Selv-optimaliserende og eksplisitte metoder for online optimalisering

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Effective Implementation of optimal operation using Off-Line Computations

Servomøtet, Trondheim, Oktober 2009

Research Sigurd Skogestad

Graduated PhDs since 2000

1. **Truls Larsson, *Studies on plantwide control*, Aug. 2000. (Aker Kværner, Stavanger)**
2. Eva-Katrine Hilmen, *Separation of azeotropic mixtures*, Des. 2000. (ABB, Oslo)
3. Ivar J. Halvorsen; *Minimum energy requirements in distillation* ,May 2001. (SINTEF)
4. **Marius S. Govatsmark, *Integrated optimization and control*, Sept. 2003. (Statoil, Haugesund)**
5. **Audun Faanes, *Controllability analysis and control structures*, Sept. 2003. (Statoil, Trondheim)**
6. Hilde K. Engelien, *Process integration for distillation columns*, March 2004. (Aker Kværner)
7. Stathis Skouras, *Heteroazeotropic batch distillation*, May 2004. (StatoilHydro, Haugesund)
8. **Vidar Alstad, *Studies on selection of controlled variables*, June 2005. (Statoil, Porsgrunn)**
9. **Espen Storkaas, *Control solutions to avoid slug flow in pipeline-riser systems*, June 2005. (ABB)**
10. **Antonio C.B. Araujo, *Studies on plantwide control*, Jan. 2007. (Un. Campina Grande, Brazil)**
11. **Tore Lid, *Data reconciliation and optimal operation of refinery processes* , June 2007 (Statoil)**
12. **Federico Zenith, *Control of fuel cells*, June 2007 (Max Planck Institute, Magdeburg)**
13. **Jørgen B. Jensen, *Optimal operation of refrigeration cycles*, May 2008 (ABB, Oslo)**
14. **Heidi Sivertsen, *Stabilization of desired flow regimes (no slug)*, Dec. 2008 (Statoil, Stjørdal)**
15. **Elvira M.B. Aske, *Plantwide control systems with focus on max throughput*, Mar 2009 (Statoil)**
16. **Andreas Linhart An aggregation model reduction method for one-dimensional distributed systems, Oct. 2009.**

Current research:

- Restricted-complexity control (self-optimizing control):
  - off-line and analytical solutions to optimal control (incl. explicit MPC & explicit RTO)
  - multivariable PID
  - batch processes
- Plantwide control. Applications: LNG, GTL

Outline

- Implementation of optimal operation
- Paradigm 1: On-line optimizing control
- Paradigm 2: "Self-optimizing" control schemes
  - Precomputed (off-line) solution
- Examples
- Control of optimal measurement combinations
  - Nullspace method
  - Exact local methom
  - Link to optimal control / Explicit MPC
- Conclusion

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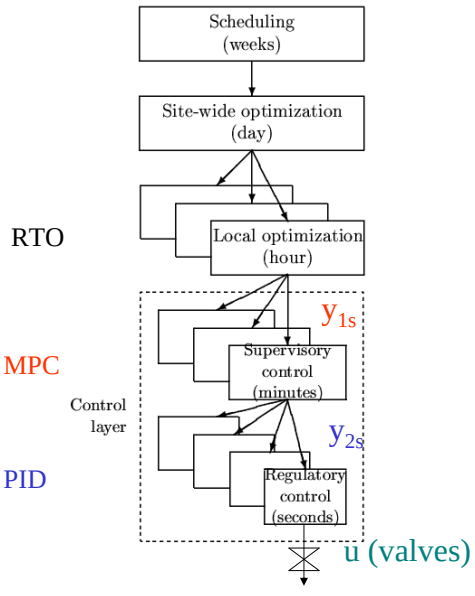
Process control: Implementation of optimal operation



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Optimal operation

- A typical dynamic optimization problem

$$\begin{aligned} \min_u \; & J(x,u,d) \\ \text{s.t. } & \dot{x} = f(x,u,d), \\ & h(x,u,d) = 0, \\ & g(x,u,d) \leq 0. \end{aligned}$$

- **Implementation:** "Open-loop" solutions not robust to disturbances or model errors
- Want to introduce feedback

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Implementation of optimal operation

- **Paradigm 1: On-line optimizing control** where measurements are used to update model and states
- **Paradigm 2: "Self-optimizing" control scheme** found by exploiting properties of the solution

