LINEARIZING EFFECT OF HIGH-GAIN FEEDBACK

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LINEARIZING EFFECT OF HIGH-GAIN FEEDBACK. Abstract

The linearizing effect of high-gain feedback has been known since Harold Black invented the feedback amplifier in the 1920s. It seemed like magic, so the idea was met with a lot of resistance - and it took 10 years to grant the patent. Even today, the effectiveness of high-gain feedback is not well known by most control engineers (who tend to think feedforward).

In the talk I go through the development of Blacks idea and how it initiated theoretical developments of stability (Nyquist). I also make the link to cascade control, where the main benefit of adding a high-gain slave loop is often to linearize the system (in addition to stabilization and disturbance rejection, which seem to be more well known).

Linearizing effect of high-gain feedback

- 100 years after Black's Feedback Amplifier (1927)
- Exrtemely important in many applications
- Was met with strong opposition
  - Because feedback is counter-intuitive
- Still not understood (and known) by todays control engineers

What is the first idea if you encounter nonlinearity?

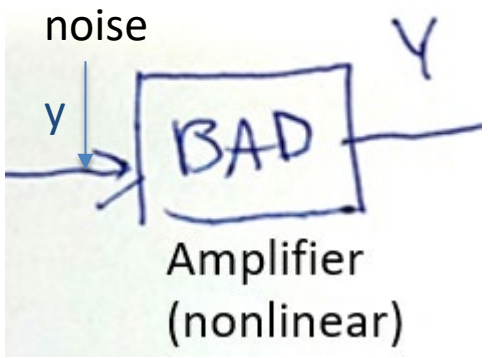
- Insert a block that inverts the nonlineary
- Perfect for static nonlinearity with known model

- But what if the model is unknown?
  - Feedforward control
  - Feedback control

«Telephone repeater» (Amplifier)

Amplification was required to send telephone signals across the US in the 1920s - and it required 12 amplifications on the way, so they better be fairly accurate.

Amplification each station: About 10 (20 dB)



$Y = G(y)\,(y+n)$

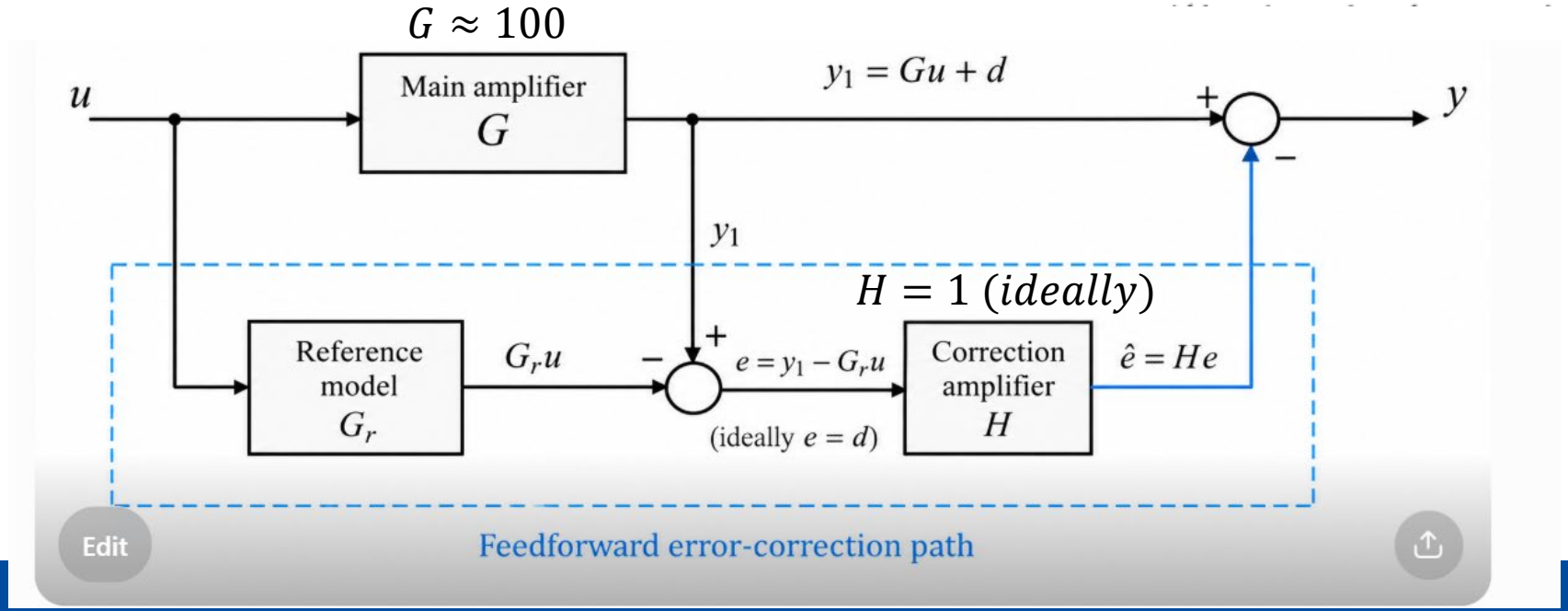
Telephone repeater station around 1930

Problem: Think of «Hviskeleken» (Telephone game)

