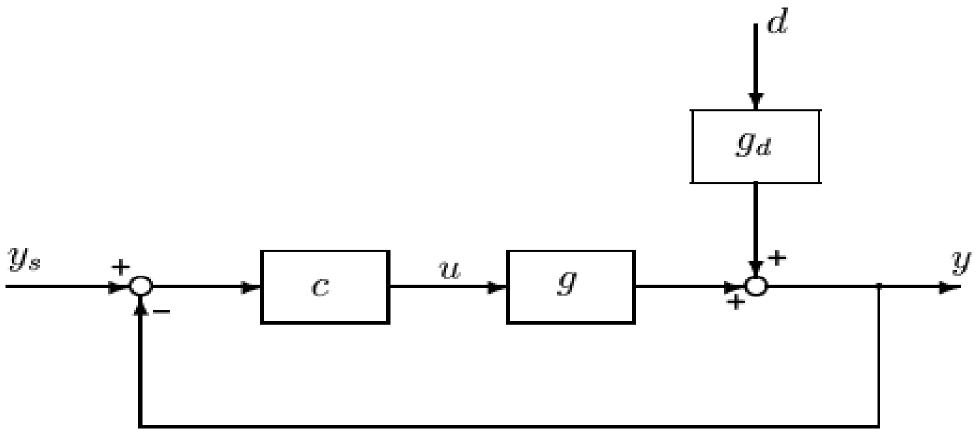
$\tau_c \geq \theta$: Desired closed-loop response time (tuning parameter)

Integral time rule combines well-known rules:
IMC (Lambda-tuning): Same as SIMC for small τ_1 ($\tau_I = \tau_1$)
Ziegler-Nichols: Similar to SIMC for large τ_1 (if we choose $\tau_c = 0$; aggressive!)

- Dominant 2nd order process. Add derivative time $\tau_D = \tau_2$ (note: this is for series PID-form)

S. Skogestad, "Simple analytic rules for model reduction and PID controller design", *J.Proc.Control*, Vol. 13, 291-309, 2003

Basis: Direct synthesis (IMC)



Closed-loop response to setpoint change:

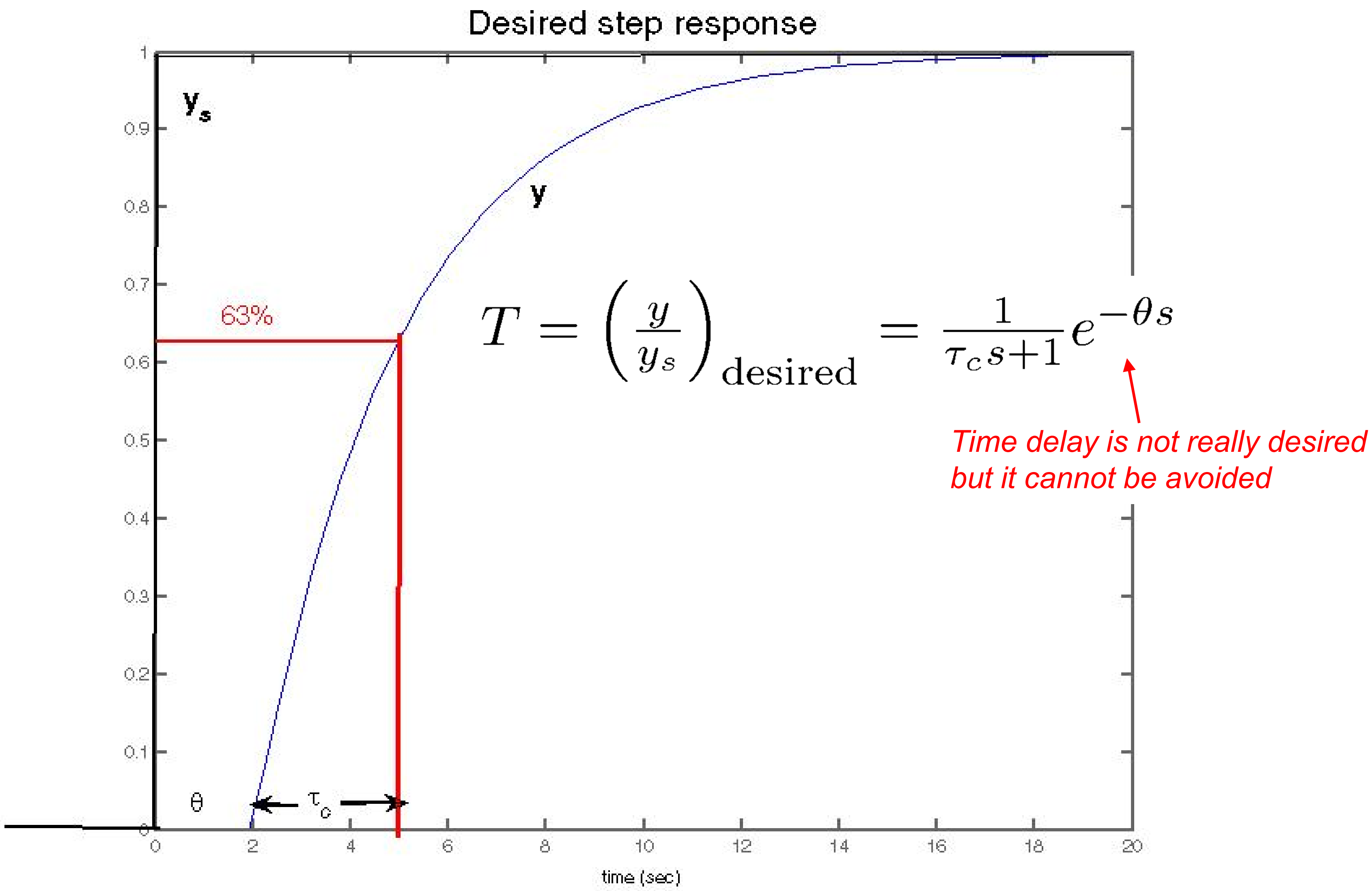
$$y = T\, y_s, \qquad T(s) = \frac{gc}{1+gc}$$

Idea: specify desired response T and from this get the controller:

$$c = \frac{1}{g} \cdot \frac{1}{\frac{1}{T} - 1}$$

SIMC-tunings

Note: Process g has time delay (θ)



$$T = \left(\frac{y}{y_s}\right)_{\text{desired}} = \frac{1}{\tau_c s + 1} e^{-\theta s}$$

NOTE: Setting the steady-state gain = 1 in T will result in integral action in the controller!

IMC Tuning = Direct Synthesis

Algebra:

- Controller: $c(s) = \frac{1}{g(s)} \cdot \frac{1}{\frac{1}{(y/y_s)_{\text{desired}}} - 1}$
- Consider second-order with delay plant: $g(s) = k\frac{e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$
- Desired first-order setpoint response: $\left(\frac{y}{y_s}\right)_{\text{desired}} = \frac{1}{\tau_c s+1}e^{-\theta s}$
- Gives a "Smith Predictor" controller: $c(s) = \frac{(\tau_1 s+1)(\tau_2 s+1)}{k}\frac{1}{(\tau_c s+1-e^{-\theta s})}$
- To get a PID-controller use $e^{-\theta s} \approx 1 - \theta s$ and derive

$$c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k}\frac{1}{(\tau_c + \theta)s}$$

  which is a cascade form PID-controller with

$$\boxed{K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta}; \quad \tau_I = \tau_1; \quad \tau_D = \tau_2}$$

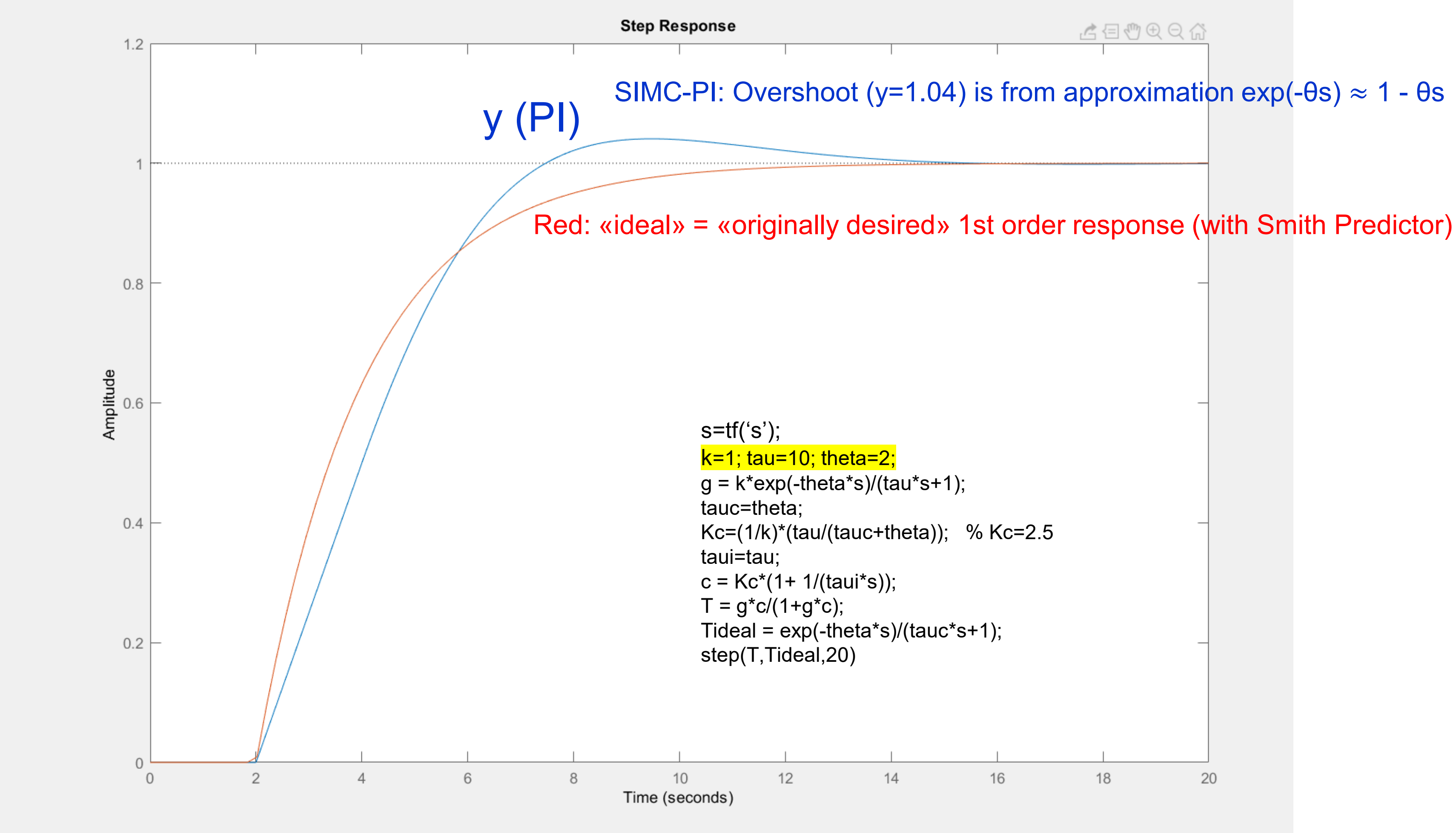
- τ_c is the sole tuning parameter

IMC-tuning is the same as "Lambda-tuning": τ_c is sometimes called λ

Surprisingly, this PID-controller is generally better, or at least more robust with respect to changes in the time delay θ, than the Smith Predictor controller from which it was derived. We are lucky ☺.

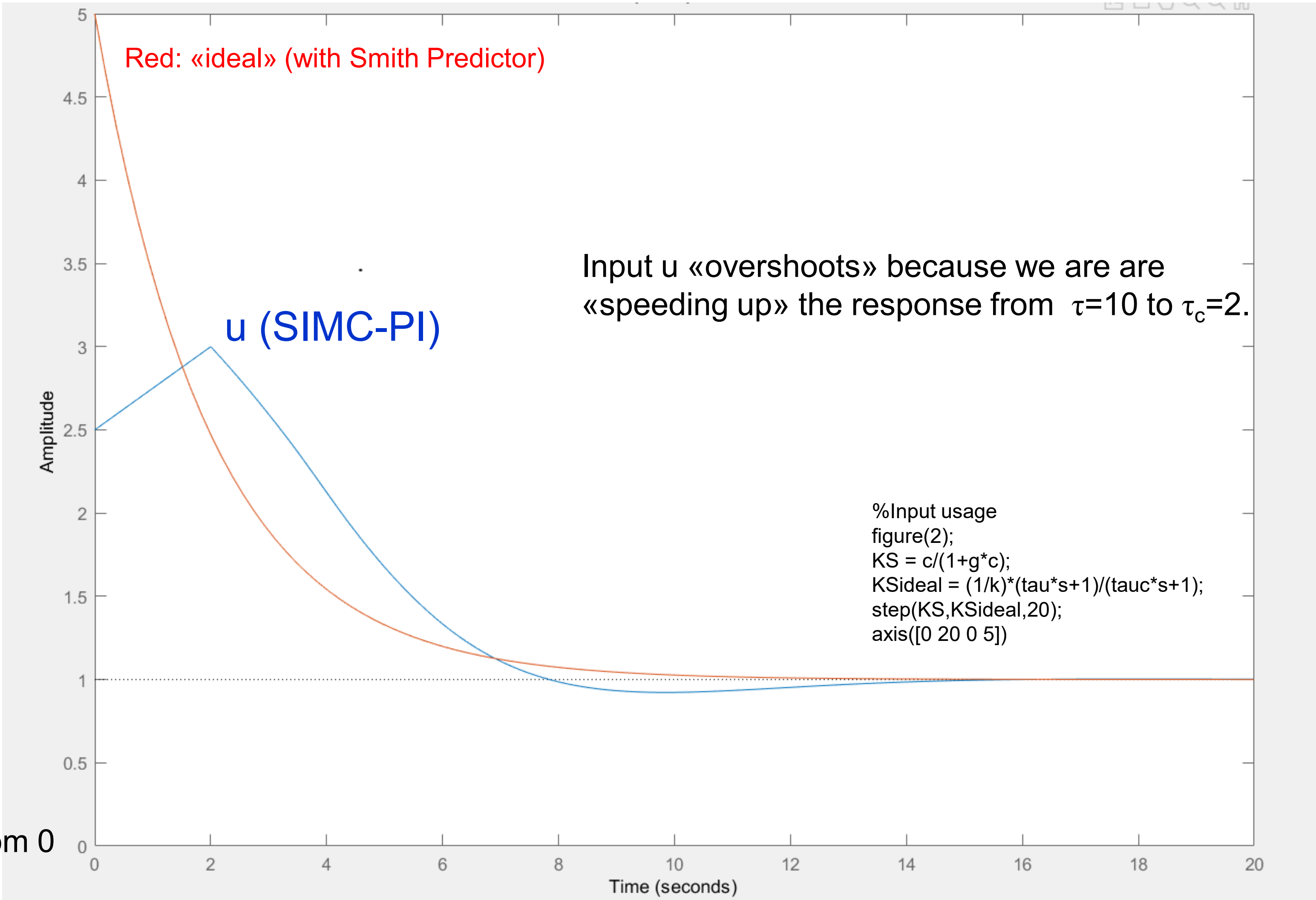
Reference: Chriss Grimholt and Sigurd Skogestad. **"Should we forget the Smith Predictor?"** **(2018)**
In 3rd IFAC conference on Advances in PID control, Ghent, Belgium, 9-11 May 2018. *In IFAC papers Online (2018)* .

