Approximate MPC based on machine learning and probabilistic verification

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Motivation

Solving NMPC problems in real time is still challenging:

- For very fast systems
- On low-cost embedded hardware

Even more challenging in the case of robust NMPC

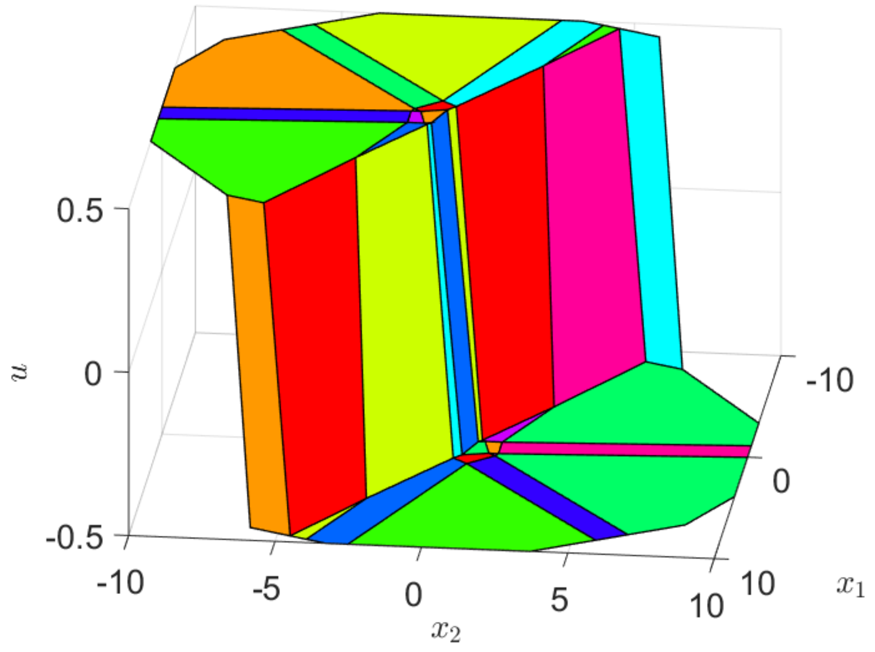
- Goal: Development of an approach that simultaneously
    - Obtains approximate optimal robust solutions
    - Has small memory footprint
    - Can be rapidly evaluated on an embedded device

Explicit MPC in the linear case

The MPC control law for LTI systems is a **piecewise-affine function**

- Depends only on the current state (and possibly parameters)
- Can be offline precomputed and stored

$$\mathcal{K}(x_{\text{init}}) = \begin{cases} K_1 x_{\text{init}} + g_1 & \text{if} \quad x_{\text{init}} \in \mathcal{R}_1, \\ & \vdots \\ K_r x_{\text{init}} + g_r & \text{if} \quad x_{\text{init}} \in \mathcal{R}_r, \end{cases}$$



Each region is described by a polyhedron:

$$\mathcal{R}_i = \{x \in \mathbb{R}^{n_x} \mid Z_i x \leq z_i\} \quad \forall i = 1, \ldots, n_r.$$

[A. Bemporad, M Morari, V. Dua, E.N. Pistikopoulos, 2002]

Reducing complexity of explicit MPC

- Optimal representations:
  - Merging of regions with same feedback law
  - Lattice representation
- Suboptimal approximations:
  - Trade-off between complexity reduction and performance
  - Simplicial partitions
  - Neural networks

[T. A. Johannsen, A. Bemporad, F. Borrelli, C. Jones, M. Morari, M. Kvanisca and others]

Using neural networks to approximate MPC

It is an old idea: already done in 1995 for nonlinear MPC

- What is new?

Common practice until recent successes in deep learning was:

- Because of **universal approximation theorem**: use only 1 layer

What are the possible **advantages of deep learning** for approximating complex MPC laws?

[T. Parisini and R. Zoppoli, 1995, Akesson and Toivonen, 2006]

Deep neural networks (DNN)

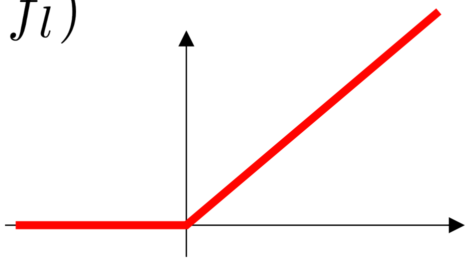
Neural network with L **hidden layers** and M neurons per layer

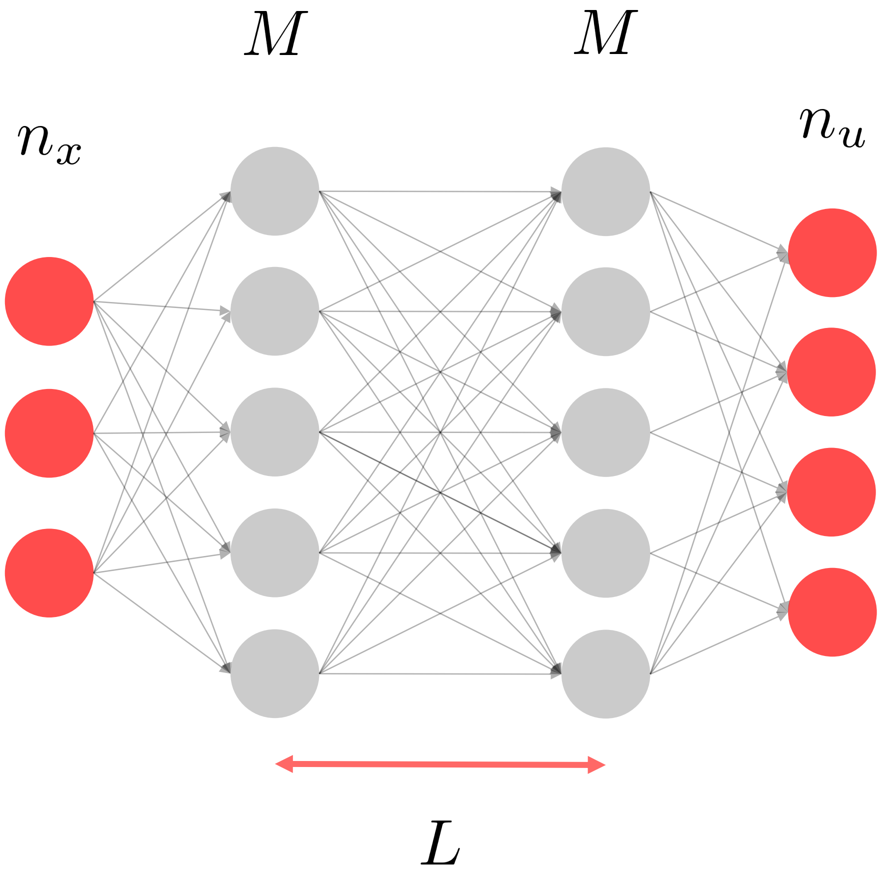
$$\mathcal{N}(x;\theta,M,L) = f_{L+1} \circ g_L \circ f_L \circ \cdots \circ g_1 \circ f_1(x)$$

- Affine transformation $\theta_l = \{W_l, b_l\}$

  $$f_l(x_{l-1}) = W_l x_{l-1} + b_l$$

- Activation function
  - tanh: $g_l(f_l) = \tanh(f_l) = \dfrac{e^{f_l} - e^{-f_l}}{e^{f_l} - e^{-f_l}}$
  - ReLU: $g_l(f_l) = \max(0, f_l)$





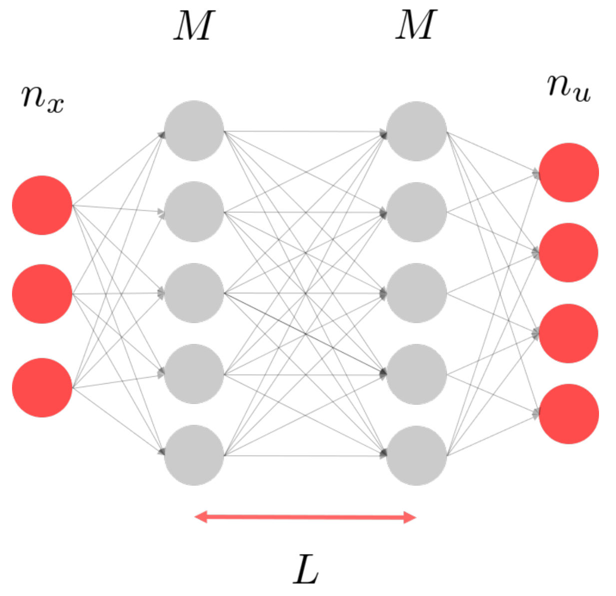
Why deep (and not shallow)?

