

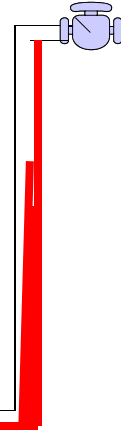
# Sammenligning av lineære og ulineære metoder for robust Anti-slug regulering

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SERVOMØTET, OKTOBER 2013

Two-phase pipe flow  
 (liquid and vapor)

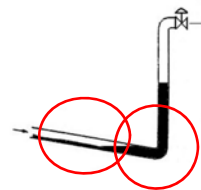
Slug (liquid) buildup



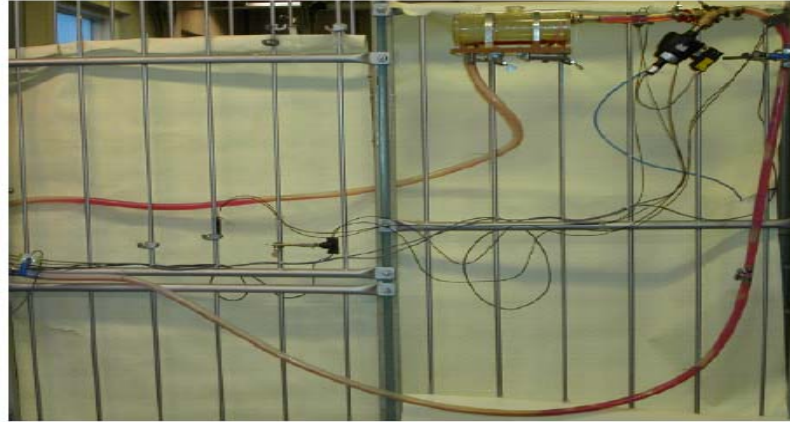
## Slug cycle (stable limit cycle)



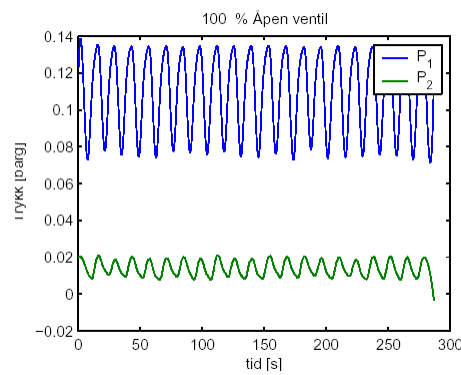
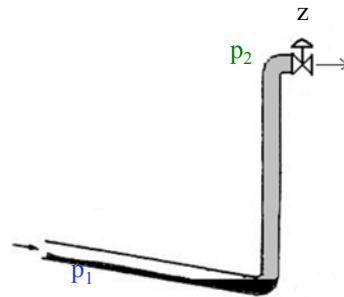
Experiments performed by the Multiphase Laboratory, NTNU



## Experimental mini-loop (2003)

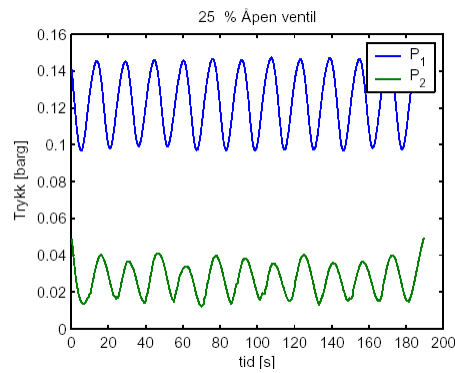
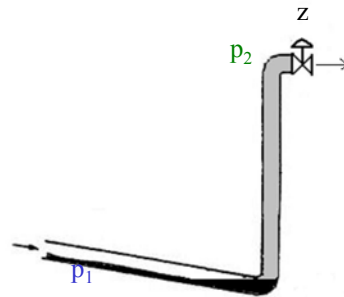


## Experimental mini-loop Valve opening ( $z$ ) = 100%



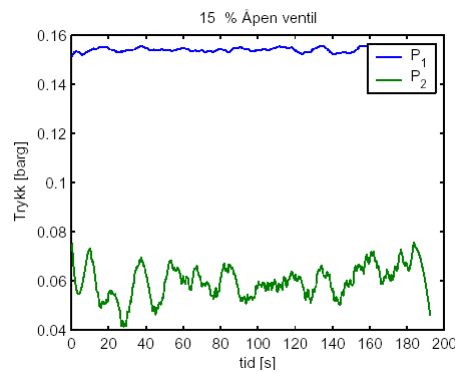
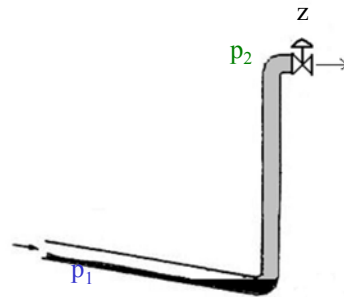
**SLUGGING**