Example step setpoint response (with choice $\tau_c = \theta = 2$)




```
s=tf('s');
k=1; tau=10; theta=2;
g = k*exp(-theta*s)/(tau*s+1);
tauc=theta;
Kc=(1/k)*(tau/(tauc+theta));   % Kc=2.5
taui=tau;
c = Kc*(1+ 1/(taui*s));
T = g*c/(1+g*c);
Tideal = exp(-theta*s)/(tauc*s+1);
step(T,Tideal,20)
```

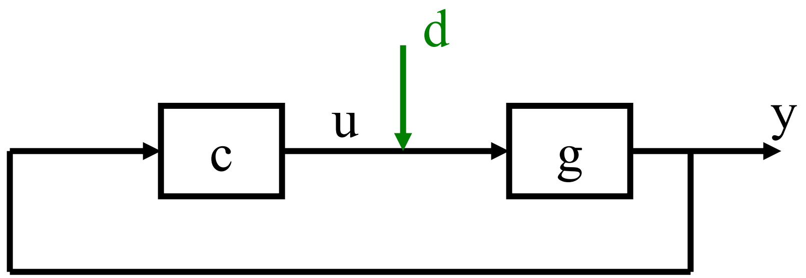
Input usage for setpoint response



Input starts from 0

Integral time

- Found: Integral time = dominant time constant ($\tau_I = \tau_1$)
- Gives P-controller for integrating process ($\tau_I = \infty$)
  - This works well for setpoint changes
  - But: τ_I needs to be modified (reduced) for integrating disturbances



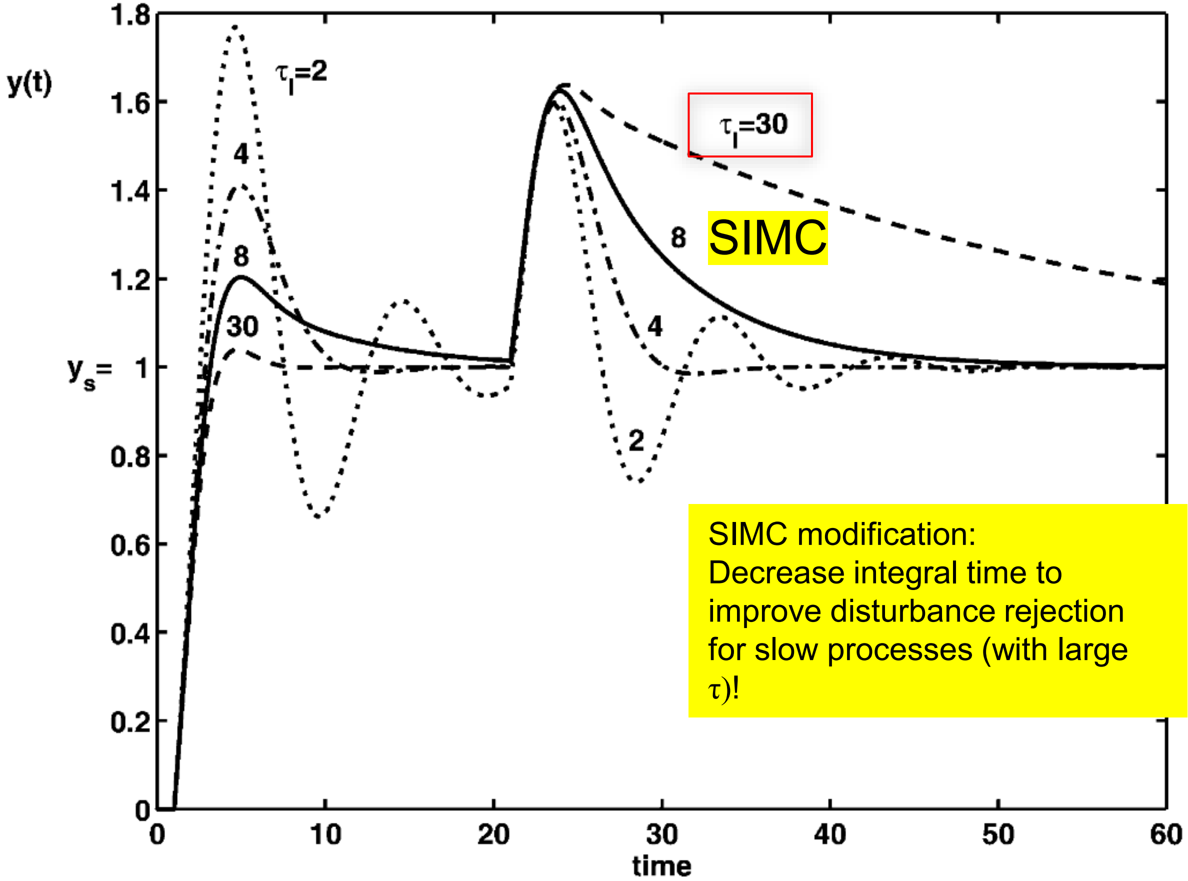
Example. “Almost-integrating process” with disturbance at input:

$G(s) = e^{-s}/(30s+1)$

Original integral time $\tau_I = 30$ gives poor disturbance response

Try reducing it!

Effect of decreasing τ_I



SIMC modification:
Decrease integral time to improve disturbance rejection for slow processes (with large τ)!

Fig. 3. Effect of changing the integral time τ_I for PI-control of "almost integrating" process $g(s) = e^{-s}/(30s + 1)$ with $K_c = 15$. Unit setpoint change at $t = 0$; load disturbance of magnitude 10 at $t = 20$.

Integral time correction

- Want to reduce the integral time for "integrating" processes
- But to avoid "slow oscillations" (not caused by the delay θ) we must require $k'K_c\tau_I \geq 4$, which with the SIMC-rule for K_c gives:

$$\tau_I \geq 4(\tau_C + \theta)$$

- Proof:

$$G(s) = k\frac{e^{-\theta s}}{\tau_1 s+1} \approx \frac{k'}{s} \text{ where } k' = \frac{k}{\tau_1};\ C(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

Closed-loop poles:

$$1 + GC = 0 \Rightarrow 1 + \frac{k'}{s}K_c\left(1 + \frac{1}{\tau_I s}\right) = 0 \Rightarrow \tau_I s^2 + k'K_c\tau_I s + k'K_c = 0$$

To avoid oscillations we must not have complex poles:

$$\boxed{B^2 - 4AC \geq 0} \Rightarrow k'^2K_c^2\tau_I^2 - 4k'K_c\tau_I \geq 0 \Rightarrow \boxed{k'K_c\tau_I \geq 4} \Rightarrow \tau_I \geq \frac{4}{k'K_c}$$

Inserted SIMC-rule for $K_c = \frac{1}{k'}\frac{1}{\tau_c+\theta}$ then gives

$$\tau_I \geq 4(\tau_c + \theta)$$

Conclusion: SIMC-PID Tuning Rules

For cascade form PID controller:

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c+\theta} = \frac{1}{k'}\cdot\frac{1}{\tau_c+\theta} \quad (1)$$

$$\tau_I = \min\{\tau_1, \frac{4}{k'\ K_c}\} = \min\{\tau_1, 4(\tau_c+\theta)\} \quad (2)$$

$$\tau_D = \tau_2 \quad (3)$$

Derivation:

1. First-order setpoint response with response time τ_c (IMC-tuning = "Direct synthesis")
2. Reduce integral time to get better disturbance rejection for slow or integrating process (but avoid slow cycling $\Rightarrow \tau_I \geq \frac{4}{k'\ K_c}$)

One tuning parameter: τ_c

Some special cases

Process	$g(s)$	K_c	τ_I	$\tau_D^{(4)}$
First-order	$k\frac{e^{-\theta s}}{(\tau_1 s+1)}$	$\frac{1}{k}\frac{\tau_1}{\tau_c+\theta}$	$\min\{\tau_1, 4(\tau_c+\theta)\}$	-
Second-order, eq.(4)	$k\frac{e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{k}\frac{\tau_1}{\tau_c+\theta}$	$\min\{\tau_1, 4(\tau_c+\theta)\}$	τ_2
Pure time delay$^{(1)}$	$ke^{-\theta s}$	0	$0^{\ (*)}$	-
Integrating$^{(2)}$	$k'\frac{e^{-\theta s}}{s}$	$\frac{1}{k'}\cdot\frac{1}{(\tau_c+\theta)}$	$4(\tau_c+\theta)$	-
Integrating with lag	$k'\frac{e^{-\theta s}}{s(\tau_2 s+1)}$	$\frac{1}{k'}\cdot\frac{1}{(\tau_c+\theta)}$	$4(\tau_c+\theta)$	τ_2
Double integrating$^{(3)}$	$k''\frac{e^{-\theta s}}{s^2}$	$\frac{1}{k''}\cdot\frac{1}{4(\tau_c+\theta)^2}$	$4\ (\tau_c+\theta)$	$4\ (\tau_c+\theta)$

Table 1: SIMC PID-settings (23)-(25) for some special cases of (4) (with τ_c as a tuning parameter).

(1) The pure time delay process is a special case of a first-order process with $\tau_1 = 0$.

(2) The integrating process is a special case of a first-order process with $\tau_1 \to \infty$.

(3) For the double integrating process, integral action has been added according to eq.(27).

(4) The derivative time is for the series form PID controller in eq.(1).

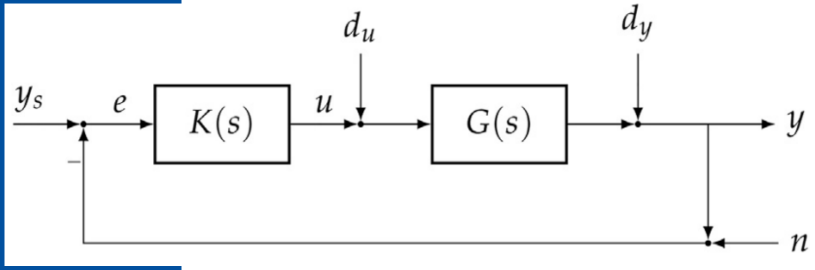
(*) Pure integral controller $c(s) = \frac{K_I}{s}$ with $K_I \stackrel{\text{def}}{=} \frac{K_c}{\tau_I} = \frac{1}{k(\tau_c+\theta)}$.

One tuning parameter: τ_c

(1)(*) Note that we get pure I-controller for static process with delay.

Choice of SIMC-tuning parameter τ_c.

1. Trade-off between robustness (Ms) and performance (J=IAE)



2.1. Performance

In this paper, we quantify performance in terms of the integrated absolute error (IAE),

$$\mathrm{IAE} = \int_0^{\infty} |y(t) - y_s(t)|\mathrm{d}t. \quad (7)$$

To balance the servo/regulatory trade-off we choose a weighted average of the IAE for a step input disturbance d_u (load disturbance) and a step output disturbance d_y:

$$J(p) = 0.5\left(\frac{\mathrm{IAE}_{dy}(p)}{\mathrm{IAE}^{\circ}_{dy}} + \frac{\mathrm{IAE}_{du}(p)}{\mathrm{IAE}^{\circ}_{du}}\right) \quad (8)$$

where $\mathrm{IAE}^{\circ}_{dy}$ and $\mathrm{IAE}^{\circ}_{du}$ are weighting factors, and p denotes the controller parameters. Note that we do not consider setpoint responses, but instead output disturbances. For the system in Fig. 1,

Ms = Peak of |S(jω)| = 1/(smallest distance to (-1)-point). Want less than 1.7

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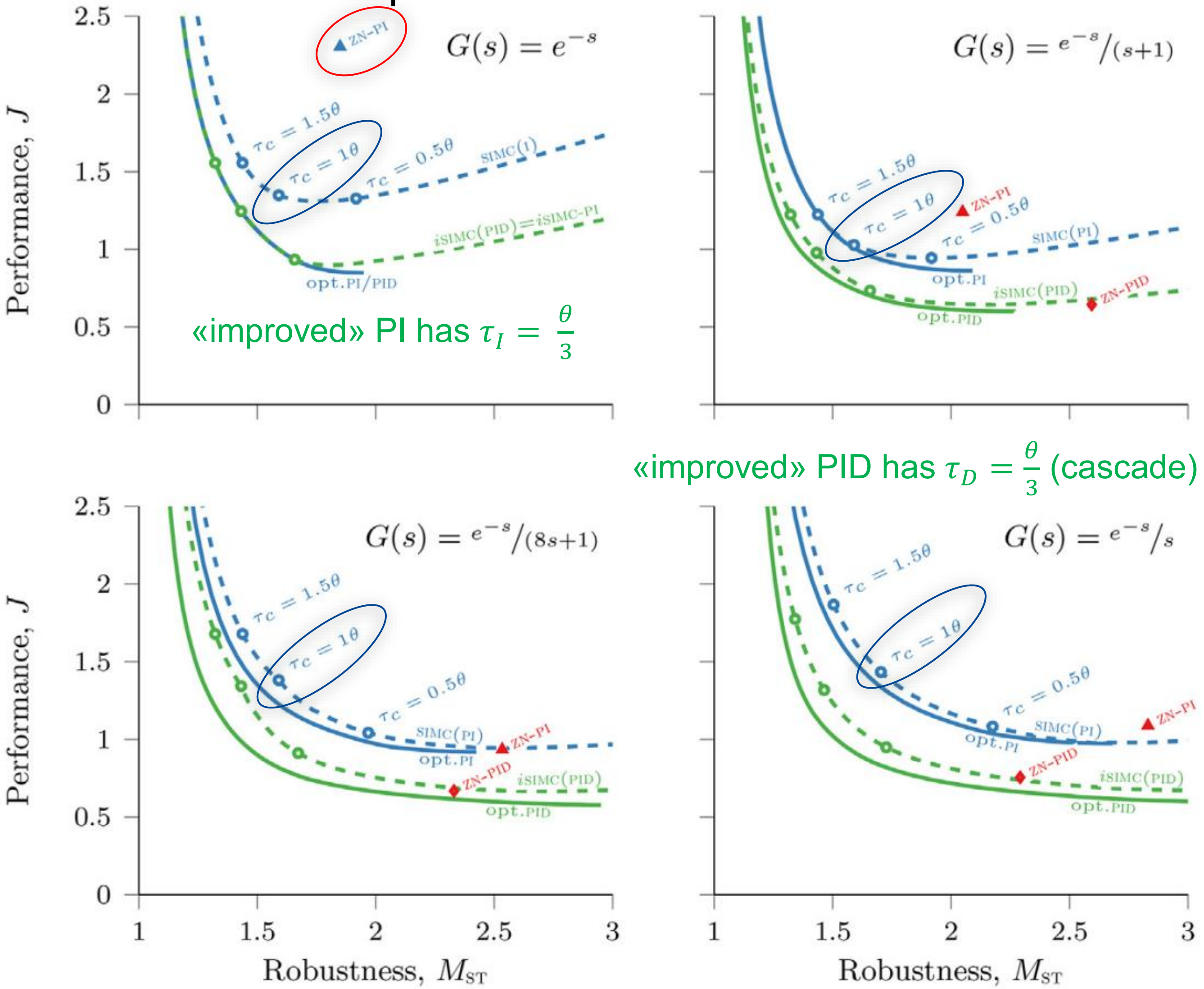
Pareto-optimal curves

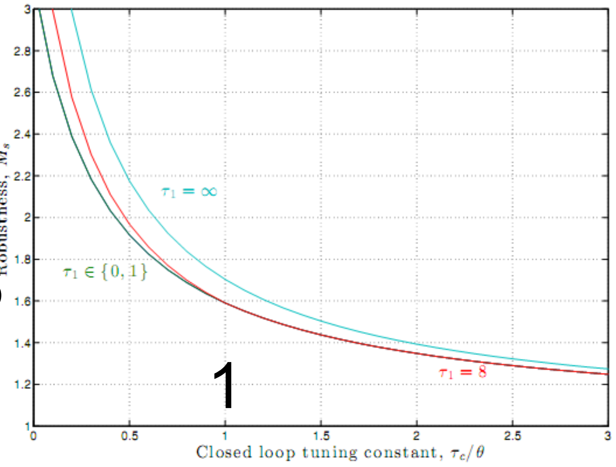


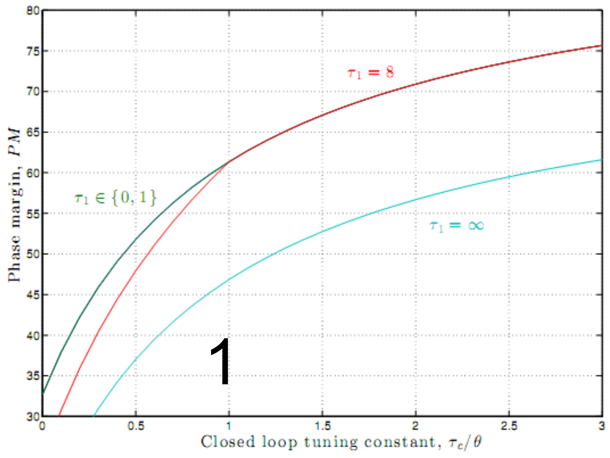
Optimal PI and PID control of first-order plus delay processes and evaluation of the original and improved SIMC rules
Chriss Grimholt, Sigurd Skogestad Journal of Process Control 70 (2018) 36–46

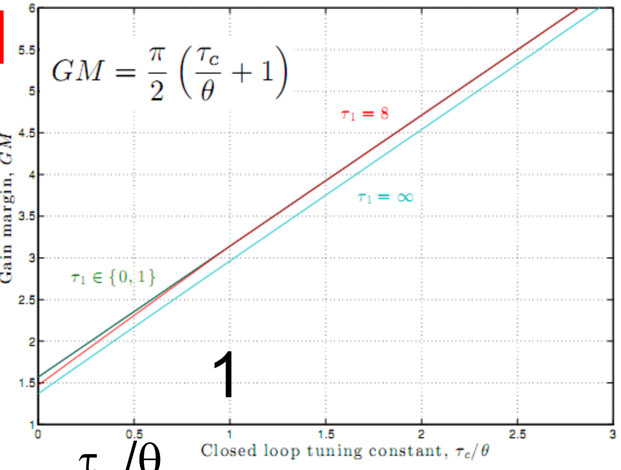
Choice of tuning SIMC-parameter τ_c.

