Model Predictive Control: Its Interesting Past and Bright Future

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Outline

1. Where do I come from
2. The last 20 years — what tools have researchers developed
3. Industrial impact of these ideas
4. Have all the questions been answered?
   - Control of large-scale systems
   - Optimizing economics
5. Conclusions and future outlook

Wisconsin

The Midwest of the United States.

Downtown Madison

Downtown Madison is bounded by Lakes Mendota and Monona.

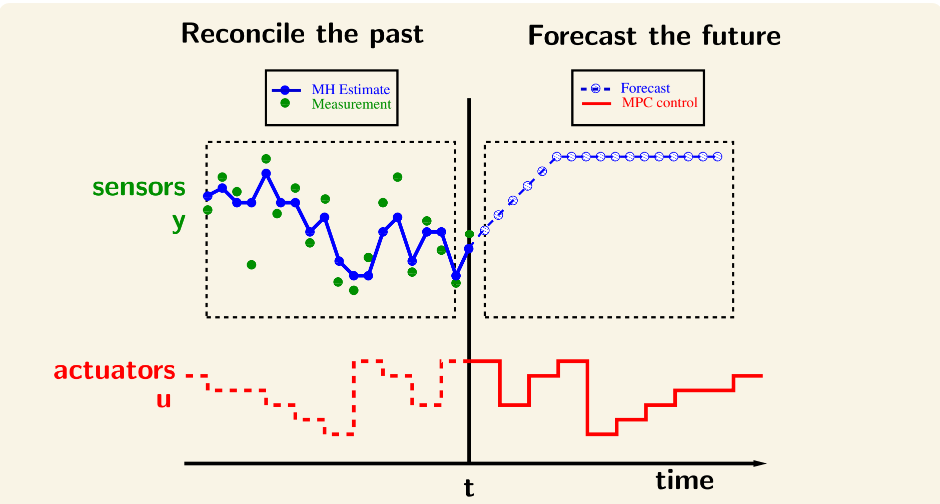
University of Wisconsin (UW)

UWMadison's campus sprawls across 933 rolling acres, with Madison at its front door, and Lake Mendota in its back yard.

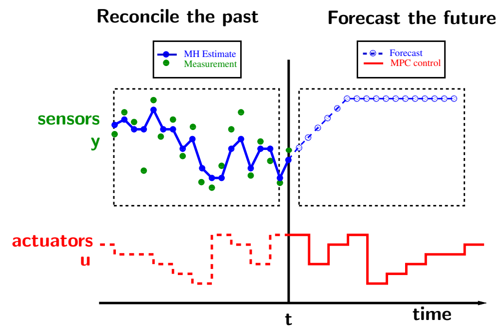
Music Hall

Yes, Madison has winter!

The model predictive control framework



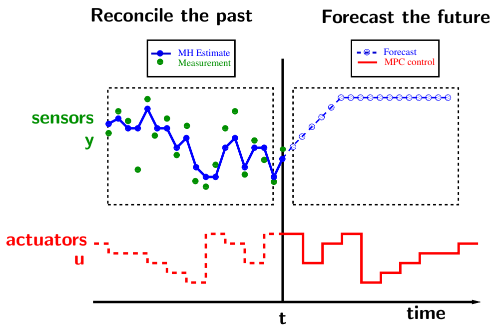
Predictive control



$$\min_{u(t)} \int_0^T |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2 \, dt$$

$$\dot{x} = f(x,u)$$
$$x(0) = x_0 \quad \text{(given)}$$
$$y = g(x,u)$$

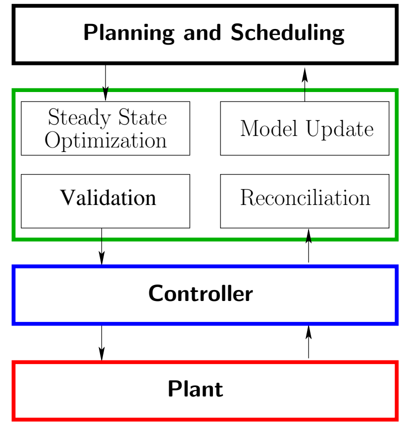
State estimation



$$\min_{x_0, w(t)} \int_{-T}^{0} |y - g(x,u)|_R^2 + |\dot{x} - f(x,u)|_Q^2 \, dt$$

$$\dot{x} = f(x,u) + w \quad \text{(process noise)}$$
$$y = g(x,u) + v \quad \text{(measurement noise)}$$

Industrial impact of the research



Two layer structure

- Steady-state layer
  - RTO optimizes steady-state model
  - Optimal setpoints passed to dynamic layer
- Dynamic layer
  - Controller tracks the setpoints
  - Linear MPC (replaces multiloop PID)

Large industrial success story!