The feedforward amplifier

- The original idea of all engineers is to think **feedforward** (Bell Labs)
- Y = G y
  - where G is the amplifier gain
  - This works well if G can be set accurately + no noise
  - But it is very difficult, especially with 1920s vacuum tube amplifiers
- So Harold Black proposed the «feedforward» amplifier
  - Idea: Add a smaller «correction amplifier» H

down to the fundamentals of a problem. As a result I restated my assignment as being that of removing distortion products from the amplifier output. I immediately observed that by reducing the output to the same amplitude as the input and subtracting one from the other, the distortion products only would remain which could then be amplified in a separate amplifier and used to cancel out the distortion products in the original amplifier output ... Thus, the Feedforward Amplifier came into being" [4, pp. 64-65].



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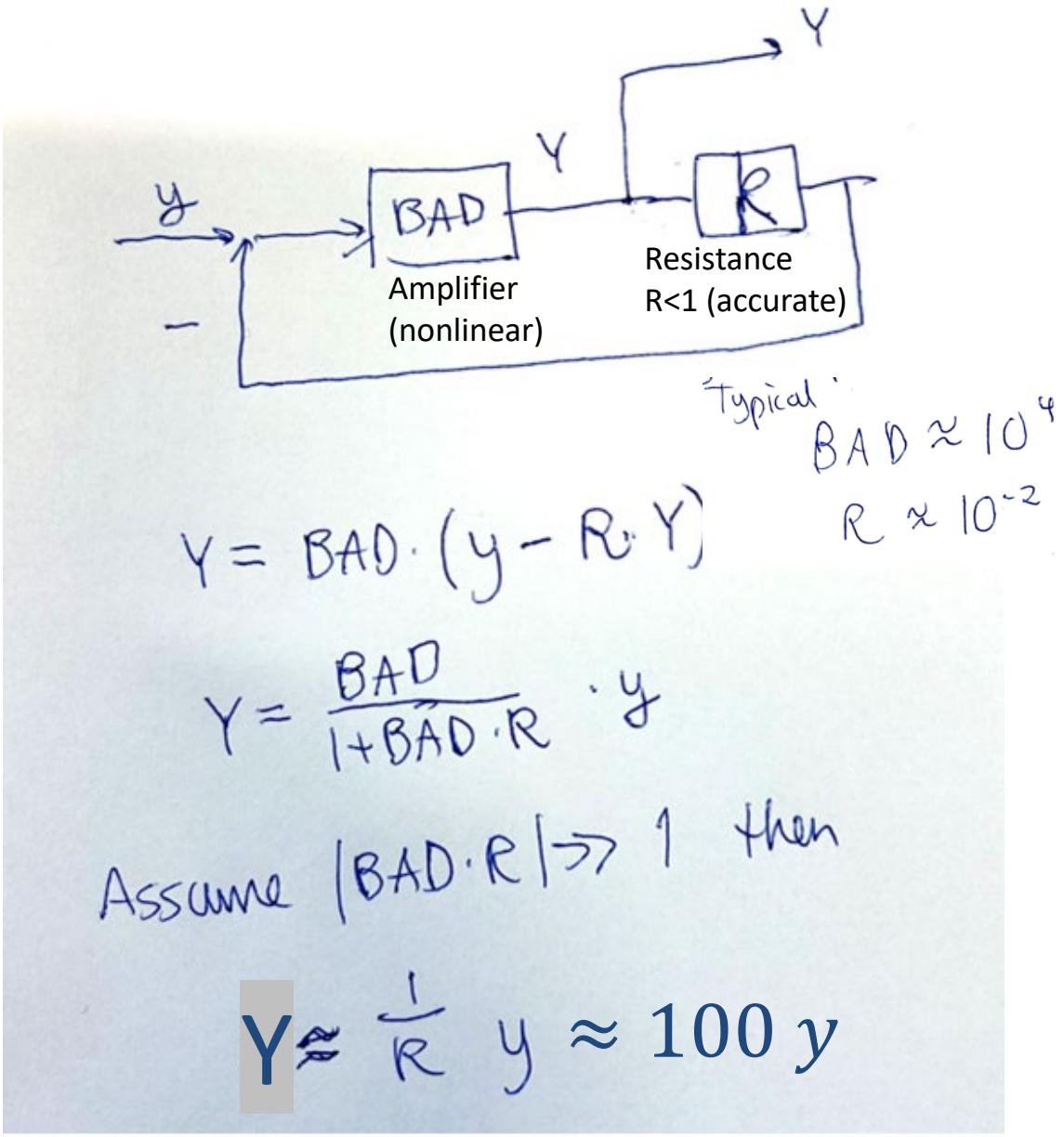
The negative feedback amplifier