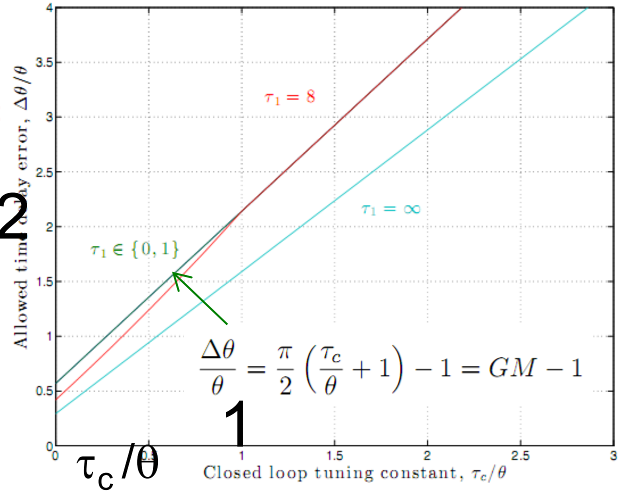
2. Relationship between τ_c and robustness (Ms, GM, PM, DM)

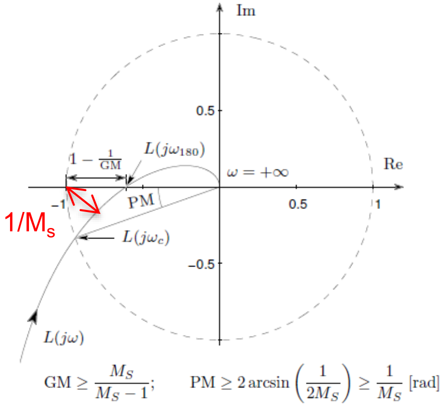
Conclusion: $\tau_c/\theta = 1$ gives a acceptable robustness (Ms=1.6, PM=60°, GM=3, DM=2)











SIMC: GM and DM increase linearly with τ_c

Chriss Grimholt and Sigurd Skogestad.
"Optimal PI-Control and Verification of the SIMC Tuning Rule".
Proceedings *IFAC conference on Advances in PID control* (PID'12), Brescia, Italy, 28-30 March 2012.

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Typical closed-loop SIMC responses with the choice $\tau_c=\theta$ (delay)

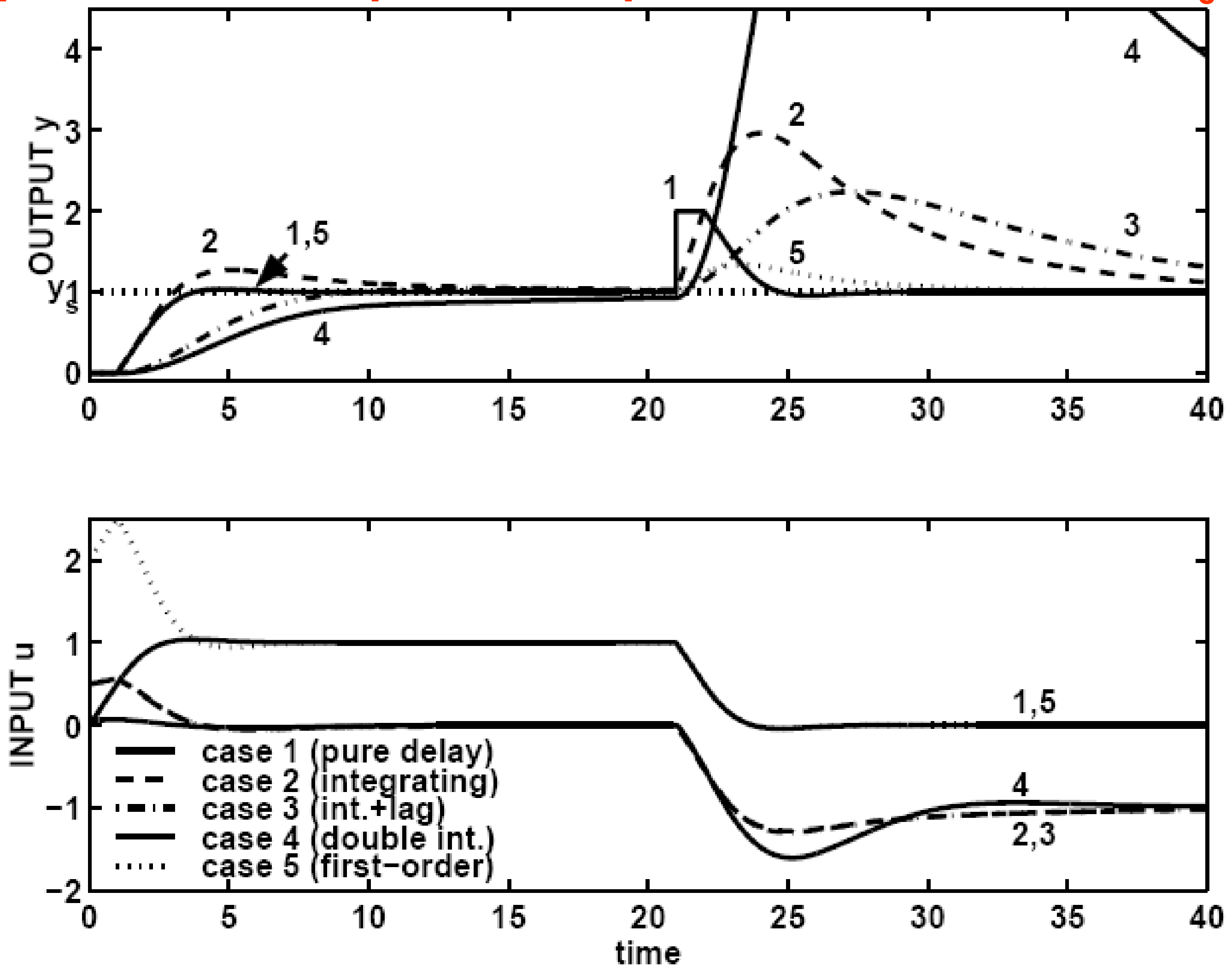


Figure 4: Responses using SIMC settings for the five time delay processes in Table 3 ($\tau_c = \theta$).
Unit setpoint change at $t = 0$; Unit load disturbance at $t = 20$.
Simulations are without derivative action on the setpoint.
Parameter values: $\theta = 1, k = 1, k' = 1, k'' = 1$.

6.3 Ideal PID controller

The settings given in this paper (K_c, τ_I, τ_D) are for the series (cascade, "interacting") form PID controller in (1). To derive the corresponding settings for the ideal (parallel, "non-interacting") form PID controller

$$\text{Ideal PID}: \quad c'(s) = K_c'\left(1 + \frac{1}{\tau_I' s} + \tau_D' s\right) = \frac{K_c'}{\tau_I' s}\left(\tau_I'\tau_D' s^2 + \tau_I' s + 1\right) \tag{35}$$

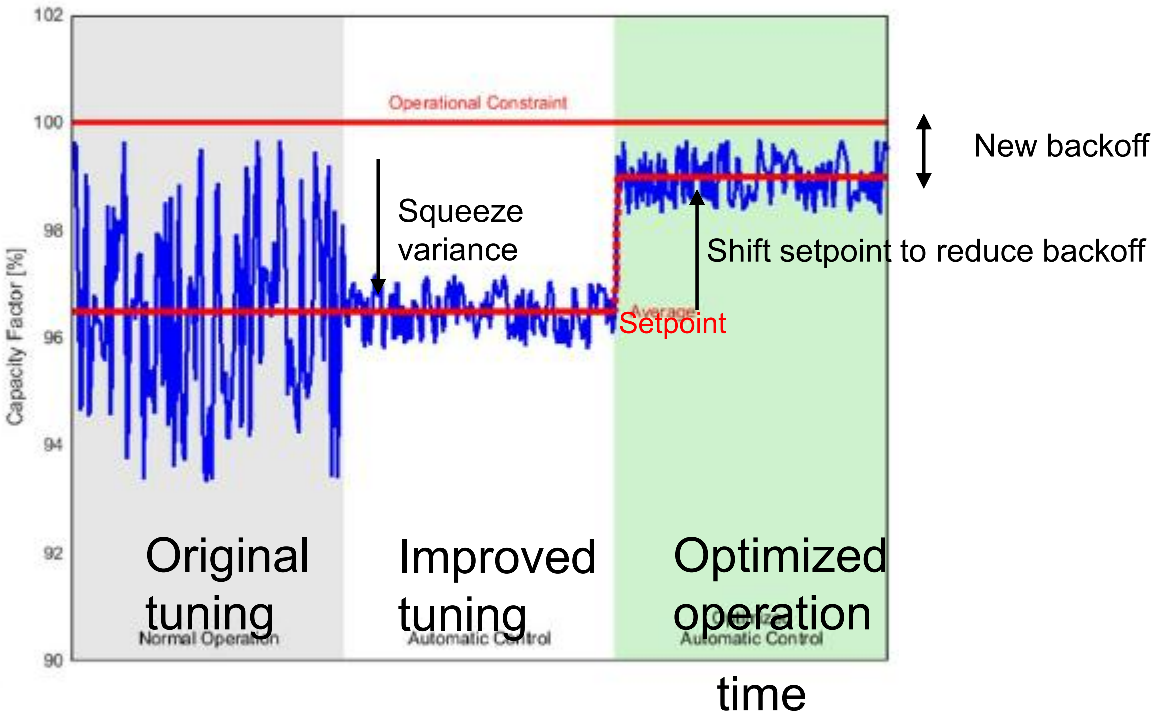
we use the following translation formulas

$$K_c' = K_c\left(1 + \frac{\tau_D}{\tau_I}\right); \quad \tau_I' = \tau_I\left(1 + \frac{\tau_D}{\tau_I}\right); \quad \tau_D' = \frac{\tau_D}{1 + \frac{\tau_D}{\tau_I}} \tag{36}$$

Example. *Consider the second-order process $g/s) = e^{-s}/(s+1)^2$ (E9) with the $k = 1, \theta = 1, \tau_1 = 1$ and $\tau_2 = 1$. The series-form SIMC settings are $K_c = 0.5$, $\tau_I = 1$ and $\tau_D = 1$. The corresponding settings for the ideal PID controller in (35) are $K_c' = 1$, $\tau_I' = 2$ and $\tau_D' = 0.5$. The robustness margins with these settings are given by the first column in Table 2.*

When do we need «tight control»?
For hard constraints where backoff is costly

«SQEEZE and SHIFT» RULE



Selection of tuning parameter τ_c

Two main cases

1. TIGHT CONTROL (τ_c small): Want "fastest possible control" subject to having good robustness
   - Want tight control of active constraints ("squeeze and shift")
   - Select $\tau_c = \theta$ (effective delay)

2. SMOOTH CONTROL (τ_c large): Want "slowest possible control" subject to acceptable disturbance rejection
   - Prefer smooth control if fast control is not required

Tuning for smooth control

- Tuning parameter: τ_c = desired closed-loop response time
- Selecting $\tau_c=\theta$ if we need "tight control" of y.
- Other cases: "Smooth control" of y is sufficient, so select $\tau_c > \theta$ for
  - slower control
  - smoother input usage
    - less disturbing effect on rest of the plant
  - less sensitivity to measurement noise
  - better robustness
- Question: Given that we require some disturbance rejection.
  - What is the largest possible value for τ_c ?
  - ANSWER: $\tau_{c,max} = 1/\omega_d$ (where ω_d is defined as the frequence where $|g_d(j\omega_d)| = y_{max}/d_{max}$)

Proof. y=Sgd d, where S=(1+L). Require |y|<ymax at all frequencies, so |S| < |gd| d/ymax at all frequencies.
The integral action takes care of most of the disturbance rejection, so usually, the «worst-case» frequency is where |S| reaches 1, which is approximately at wc=1/tauc.
So define wd as the frequency where (gd/g) d/ymax = 1 and we must require wc > wd or equivalently tauc < 1/wd. Thus we have tauc.maxc=1(wd.

This bound may be optimistic if there are disturbances with two or more «slow» poles, because then the worst-case frequency may be lower than wc.

Comment: An simpler (but sometimes conservative) answer is to select **Kc,min =|ud|/|ymax|** where |ud| is the input magnitude to reject the maximum disturbance. (Given Kc,min we may obtain the corresponding tauc,max using the SIMC-rule for Kc).
.
More detailed proof: S. Skogestad, ``Tuning for smooth PID control with acceptable disturbance rejection'', *Ind.Eng.Chem.Res*, **45** (23), 7817-7822 (2006).

Level control

- Level control often causes problems
- Typical story:
  - Level loop starts oscillating
  - Operator detunes by decreasing controller gain
  - Level loop oscillates even more
  - ......
- ???
- Explanation: Level is by itself unstable and requires control.

Level control (integrating process): Can have both fast and slow oscillations

- Fast oscillations (K_c too high): $P < \pi \tau_I$
  - Caused by (effective) time delay
- Slow oscillations (K_c too low): $P > \pi \tau_I$
  - Caused by integral action in controller
  - Avoid slow oscillations: $k' K_c \tau_I \geq 4$.

P=period of oscillations = $2\pi/\omega$

How avoid slowly oscillating levels?

- Simplest: Use P-control only (no integral action)
- If you insist on integral action, then make sure the controller gain is sufficiently large
- If you have a level loop that is oscillating then use *Sigurds rule* (can be derived):

To avoid oscillations, increase $K_c \cdot \tau_I$ by factor

$$f = 0.1 \cdot (P_0/\tau_{I0})^2$$

where

P_0 = period of oscillations [s]

τ_{I0} = original integral time [s]

$0.1 \approx 1/\pi^2$

$$P_0 = \frac{2\pi}{\sqrt{1-\zeta^2}}\tau_0 = \frac{2\pi}{\sqrt{1-\zeta^2}}\sqrt{\frac{\tau_{I0}}{k' K_{c0}}} \approx 2\pi\sqrt{\frac{\tau_{I0}}{k' K_{c0}}} \tag{39}$$

where we have assumed $\zeta^2 << 1$ (significant oscillations). Thus, from (39) the product of the original controller gain and integral time is approximately

$$K_{c0} \cdot \tau_{I0} = (2\pi)^2 \frac{1}{k'}\left(\frac{\tau_{I0}}{P_0}\right)^2$$

To avoid oscillations ($\zeta \geqslant 1$) with the new settings we must from (21) require $K_c\tau_I \geqslant 4/k'$, that is, we must require that

$$\frac{K_c \tau_I}{K_{c0}\tau_{I0}} \geqslant \frac{1}{\pi^2} \cdot \left(\frac{P_0}{\tau_{i0}}\right)^2 \tag{40}$$

Here $1/\pi^2 \approx 0.10$, so we have the **rule**:

- To avoid "slow" oscillations of period P_0 the product of the controller gain and integral time should be increased by a factor $f \approx 0.1(P_0/\tau_{I0})^2$.

$$\textit{Avoid slow oscillations: } k' K_C \tau_I \geq 4$$

Case study oscillating level

- We were called upon to solve a problem with oscillations in a distillation column
- Closer analysis: Problem was oscillating reboiler level in upstream column
- Use of Sigurd’s rule solved the problem

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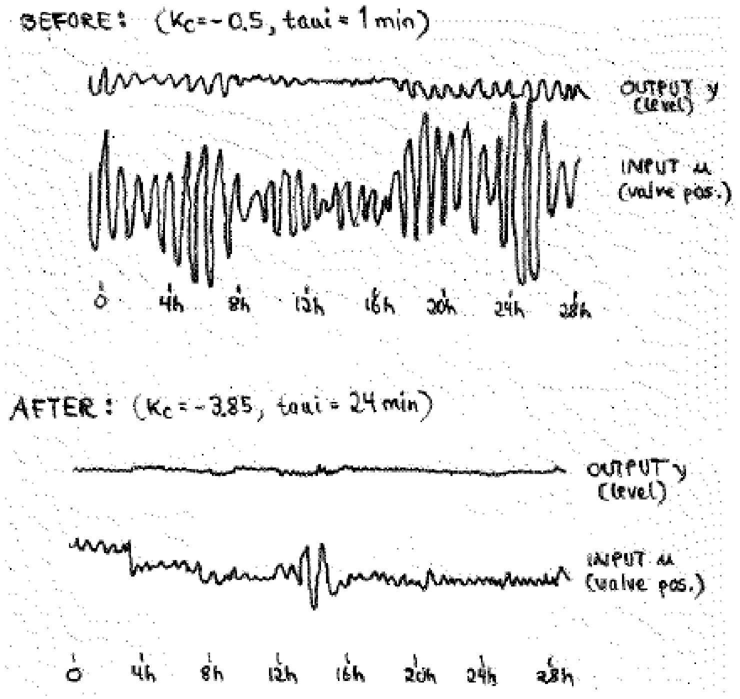
APPLICATION: RETUNING FOR INTEGRATING PROCESS

To avoid "slow" oscillations the product of the controller gain and integral time should be increased by factor $f \approx 0.1(P_0/\tau_{I0})^2$.