Linear MPC and ethylene manufacturing

- Number of MPC applications in ethylene: 800 to 1200
- Credits 500 to 800 M$/yr (2007)
- Achieved primarily by increased on-spec product, decreased energy use

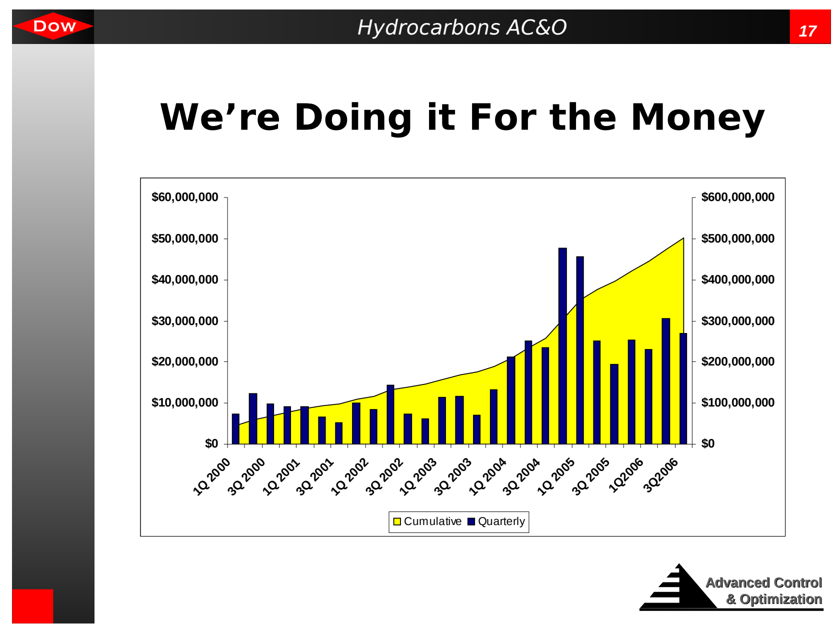
Eastman Chemical experience with MPC

- First MPC implemented in 1996
- Currently 55-60 MPC applications of varying complexity
- 30-50 M$/year increased profit due to increased throughput (2008)

Praxair experience with MPC

- Praxair currently has more than 150 MPC installations
- 16 M$/year increased profit (2008)

Impact for 13 ethylene plants (Starks and Arrieta, 2007)



Broader industrial impact (Qin and Badgwell, 2003)

Area	Aspen Technology	Honeywell Hi-Spec	Adersa	PCL	MDC	Total
Refining	1200	480	280	25		1985
Petrochemicals	450	80	-	20		550
Chemicals	100	20	3	21		144
Pulp and Paper	18	50	-	-		68
Air & Gas	-	10	-	-		10
Utility	-	10	-	4		14
Mining/Metallurgy	8	6	7	16		37
Food Processing	-	-	41	10		51
Polymer	17	-	-	-		17
Furnaces	-	-	42	3		45
Aerospace/Defense	-	-	13	-		13
Automotive	-	-	7	-		7
Unclassified	40	40	1045	26	450	1601
Total	1833	696	1438	125	450	4542
First App.	DMC:1985 IDCOM-M:1987 OPC:1987	PCT:1984 RMPCT:1991	IDCOM:1973 HIECON:1986	PCL: 1984	SMOC: 1988	
Largest App	603x283	225x85	-	31x12	-	

A brief look into MPC's past

Comments before we jump in.

- Let's avoid the trap of *presentism*.
- What we'll find is that today's understanding, which we use to examine the inadequacies and errors of the past, will itself not hold up to future scrutiny.
- Nothing is as uncertain as the future ... *except the past*.

It wasn't always so obvious that MPC would be a success

Disturbance models

–F.G. Shinskey

Feedback Controllers for the Process Industries, 1994

> *The DMC is capable of outperforming the PID controller on setpoint changes but not on load changes introduced upstream of a dominant lag.*
>
> *In fact, the load always enters upstream of the dominant process time constant and, in many cases, at the same point as the manipulated variable.*

...Or *why* MPC would be a success

Input *constraints*?

–C.R. Cutler and B.L. Ramaker, 1980

Joint Automatic Control Conference

> *The set of equations is over-determined which prevents direct solution, but can be solved using a least squares criterion.*
> *The preceding description of the DMC calculations can be expressed conveniently in the following matrix notation:*

$$\vec{I} = -(A^T A)^{-1} A^T \vec{E} \qquad (i.e., \quad u = Kx)$$

What was new about MPC?