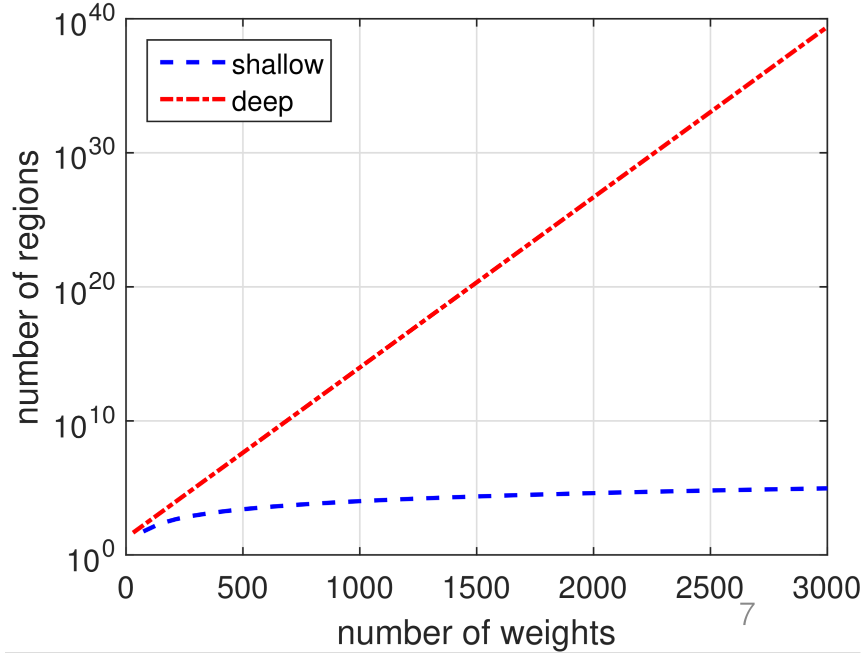
Number of linear regions represented by a ReLU network of depth L and width M

$$n_{\mathrm{r}} = \left(\prod_{l=1}^{L-1} \left\lfloor \frac{M}{n_x} \right\rfloor^{n_x} \right) \sum_{j=0}^{n_x} \binom{L}{j}$$

- **Exponential** growth of regions w.r.t. depth
- Greater expressiveness with same amount of weights

[Montufar et al., 2014]





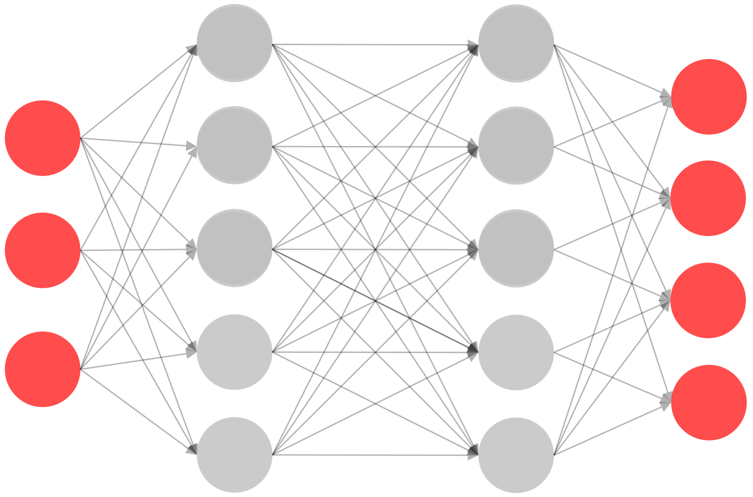
Proposed approach

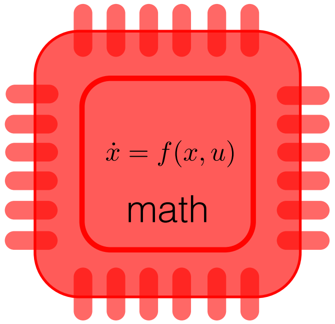
1: Generate training samples by solving many MPC problems

(x_0, u_0^*)

2: Offline training of the deep neural network

Place here your preferred (overcomplicated) **robust NMPC Method**





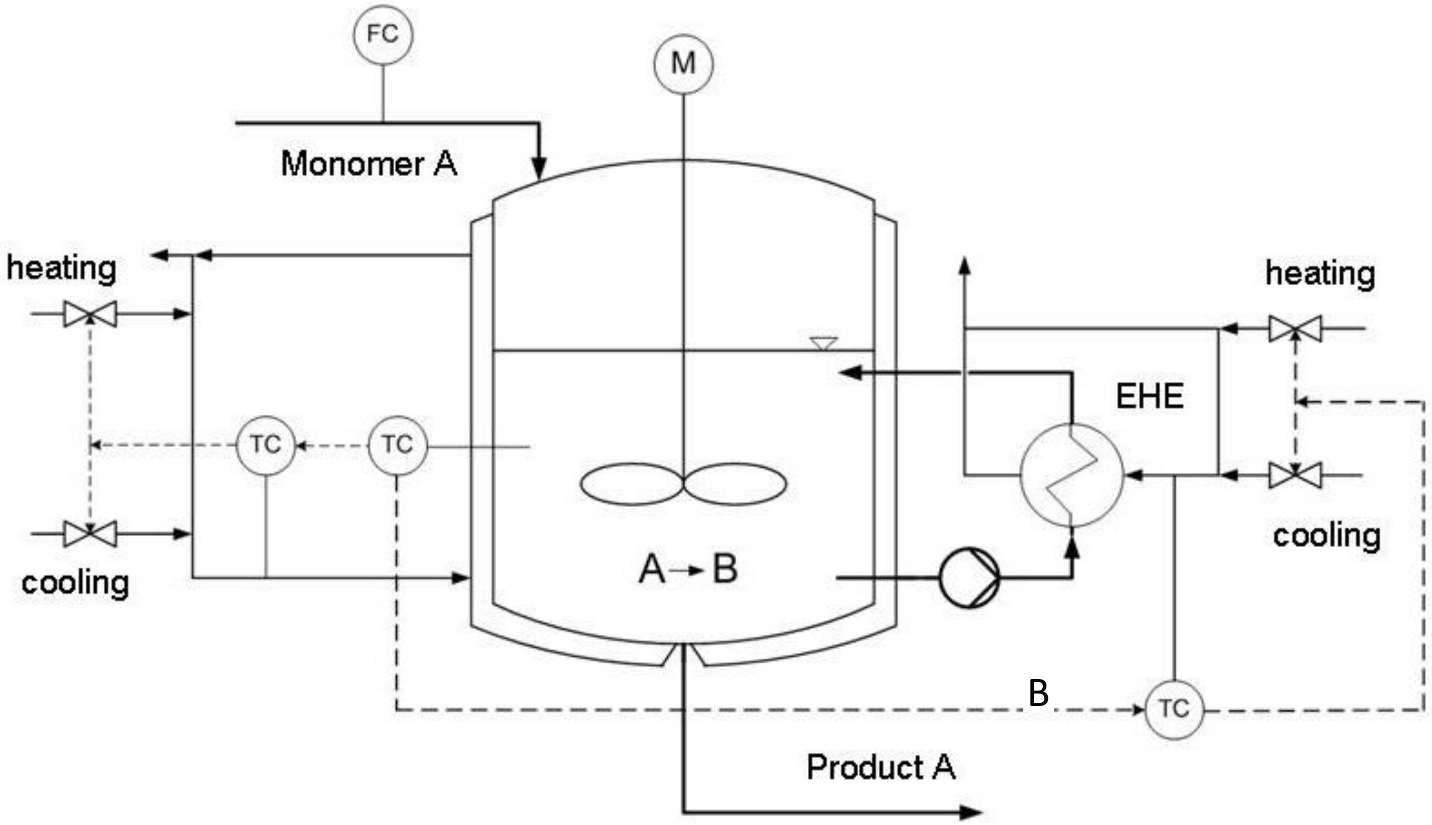
3: High performance implementation on low-cost embedded hardware