Real Plant data:

$$\text{Period of oscillations } P_0 = 0.85h = 51min \Rightarrow f = 0.1 \cdot (51/1)^2 = 260$$



Default PI tunings

- In many cases the PI controller can be tuned without any data

Default tuning: 5&5 PI-rule for level controller (LC)

- **Scale both flow (u) and level (y) in range 0-100%**
- **Rule: K_c=5 %/%,** $\boldsymbol{\tau_I = 5\tau}$ (larger integral time τ_I than normal $\tau_I = 0.8\,\tau$ for SIMC)
  - $\boldsymbol{\tau}$ **= residence time** at max (100%) flow and full (100%) tank = Vmax/Fmax (= 10 min typical)
  - K_c=5%/% means: 1% change in level (y) gives 5% change in flowrate (u).

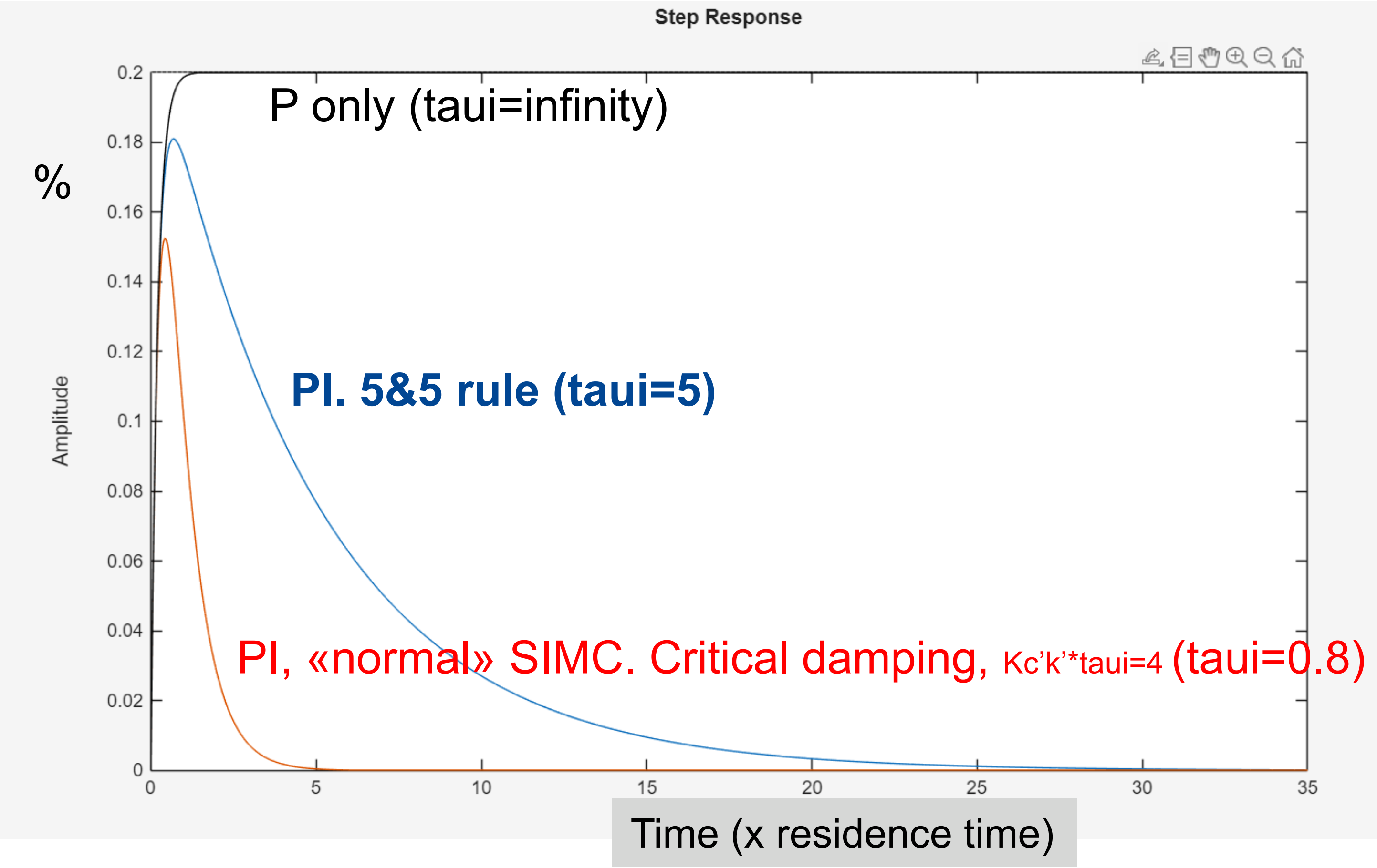
Note:

- K_c=5 corresponds to $\tau_c = \frac{\tau}{K_c} = 0.2\,\tau$
- and gives $K_c k' \tau_I = 25$ (which is well above the lower SIMC-limit of 4 to avoid oscillations)

Proof of notes: With this scaling, the model from flow [%] to level [%] is G(s) = k'/s where k'=1/tau.
SIMC: Kc=(1/k')(1/tauc) -> tauc=(1/(k'Kc)) = 0.2 tau (with Kc=5).

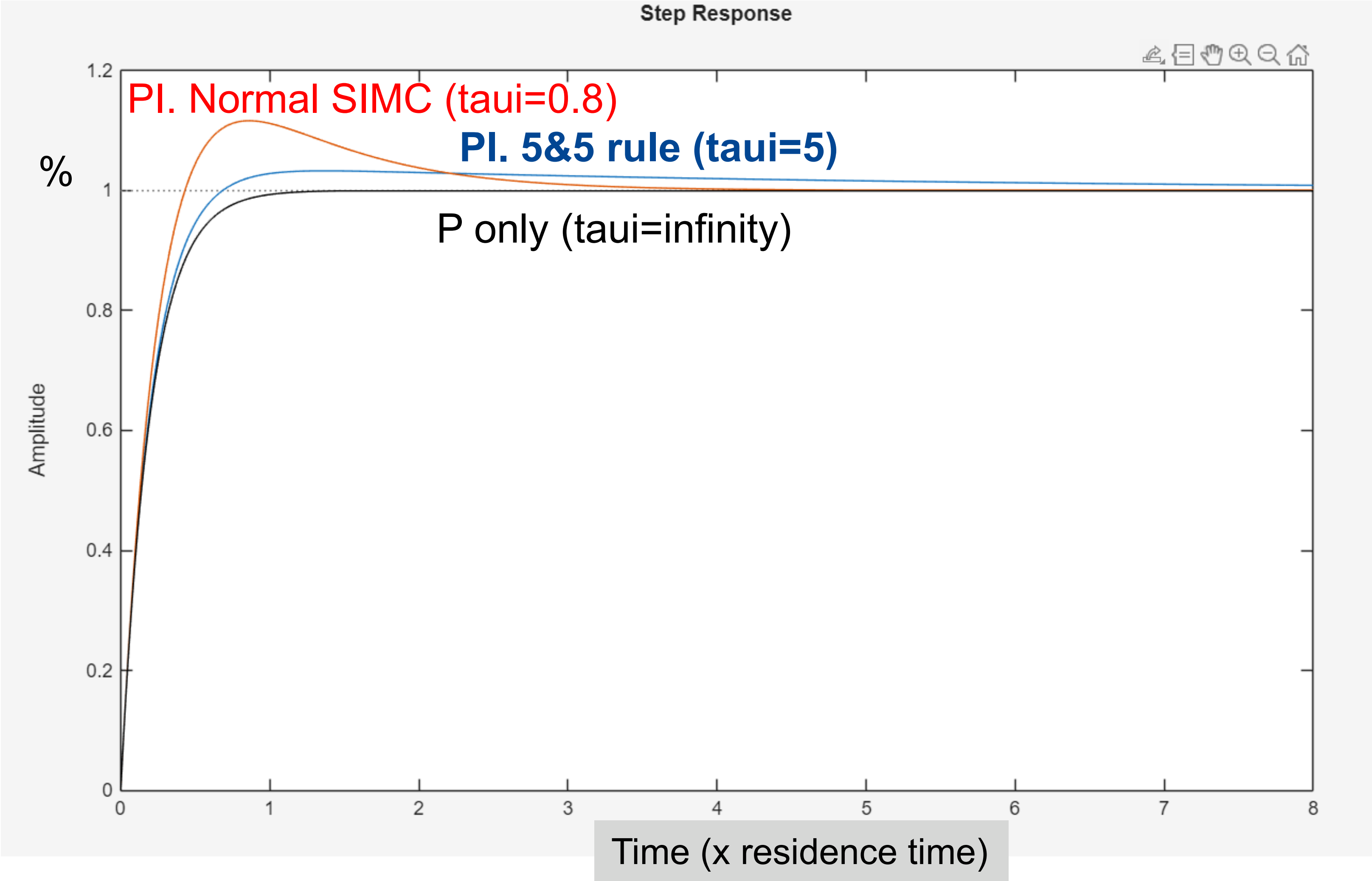
Level (y) in response to step disturbance (1% increase inflow)

In all cases, Kc=5%/%.



Input (u) (outflow) response to disturbance

= Level (y) Setpoint response = (both have same transfer function T)



$y = T\, y_s$
$u = T\, d_u$

Default tuning: Flow controller (FC)*

- $y = G(s)\,u$, u=valve position (0-100%), y=flowrate (0-100%)
- Usually fast dynamics, $G(s) \approx k$, where k=1 %/% (linear valve)
  - SIMC* gives pure I-controller, $C(s) = \frac{K_I}{s}$ $with\ K_I = \frac{K_c}{\tau_I} = \frac{1}{k\,(\tau_c+\theta)} = \frac{1}{\tau_c}$
    - τ_c = closed-loop response time: should be significantly larger than any delay θ to avoid instability
  - So the I-action is the most important for flow control ($\Rightarrow K_c$-value not so important)
    - Comment: Whereas for level control, the P-action is most important ($\Rightarrow \tau_I$ -value not so important)
- FC with PI-control (pick a $K_c \neq 0$): From SIMC K_I-value use: $\boxed{\tau_I = K_c k\,\tau_c = K_c\,\tau_c}$
  - $K_c = 0.2$. $Gives\ \tau_I = 0.2\,\tau_c$ $(= 1\,s\ with\ \tau_c = 5s)$.
  - $K_c = 0.5$ $Gives\ \tau_I = 0.5\,\tau_c$ $(= 2.5\,s\ with\ \tau_c = 5s)$.
  - $K_c = 1^{**}$ $Gives\ \tau_I = \tau_c$ $(= 5\,s\ with\ \tau_c = 5s)$.
  - According to SIMC, a larger Kc is for cases with a relatively large valve time constant τ.

Question to ABB: Does this seem reasonable?

- Note: From improved SIMC-PI, a better alternative is to fix $\tau_I = \frac{\theta}{3} + \tau$ and use $K_c = \frac{\tau_I}{k\,\tau_c}$

«Improved» SIMC PI-rule for $G(s) = k\frac{e^{-\theta s}}{\tau\,s+1}$: $K_c = \frac{1}{k}\frac{\tau+\frac{\theta}{3}}{\tau_c+\theta}, \tau_I = \tau + \frac{\theta}{3}$.

Chriss Grimholt and Sigurd Skogestad, "Optimal PI and PID control of first-order plus delay processes and evaluation of the original and improved SIMC rules" ,
Published in: J. Process Control, vol. 70 (2018), 36-46.

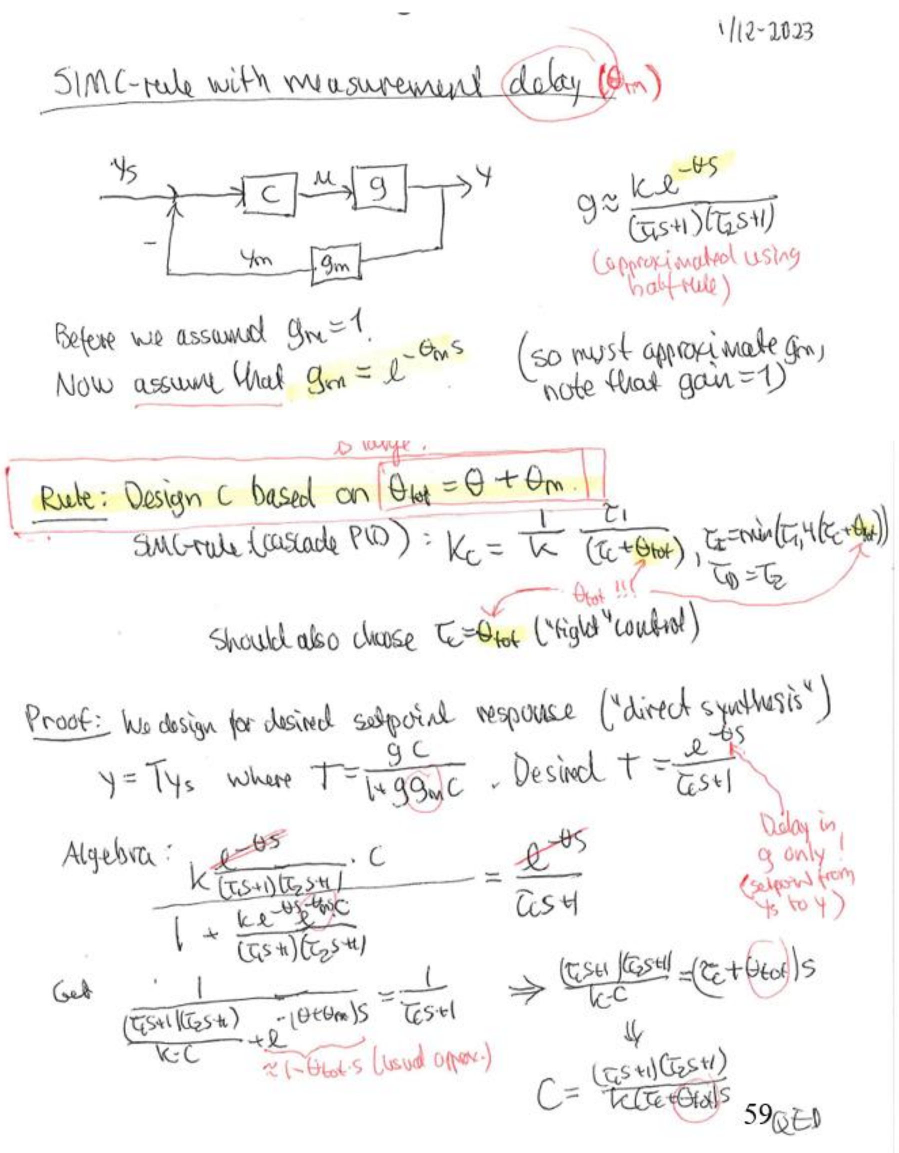
- A little more on PID control

SIMC-rule with measurement dynamics

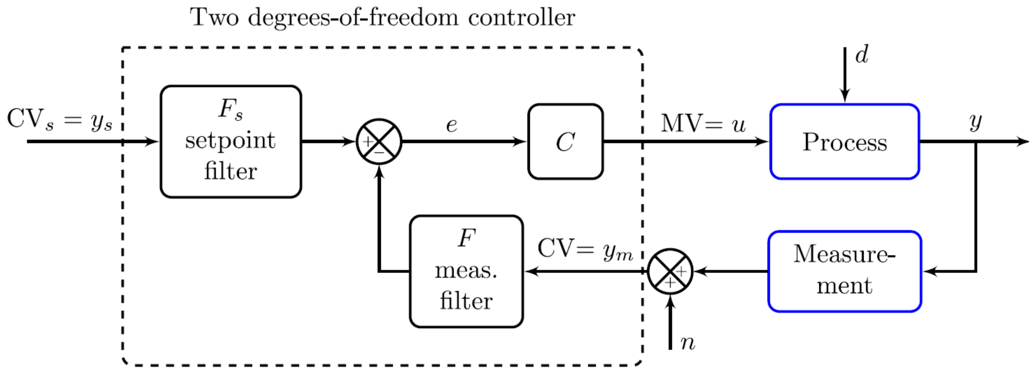
This is simple: Combine the measurement dynamics $g_m(s)$ and the process model $g(s)$ and apply the SIMC-rules on gg_m.

This applies both to the model approximation (half rule) to get a 1st or 2nd model and to the PI- or PID-tuning, including the choice of τ_c.

See also the handwritten note for a «proof», for example, that the total delay also includes the delay in the measurement g_m.