Experimental mini-loop  
Valve opening ( $z$ ) = 25%



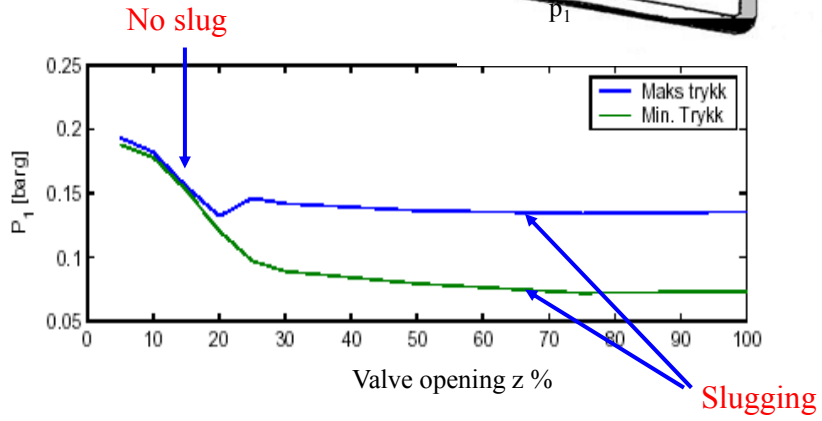
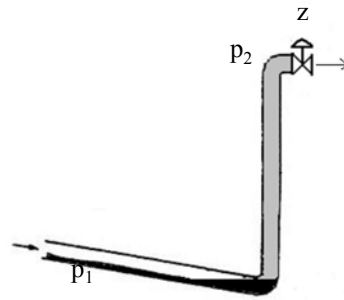
SLUGGING

Experimental mini-loop  
Valve opening ( $z$ ) = 15%



NO SLUG

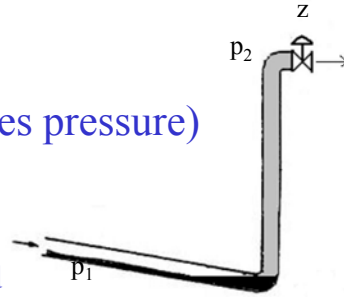
### Experimental mini-loop: Bifurcation diagram



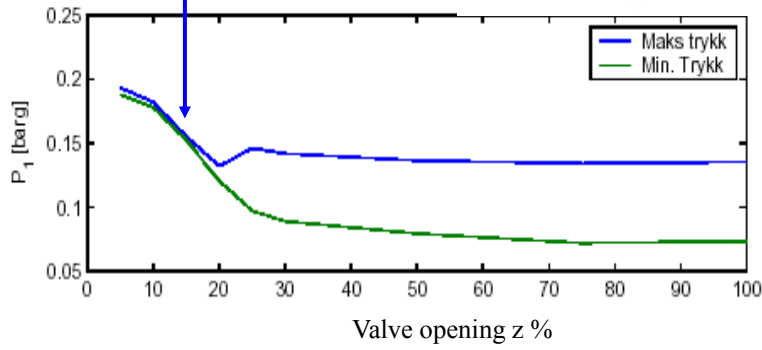
### How to avoid slugging?

Design change

Avoid slugging:  
 1. Close valve (but increases pressure)