- Y = G y
  - where G=BAD is the amplifier gain

Inspiration on the Lackawanna Ferry

Then came the morning of Tuesday, August 2, 1927, when the concept of the negative feedback amplifier came to me in a flash while I was crossing the Hudson River on the Lackawanna Ferry, on my way to work. For more than 50 years I have pondered how and why the idea came, and I can't say any more today than I could that morning. All I know is that after several years of hard work on the problem, I suddenly realized that if I fed the amplifier output back to the input, in reverse phase, and kept the device from oscillating (singing, as we called it then), I would have exactly what I wanted: a means of canceling out the distortion in the output. I opened my morning newspaper and on a page of *The New York Times* I sketched a simple canonical diagram of a negative feedback amplifier plus the equations for the amplification with feedback (Fig. 2). I signed the sketch, and 20 minutes later, when I reached the laboratory at 463 West Street, it was witnessed, understood, and signed by the last Earl C. Blessing.

Harold Black (on a New York ferry, 02 August 1927):



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The Bell System Technical Journal

January, 1934

Stabilized Feedback Amplifiers*

By H. S. BLACK

This paper describes and explains the theory of the feedback principle and then demonstrates how stability of amplification and reduction of modulation products, as well as certain other advantages, follow when stabilized feedback is applied to an amplifier. The underlying principle of design by means of which singing is avoided is next set forth. The paper concludes with some examples of results obtained on amplifiers which have been built employing this new principle.

The carrier-in-cable system dealt with in a companion paper [1] involves many amplifiers in tandem with many telephone channels passing through each amplifier and constitutes, therefore, an ideal field for application of this feedback principle. A field trial of this system was made at Morristown, New Jersey, in which seventy of these amplifiers were operated in tandem. The results of this trial were highly satisfactory and demonstrated conclusively the correctness of the theory and the practicability of its commercial application.

Conclusion

The feedback amplifier dealt with in this paper was developed primarily with requirements in mind for a cable carrier telephone system, involving many amplifiers in tandem with many telephone channels passing through each amplifier. Most of the examples of feedback amplifier performance have naturally been drawn from amplifiers designed for this field of operation. In this field, vacuum tube amplifiers normally possessing good characteristics with respect to stability and freedom from distortion are made to possess superlatively good characteristics by application of the feedback principle.

However, certain types of amplifiers in which economy has been secured by sacrificing performance characteristics, particularly as regards distortion, can be made to possess improved characteristics by the application of feedback. Discussion of these amplifiers is beyond the scope of this paper.

*Presented at Winter Convention of A. I. E. E., New York City, Jan. 23–26, 1934. Published in *Electrical Engineering*, January, 1934.

[1] "Carrier in Cable" by A. B. Clark and B. W. Kendall, presented at the A. I. E. E. Summer Convention, Chicago, Ill., June, 1933; published in *Electrical Engineering*, July, 1933, and in *Bell Sys. Tech. Jour.*, July, 1933.

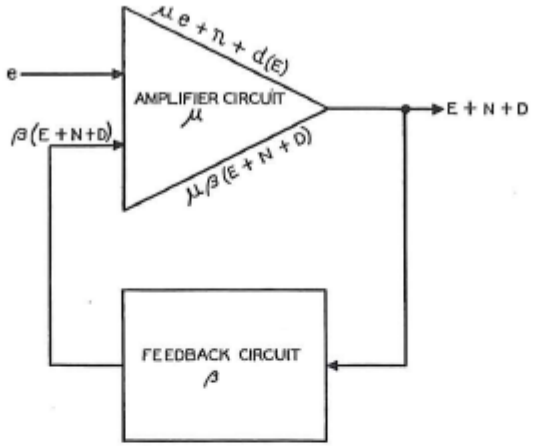

Fig. 1—Amplifier system with feedback.

e—Signal input voltage.
μ—Propagation of amplifier circuit.
μe—Signal output voltage without feedback.
n—Noise output voltage without feedback.
$d(E)$—Distortion output voltage without feedback.
β—Propagation of feedback circuit.
E—Signal output voltage with feedback.
N—Noise output voltage with feedback.
D—Distortion output voltage with feedback.