Increasingly popular

In many cases including strategies to have some guarantees:

- Chen et al., ACC 2018 (Projection at the output to achieve guarantees)
- Hertneck et al., IEEE Control System Letters 2018 (Hoeffdings inequality)
- Zhang, Bujarbaruah and Borrelli, ACC 2019 (Statistical validation)
- Drgona et al., Applied Energy 2018 (application on building control)
- Karg and Lucia, ECC 2018, NMPC 2018, ECC 2019 (applications, validation)

It works well in practice



[Lucia, Andersson, Brandt, Diehl and Engell. JPC 2014]

An industrial polymerization reactor

8 differential states
3 control inputs
2 uncertain parameters

$$\dot{m}_W = \dot{m}_{W,F}$$

$$\dot{m}_A = \dot{m}_{A,F} - k_{R1} m_{A,R} - \frac{p_1 k_{R2} m_{AWT} m_A}{m_{ges}}$$

$$\dot{m}_P = k_{R1} m_{A,R} + \frac{p_1 k_{R2} m_{AWT} m_A}{m_{ges}}$$

$$\dot{T}_R = \frac{1}{c_{p,R} m_{ges}} \left[\dot{m}_F c_{p,F} (T_F - T_R) + \Delta H_R k_{R1} m_{A,R} - k_K A (T_R - T_S) - \dot{m}_{AWT} c_{p,R} (T_R - T_{EK}) \right]$$

$$\dot{T}_S = 1/(c_{p,S} m_S)[k_K A (T_R - T_S) - k_K A (T_S - T_M)]$$

$$\dot{T}_M = \frac{1}{c_{p,W} m_{M,KW}} \left[\dot{m}_{M,KW} c_{pW} (T_M^{IN} - T_M) + k_K A (T_S - T_M) \right]$$

$$\dot{T}_{EK} = \frac{1}{c_{p,R} m_{AWT}} \left[\dot{m}_{AWT} c_{p,W} (T_R - T_{EK}) - \alpha (T_{EK} - T_{AWT}) + \frac{p_1 k_{R2} m_A m_{AWT} \Delta \mathrm{H}_R}{m_{ges}} \right]$$

$$\dot{T}_{AWT} = \frac{1}{c_{p,W} m_{AWT,KW}} \left[\dot{m}_{AWT,KW} c_{p,W} (T_{AWT}^{IN} - T_{AWT}) - \alpha (T_{AWT} - T_{EK}) \right]$$

$$k_{R1} = k_0 e^{-\frac{E_a}{R T_R}} (k_{U1}(1 - U) + k_{U2} U)$$

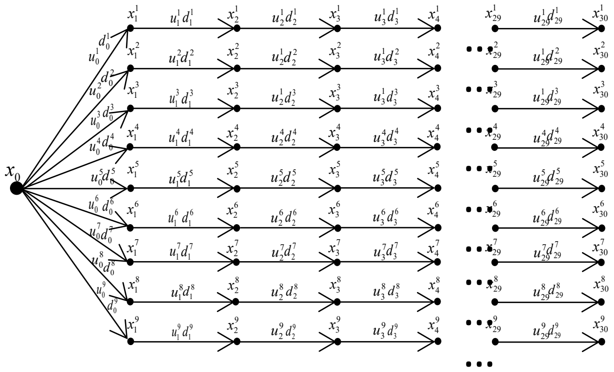
$$k_{R2} = k_0 e^{-\frac{E_a}{R T_{EK}}} (k_{U1}(1 - U) + k_{U2} U)$$

Simulation results for multi-stage NMPC

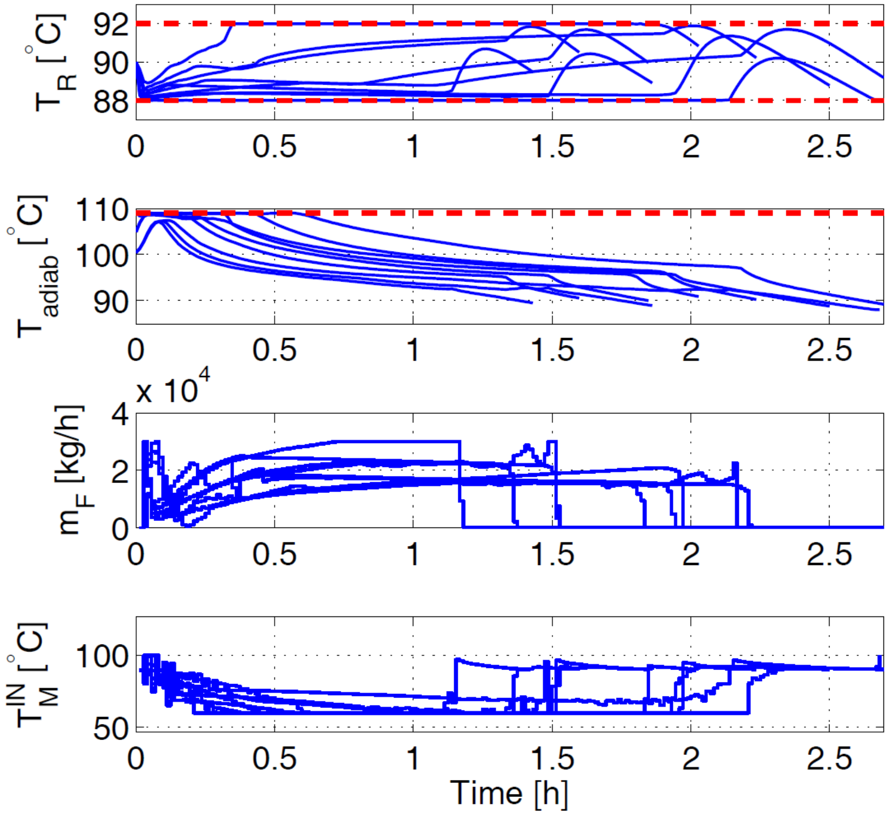
Simple scenario tree

- Extreme values of the uncertainty
- Branch the tree only one stage

- Economic cost function



Multi-stage NMPC



Simulations for different values of k and ΔH ($\pm$30%)

Proposed approach

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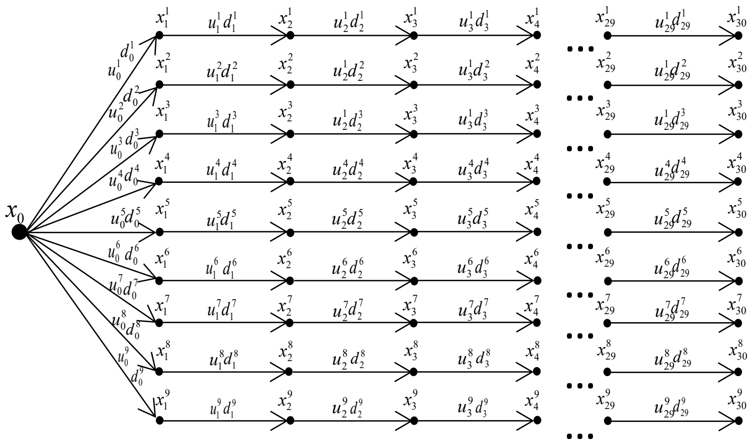
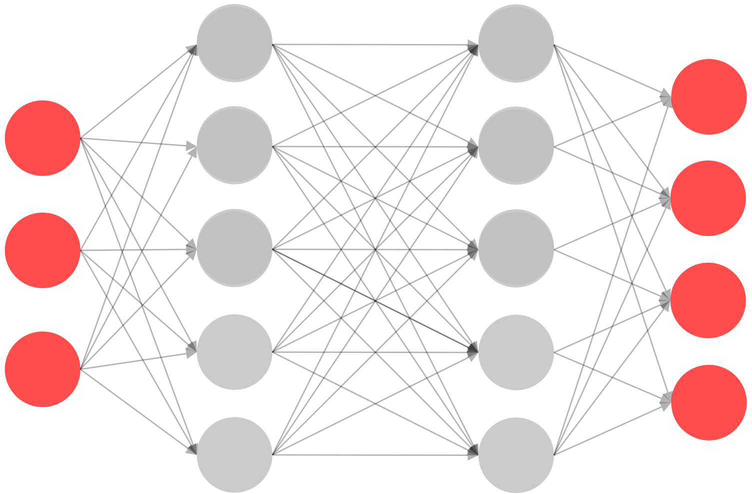
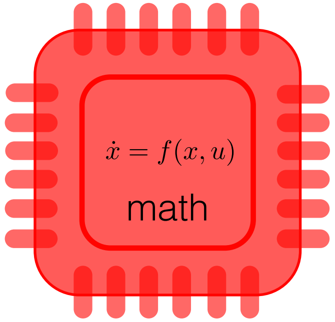