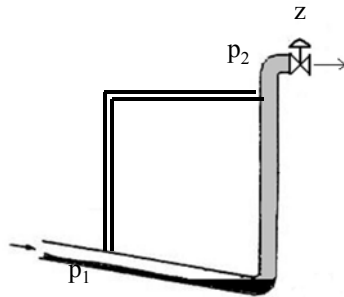


No slugging when valve is closed



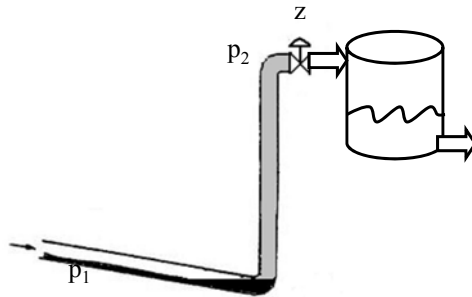
Design change

Avoid slugging:  
 2. Design change to avoid slugging



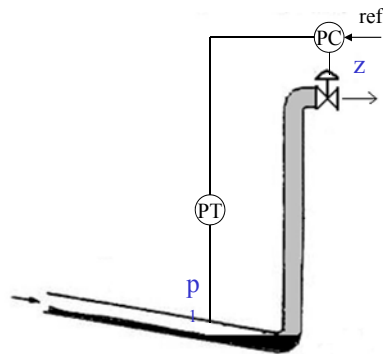
Design change

### Minimize effect of slugging: 3. Build large slug-catcher



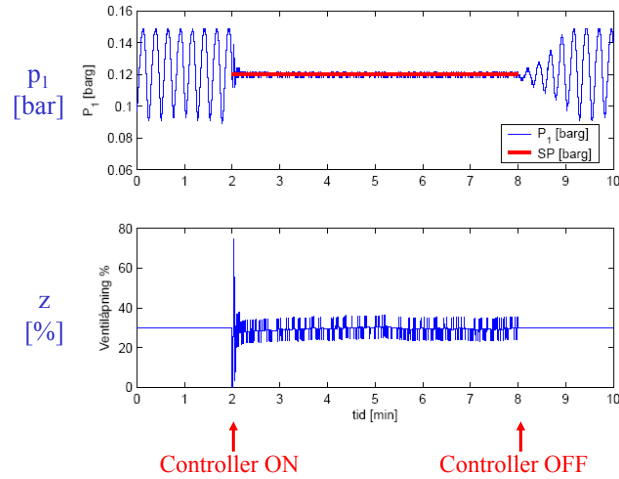
- Most common strategy in practice

### Avoid slugging: 4. "Active" feedback control

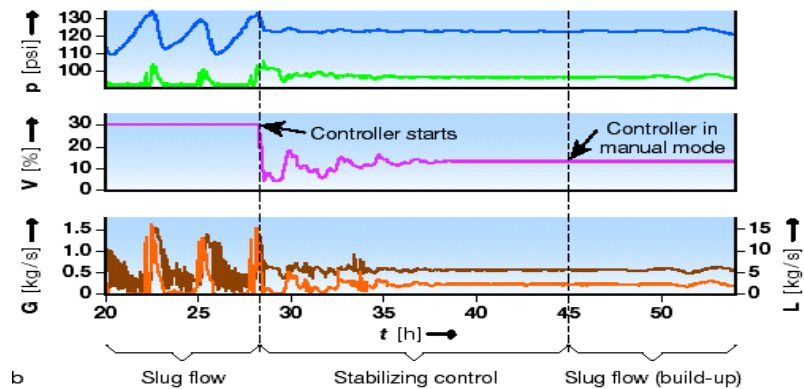


Simple PI-controller

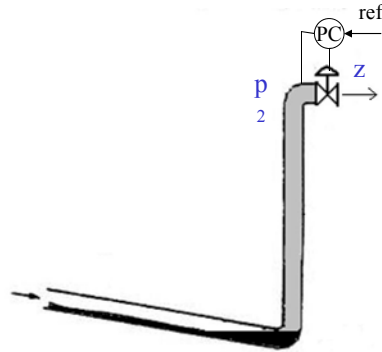
## Anti slug control: Mini-loop experiments



## Anti slug control: Full-scale offshore experiments at Hod-Vallhall field (Havre, 1999)



## Avoid slugging: 5. "Active" feedback control with topside measurement?



Control is difficult  
(Inverse response = Unstable zero dynamics)

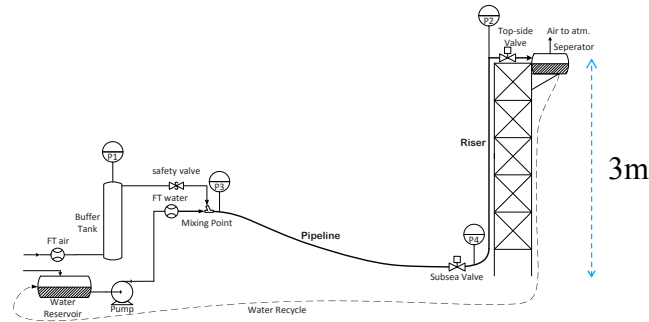
## Summary anti slug control (2008)\*

- Stabilization of desired non-slug flow regime = \$\$\$\$!
- Stabilization using downhole pressure simple
- Stabilization using topside measurements difficult
- Control can make a difference!
- "Only" problem: Not sufficiently robust

\*Thanks to: Espen Storkaas + Heidi Sivertsen + Håkon Dahl-Olsen + Ingvald Bårdsen

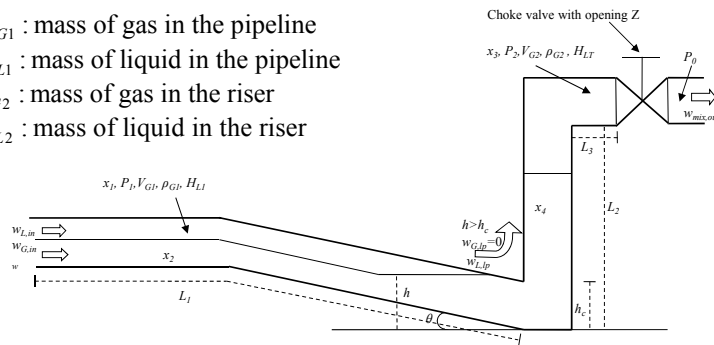


## New Experimental mini-rig



## 1<sup>st</sup> step: Developed new simple 4-state model\*

- $m_{G1}$  : mass of gas in the pipeline
- $m_{L1}$  : mass of liquid in the pipeline
- $m_{G2}$  : mass of gas in the riser
- $m_{L2}$  : mass of liquid in the riser