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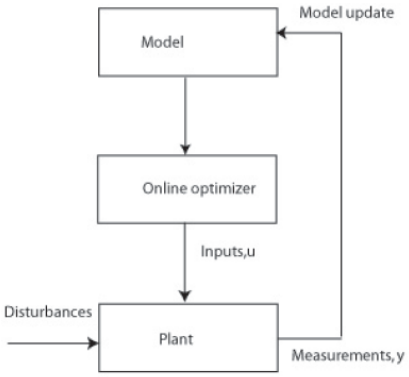
Implementation: Paradigm 1

- Paradigm 1: **Online** optimizing control
- Measurements are primarily used to update the model
- The optimization problem is resolved online to compute new inputs.
- Example: Conventional MPC
- This is the "obvious" approach (for someone who does not know control)



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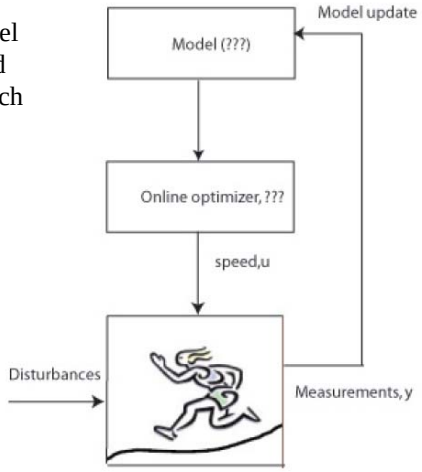
Example paradigm 1: On-line optimizing control of Marathon runner

- Even getting a reasonable model requires > 10 PhD's ☺ … and the model has to be fitted to each individual….
- Clearly impractical!



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Implementation: Paradigm 2

- **Paradigm 2:** Precomputed solutions based on **off-line** optimization
- Find properties of the solution suited for simple and robust on-line implementation
- **Proposed method**: Turn optimization into feedback problem.
  - Find regions of active constraints and in each region:
    1. **Control active constraints**
    2. **Control "self-optimizing " variables for the remaining unconstrained degrees of freedom**
       - "inherent optimal operation"

- Examples
  - Marathon runner
  - Hierarchical decomposition
  - Optimal control
  - Explicit MPC

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Optimal operation - Runner

Solution 2 – Feedback (Self-optimizing control)

- What should we control?

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Self-optimizing control: Sprinter (100m)

- 1. Optimal operation of Sprinter, J=T
  - Active constraint control:
    - Maximum speed (”no thinking required”)

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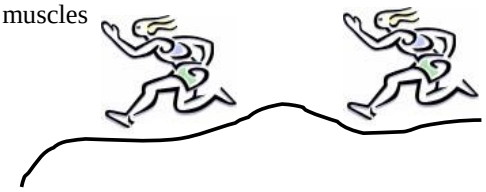
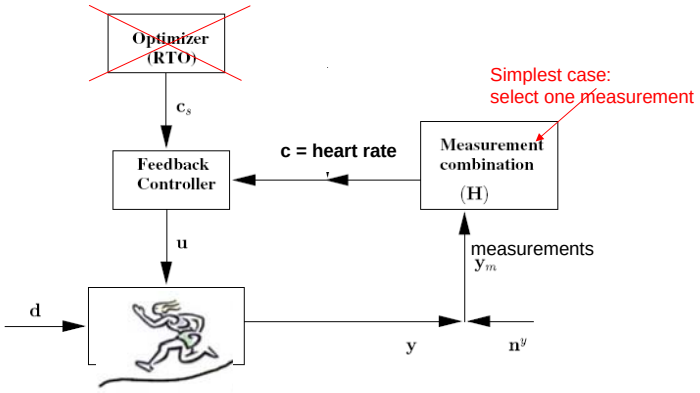
Self-optimizing control: Marathon (40 km)

- Optimal operation of Marathon runner, J=T
- Any self-optimizing variable c (to control at constant setpoint)?
  - c_1 = distance to leader of race
  - c_2 = speed
  - c_3 = heart rate
  - c_4 = level of lactate in muscles



Implementation paradigm 2: Feedback control of Marathon runner



- Simple and robust implementation
- Disturbances are indirectly handled by keeping a constant heart rate
- May have infrequent adjustment of setpoint (heart rate)

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Further examples self-optimizing control

- Marathon runner
- Central bank
- Cake baking
- Business systems (KPIs)
- Investment portifolio
- Biology
- Chemical process plants