$$(x_0, u_0^*)$$

2: Offline training of the deep neural network

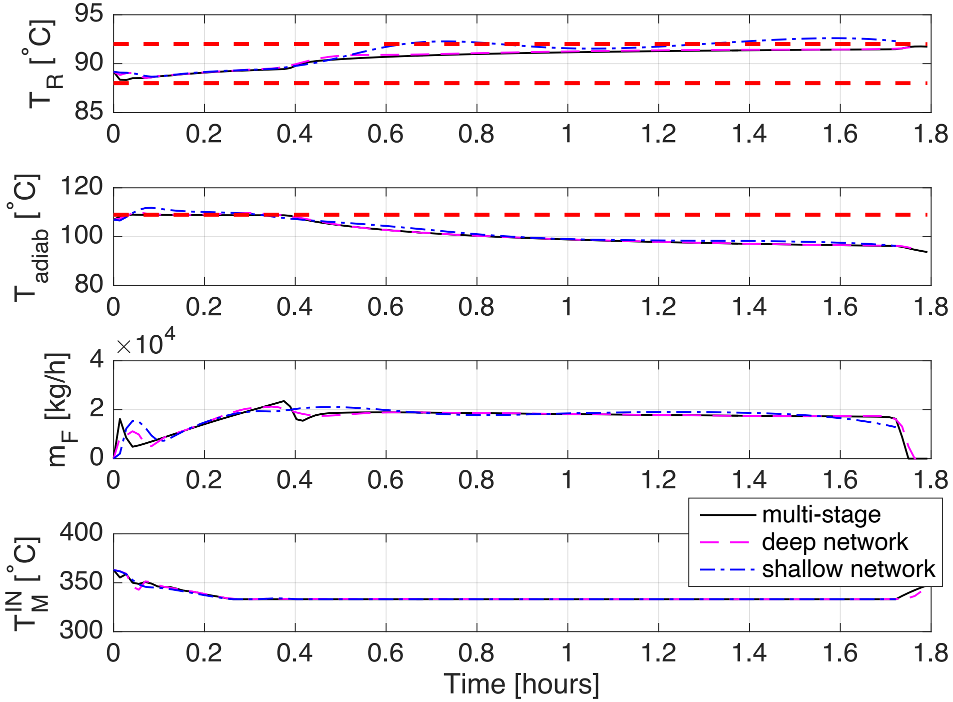




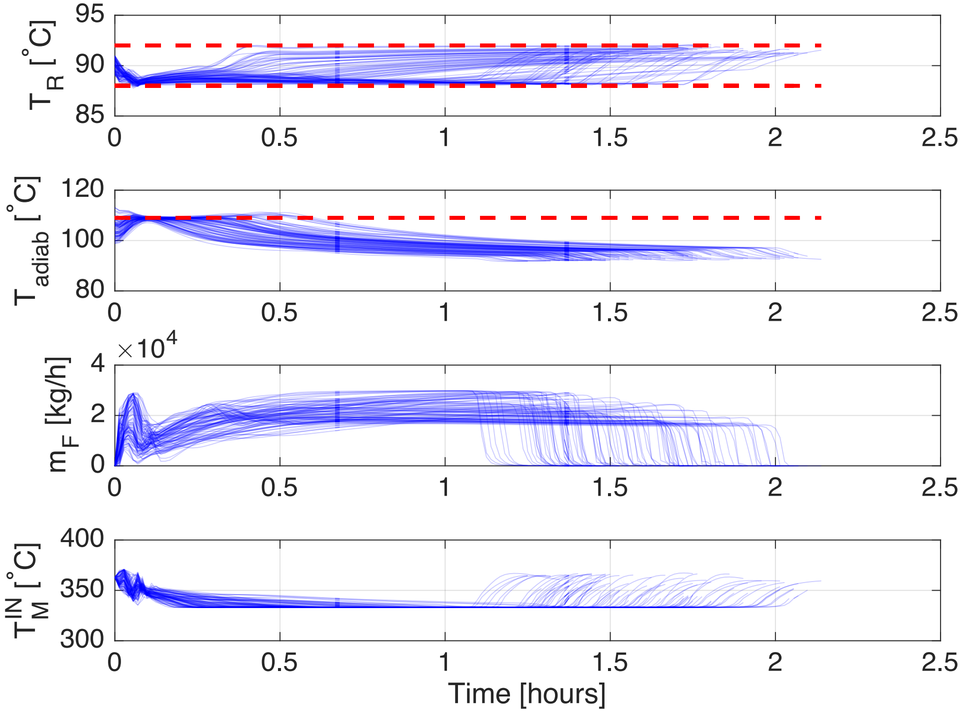
3: High performance implementation on low-cost embedded hardware

Performance of deep-learning based ms-NMPC

Exact vs. deep vs. shallow multi-stage NMPC



Deep-learning based multi-stage NMPC



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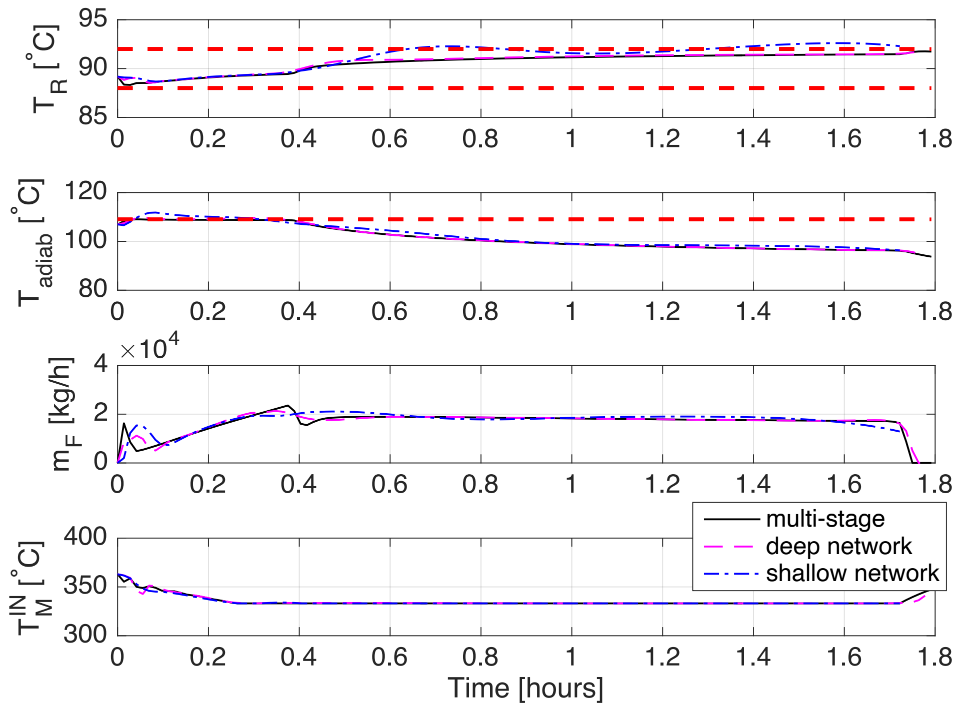
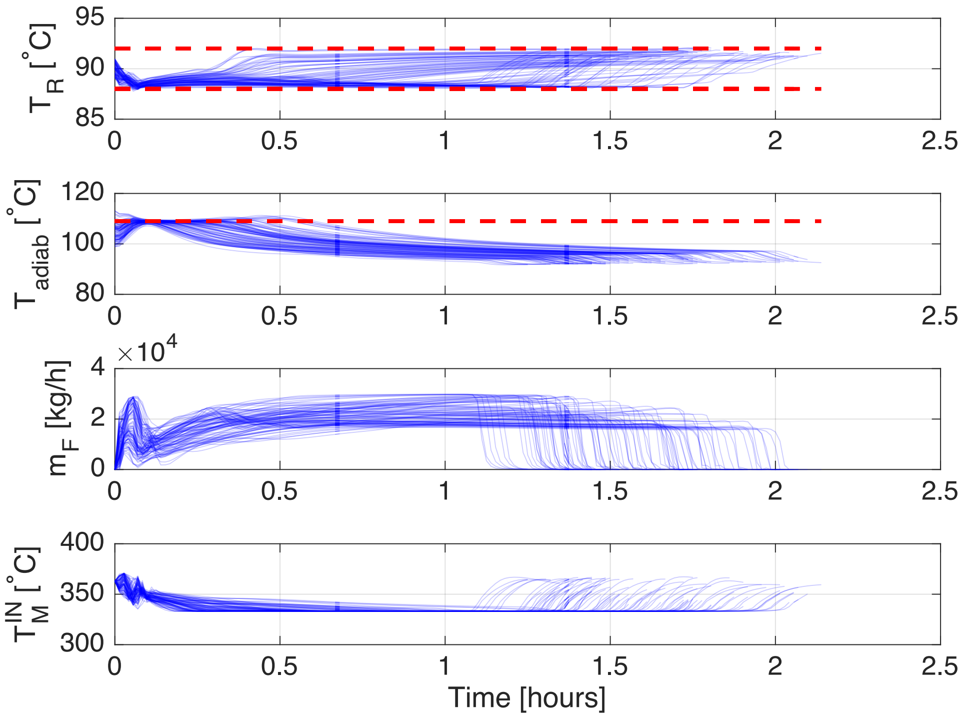