Measurement filter



Here C is the feedback controller (e.g., PID), whereas F_s and F typically are lead–lag transfer functions, with a steady-state gain of 1. In process control, we often use $F = 1$ (no measurement filter) or a first-order filter,

$$F(s) = \frac{1}{\tau_F s + 1} \qquad \text{(A.3)}$$

Here τ_F is the measurement filter time constant, and the inverse ($\omega_F = 1/\tau_F$) is known as the cutoff frequency. However, one should be careful about selecting a too large filter time constant τ_F as it acts as a effective delay as seen from the controller C; see also the recommendation $\tau_F \le \tau_c/2$ in (C.17).

Filter on D-action: Traditionally (in most older books), a first-order filter is put on the D-action (often with $\tau_F = \epsilon\tau_D$ with $\epsilon = 0.1$), but I recommend to use instead the above approach where the filter is on y (so also on the PI-part)

- For cascade PID it is equivalent (because here the filter multiplies the whole controller)
- But for ideal PID the filtering of only the D-action is difficult to interpret and not recommended (see paper by Hägglund).
  - So don't use $C(s) = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\epsilon\tau_D s+1}\right)$
  - use instead $C(s) = \frac{K_c}{\tau_F s+1}\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$

Simple analysis of choice of filter time constant τ_F.

Assume the loop transfer function is $L = C\ G\ G_m F = \frac{1}{\tau_c s}\frac{1}{\tau_F s+1}$

The closed-loop polynomial (set 1+L(s)=0) becomes $\tau_c\tau_F s^2 + \tau_c s + 1$.

- $\tau_F \le 0.25\ \tau_c$: Real poles. $\tau_c\tau_F s^2 + \tau_c s + 1 = (\tau_1 s + 1)(\tau_2 s + 1)$
  - $\tau_F \ll \tau_c$: $\tau_1 \approx \tau_c$, $\tau_2 \approx \tau_F$
  - $\tau_F = 0.1\ \tau_c$: $\tau_1 = 0.89\ \tau_c$, $\tau_2 = 0.113\ \tau_c = 1.13\ \tau_F$
  - $\tau_F = 0.25\ \tau_c$: $\tau_1 = \tau_2 = 0.5\ \tau_c = 2\ \tau_F$
- $\tau_F > 0.25\ \tau_c$: Complex poles. $\tau_c\tau_F s^2 + \tau_c s + 1 = \tau^2 s^2 + 2\zeta\tau s + 1$
  - $\tau_F = 0.5\ \tau_c$: $\tau = 0.71\ \tau_c$, $\zeta = 0.71$
  - Note: $\tau_F = 0.5\ \tau_c$ is the maximum recommended value, but I prefer at least a factor 2 smaller (then it does not oscillate and robustness is better).

Setpoint filter F_s

- Can also be useful!
- Simpler alternative: Beta-factor on setpoint for P-action.
  - Same as selecting $F(s) = \frac{\beta\tau_I s+1}{\tau_I s+1}$

CONCLUSION

Tuning of PID controllers

- SIMC tuning rules ("Skogestad IMC")(*)
- Main message: Can usually do much better by taking a systematic approach
- Key: Look at initial part of step response

  Initial slope: $k' = k/\tau_1$
- One tuning rule!

For cascade-form PID controller:

$$K_c = \frac{1}{k'} \cdot \frac{1}{(\theta+\tau_c)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$
$$\tau_D = \tau_2$$

- τ_c: desired closed-loop response time (tuning parameter)
- For robustness select: $\tau_c \geq \theta$

Note: The delay θ includes any measurement delay

Reference: S. Skogestad, "Simple analytic rules for model reduction and PID controller design", *J.Proc.Control*, Vol. 13, 291-309, 2003
(Also reprinted in MIC)
(*) "Probably the best simple PID tuning rules in the world"

Model
(read yourself)

Need a model for tuning

- Model: Dynamic effect of change in input u (MV) on output y (CV)

- First-order + delay model for PI-control

$$G(s) = \frac{k}{\tau_1 s + 1} e^{-\theta s}$$

- Second-order model for PID-control

$$G(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s}$$

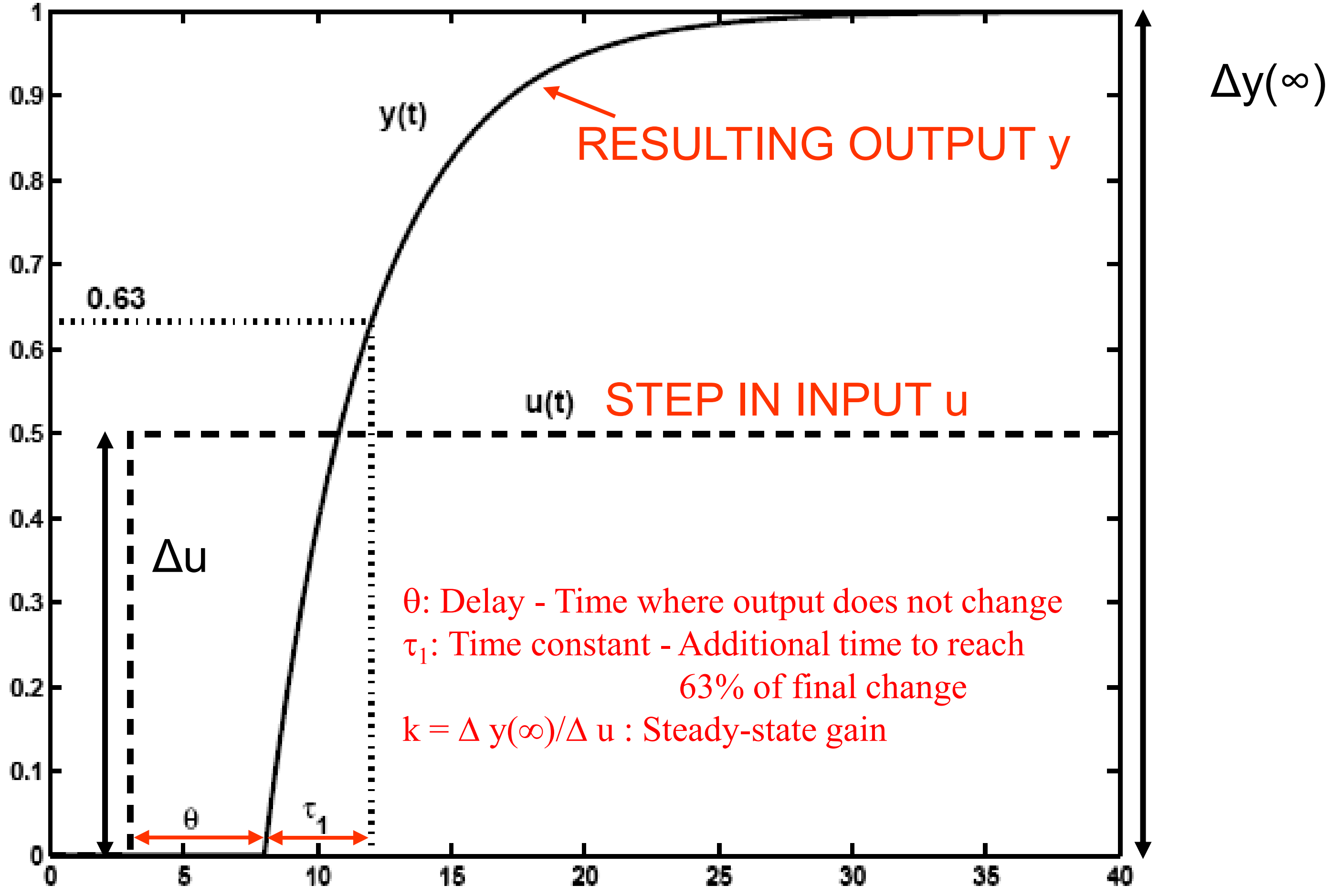
  – Recommend: Use second-order model only if $\tau_2 \geq \theta$

1. Step response experiment

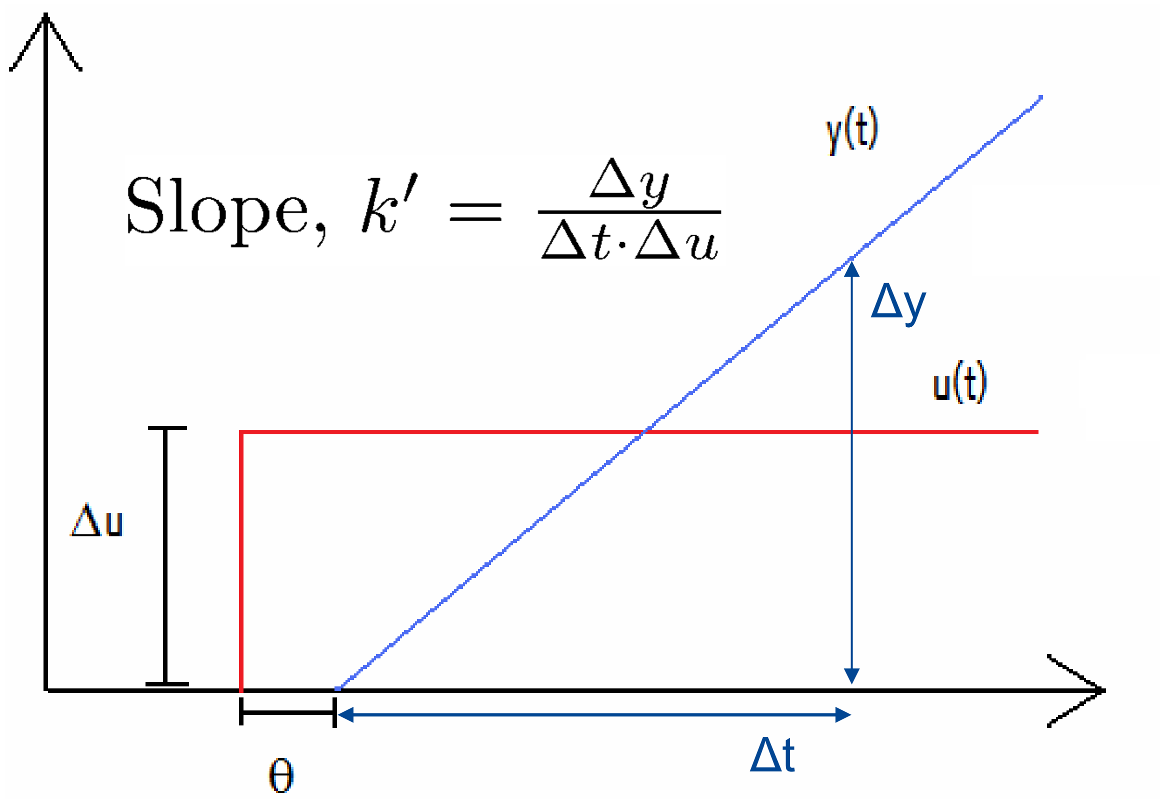
- Make step change in one u (MV) at a time

- Record the output (s) y (CV)

1A. Open-loop setting



Step response of integrating process

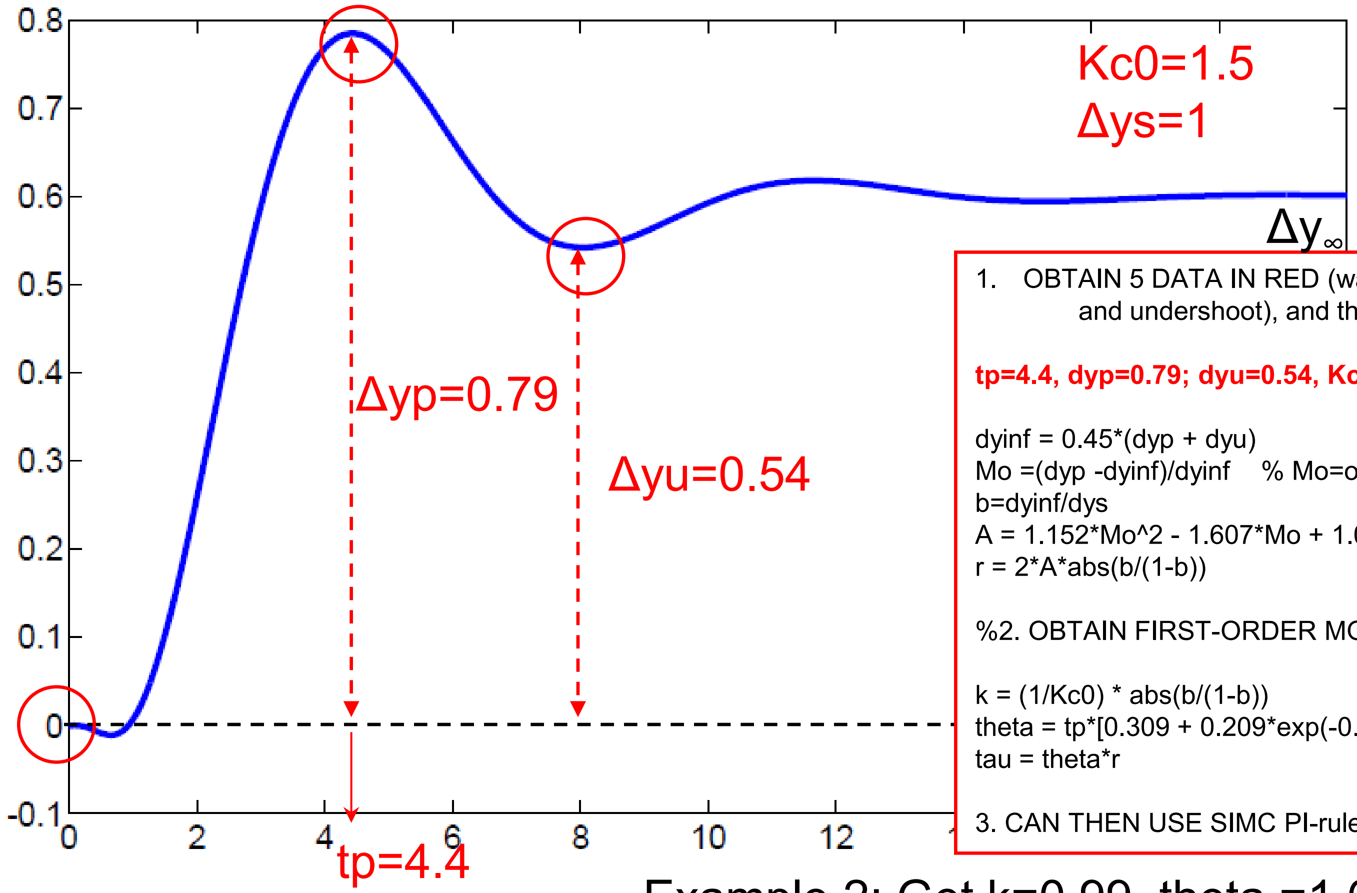


Imagine this as a 1st order with "infinite" τ_1:

$$G(s) = \frac{k}{\tau_1 s + 1} \approx \frac{k}{\tau_1 s} = \frac{k'}{s}$$

1B. Closed-loop setpoint response

- Shams' method: P-controller with about 20-40% overshoot



1. OBTAIN 5 DATA IN RED (wait for first overshoot and undershoot), and then read off:

tp=4.4, dyp=0.79; dyu=0.54, Kc0=1.5, dys=1

dyinf = 0.45*(dyp + dyu)
Mo =(dyp -dyinf)/dyinf   % Mo=overshoot (about 0.3)
b=dyinf/dys
A = 1.152*Mo^2 - 1.607*Mo + 1.0
r = 2*A*abs(b/(1-b))

%2. OBTAIN FIRST-ORDER MODEL:

k = (1/Kc0) * abs(b/(1-b))
theta = tp*[0.309 + 0.209*exp(-0.61*r)]
tau = theta*r

3. CAN THEN USE SIMC PI-rule

Example 2: Get k=0.99, theta =1.68, tau=3.03

Ref: Shamssuzzoha and Skogestad (JPC, 2010)
+ modification by C. Grimholt (Project, NTNU, 2010; see also PID-book 2012,
Sigurd Skogestad and Chriss Grimholt. "The SIMC Method for Smooth PID Controller Tuning" (2012))

2. Model reduction

- Start with complicated stable model on the form

$$G_0(s) = k_0 \frac{(T_{10}s + 1)(T_{20}s + 1) \dots}{(\tau_{10}s + 1)(\tau_{20}s + 1) \dots} e^{-\theta_0 s}$$

- Want to get a simplified model on the form

$$G(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s}$$

- Most important parameter is the "effective" delay θ

OBTAINING THE EFFECTIVE DELAY θ

Basis (Taylor approximation):

$$e^{-\theta s} \approx 1 - \theta s \quad \text{and} \quad e^{-\theta s} = \frac{1}{e^{\theta s}} \approx \frac{1}{1 + \theta s}$$

Effective delay =

"true" delay

+ inverse reponse time constant(s)

+ $\boxed{\text{half}}$ of the largest neglected time constant (the "half rule") (this is to avoid being too conservative)

+ all smaller high-order time constants

The "other half" of the largest neglected time constant is added to τ_1 (or to τ_2 if use second-order model).