The output voltage with feedback is $E + N + D$ and is the sum of $\mu e + n + d(E)$, the value without feedback plus $\mu\beta[E + N + D]$ due to feedback.

$$E + N + D = \mu e + n + d(E) + \mu\beta[E + N + D]$$

$$[E + N + D](1 - \mu\beta) = \mu e + n + d(E)$$

$$E + N + D = \frac{\mu e}{1 - \mu\beta} + \frac{n}{1 - \mu\beta} + \frac{d(E)}{1 - \mu\beta}$$

If $|\mu\beta| \gg 1$, $E \doteq -\frac{e}{\beta}$. Under this condition the amplification is independent of μ but does depend upon β. Consequently the over-all characteristic will be controlled by the feedback circuit which may include equalizers or other corrective networks.

Experiment

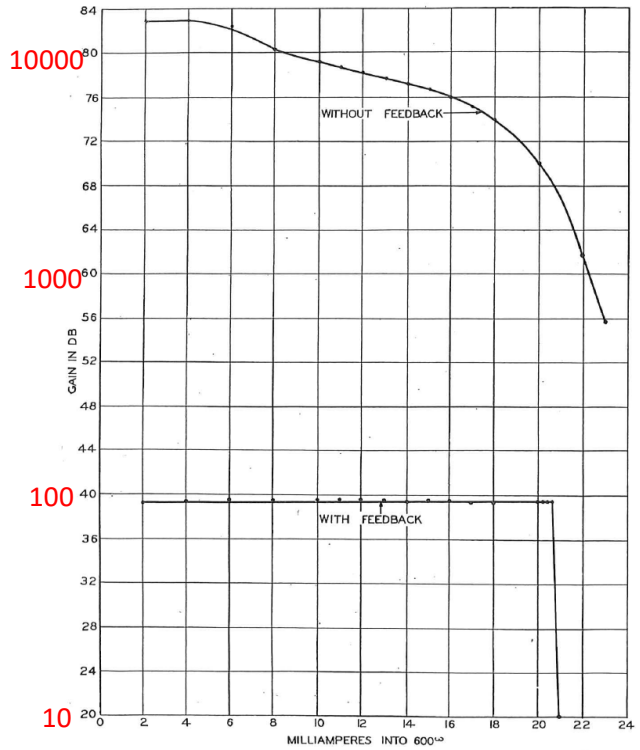

Fig. 10—Gain-load characteristic with and without feedback for a low level amplifier designed to amplify frequencies from 3.5 to 50 kc.

BAD amplifier:

$$\mu \approx 10^4 \quad (80\ dB)$$

Accurate resistance:

$$\beta \approx 10^{-2}$$

Get

$$\mu\beta \approx 100 \gg 1$$

Resulting amplification (negative feedback)