Deep-learning based multi-stage NMPC



Average performance over random 100 batches

Algorithm	Batch time [h]	Cons. Viol. [°C/h]
Exact	1.6459	0.0058
Shallow	1.7328	0.3087
Deep	1.6297	0.0549

Two main advantages

Enable low-cost emb. implementation

- 32 bit ARM Cortex M0+
- 48 MHz with 32 kB RAM

Approx. robust NMPC:

- Memory footprint: 27 kB
- Evaluation time on a uC: 37 ms
- Trivial code-generation (uC, FPGA)

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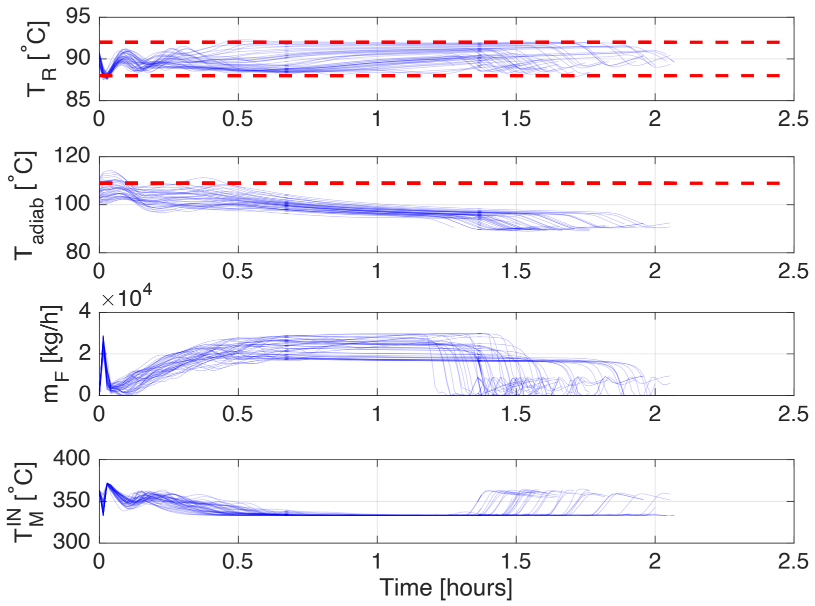
Enable large(r)-scale systems

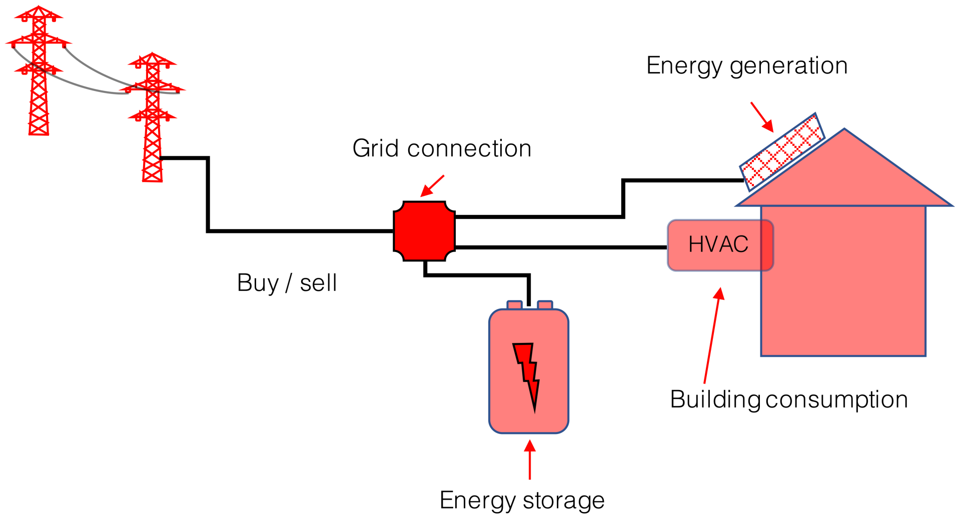
Problem with 5 uncertainties

- 243 scenarios
- ~115,000 variables and constraints



Mixed-integer case: Energy management system

- You can learn a **global optimal** solution



$$
\begin{aligned}
\underset{(x,u)}{\text{minimize}} \quad & \sum_{k=0}^{N-1} (P_{\text{grid}}^k + \gamma (E_{\text{bat}}^k - E_{\text{bat}}^{\text{ref}})^2) \\
\text{subject to} \quad & x_{k+1} = Ax_k + Bu_k + Ed_k, \\
& x_{\text{lb}} \le x_k \le x_{\text{ub}}, \\
& u_{\text{lb}} \le u_k \le u_{\text{ub}}, \\
& \alpha \in \{0, 1\}, \\
& m_{\text{lb}} \le Du_k + Gd_k \le m_{\text{ub}}.
\end{aligned}
$$

$$
A = \begin{bmatrix} 0..8511 & 0.0541 & 0.0707 & 0 \\ 0.1293 & 0.8635 & 0.0055 & 0 \\ 0.0989 & 0.0003 & 0.0002 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0035 \\ 0.0003 \\ 0.0002 \\ -5 \end{bmatrix},
$$

$$
E = 10^{-3} \begin{bmatrix} 22.217 & 1.7912 & 42.212 \\ 1.5376 & 0.6944 & 2.9214 \\ 103.18 & 0.1032 & 196.04 \\ 0 & 0 & 0 \end{bmatrix}
$$