Details:

- **Half rule**: the largest neglected (denominator) time constant (lag) is distributed evenly to the effective delay and the smallest retained time constant.

In summary, let the original model be in the form

$$\frac{\prod_j \left(-T_{j0}^{\text{inv}} + 1\right)}{\prod_i \tau_{i0} s + 1} e^{-\theta_0 s} \tag{9}$$

where the lags τ_{i0} are ordered according to their magnitude, and $T_{j0}^{\text{inv}} > 0$ denote the inverse response (negative numerator) time constants. Then, according to the half-rule, to obtain a first-order model $e^{-\theta s}/(\tau_1 s + 1)$, we use

$$\tau_1 = \tau_{10} + \frac{\tau_{20}}{2}; \qquad \theta = \theta_0 + \frac{\tau_{20}}{2} + \sum_{i \geqslant 3} \tau_{i0} + \sum_j T_{j0}^{\text{inv}} + \frac{h}{2} \tag{10}$$

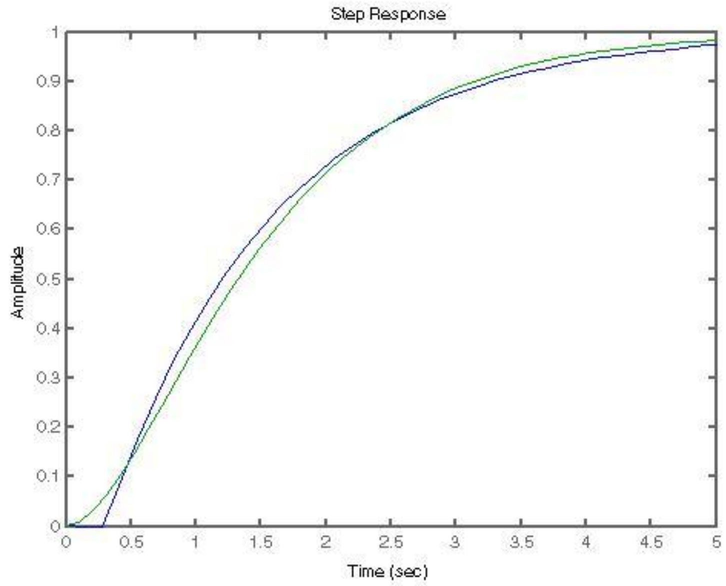
and, to obtain a second-order model (4), we use

$$\tau_1 = \tau_{10}; \qquad \tau_2 = \tau_{20} + \frac{\tau_{30}}{2};$$
$$\theta = \theta_0 + \frac{\tau_{30}}{2} + \sum_{i \geqslant 4} \tau_{i0} + \sum_j T_{j0}^{\text{inv}} + \frac{h}{2} \tag{11}$$

where h is the sampling period (for cases with digital implementation).

The main basis for the empirical half-rule is to maintain the robustness of the proposed PI- and PID-tuning rules, as is justified by the examples later.

Example 1



The second-order process

$$g_0(s) = \frac{1}{(1s+1)(0.6s+1)}$$

Half rule

is approximated as a first-order with delay process

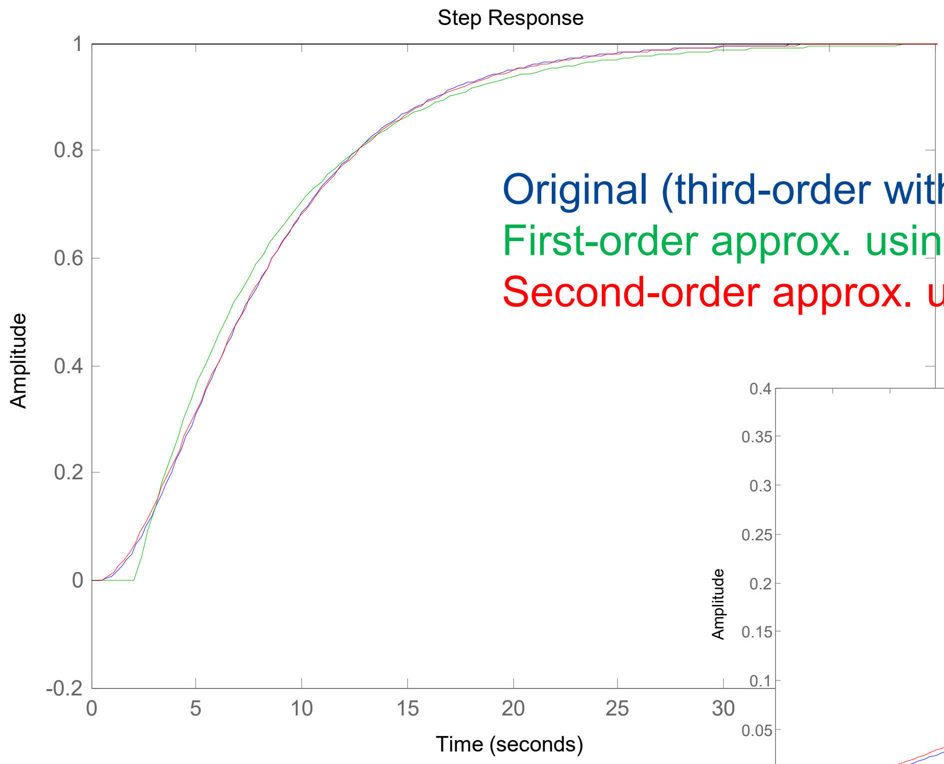
$$g(s) = k\frac{e^{-\theta s}}{\tau_1 s + 1}$$

with

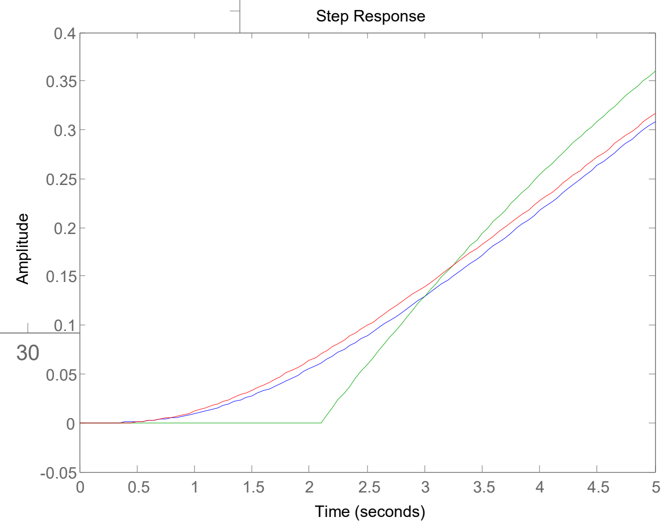
$$\boxed{k = 1; \quad \tau_1 = 1 + 0.6/2 = 1.3; \quad \theta = 0.6/2 = 0.3;}$$

Example 2

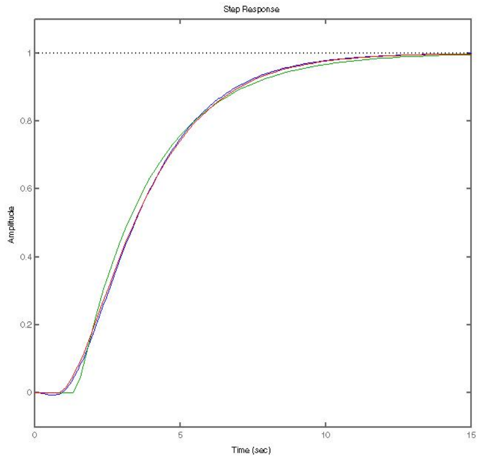
```
s=tf('s')
g=(-0.1*s+1)/[(5*s+1)*(3*s+1)*(0.5*s+1)]
g1 = exp(-2.1*s)/(6.5*s+1)
g2 = exp(-0.35*s)/[(5*s+1)*(3.25*s+1)]
step(g,g1,g2)
```



Original (third-order with inverse response)
First-order approx. using half rule
Second-order approx. using half rule



MODEL, Approach 2



Example 3

$$g_0(s) = k\frac{(-0.3s+1)(0.08s+1)}{(2s+1)(1s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

half rule

is approximated as a first-order delay process with

$$\tau_1 = 2 + 1/2 = 2.5$$
$$\theta = 1/2 + 0.4 + 0.2 + 3 \cdot 0.05 + 0.3 - 0.08 = 1.47$$

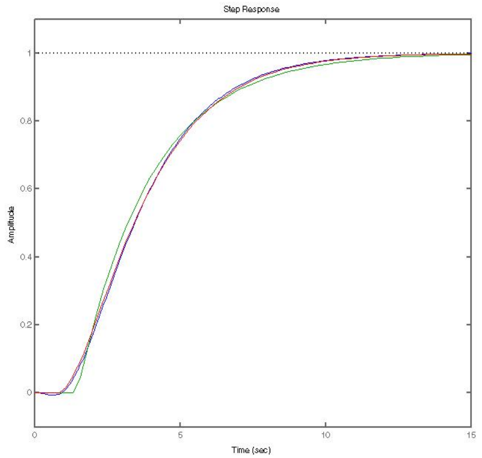
or as a second-order delay process with

$$\tau_1 = 2$$
$$\tau_2 = 1 + 0.4/2 = 1.2$$
$$\theta = 0.4/2 + 0.2 + 3 \cdot 0.05 + 0.3 - 0.08 = 0.77$$

Comment: The subtraction of T0=0.08 from the effective delay follows from the approximation (0.08s+1)/(0.2s+1) $\approx \frac{1}{(0.2-0.08)s+1}$ (rule T3).

28 Nov. 2024

Alternatively, we could have used the approximation (0.08s+1)/(0.05s+1) ≈ 1 (rule T1b) which would reduce the effective delay by 0.05 (instead of 0.08). In any case, it only has a small effect om the effective delay, so it does not matter much for the final result.



Example 3

$$g_0(s) = k\frac{(-0.3s+1)(0.08s+1)}{(2s+1)(1s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

half rule

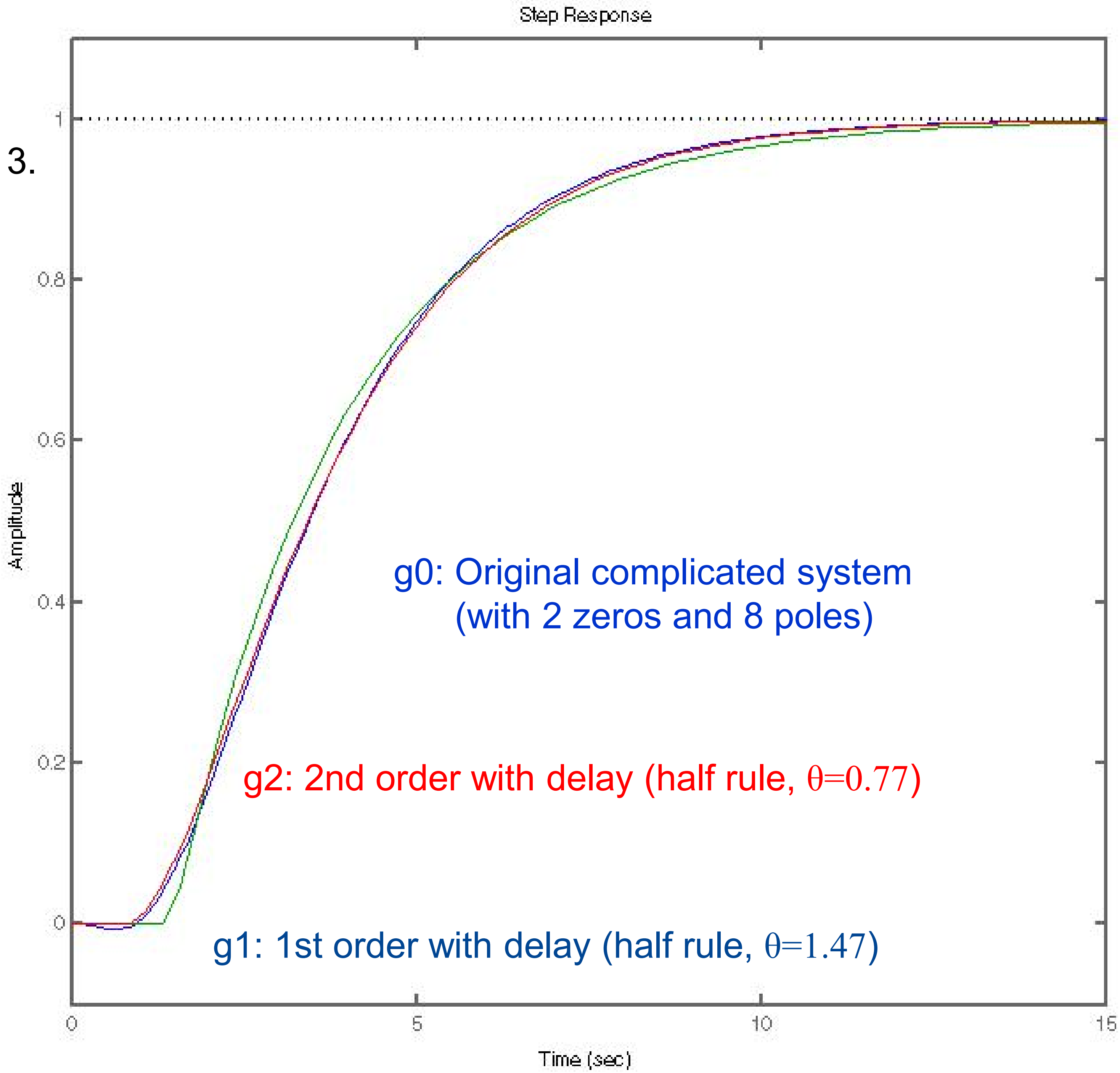
is approximated as a first-order delay process with

$$\tau_1 = 2 + 1/2 = 2.5$$
$$\theta = 1/2 + 0.4 + 0.2 + 3 \cdot 0.05 + 0.3 - 0.08 = 1.47$$

or as a second-order delay process with

$$\tau_1 = 2$$
$$\tau_2 = 1 + 0.4/2 = 1.2$$
$$\theta = 0.4/2 + 0.2 + 3 \cdot 0.05 + 0.3 - 0.08 = 0.77$$

Example 3.



Step Response
1
0.8
0.6
0.4
0.2
0
Amplitude
0
5
10
15
Time (sec)
g0: Original complicated system
(with 2 zeros and 8 poles)
g2: 2nd order with delay (half rule, θ=0.77)
g1: 1st order with delay (half rule, θ=1.47)

Example 4. Integrating process

$$g_0(s) = \frac{k'}{s(\tau_{20}s+1)}$$

Half rule gives

$$g(s) = \frac{k' e^{-\theta s}}{s} \text{ with } \theta = \frac{\tau_{20}}{2}$$

Proof:

Note that integrating process corresponds to an infinite time constant

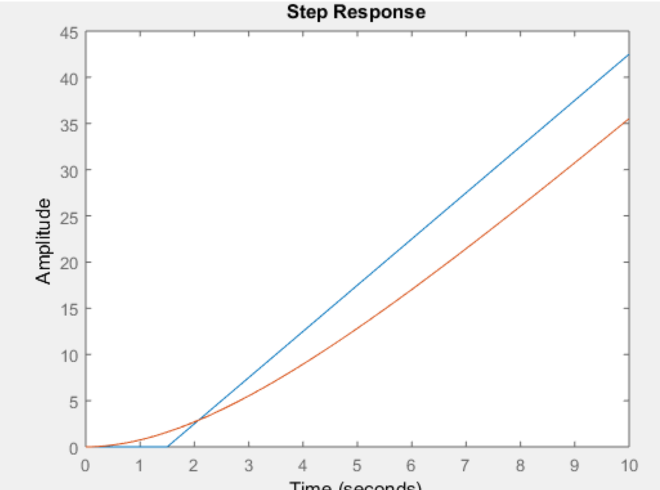
Write

$$g_0(s) = \frac{k'\tau_1}{\tau_1 s(\tau_{20}s+1)} = \frac{k'\tau_1}{(\tau_1 s+1)(\tau_{20}s+1)} \text{ where } \tau_1 \to \infty$$

and then apply half rule as normal, noting that $\tau_1 + \frac{\tau_{20}}{2} \approx \tau_1$:

$$g(s) \approx \frac{k'\tau_1 e^{-\frac{\tau_{20}}{2}s}}{(\tau_1+\frac{\tau_{20}}{2})s} = k' \frac{e^{-\frac{\tau_{20}}{2}s}}{s}$$

Example. g0 = 5/(s*(3*s+1)),
g = 5*exp(-1.5*s)/s,
step(g,g0,10)



Doesn't look so good
But it's OK

Approximation of LHP-zeros

$$\frac{T_0 s+1}{\tau_0 s+1} \approx \begin{cases} T_0/\tau_0 & \text{for } T_0 \geq \tau_0 \geq \tau_c & \text{(Rule T1)}, \\ T_0/\tau_c & \text{for } T_0 \geq \tau_c \geq \tau_0 & \text{(Rule T1a)}, \\ 1 & \text{for } \tau_c \geq T_0 \geq \tau_0 & \text{(Rule T1b)}, \\ T_0/\tau_0 & \text{for } \tau_0 \geq T_0 \geq 5\tau_c & \text{(Rule T2)}, \\ \frac{(\tilde{\tau}_0/\tau_0)}{(\tilde{\tau}_0 - T_0)s+1} & \text{for } \tilde{\tau}_0 \stackrel{\text{def}}{=} \min(\tau_0, 5\tau_c) \geq T_0 & \text{(Rule T3)}. \end{cases}$$

$$\frac{T_0 s+1}{(\tau_{10}s+1)(\tau_{20}s+1)} \approx \frac{1}{\tau s+1} \qquad \tau = \frac{\tau_{10}\tau_{20}}{T_0}$$ (Rule T4, 28 Nov. 2024)

We should approximate T_0 by a "close-by" τ_0 .