$$\frac{\mu}{1 + \mu\beta} \approx \frac{1}{\beta} = 100 \quad (40\ dB)$$

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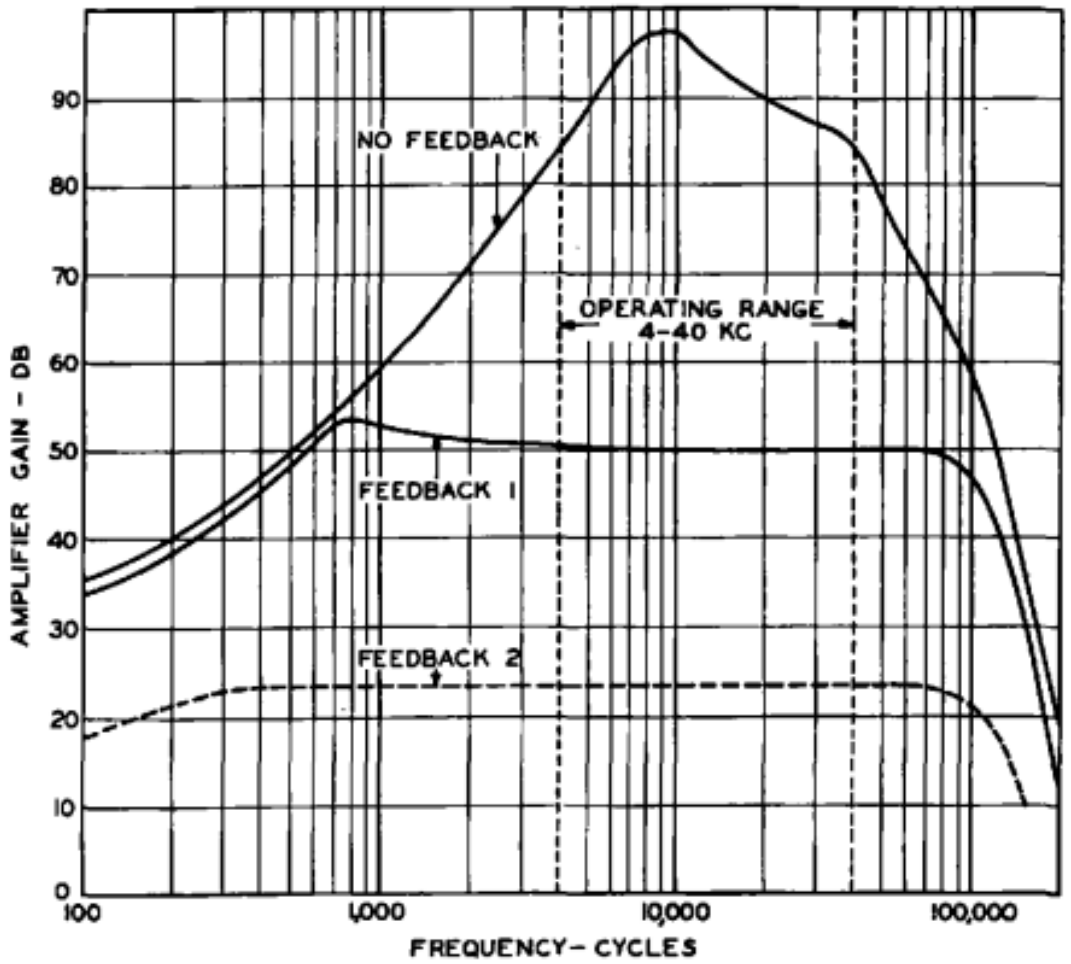


Fig. 6—Gain frequency characteristics with and without feedback of amplifier of

BAD amplifier:

$$G=BAD \approx 10^4 \quad (80\,dB)$$

Accurate resistance:

$$R \approx 10^{-2}$$

Get loop gain

$$GR \approx 100 \gg 1$$

Resulting amplification (negative feedback)

$$\frac{G}{1+GR} \approx \frac{1}{R} = 100 \quad (40\,dB)$$

Patents

- **1925:** Black filed his patent application for the feedforward amplifier.
- **1927:** He invented the negative-feedback amplifier while riding the ferry to Manhattan.
- **1928:** His feedforward patent, US 1,686,792, was granted.
- **1937:** His negative-feedback patent was finally granted.

Nine years in the Patent Office

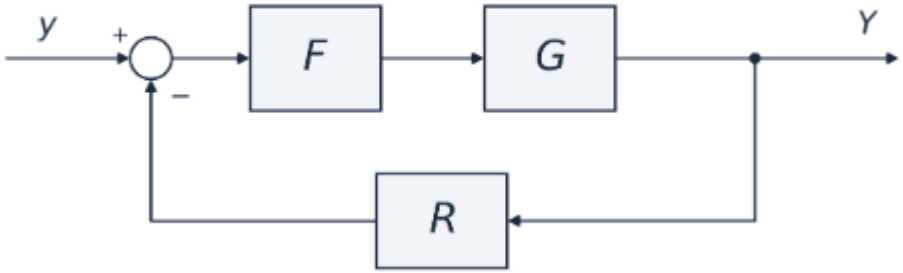
Although the invention had been submitted to the U.S. Patent Office on August 8, 1928, more than nine years would elapse before the patent was issued on December 21, 1937 (No. 2 102 671). One reason for the delay was that the concept was so contrary to established beliefs that the Patent Office initially did not believe it would work. The Office cited technical papers, for example, that maintained the output could not be connected back to the input unless the loop gain was less than one, whereas mine was between 40 and 50 dB. In England, our patent application was treated in the same manner as one for a perpetual-motion machine. The British patent office also claimed the amplifier would not work and asked me to submit a working model! Harry Burgess was eventually able to overcome all these objections by submitting evidence that 70 amplifiers were working successfully in the telephone building at Morristown. But this process took several years.