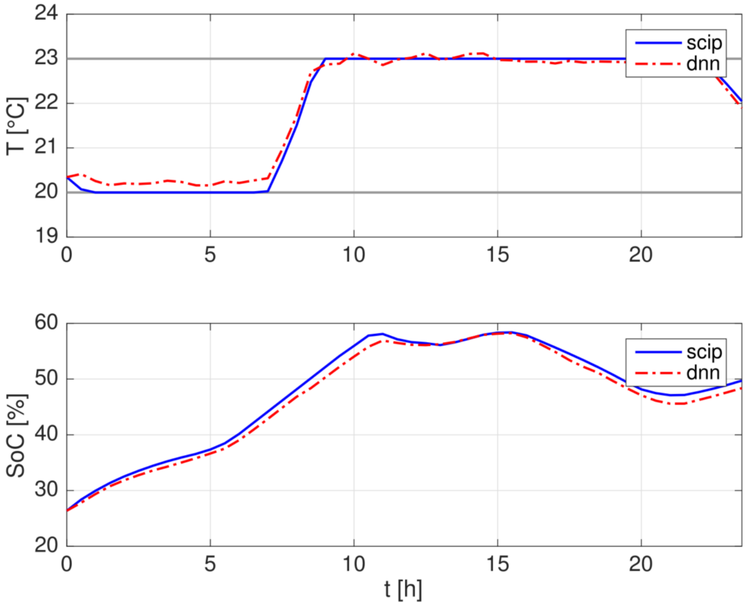
$$x = [T_{\text{r}}, T_{\text{w,i}}, T_{\text{w,e}}, E_{\text{bat}}]^T$$

$$u = [P_{\text{grid}}, P_{\text{hvac}}, P_{\text{bat}}, \alpha]^T$$

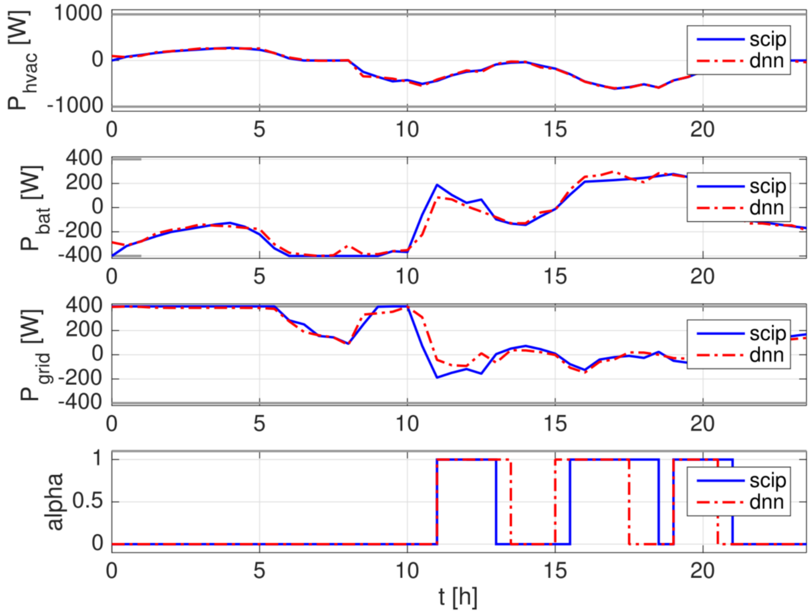
$$d = [d_{\text{amb}}, d_{\text{sol}}, d_{\text{int}}]^T$$

Mixed-integer case

states



inputs

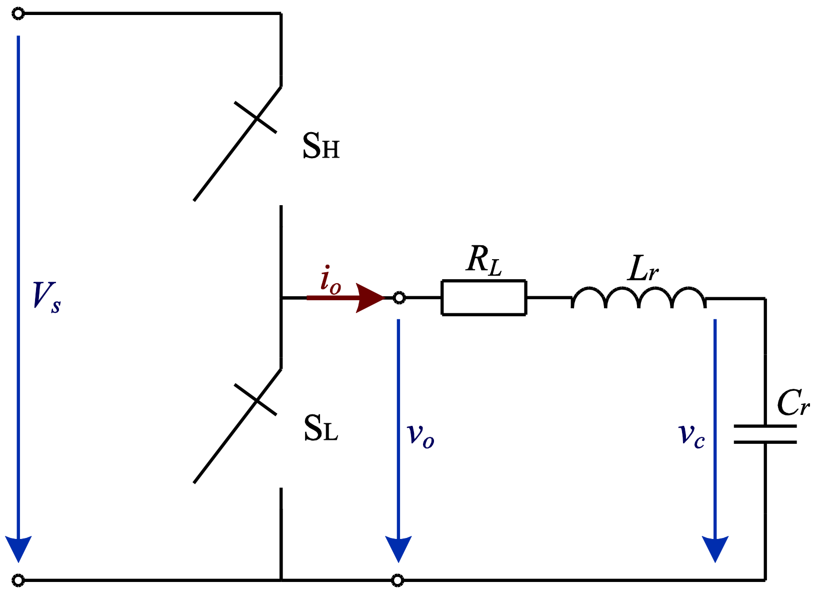


Controller	Average integrated cost	Average violations
SCIP	1.469e3	0
DNN	1.463e3	1.46e-2

[Karg and Lucia, ECC 2018]

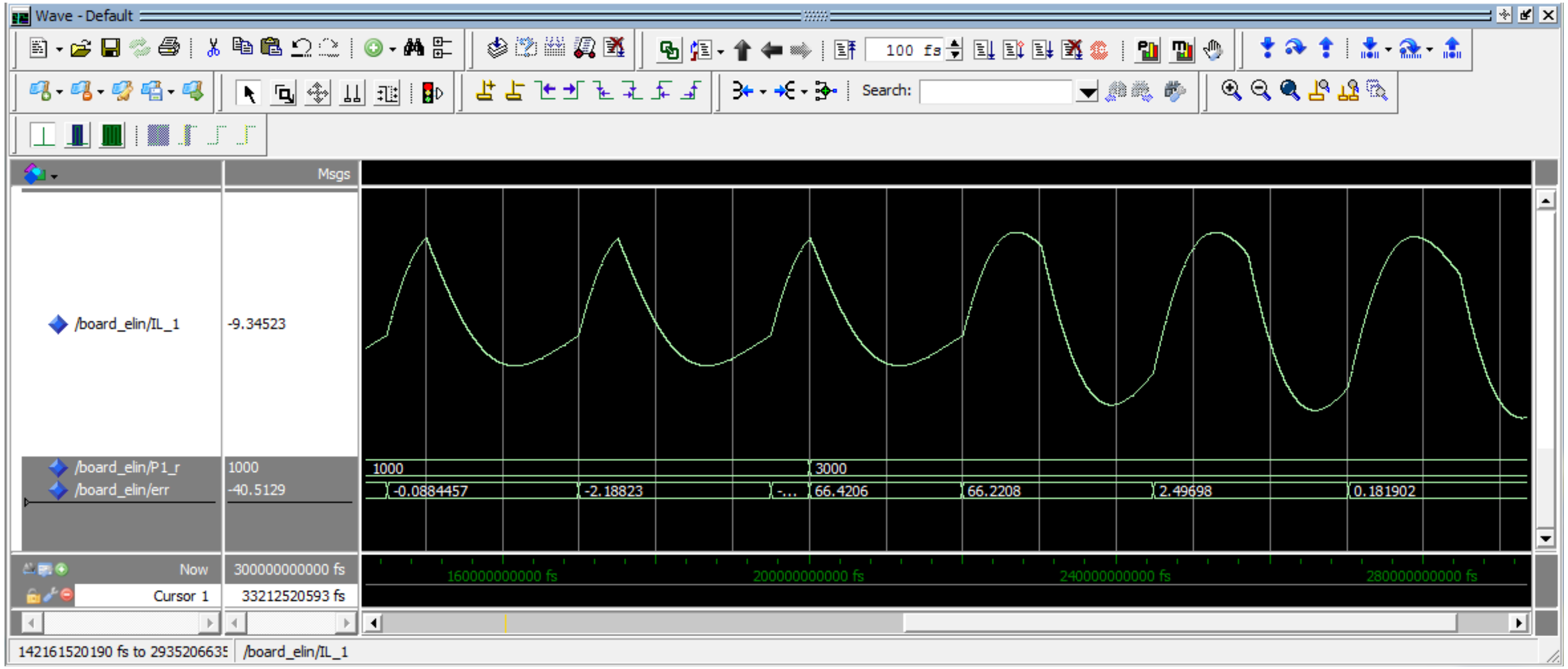
Robust NMPC in 1 microsecond

Induction heating is currently used in many industrial and domestic applications



Control switching frequency and duty cycle. Satisfy constraints under uncertainty