State equations (mass conservations law):

$$\dot{m}_{G1} = w_{G,in} - w_{G,lp}$$

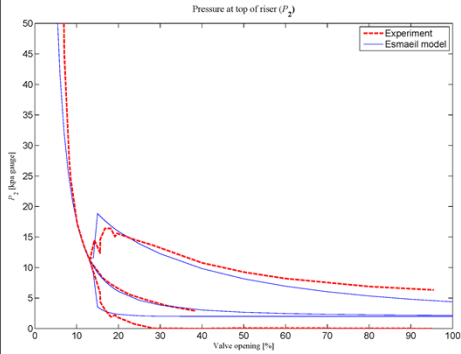
$$\dot{m}_{L1} = w_{L,in} - w_{L,lp}$$

$$\dot{m}_{G2} = w_{G,lp} - w_{G,out}$$

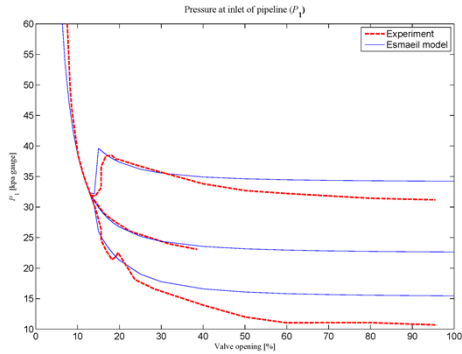
$$\dot{m}_{L2} = w_{L,lp} - w_{L,out}$$

New 4-state model. Comparison with experiments:

Top pressure



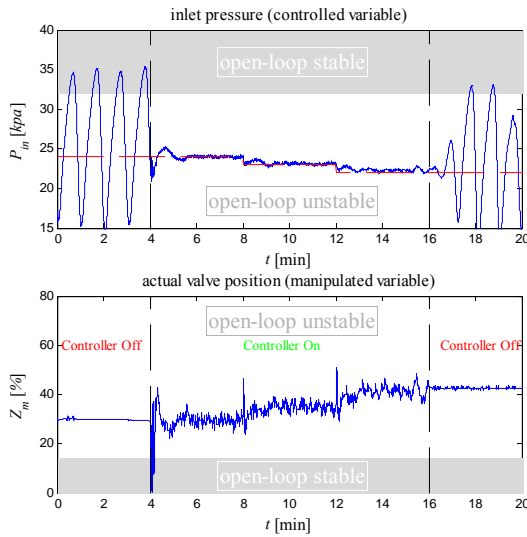
Subsea pressure



Linear Control Solutions

### Solution 1: $H_\infty$ control based on linearizing new model

Experiment, mixed-sensitivity design



$$\min_K \|N(K)\|_\infty, \quad N \triangleq \begin{bmatrix} W_u K S \\ W_T T \\ W_P S \end{bmatrix}$$

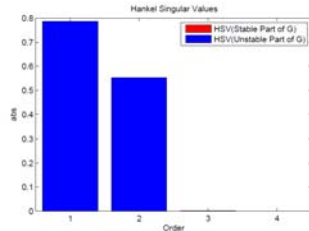
$$K = \frac{-257.77(s^2 + 0.0046s + 0.002)}{s(s + 0.26)(s + 4.9)}$$

### Experimental linear model (new approach)

Fourth-order mechanistic model:

$$G(s) = \frac{\theta_1(s + \theta_2)(s + \theta_3)}{(s^2 - \theta_4 s + \theta_5)(s^2 + \theta_6 s + \theta_7)}$$

Hankel Singular Values:

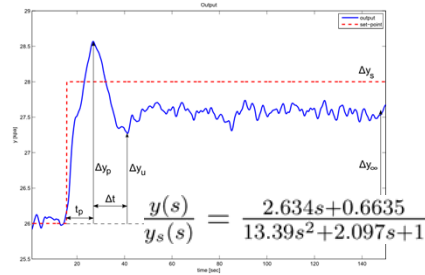
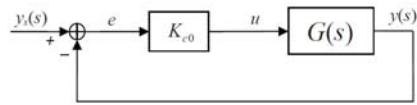


Model reduction:

$$G(s) = \frac{b_1 s + b_0}{s^2 - a_1 s + a_0}$$

4 parameters need to be estimated

Closed-loop step-response with P-control



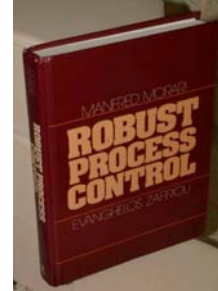
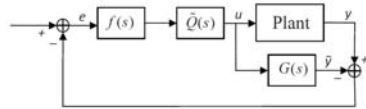
$$\frac{y(s)}{y_s(s)} = \frac{2.634s + 0.6635}{13.39s^2 + 2.097s + 1}$$

$$G(s) = \frac{-0.0098s - 0.0025}{s^2 - 0.0401s + 0.0251}$$

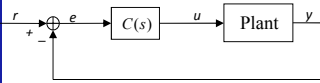
## Solution 2: IMC based on identified model

IMC design

Block diagram for Internal Model Control system



IMC for unstable systems:



$$C(s) = \frac{\tilde{Q}(s)f(s)}{1-G(s)\tilde{Q}(s)f(s)}$$

Model:

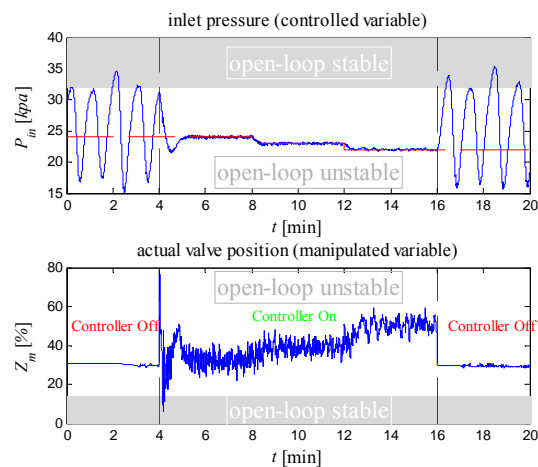
$$G(s) = \frac{-0.0098s - 0.0025}{s^2 - 0.0401s + 0.0251}$$

IMC controller:  $\lambda = 8 \text{ s}$

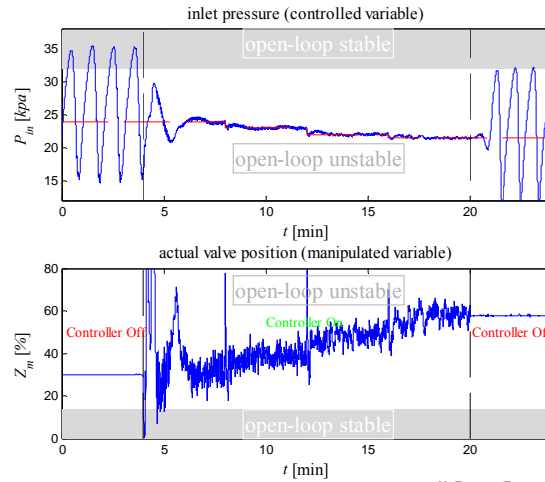
$$C(s) = \frac{-42.20(s^2 + 0.052s + 0.0047)}{s(s + 0.251)}$$

## Solution 2: IMC based on identified model

Experiment



### Solution 3: «Robustified» IMC using $H_\infty$ loop shaping



$$\gamma_K = \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_\infty \leq \frac{1}{\epsilon}$$

### Comparing linear controllers (subsea pressure)

Comparison between IMC-PID controller and  $H_\infty$  controllers

|                 | IMC-PID | $H_\infty$ mixed-sensitivity | $H_\infty$ loop-shaping |
|-----------------|---------|------------------------------|-------------------------|
| Gain Margin     | 0.38    | 0.46                         | 0.29                    |
| GM Frequency    | 0.19    | 0.18                         | 0.16                    |
| Phase Margin    | 61.68   | 50.63                        | 64.44                   |
| PM Frequency    | 0.52    | 0.43                         | 0.64                    |
| Delay Margin    | 2.07    | 2.07                         | 1.77                    |
| $\ S\ _\infty$  | 1.02    | 1.17                         | 1.14                    |
| $\ T\ _\infty$  | 1.63    | 1.86                         | 1.41                    |
| $\ KS\ _\infty$ | 43.17   | 36.86                        | 55.67                   |
| $Z_{max}^*$     | 50%     | 50%                          | 58%                     |

\*Controllers tuned at 30% valve opening

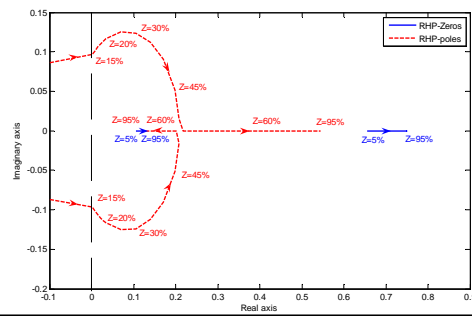
# Fundamental limitation – top pressure

Measuring topside pressure we can stabilize the system only in a limited range

Unstable (RHP) zero dynamics of top pressure

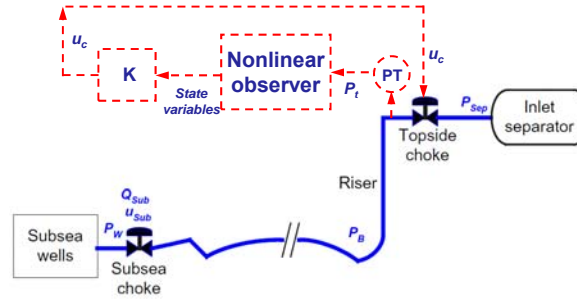
$$M_{S,min} = \prod_{i=1}^{N_p} \frac{|z + p_i|}{|z - p_i|}$$

|             | Z = 20% | Z = 40% |
|-------------|---------|---------|
| $M_{S,min}$ | 2.1     | 7.0     |



## Nonlinear Control Solutions

## Solution 1: observer & state feedback



## High-Gain Observer

$$\hat{\dot{z}}_1 = f_1(\hat{z})$$

$$\hat{\dot{z}}_2 = f_2(\hat{z})$$

$$\hat{\dot{z}}_3 = f_3(\hat{z}) + \frac{1}{\varepsilon}(y_m - \hat{y})$$

$$\hat{\dot{z}}_4 = f_4(\hat{z})$$

$z_1$  : mass of gas in the pipeline ( $m_{gp}$ )