Define optimal operation (J) and look for ”magic” variable (c) which when kept constant gives acceptable loss (self-optimizing control)

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More on further examples

- Central bank. J = welfare. u = interest rate. c=inflation rate (2.5%)
- Cake baking. J = nice taste, u = heat input. c = Temperature (200C)
- Business, J = profit. c = ”Key performance indicator (KPI), e.g.
  - Response time to order
  - Energy consumption pr. kg or unit
  - Number of employees
  - Research spending

  Optimal values obtained by ”benchmarking”
- Investment (portofolio management). J = profit. c = Fraction of investment in shares (50%)
- Biological systems:
  - ”Self-optimizing” controlled variables c have been found by natural selection
  - Need to do ”reverse engineering” :
    - Find the controlled variables used in nature
    - From this possibly identify what overall objective J the biological system has been attempting to optimize

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Example paradigm 2: Optimal operation of chemical plant

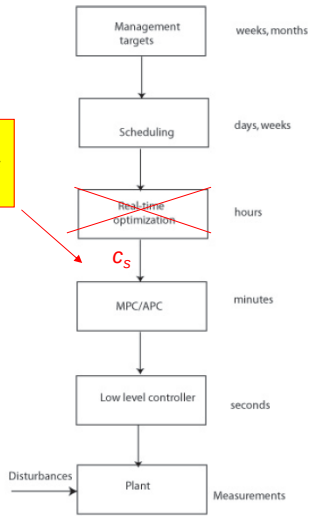
- Hierarchial decomposition based on time scale separation

Self-optimizing control: *Acceptable operation (=acceptable loss) achieved using constant set points (c_s) for the controlled variables c*

Controlled variables c

1. Active constraints
2. “Self-optimizing” variables c
   - for remaining unconstrained degrees of freedom (u)

- No or infrequent online optimization.
- Controlled variables c are found based on **off-line** analysis.



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Summary feedback approach: Turn optimization into setpoint tracking

Issue: What should we control to achieve indirect optimal operation ?

Primary controlled variables (CVs):

1. **Control active constraints!**
2. **Unconstrained CVs: Look for "magic" self-optimizing variables!**

Need to identify CVs for each region of active constraints

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"Magic" self-optimizing variables: How do we find them?

- *Intuition:* "Dominant variables" (Shinnar)
- Is there any systematic procedure?

A. Senstive variables: "Max. gain rule" (Gain= Minimum singular value)
B. "Brute force" loss evaluation
C. Optimal linear combination of measurements, c = Hy

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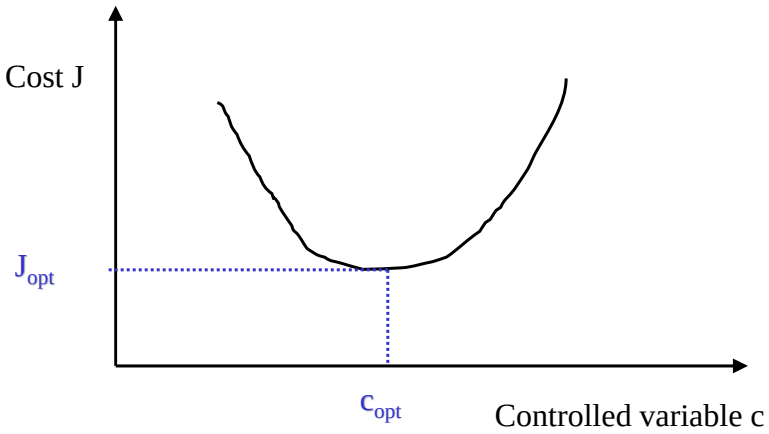
Unconstrained optimum

Optimal operation



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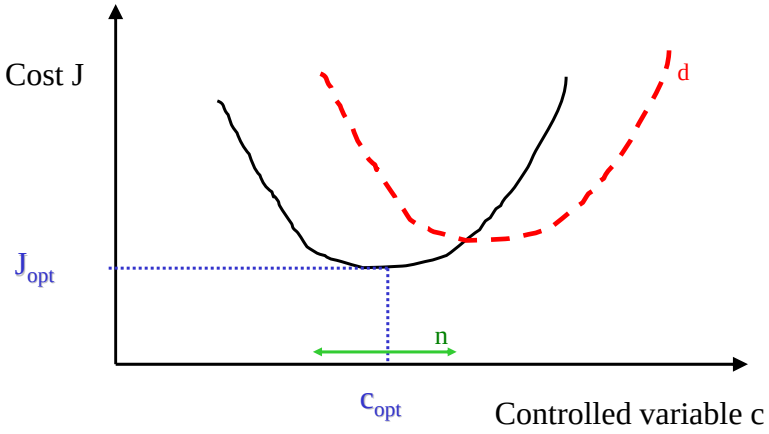
Unconstrained optimum