Hardware-in-the-loop implementation



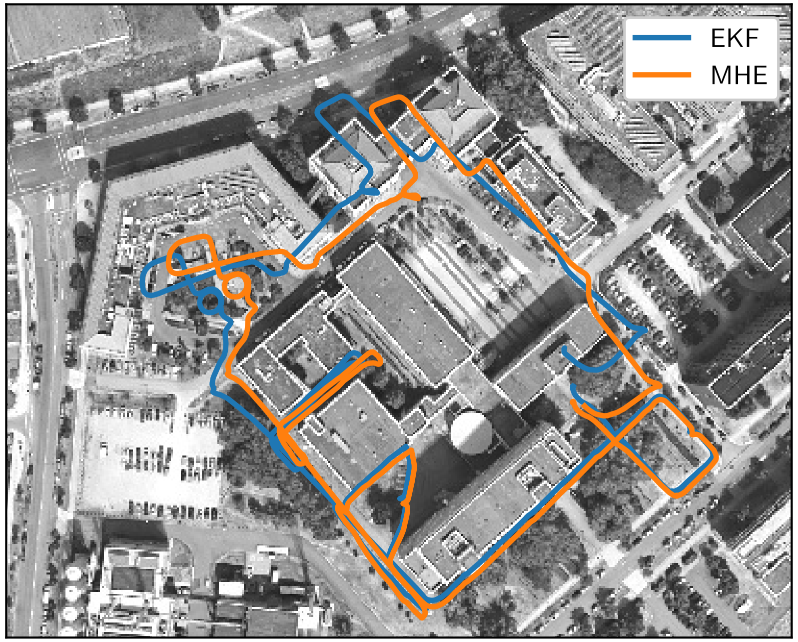
Advanced approximate optimization-based control in 1 µs (on an FPGA).

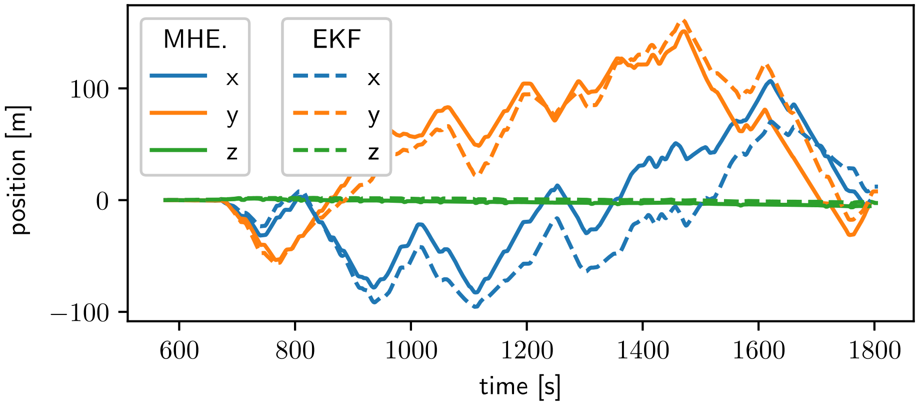
Easy to optimize FPGA implementation

Moving horizon estimation for sensor fusion

Fusing visual information and inertial sensors

- Common problem in autonomous driving, robotics
- Usually many assumptions to simplify online optimization (or EKF)





Fiedler et al., ECC 2020

Wait a minute... guarantees?

Compute the maximum approximation error

$$d = \max_{x_0} |\pi_{\mathrm{NN}}(x_0) - \pi_{\mathrm{MPC}}(x_0)|$$

Design a controller that is robust against d and iterate

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- Computing the maximum is often not possible
  - Probabilistic Validation

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Design a controller that is robust against d and iterate

- Computing the maximum is often not possible
  - Probabilistic Validation
    - Hertneck et al., IEEE CSL 2018:
      - Based on Hoeffdings inequality and indicator (binary functions)
    - Karg and Lucia, arXiv:1806.10644, (2018), ECC 2019
      - Based on Hoeffdings inequality and temporal logic with finite-time simulations
    - Zhang et al., ACC 2019:
      - Based on prob. validation results (Tempo, Bai, Dabbene, 1997) to achieve primal and dual guarantees

Probabilistic validation with performance indicators

A general (not necessarily binary) finite-time performance indicator

$$\phi(w; N_{\rm sim}, \kappa) = \phi(x(0), \hat{x}(0), \kappa(\hat{x}(0)), d(0), x(1), \kappa(\hat{x}(1)), d(1), \ldots, x(N_{\rm sim})).$$

Given a controller κ, a final simulation step $N_{\rm sim}$ and N i.i.d samples

$$w^{(j)} = \{x^{(j)}(0), \hat{x}^{(j)}(0), d^{(j)}(0), \ldots, d^{(j)}(N_{\rm sim})\},\ j = 1, \ldots, N,$$

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Given a controller κ, a final simulation step N_{sim} and N i.i.d samples

$$w^{(j)} = \{x^{(j)}(0), \hat{x}^{(j)}(0), d^{(j)}(0), \ldots, d^{(j)}(N_{\text{sim}})\},\ j = 1, \ldots, N,$$

With probability no smaller than δ

$$\text{Prob}\{\phi_i(w) > \psi_N^{\phi}(r)\} \leq \epsilon,\ i = 1, \ldots, M,$$

$\psi_N^{\phi}(r)$ is the maximum value of simulated $\phi_i(w)$ among all N, after removing the largest r elements

Provided that: $$N \geq \frac{1}{\epsilon}\left(r - 1 + \ln\frac{M}{\delta} + \sqrt{2(r-1)\ln\frac{M}{\delta}}\right).$$

Differences with previous works

- Validation on general closed-loop performance guarantees
  - Not only binary functions
  - Not only error in the controller. Validation includes e.g. estimation errors

- Discard the r largest values to facilitate successful validations

- Simultaneous design of several controllers (finite families)

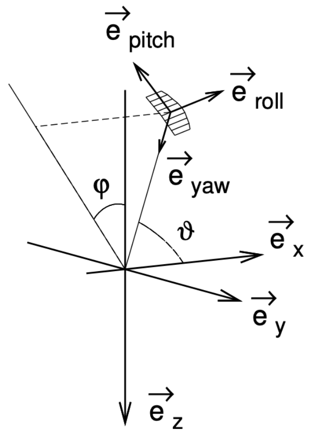
More details in:

Karg, Alamo and Lucia, Probabilistic performance validation of deep learning-based robust NMPC controllers. arXiv:1910.13906 (2019)

Some further results

Probabilistically safe, embedded robust output-feedback NMPC

$$\dot{\theta}_{\text{kite}} = \frac{v_{\text{a}}}{L_{\text{T}}}\left(\cos\psi_{\text{kite}} - \frac{\tan\theta_{\text{kite}}}{E}\right),$$
$$\dot{\phi}_{\text{kite}} = -\frac{v_{\text{a}}}{L_{\text{T}}\sin\theta_{\text{kite}}}\sin\psi_{\text{kite}},$$
$$\dot{\psi}_{\text{kite}} = \frac{v_{\text{a}}}{L_{\text{T}}}\tilde{u} + \dot{\phi}_{\text{kite}}\cos\theta_{\text{kite}},$$