Proving that Black's invention worked also initiated the idea of frequency analysis(Nyquist)

Stability (Avoid «singing»):
Harry Nyquist (also at Bell lab) solved the problem

Within a few years, Harry Nyquist would publish his generalized rule for avoiding instability in a feedback amplifier, and Heinrich W. Bode would spearhead the development of systematic techniques of design whereby one could get the most out of a specified situation and still satisfy Nyquist's criterion. Through their work, as well as the efforts of many others, the feedback amplifier was launched on the road that within 25 years would lead to a report by Dr. Kelly that "the negative feedback principle is now applied almost universally to amplifiers used for any purpose."

Stability (Avoid «singing»)



$$Y = \frac{GF}{1 + GFR} y = T_{cl}\, y$$

G = amplifier = 1.e4 + dynamics (delay)

R = gain = 1/100

F = $\frac{1}{\tau s+1}$ – filter to avoid «singing»

- Dynamics in each amplifier 1/(tau*s+1), tau=1.e-6 s (1 µs)
- For amplifier gain e4 typically need 3 stages, G = k/(tau*s+1)^3, k=e4
- Get phase lag -270° with n=3 we - so max. gain L(0) = G(0)R =8 to have stability (with n=4: L(0)=2).
- So we clearly need some dynamics (F) in the loop.
- The simplest is first-order dynamics
- Represent the plant as a delay theta = 3 e-6 s. Then we need to bring down the gain to 1, approximately at 1/3e-3s. So if the amplification is 100 we need to have tau=300 e-6 s = 3 e-4 s = 0.3 ms. So have some stability margin, probably about 1 ms (corresponding to frequency w=e3 rad/s = 159 Hz).
- So the loop gain is 100 at w=e3 and drops to 1 at w=e5 (15915 Hz),
- BUT if we put the filter in C then Tcl = C/(1+GC) = T/C (D-action) so we will amplify high frequencies more .
- SO: The “filter” must be in the feedforward path (before G)

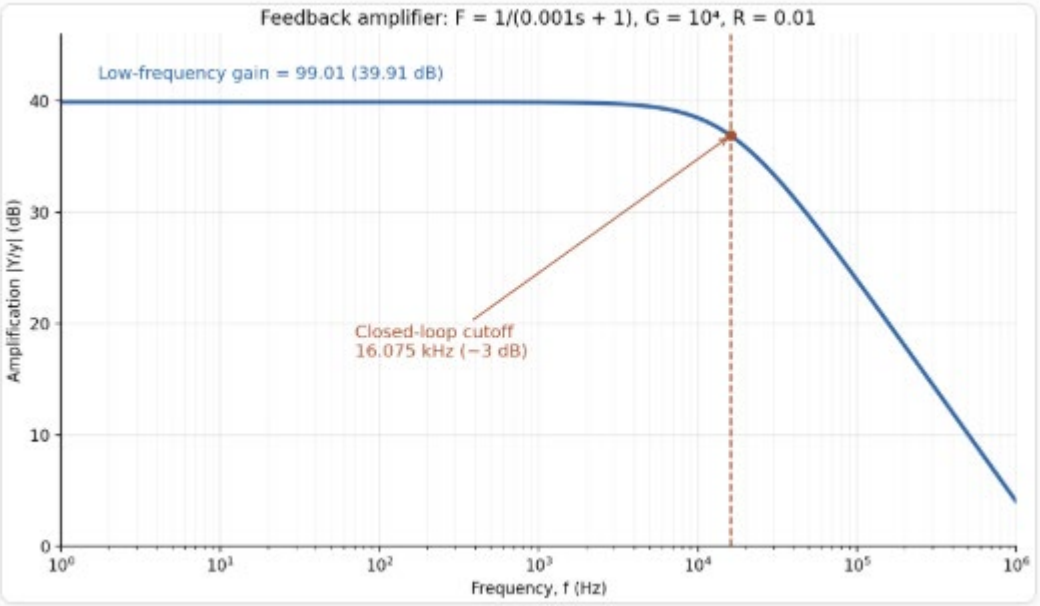
Previous: G = BAD

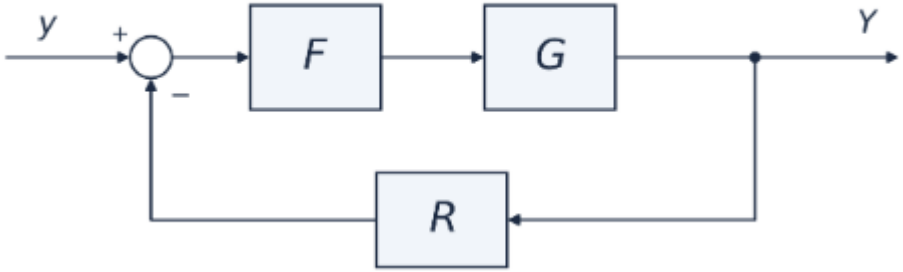
Performance

But the filter is not good for performance…
Neglect dynamics in G:

$$T_{cl} \approx \frac{100}{\tau_{Cl} s + 1}$$
$$\tau_{cl} = \frac{\tau}{RG} = \frac{\tau}{100}$$

With τ=1 ms we get τ_{cl}= 1.e-5 s (or 15920 Hz = 0.16 MHz), so OK for speech.