- BUT: The goal is to use the model for control purposes, so we would like to keep (i.e., not approximate) the τ which is closest to the desired τ_c.

τ_c = desired closed-loop time constant

Example E3. *For the process (Example 4 in (Astrom et al. 1998))*

g0

$$g_0(s) = \frac{2(15s+1)}{(20s+1)(s+1)(0.1s+1)^2} \tag{13}$$

we first introduce from Rule T2 the approximation

$$\frac{15s+1}{20s+1} \approx \frac{15s}{20s} = 0.75$$

(Rule T2 applies since $T_0 = 15$ is larger than 5θ, where θ is computed below). Using the half rule, the process may then be approximated as a first-order time delay model with

g1

$$k = 2 \cdot 0.75 = 1.5; \quad \theta = 0.1 + \frac{0.1}{2} = 0.15; \quad \tau_1 = 1 + \frac{0.1}{2} = 1.05$$

or as a second-order time delay model with

g2

$$k = 1.5; \quad \theta = \frac{0.1}{2} = 0.05; \quad \tau_1 = 1; \quad \tau_2 = 0.1 + \frac{0.1}{2} = 0.15$$

PID-controller (from 2nd order model) will give performance improvement because $\tau_2 > \theta$

In Example E3, we have two possible values for τ_0 , namely 20 and 1. Since T_0=15, it seems clear that we should select the closest τ_0 = 20 and use rule T2. Get (Rule T2): $\frac{15s+1}{(20s+1)(s+1)} \approx \frac{0,.75}{s+1}$

- But what if T_0=2, maybe selecting τ_0 = 1 is better (and using rule T1)?
- No, this is not clear. Since τ_c is between 0.05 (PID) and 0.15 (PI), we may want to keep τ =1 which is closest to τ_c ,that is, also in this case select τ_0 = 20 (and use rule T2)
- This may seem surprising, but it turns out that it will not matter very much in the case for the PI/PID-tunings (try!), because k/tau1 (and thus Kc) will not change much and because tauI = min(tau,4(tauc+theta)).
- Of course, if T_0 gets much closer to 1, then we should select τ_0 = 1.
- Rule T4 gives: $\frac{15s+1}{(20s+1)(s+1)} \approx \frac{1}{1.33s+1}$ **(g1a)**

Generally, the LHP-zeros approximation rules results in acceptable (robust) PI/PID-settings, but not necessarily the "optimal" settings.

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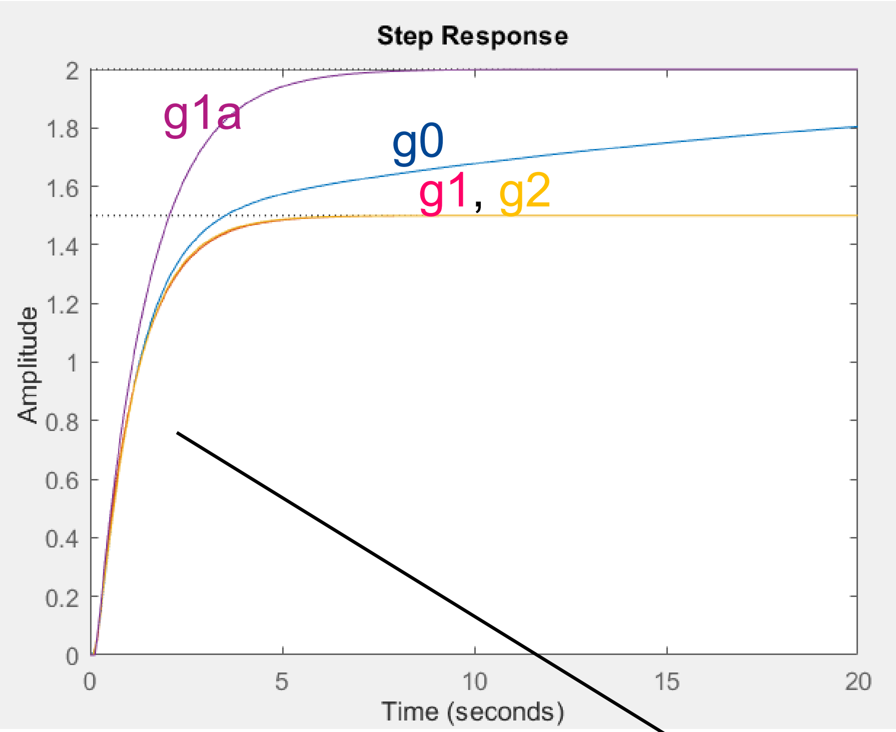
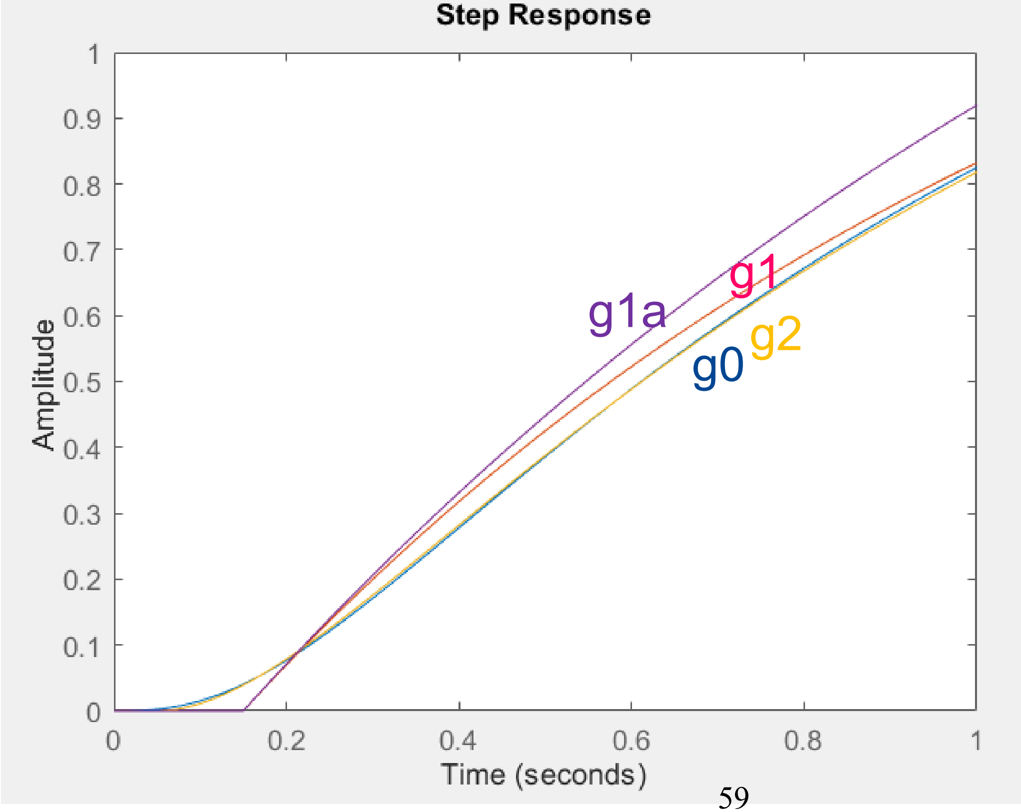
Step response (without control)

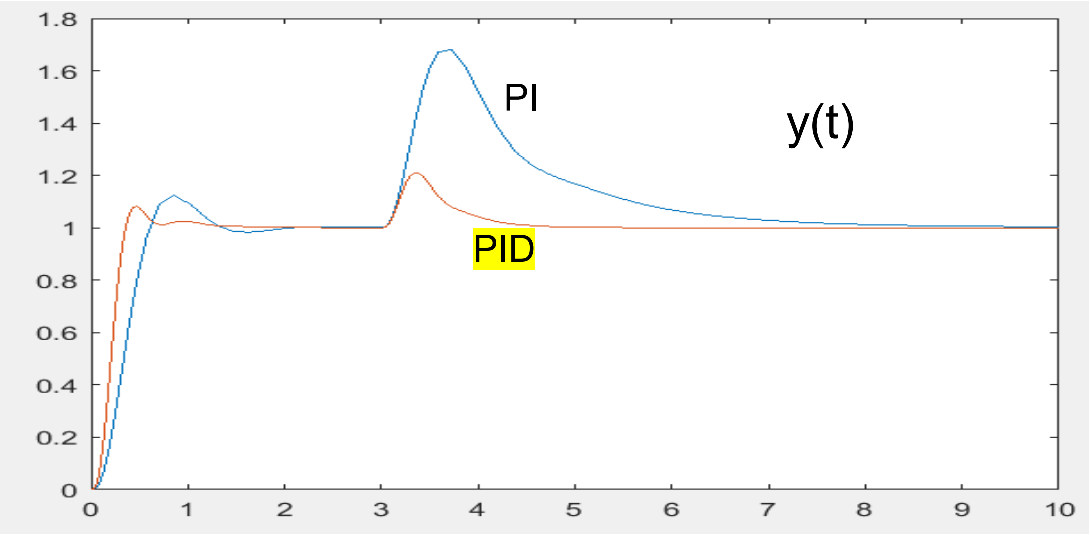


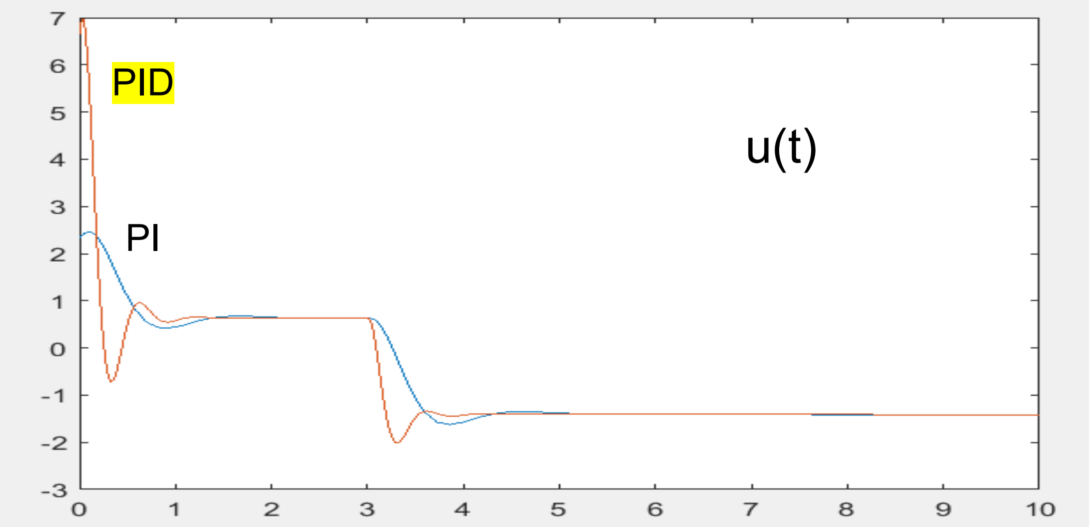
g0 = 2*(15*s+1)/((20*s+1)*(s+1)*(0.1*s+1)^2)
g1=1.5*exp(-0.15*s)/(1.05*s+1)
g2=1.5*exp(-0.05*s)/((s+1)*(0.15*s+1))
g1a = 2*exp(-0.15*s)/(1.38*s+1)
step(g0,g1,g2,g1a,1)



Note: It's the initial response that matters for feedback control (time from 0 to about 5*tauc)

Simulation with control is as expected: Better performance with controller based on g_2 (PID) than g_1 (PI) but more input usage





g = 2*(15*s+1)/((20*s+1)*(s+1)*(0.1*s+1)^2)
gd=g

Using g1:
PI. tauc=0.15
Kc=(1/1.5)*1.05/(2*0.15)=2.33
taui=min(1.05,8*tauc)=1.05
taud=0

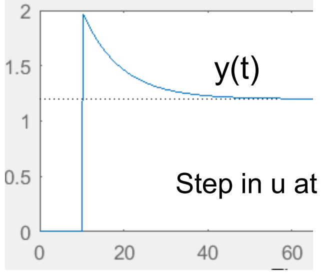
Using g2
PID. tauc=0.05
Kc=(1/1.5)*1.00/(2*0.05)=6.66
taui=min(1.05,8*tauc)=0.4
taud=0.15, tauf=0.015

60

Extra slide

Example: Approximation of zero for flow control

– $G_0(s) = 1.2\left(\frac{15s+1}{9\,s+1}\right)$



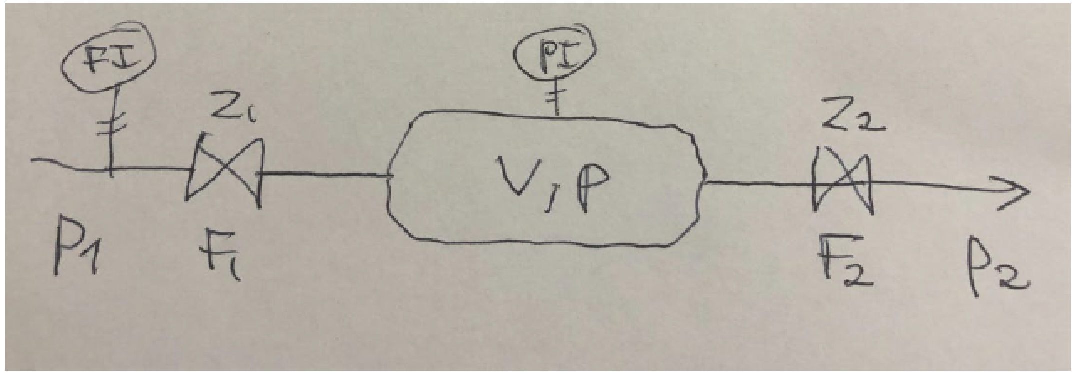
- Note that $T_0 = 15 > \tau_0 = 9$ so we get an overshoot in the step response
- How should we approximate this as a first-order with delay model?
  It will depend on the value for tauc. If we apply the LHP-zero approximation rules then we get:
  1. Small tauc (tauc<9): (15s+1)/(9s+1) ≈ 15/9 (Rule T1) ⇒ G(s)= k = 1.2*15/9 = 2
  2. Intermediate tauc (9<tauc<15): (15s+1)/(9s+1) ≈ 15/tauc (Rule T1a) ⇒ G(s)= k = 1.2*15/tauc=18/tauc
  3. Large tauc (tauc>15) . (15s+1)/(9s+1) ≈ 1 (Rule T1b) ⇒ G(s)= k = 1.2
- In all three cases we get $G(s) = k$ so we get $\tau_1 = 0$ and in all three cases the SIMC PI-controller becomes a pure I-controller C(s)=KI/s where KI = 1/(k*tauc). Here tauc is free to choose.
- **Flow controller,** The transfer function $G_0(s)$ is typical for a control valve where *u=z=valve position and y=F =flow.* Consider a typical valve equation $F = Cz\sqrt{p_1 - p_2}$. Following a step change in z, F will immediately jump (to 1.2*15/9=2), but then it will drop down again (to 1.2) because of the reduction in the pressure drop $p_1 - p_2$ which for gases may take some time ($\tau_0 = 9$ in this case). (See Exam 2022, Problem 5 for how to derive G_0)
  – For liquids the dynamics are fast because of small compressibility and can be neglected. Thus, for liquids we always have case 3 (tule T1b). However, the short-term flow overshoot may result in the phenomena of "water hammering".
  – For gases, also cases 1 or 2 may happen if the valve is close to a large gas holdup (large tank or large pipeline).

For a flow controller, a typical value is tauc=10s.