Optimal operation



Two problems:

- 1. Optimum moves because of disturbances d: $c_{opt}(d)$
- 2. Implementation error, $c = c_{opt} + n$

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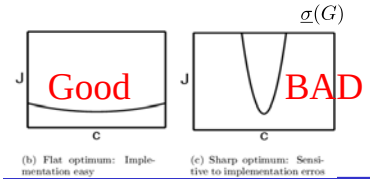
Candidate controlled variables c for self-optimizing control

Intuitive

1. The *optimal value* of c should be *insensitive* to disturbances (avoid problem 1)

2. Optimum should be flat (avoid problem 2 – implementation error).
   Equivalently: *Value* of c should be *sensitive* to degrees of freedom u.
   - "Want large gain", |G|
   - Or more generally: Maximize minimum singular value, $\underline{\sigma}(G)$



(b) Flat optimum: Implementation easy (c) Sharp optimum: Sensitive to implementation erros

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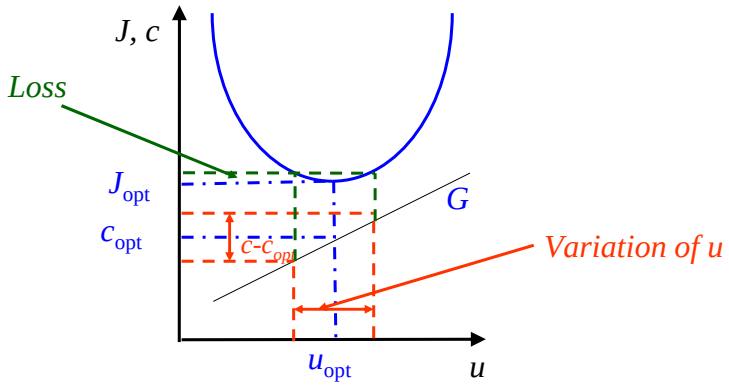
Quantitative steady-state: Maximum gain rule

Maximum gain rule *(Skogestad and Postlethwaite, 1996): Look for variables that maximize the scaled gain* $\underline{\sigma}(G_s)$ *(minimum singular value of the appropriately scaled steady-state gain matrix* G_s *from u to c)*

$$G_s = SGJ_{uu}^{1/2}, \quad S = diag\{1/\text{span}c_i\}$$

$$\text{Loss} \approx \frac{1}{2} \cdot \frac{1}{\underline{\sigma}(G_s)^2}$$

Why is Large Gain Good?



With large gain *G:* Even large implementation error *n in c* translates into small deviation of *u* from $u_{opt}(d)$ - leading to lower loss

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Unconstrained degrees of freedom:

"Self-optimizing" variable *combinations*

- **Operational objective: Minimize cost function J(u,d)**
- The ideal "self-optimizing" variable is the gradient (first-order optimality condition (ref: Bonvin and coworkers)):

$$c = \alpha J_u; \qquad J_u = \frac{\partial J}{\partial u}$$

  - Optimal setpoint = 0
- BUT: Gradient can not be measured in practice
- Possible approach: Estimate gradient J_u based on measurements y

- Here alternative approach: Find **optimal linear measurement combination**

$$c = Hy$$

which when kept constant ($\pm$ n) minimize the effect of d on loss.

Loss = J(u,d) – $J(u_{opt},d)$; where input u is used to keep c = constant $\pm$ n

- Candidate measurements (y): Include also inputs u

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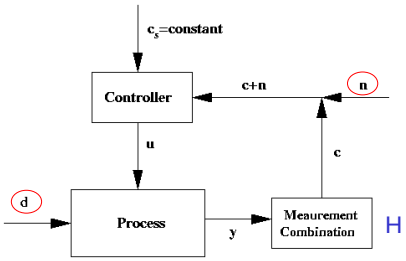
Unconstrained degrees of freedom:

Optimal measurement combination

$$\Delta c = h_1 \Delta y_1 + h_2 \Delta y_2 + \cdots = H \Delta y$$



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Unconstrained degrees of freedom:

Optimal measurement combination

$$\Delta c = h_1 \Delta y_1 + h_2 \Delta y_2 + \cdots = H \Delta y$$

***1. Nullspace method** for n = 0 (Alstad and Skogestad, 2007)*

Basis: Want optimal value of c to be independent of disturbances

$\Rightarrow \quad \Delta c_{\text{opt}} = 0 \cdot \Delta d$

- Find optimal solution as a function of d: $u_{opt}(d)$, $y_{opt}(d)$
- Linearize this relationship: $\Delta y_{opt} = F \, \Delta d$
- Want: $\Delta c_{opt} = H \Delta y_{opt} = HF \Delta d = 0$
- To achieve this for all values of Δ d:

$$HF = 0 \;\Rightarrow\; H \in \mathcal{N}(F^T)$$

- Always possible to find H that satisfies HF=0 provided

$$n_y \geq n_u + n_d$$

- *Optimal* when we disregard implementation error (n)

Amazingly simple!

Sigurd is told by Vidar Alstad how easy it is to find H

V. Alstad and S. Skogestad, ``Null Space Method for Selecting Optimal Measurement Combinations as Controlled Variables", Ind.Eng.Chem.Res, 46 (3), 846-853 (2007).

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Unconstrained degrees of freedom:

Optimal measurement combination

$$\Delta c = h_1 \Delta y_1 + h_2 \Delta y_2 + \cdots = H \Delta y$$

2. "Exact local method"
(Combined disturbances and implementation errors)

Theorem 1. Worst-case loss for given H (Halvorsen et al, 2003):

$$L_{wc} = \max_{\left\| \begin{bmatrix} \mathbf{d}' \\ \mathbf{n}^{y\prime} \end{bmatrix} \right\|_2 \leq 1} L = \frac{1}{2}(\bar{\sigma}[\mathbf{M}])^2$$

$$\mathbf{M} \triangleq [\mathbf{M_d} \quad \mathbf{M_{n^y}}]$$

$$\mathbf{M}_d = -\mathbf{J}_{uu}^{1/2}(\mathbf{HG}^y)^{-1}\mathbf{HFW}_d$$

$$\mathbf{M}_{n^y} = -\mathbf{J}_{uu}^{1/2}(\mathbf{HG}^y)^{-1}\mathbf{HW}_{n^y}$$

$$\Delta \mathbf{y}^{opt} = \underbrace{-(\mathbf{G}^y\mathbf{J}_{uu}^{-1}\mathbf{J}_{ud} - \mathbf{G}_d^y)}_{\mathbf{F}} \Delta \mathbf{d}$$

Applies to any H (selection/combination)

Optimization problem for optimal combination: $\mathbf{H} = \arg\min_{\mathbf{H}} \bar{\sigma}(\mathbf{H}\tilde{\mathbf{F}})$ subject to $\mathbf{HG}^y = \mathbf{J}_{uu}^{1/2}$

V. Alstad, S. Skogestad and E.S. Hori, ``Optimal measurement combinations as controlled variables", Journal of Process Control, 19, 138-148 (2009).

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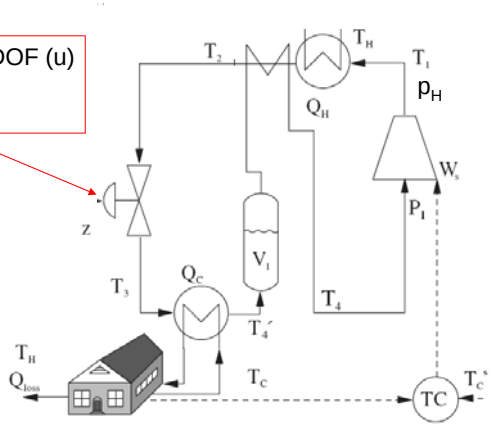
Example: CO2 refrigeration cycle

Unconstrained DOF (u)
Control what?
c=?



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CO2 refrigeration cycle

Step 1. One (remaining) degree of freedom (u=z)

Step 2. Objective function. J = W_s (compressor work)

Step 3. Optimize operation for disturbances (d_1=T_C, d_2=T_H, d_3=UA)

- Optimum always unconstrained

Step 4. Implementation of optimal operation

- No good single measurements (all give large losses):
  - p_h, T_h, z, …
- Nullspace method: Need to combine n_u+n_d=1+3=4 measurements to have zero disturbance loss
- Simpler: Try combining two measurements. Exact local method:
  - $c = h_1 p_h + h_2 T_h = p_h + k T_h$; k = -8.53 bar/K
- Nonlinear evaluation of loss: OK!