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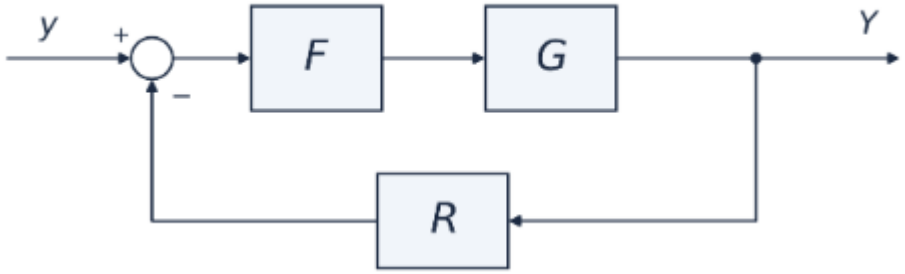
G = amplifier = 1.e4 + dynamics (delay)
R = gain = 1/100
F = $\frac{1}{\tau s+1}$ – filter to avoid «singing»

BUT: This was with fixed resistance gain R (phone)

- The stability problems is actually most critical when the amplification 1/R is small (so when R is large, say R=1).
- So if R is adjustable (electric guitar), we need to use a larger filter time constant to avoid stabilitty problems – and maybe add a minimum amplification 1/R.

Related effect of high-gain feedback: Removes sensitivity to gain variations



$$Y = \frac{GF}{1 + GFR} y = T_{cl}\, y$$

T_{cl}= closed-loop transfer function
= gain of feedback amplifier

- Sensitivity function: $S = \frac{1}{1+L}$ where $L = GFR$
- $T_{cl} = GFS$

Sensitivity to changes in the amplifier gain G

$$\frac{dT_{cl}/T_{cl}}{dG/G} = S$$

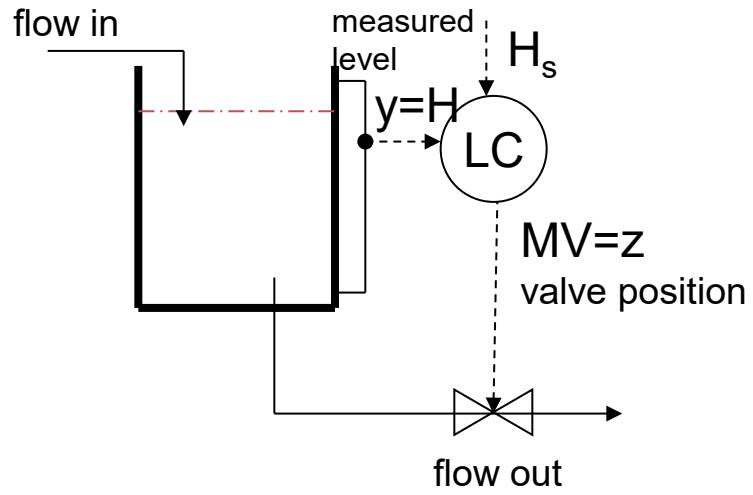
So feedback reduces the effect of uncertainty as long as $|S| < 1$.

Comments: 1. Same result if T = GFRS = GCS
2. Black proved in (7) the related relationship dTcl/Tcl / dC/C = T (since $T \approx 1$ this means that C=R needs to be accurate)