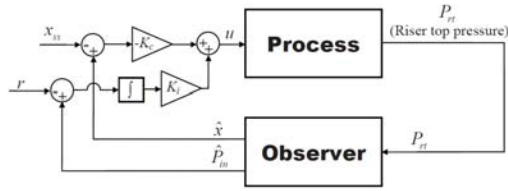
$z_2$  : mass of liquid in the pipeline ( $m_{lp}$ )

$z_3$  : pressure at top of the riser ( $P_{r,t}$ )

$z_4$  : mass of liquid in the riser ( $m_{lr}$ )

$$P_{r,t} = \frac{m_{gr}RT_r}{M_G(V_r - \frac{m_{Lr}}{\rho_l})}$$

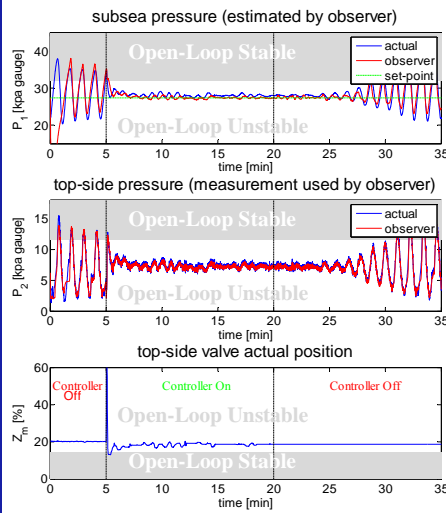
# State Feedback



$$u(t) = -K_c(\hat{x}(t) - x_{ss}) + K_i \int_0^t (\hat{P}_m(\tau) - r) d\tau$$

$K_c$ : linear optimal gain calculated by solving *Riccati equation*  
 $K_i$ : small integral gain (e.g.  $K_i = 10^{-3}$ )

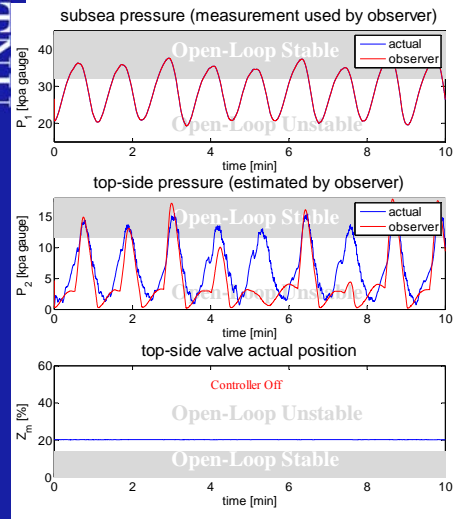
# High-gain observer – top pressure



measurement: topside pressure  
 valve opening: 20 %



# High-gain observer – subsea pressure

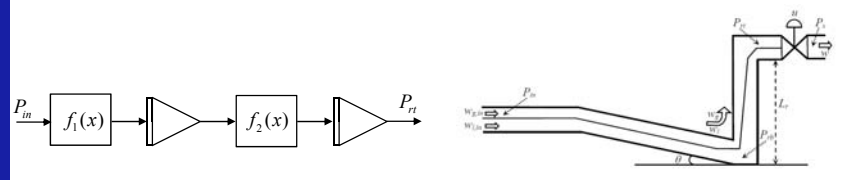


measurement: subsea pressure  
valve opening: 20 %

Not working ??!

# Chain of Integrators

- Fast nonlinear observer using subsea pressure: Not Working??!
- Fast nonlinear observer (High-gain) acts like a differentiator
- Pipeline-riser system is a chain of integrator
- Measuring top pressure and estimating subsea pressure is differentiating
- Measuring subsea pressure and estimating top pressure is integrating



## Nonlinear observer and state feedback

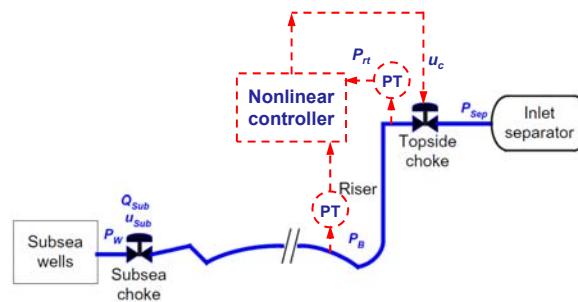
### Summary

- Anti-slug control with top-pressure is possible using fast nonlinear observers
- The operating range of top pressure is still less than subsea pressure
- Surprisingly, nonlinear observer is not working with subsea pressure, but a (simpler) linear observer works very fine.

|                    | Subsea pressure | Top Pressure |
|--------------------|-----------------|--------------|
| Nonlinear Observer | Not Working !?  | Working*     |
| Linear Observer    | Working         | Not Working  |
| PI Control         | Working         | Not Working  |
| Max. Valve         | 60%             | 20%          |