Erhard and Strauch, 2012

Objective is to maximize thrust

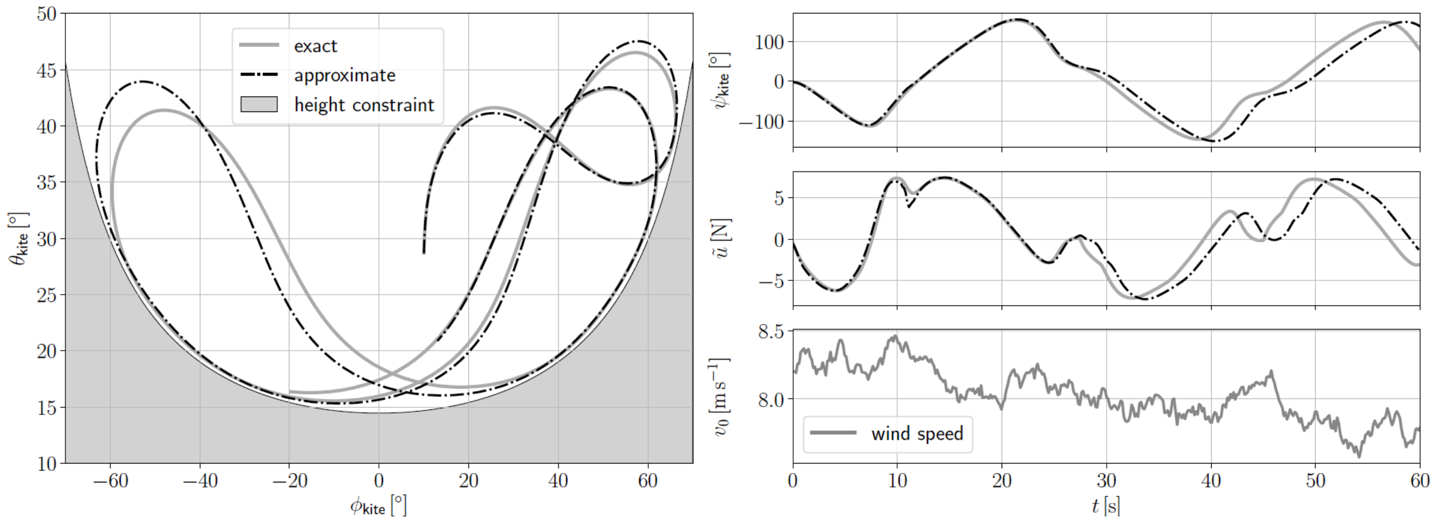
Two states can be measured, EKF to estimate

Uncertain aerodinamic coefficients and wind parameters

Results

Embedded real-time implementation on an ARM-Cortex M3

- 96 kB memory footprint, 32 ms running time for DNN and 28 ms for EKF



controller	$\kappa_{\text{dnn},0}$	$\kappa_{\text{dnn},2}$	$\kappa_{\text{dnn},4}$	$\kappa_{\text{dnn},6}$
feasible trajectories	660/1388	1380/1388	1385/1388	1387/1388
$\psi(\mathbf{v}, 4)$ [m]	1.682	0.273	-0.316	-1.818
T_{F} (avg.) [kN]	227.516	225.997	224.185	222.179
probabilistically safe	No	No	Yes	Yes

Summary

1. Efficient approximation of the MPC control law using deep learning
   - Enables simple embedded implementation with very low memory footprint
   - Enables real-time robust NMPC of large complex systems

2. Statistical verification can be used to achieve guarantees

3. Recently good results for many different applications

Some material for discussion

- What is better:
  - First approximate then solve (usual path)
  - First solve (as complex as you can) then approximate

- What is more rigorous:
  - A priori guarantees (assumes knowledge of reality, including unc. description)
  - Probabilistic validation (assumes a reality simulator exists)
  - Many approaches use safety sets / backup controllers:
    - Nonlinear optimization running online -> one probably needs safety checks anyway…

- Hierarchy: learn a new controller when *something* changes

- Are finite-time guarantees acceptable? (even t is large?)

Open Invited Track at IFAC WC 2020 in Berlin

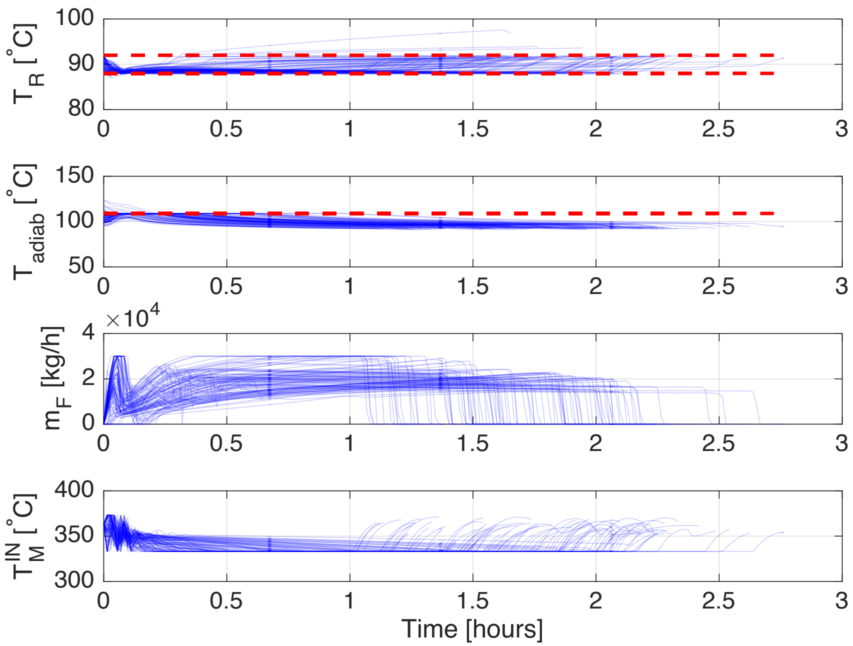
- Together with Ali Mesbah (UC Berkeley)
- Open Invited Track on „Machine Learning and MPC“
- Use submission code a1d55

- Deadline just extended to November 18th

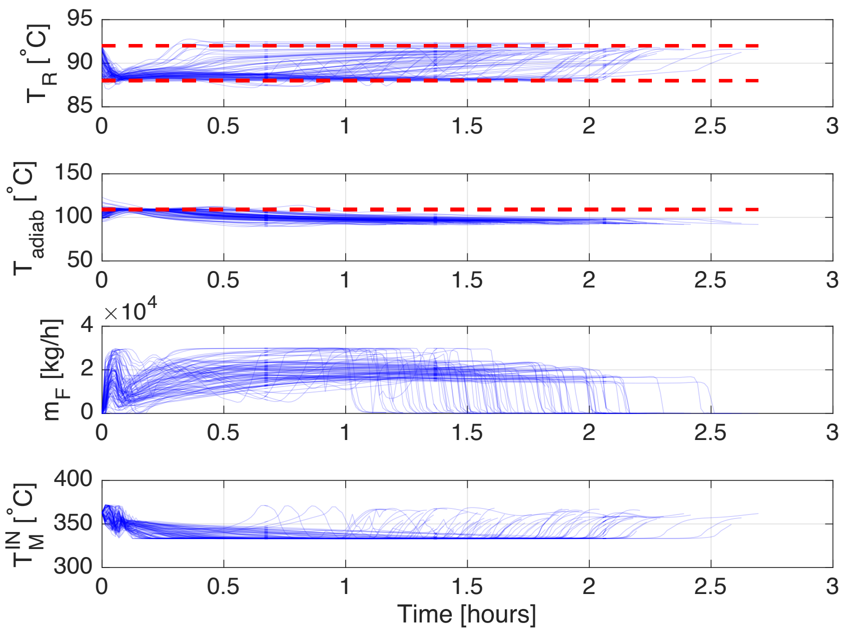
Graceful performance degradation

Larger set of random initial conditions and uncertain parameters (±40%)

Exact multi-stage NMPC



Deep-learning based multi-stage NMPC



Algorithm	Batch time [h]	Cons. Viol. [°C/h]
Exact	1.7882	0.1007
Deep	1.7800	0.1502

Training

Samples generated solving multi-stage NMPC (CasADi + IPOPT)

100 batches of data (with random initial cond. and uncertain param.)

- 50 s sampling time
- Total of 21050 samples

- Training with Keras / Tensorflow

$$\min_{W_l, b_l} \frac{1}{n_{\mathrm{tr}}} \sum_{i=1}^{n_{\mathrm{tr}}} \left(\hat{u}(x_{\mathrm{tr},i}) - u^*(x_{\mathrm{tr},i})\right)^2$$

Output neural network Output multi-stage NMPC

n_l	n_n	n_tot	n_w	MSE_train	MSE_test
1	90	90	1263	0.0043	0.0046
2	45	90	2703	0.0024	0.0024
6	15	90	1413	0.0015	0.0014
9	10	90	1023	0.0014	0.0014

Summary

- Scheme to approximate **complex model predictive controllers**
- Efficient approximation of the MPC control law using **deep learning**
- Two main advantages
  - Enable embedded implementation with very low memory footprint
  - Enable real-time robust NMPC of large complex systems

- Some kind of **safety net** is necessary to have guarantees
  - (don‘t we always need this in reality, at least for the complex nonlinear case?)
- Other problems: adaptation and RL, optimal training, optimal structure