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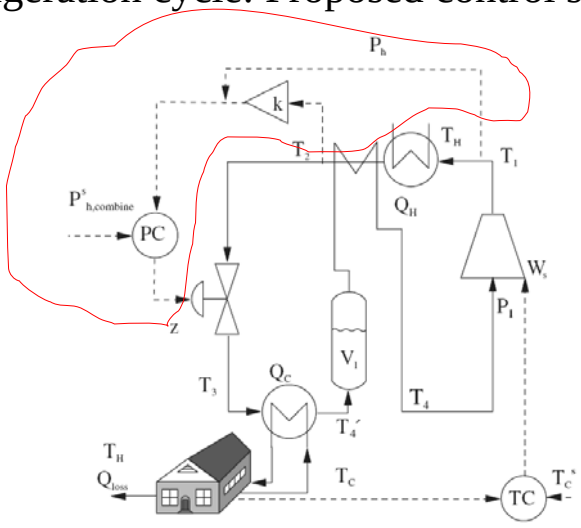
Refrigeration cycle: Proposed control structure



Control c= “temperature-corrected high pressure”

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Summary:
Procedure selection controlled variables

1. Define economics (cost J) and operational constraints
2. Identify degrees of freedom and important disturbances
3. Optimize for various disturbances
4. Identify active constraints regions (off-line calculations)

For each active constraint region do step 5-6:

5. Identify "self-optimizing" controlled variables for remaining degrees of freedom
6. Identify switching policies between regions

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What about optimal control and MPC (model predictive control)?

Paradigm 1: On-line optimizing control where measurements are used to update model and states — *MPC*

Paradigm 2: "Self-optimizing" control scheme found by exploiting properties of the solution — *Optimal control = "Explicit MPC"*

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Example paradigm 2: Feedback implementation of optimal control (LQ)

- Optimal solution to infinite time dynamic optimization problem
- Originally formulated as a "open-loop" optimization problem (no feedback)
- "By chance" the optimal u can be generated by simple state feedback

  $u = K_{LQ}\, x$
- K_{LQ} is obtained **off-line** by solving Riccatti equations

- Explicit MPC: Extension using different K_{LQ} in each constraint region

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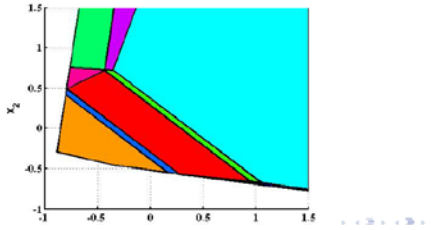
Example paradigm 2: Explicit MPC

The optimal solution $U^*(x)$ is a Piece-Wise Affine function of the current state x_t: (Bemporad et al., 2002)

$$U^*(x) = \begin{cases} K_1 x + g_1, \ if, \ x \in X_1 \\ K_2 x + g_2, \ if \ x \in X_2 \\ \vdots \\ K_n x + g_n, \ if \ x \in X_n \end{cases}$$



- **Summary: Two paradigms MPC**
  1. Conventional MPC: On-line optimization
  2. Explicit MPC: Off-line calculation of K_{LQ} for each region (must determine regions online)

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A. Bemporad, M. Morari, V. Dua, E.N. Pistikopoulos, "The Explicit Linear Quadratic Regulator for Constrained Systems", Automatica, vol. 38, no. 1, pp. 3-20 (2002).

Summary Paradigm 2: Precomputed on-line solutions based on **off-line** optimization

Issues (expected research results for specific application):

1. Find analytical or precomputed solutions suitable for on-line implementation
2. Find structure of optimal solution for specific problems
   - Typically, identify **regions** where different set of constraints are active
3. Find good "self-optimizing" variables c to control in each **region:**
   - Active constraints
   - Good variables or variable *combinations* (for remaining unconstrained)
4. Find optimal values (or trajectories) for unconstrained variables
5. Determine a switching policy between different **regions**

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Conclusion

- Simple control policies are always preferred in practice (if they exist and can be found)
- **Paradigm 2**: Use off-line optimization and analysis to find simple near-optimal control policies suitable for on-line implementation
- Current research: Several interesting extensions
  - Optimal region switching
  - Dynamic optimization
  - Explicit MPC

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