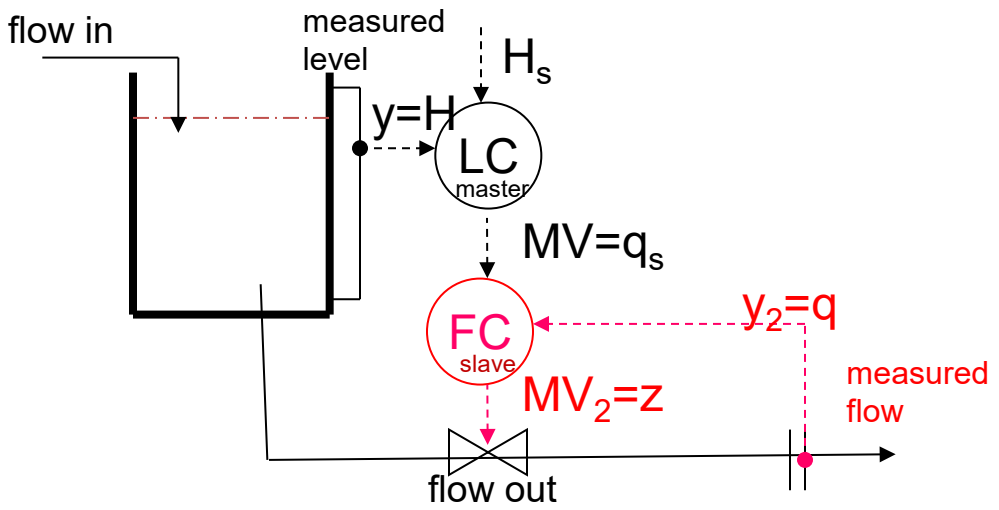
Linearization of valve using cascade control

- Benefits: 1. Local distrurbance rejection, 2. Linearization
- Does nonlinearity disappear?

WITHOUT CASCADE



WITH CASCADE (2 controllers)



No, it moves to the time constant for slave loop – OK if we we have time scale separation between master and slave

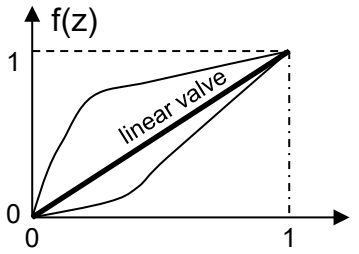
Nonlinear valve with varying gain k_2: $G_2(s) = k_2(z) / (\tau_2 s+1)$

- Slave (flow) controller K_2: PI-controller with gain K_{c2} and integral time $\tau_I = \tau_2$ (SIMC-rule). Get

$$L_2 = K_2(s)G_2(s) = \frac{K_{c2}k_2}{\tau_2 s}$$

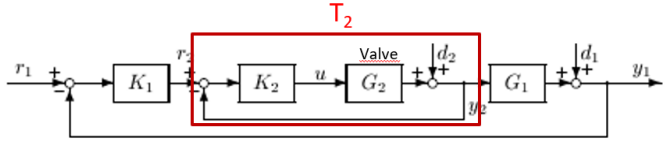
- With slave controller: Transfer function T_2 from y_{2s} to y_2 (as seen from master loop):

$T_2 = L_2/(1+L_2) = 1/(\tau_{C2}\, s + 1)$, where $\tau_{C2} = \tau_2/(k_2\, K_{c2})$



$k_2(z)$ = slope = df/dz

- Linearization: Gain for T_2 is always 1 (independent of k_2) because of intergal action in the inner (slave) loop
- But: Gain variation in k_2 K_{c2}(inner loop) translates into variation in closed-loop time constant τ_{C2}.
- But τ_{C2} is like an effective delay in the master loop, and this delay gets larger if the slave loop is slow (which heppens in regions where k2 is small).



G_1T_2 = «Process» for tuning master controller K_1

Comment: For a purely static valve with τ_2=0, we can use a pure I-controller, $K_2(s)=K_{I2}/s$, where K_{I2} is the integral controller gain. We then get $L_2(s) = \frac{K_{I2}k_2}{s}$ and $\tau_{C2} = 1/(k_2\, K_{I2})$.
So the conclusion is the same: Gain variation in k_2K_{I2}(inner loop) translates into variation in the closed-loop time constant τ_{C2}.

17

Aside: Stabilization

- The only way we can change dynamics (poles) and stabilize is by feedback
- So: Stabilization with feedforward does NOT work
  - Example: level control
    - G(s) = k'/s
    - Integrating process with pole at s=0. At the limit to instability
    - It is practically impossible to control level by trying to set $q_{out}=q_{in}$ using feedforward.
    - We get «internal instability»: Level will eventually go out of bound
- But we need to be careful: Feedback often causes oscillations and even instability

Conclusion I

- High-gain feedback linearizes the system
- Stability: be careful about too high gain (Bode)
- Stability of outer (master) loop: Be careful about too small gain (effective delay)

Conclusion II (thank you, ChatGPT!)

- Modern control education often emphasizes models, prediction, optimization, and feedforward compensation. The underlying idea is that if we have a sufficiently good model, we can calculate the correct control action.
- Black's philosophy was almost the opposite:
  - **We do not need an accurate model of the process if we have sufficiently large feedback gain.**

- **The primary purpose of feedback is to reduce the dependence of the controlled system on uncertainties in the process itself.**
- This also explains why simple PI and PID controllers remain so successful in industry despite the development of sophisticated model-based control methods.