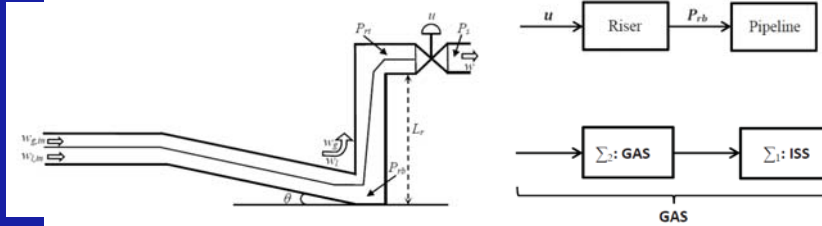
\*but only for small valve openings

## Solution 2: feedback linearization



## Solution 2: feedback linearization

Cascade system



## Output-linearizing controller

Stabilizing controller for riser subsystem

System in normal form:

$\xi$ : riser-base pressure      $\eta$ : top pressure

$$\dot{\xi} = (w_g + w_l) [F(\xi, \eta) + c] - [F(\xi, \eta) + c] w$$

$$\dot{\eta} = (w_g + w_l) F(\xi, \eta) - F(\xi, \eta) w,$$

Linearizing controller:

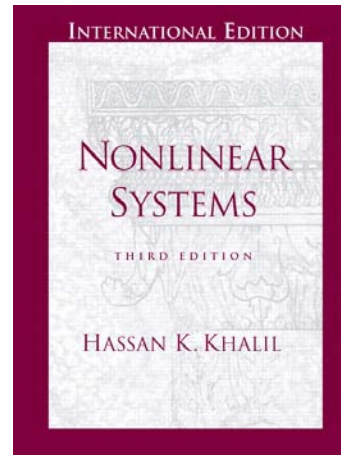
$$w = \frac{w_{in}(F(y)+c)+K_1(y_1-\bar{y}_1)}{F(y)+c}$$

$$\dot{\xi} = -K_1 \xi$$

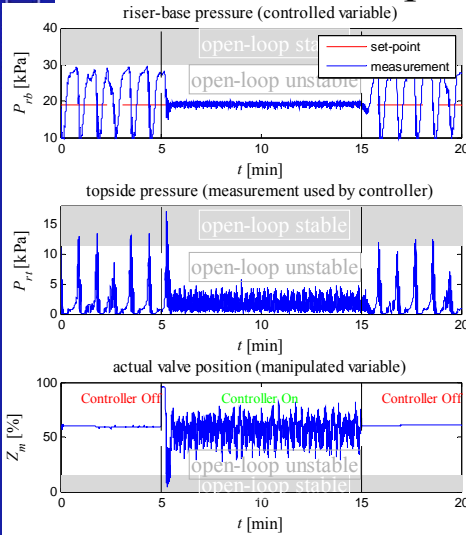
$\eta$  dynamics bounded

Control signal to valve:

$$u = \text{sat} \left( \frac{w}{C_v \sqrt{\rho_{rt}(y_2 - P_s)}} \right)$$



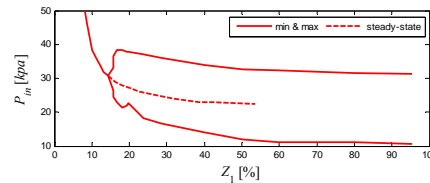
## CV: riser-base pressure ( $y_1$ ), $Z=60\%$



$$w = \frac{w_{in}(F(y)+c)+K_1(y_1-\bar{y}_1)}{F(y)+c}$$

$$u = \text{sat} \left( \frac{w}{C_v \sqrt{\rho_{rt}(y_2 - P_s)}} \right)$$

Gain:  $\frac{\partial y}{\partial u} = \frac{\partial P_{in}}{\partial Z}$



## Solution 3: Adaptive PI Tuning

Static gain:  $\frac{\partial y}{\partial u} = \frac{\partial P_{in}}{\partial Z}$

$$k(z) = \frac{-2\bar{a} \frac{\partial f(z)}{\partial z}}{f(z)^3}$$

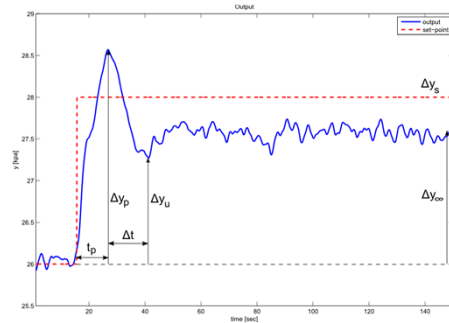
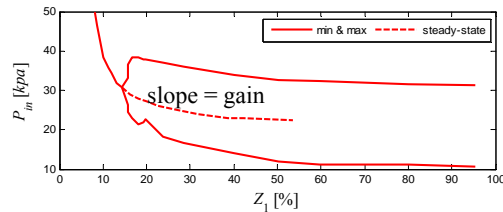
Linear valve:  $k(z) = \frac{-2\bar{a}}{z^3}$