*Some commercial controllers do not allow a pure I-controller. In this case, select taui as some small value (say taui=1s) and use Kc=KI*taui, that is, Kc=(1/k)*(taui/tauc) .*

*However, if the dynamics for changing z or measuring F are slow compared to the desired closed-loop response time tauc, then a better approximatiom of the valve may be G=k/(tau1*s+1). In this case a PI-controller with tauc=tu1 is recommened (SIMC-rule).*

Problem 5 (25%). Modelling and control of flow and pressure



Consider a gas pipeline with two valves. We have measurements of the inflow F_1 and the intermediate pressure p and these should be controlled. The volume of the pipeline can be represented as a tank with volume V as shown in the figure above.

Steady-state data: F_1=1 kg/s, z_1=z_2=0.5, p_1=2 bar, p=1.88 bar, p_2=1.8 bar, V=130 m^3, T=300 K, Parameters: R=8.31 J/K.mol, M_W=18e-3 kg/mol (so the gas is steam).

The following model equations are suggested to describe the system.

(1) $dm/dt = F_1 - F_2$
(2) $m = k_p\, p$ where $k_p = VM_w/(RT)$
(3) $F_1 = C_1 z_1 \sqrt{p_1 - p}$
(4) $F_2 = C_2 z_2 \sqrt{p - p_2}$

(a) (3%) Explain what the variables and equations represent. What assumptions have been made?
(b) (3%) Determine the parameters in the model (C_1, C_2, k_p). What is the steady-state value of m? What is the residence time of the gas, m/F_1?
(c) (12%) Linearize the model and find the 2x2 transfer function model from z_1 and z_2 (inputs) to F_1 and p (controlled variables). (Note: To simplify, you can assume p_1 and p_2 are constant)
(d) (4%) What pairings do you suggest for single-loop control (with u = [z_1 z_2], y= [F_1 p])? How could control be improved?
(e) (3%) (This can be answered without solving parts a-d). What control structure would you propose if we instead of p want to control the downstream pressure p_2? Thus, we have u=[z_1 z_2] and y = [F_1 p_2] .

Problem 5 (25%)

a) Model equations and assumptions.

(1) is the mass balance for the pipeline section [kg/s]

(2) is the ideal gas equation on mass basis with the temperature T is assumed constant.

(3) and (4) are the assumed valve equations. Note that we have assumed a linear valve characteristic.

Variables:

F_1: inlet flow
F_2: outlet flow
z_1: inlet valve opening
z_2: outlet valve opening
C_1: inlet valve constant
C_2: outlet valve constant
m: mass of gas in the pipeline
p: pressure of gas in the pipeline
p_1: pressure of gas at the inlet
p_2: pressure of gas at the outlet
V: volume of pipeline
T: temperature of the system
R: ideal gas constant
M_w: molar mass of gas

b) At steady state, $F_1 = F_2$, and therefore:

$$C_1 = \frac{F_1}{z_1\sqrt{p_1 - p}} = \frac{1}{0.5 \times \sqrt{2 - 1.88}} = 5.773\ kg/s \cdot bar^{1/2}$$

$$C_2 = \frac{F_2}{z_2\sqrt{p - p_2}} = \frac{1}{0.5 \times \sqrt{1.88 - 1.8}} = 7.071\ kg/s \cdot bar^{1/2}$$

$$k_p = \frac{VM_w}{RT} = \frac{130 \times 18 \times 10^{-3}}{8.31 \times 300} \frac{m^3 \times \frac{kg}{mol}}{\frac{J}{mol\ K}\ K} = 9.386 \times 10^{-4}\ kg/Pa = 93.86\ kg/bar$$

$$m = k_p\, p = 93.86 \times 1.88 = 176.457\ kg$$

Residence time: $t_R = m/F_1 = 176.457\ s$

c) Linearizing the model. First linearize the two static valve equations (3) and (4):

$$y_1 = \Delta F_1 = \left(C_1\sqrt{p_1 - p}\right)\Big|_* \Delta z_1 + \left(-\frac{C_1 z_1}{2\sqrt{p_1 - p}}\right)\Big|_* \Delta p = 2\,\Delta z_1 - 4.166\,\Delta p$$

$$\Delta F_2 = \left(C_2\sqrt{p - p_2}\right)\Big|_* \Delta z_2 + \left(\frac{C_1 z_2}{2\sqrt{p - p_2}}\right)\Bigg|_* \Delta p = 2\,\Delta z_2 + 6.250\,\Delta p$$

From (2) the mass balance (1) becomes $k_p\, dp/dt = F_1 - F_2$ which gives the linearized model for $y_2 = \Delta p$:

$$k_p \frac{d\Delta p}{dt} = \Delta F_1 - \Delta F_2 = 2\,\Delta z_1 - 2\,\Delta z_2 - 10.416\,\Delta p$$

$$\Rightarrow 93.86 \frac{d\Delta p}{dt} + 10.416\,\Delta p = 2\,\Delta z_1 - 2\,\Delta z_2$$

$$\Rightarrow 9.011 \frac{d\Delta p}{dt} + \Delta p = 0.192\,\Delta z_1 - 0.192\,\Delta z_2$$

Applying the Laplace transform to the last expression gives the transfer function for $y_2 = \Delta p$:

$$\Delta p(s) = \frac{0.1925}{9.011\,s + 1}\Delta z_1 - \frac{0.1925}{9.011\,s + 1}\Delta z_2$$

The expression for $y_1 = \Delta F_1$ then becomes

$$\Delta F_1 = \left(2 - 4.166 \times \frac{0.1925}{9.011\,s + 1}\right)\Delta z_1 - 4.166 \times \left(\frac{-0.1925}{9.011\,s + 1}\right)\Delta z_2$$

$$= \left(\frac{2 \times (9.011\,s + 1) - 4.166 \times 0.1925}{9.011\,s + 1}\right)\Delta z_1 + \frac{0.800}{9.011\,s + 1}\Delta z_2$$

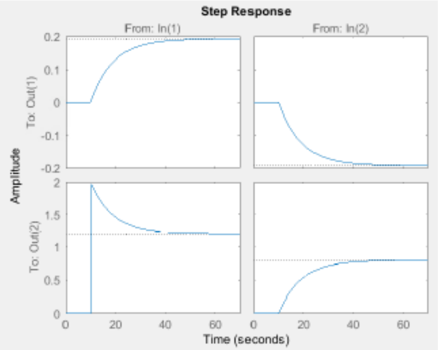
$$= \left(\frac{18.022\,s + 1.200}{9.011\,s + 1}\right)\Delta z_1 + \frac{0.8}{9.011\,s + 1}\Delta z_2 = 1.2\left(\frac{15.018\,s + 1}{9.011\,s + 1}\right)\Delta z_1 + \frac{0.8}{9.011\,s + 1}\Delta z_2$$

Conclusion

$$\begin{bmatrix}\Delta p \\ \Delta F_1\end{bmatrix} = G(s)\begin{bmatrix}\Delta z_1 \\ \Delta z_2\end{bmatrix}, \qquad G(s) = \begin{bmatrix} \frac{0.1925}{9.011\,s + 1} & \frac{-0.1925}{9.011\,s + 1} \\ 1.2\left(\frac{15.018\,s + 1}{9.011\,s + 1}\right) & \frac{0.8}{9.011\,s + 1}\end{bmatrix}$$

Note that the time constant of 9s is much smaller than the residence time of 176s. This is typical for gas systems. Also note that u1=z1 has a direct effect on y2=F1(as expected from physics:; see element g21 in step response below which has an overshoot because of the zero).

```
s=tf('s')
g11=0.1925/(9*s+1); g12=-g11;
g21=1.2*(15*s+1)/(9*s+1);
g22=0.8/(9*s+1);
G=[g11 g12; g21 g22];
step(G*exp(-10*s)) % To make plot
clearer I put in a delay so that step is
at t=10
```

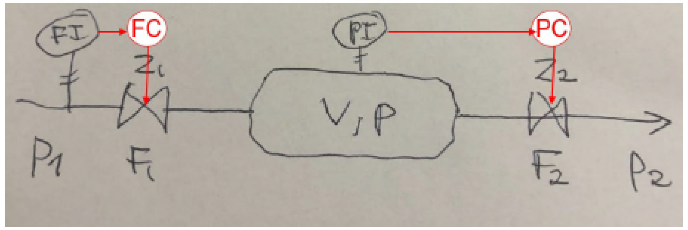


d)

Steady-state gain matrix: $G(0) = \begin{bmatrix} 0.1925 & -0.1925 \\ 1.2 & 0.8 \end{bmatrix} \rightarrow$

steady-state RGA matrix: $\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$ where $\lambda = \frac{0.1925 \times 0.8}{0.1925 \times 0.8 + 0.1925 \times 1.2} = 0.4.$

From the steady-state RGA, the recommended pairing is then the off-diagonal pairing, that is, $F_1 - z_1$ and $p - z_2$. This happens to coincide with the intuitive pairing ("pair-close rule") since z1 has a direct effect on F1. It also agrees with what we get from the RGA if we consider the initial response (high frequency).



However, high steady-state interaction is to be expected, since Λ is far from the ideal case (identity matrix). Possible solutions are the implementation of a decoupler (probably steady-state decoupler is OK), or separating the timescales of the two loops.

Since the flow control has a direct effect from z1 to F1, this should probably be the fast loop, and then the pressure loop can be about 5 times slower. But if both loops should be equally fast, a decoupler is preferred.

What about the tuning of the flow loop? What model should we use? We have that

$$G_0(s) = 1.2\left(\frac{15s + 1}{9\,s + 1}\right)$$

Note that T0=15 > tau0=9. How should we approximate this as a first-order with delay model? It will depend on the value for tauc. If we apply the LHP-zero approximation rules then we get.

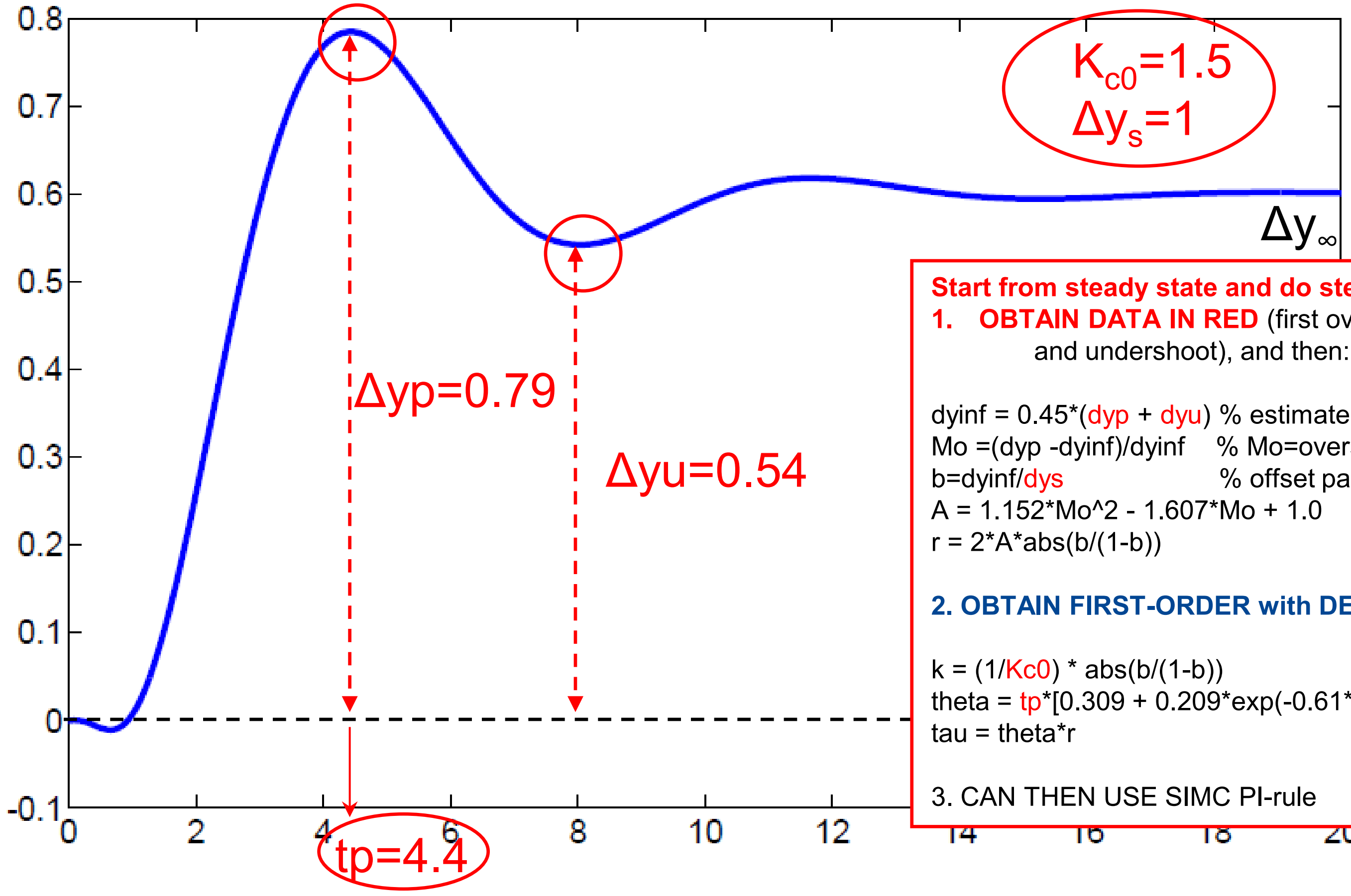
Small tauc (tauc<9): (15s+1)/(9s+1) ≈ 15/9 (Rule T1) ⇒ G(s)=1.2*15/9 = 2
Intermediate tauc (9<taux<15): (15s+1)/(9s+1) ≈ 15/tauc (Rule T1a) ⇒ G(s)=18/tauc
Large tauc (tauc>15) . (15s+1)/(9s+1) ≈ 1 (Rule T1b) ⇒ G(s)=1.2

In all these three case the SIMC PI-controller becomes a pure I-controller C(s)=KI/s with KI = 1/(k*tauc). Note that for the intermediate tauc we get KI=1/18 (independent of Kc).

e) This is a trick question, because it will not work. This control strategy would not be consistent, as we can see that that is does not follow the radiation rule. In general, the control of pressures that are external to the process is equivalent to a flow specification (TPM), which in this case would conflict with the specification of F_1.

Alternative to Ziegler-Nichols closed-loop experiment: Obtains more information and avoids cycling.

Shams' method: Closed-loop setpoint response with P-controller with about 20-40% overshoot



Start from steady state and do step P-response

1. **OBTAIN DATA IN RED** (first overshoot and undershoot), and then:

```
dyinf = 0.45*(dyp + dyu) % estimate dyinf (so don't wait)
Mo =(dyp -dyinf)/dyinf    % Mo=overshoot (about 0.3)
b=dyinf/dys               % offset parameter
A = 1.152*Mo^2 - 1.607*Mo + 1.0
r = 2*A*abs(b/(1-b))
```

2. OBTAIN FIRST-ORDER with DELAY MODEL:

```
k = (1/Kc0) * abs(b/(1-b))
theta = tp*[0.309 + 0.209*exp(-0.61*r)]
tau = theta*r
```

3. CAN THEN USE SIMC PI-rule

Example 2: Get k=0.99, theta =1.67, tau=3.0

Ref: Shamssuzzoha and Skogestad (JPC, 2010) + modification by C. Grimholt (PID-book 2012)

Example E2 (Further continued) We want to derive PI- and PID-settings for the process

$$g_0(s) = \frac{(-0.3s+1)(0.08s+1)}{(2s+1)(1s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

using the SIMC tuning rules with the "default" recommendation $\tau_c = \theta$. From the closed-loop setpoint response, we obtained in a previous example a first-order model with parameters $k = 0.994, \theta = 1.67, \tau_1 = 3.00$ (5.10). The resulting SIMC PI-settings with $\tau_c = \theta = 1.67$ are

$$\mathrm{PI}_{cl}: \quad K_c = 0.904, \qquad \tau_I = 3.$$

From the full-order model $g_0(s)$ and the half rule, we obtained in a previous example a first-order model with parameters $k = 1, \theta = 1.47, \tau_1 = 2.5$. The resulting SIMC PI-settings with $\tau_c = \theta = 1.47$ are

$$\mathrm{PI}_{\text{half-rule}}: \quad K_c = 0.850, \qquad \tau_I = 2.5.$$

From the full-order model $g_0(s)$ and the half rule, we obtained a second-order model with parameters $k = 1, \theta = 0.77, \tau_1 = 2, \tau_2 = 1.2$. The resulting SIMC PID-settings with $\tau_c = \theta = 0.77$ are

$$\text{Series PID}: \quad K_c = 1.299, \qquad \tau_I = 2, \qquad \tau_D = 1.2.$$

The corresponding settings with the more common ideal (parallel form) PID controller are obtained by computing $f = 1 + \tau_D/\tau_I = 1.60$, and we have

$$\text{Ideal PID}: \quad K_c' = K_c f = 1.69, \qquad \tau_I' = \tau_I f = 3.2, \qquad \tau_D' = \tau_D/f = 0.75. \tag{5.30}$$