PI Tuning:

$$\beta = \frac{-\ln \left( \frac{\Delta y_{\infty} - \Delta y_u}{\Delta y_p - \Delta y_{\infty}} \right)}{2\Delta t} + \frac{K_{c0}k(z_0) \left( \frac{\Delta y_p - \Delta y_{\infty}}{\Delta y_{\infty}} \right)^2}{4t_p}$$

$$K_c(z) = \frac{\beta T_{osc}}{k(z) \sqrt{z/z^*}}$$

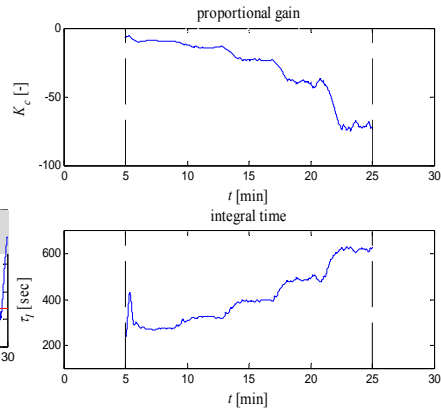
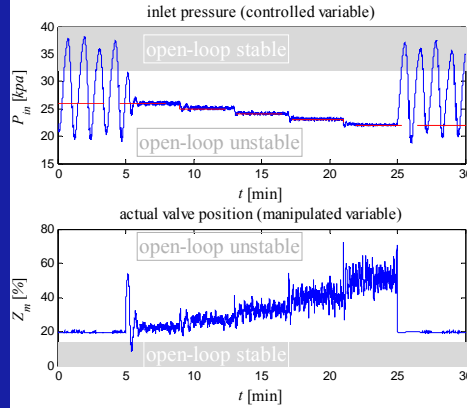
$$\tau_I(z) = 3T_{osc}(z/z^*)$$



# Solution 3: Adaptive PI Tuning

## Experiment

Experiment



# Solution 4: Gain-Scheduling IMC

Three identified model from step tests:

$$Z=20\%: G_1(s) = \frac{-0.015(s + 0.26)}{s^2 - 0.045s + 0.0094}$$

$$Z=30\%: G_2(s) = \frac{-0.0098(s + 0.25)}{s^2 - 0.040s + 0.025}$$

$$Z=40\%: G_3(s) = \frac{-0.0056(s + 0.27)}{s^2 - 0.017s + 0.096}$$

Three IMC controllers:

$$C_1(s) = \frac{-16.15(s^2 + 0.016s + 0.0012)}{s(s + 0.26)}$$

$$C_2(s) = \frac{-42.20(s^2 + 0.052s + 0.0047)}{s(s + 0.25)}$$

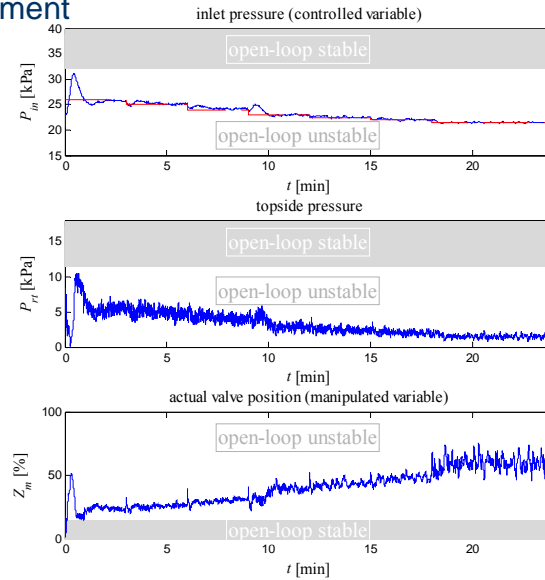
$$C_3(s) = \frac{-115.11(s^2 + 0.052s + 0.014)}{s(s + 0.27)}$$

Gain-scheduling logic

| Pressure set-point                            | Controller |
|---|------------|
| $P_{set} \geq 24 \text{ kPa}$                 | $C_1(s)$   |
| $24 \text{ kPa} > P_{set} > 21.5 \text{ kPa}$ | $C_2(s)$   |
| $P_{set} \leq 21.5 \text{ kPa}$               | $C_3(s)$   |

## Solution 4: Gain-Scheduling IMC

### Experiment



## Comparison of Nonlinear Controllers

- Gain-scheduling IMC is the most robust solution
- Adaptive PI controller is the second-best
- Controllability remarks:
  - Fundamental limitation control: gain of the system goes to zero for fully open valve
  - Additional limitation top-side pressure: Inverse response (non-minimum-phase)

|                      | CV                  | $\theta = 0$ | $\theta = 1$ | $\theta = 2$ |
|----------------------|---------------------|--------------|--------------|--------------|
| Gain-scheduling IMC  | $P_{in}$            | 60%          | 60%          | 50%          |
| Adaptive PI          | $P_{in}$            | 60%          | 50%          | 32%          |
| Output linearization | $P_{rb}$ & $P_{rt}$ | 60%          | 40%          | 25%          |
| Nonlinear observer   | $P_{rt}$            | 28%          | 24%          | 22%          |

## Conclusions

- A new simplified model verified by OLGA simulations and experiments
- Best: Anti-slug control using a subsea valve close to riser-base
- New robust controllers have been developed
- Main point: Increase controller gain for large valve openings

### Acknowledgements

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