Input constraints

- Hard input constraints were added to DMC only later (García and Morshedi, 1986).
- Required the online solution of a quadratic program rather than an offline feedback gain.
- The addition of hard input constraints makes the controller nonlinear, which is the only feature that distinguished industrial MPC proposals from the already available linear quadratic regulator theory.

Other confusing side issues—model representation

Impulse and step response models

–C.E. Garcia and M. Morari, 1982,
Internal Model Control. 1. A Unifying Review and Some New Results.

> *The use of an impulse response model is advantageous because a structural model identification is not required and the nonminimal representation adds robustness to the scheme.*

Convolution models and constraints

IDCOM

–Mehra, Rouhani, Eterno, Richalet, and Raul, 1982,
Model Algorithmic Control (MAC); Review and Recent Developments.

> *One of the most useful properties of MAC is its robustness, which appears at least partially to result from the use of impulse response model for plant representation.*
>
> *The exact satisfaction of control and state constraints is absolutely essential in many applications. Such constraints are handled much more easily in IDCOM, thanks to the impulse response representation of the system.*

Checking in again a few years later

Convolution models and robustness

–C.E. Garcia, D.M. Prett and M. Morari, 1989,
Model Predictive Control: Theory and Practice—a Survey.

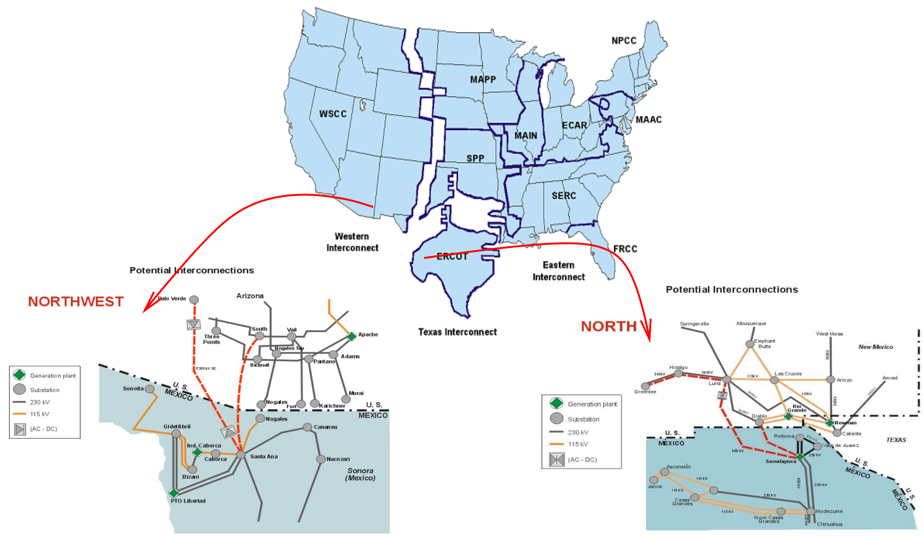
> *MPC is not inherently more or less robust than classic feedback as has been falsely claimed (Mehra et al. 1982). Also the large number of parameters in a step response model ... does not add any robustness. By that argument one could include some parameters in the model which have no effect on the input-output description at all and obtain even more robustness.*

Turning to the future

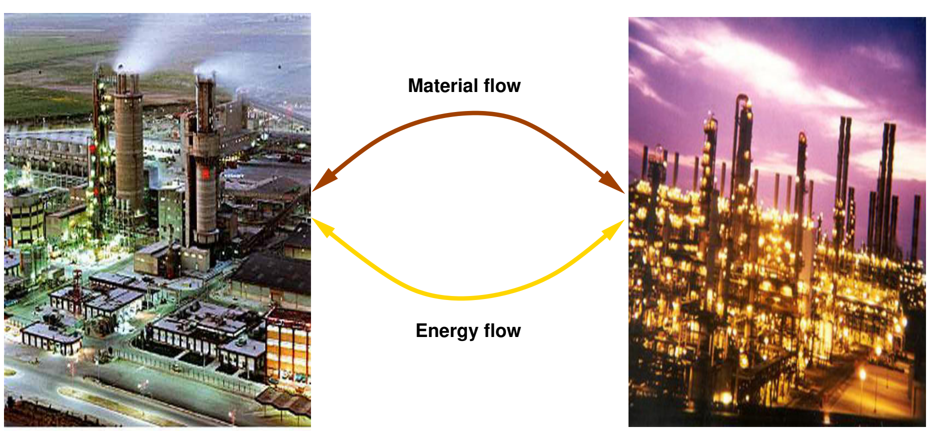
Are all the problems solved?

- How do we best decompose *large-scale systems* into manageable problems?
- How do we optimize dynamic *economic* operation?

Electrical power distribution



Chemical plant integration



MPC at the large scale

Decentralized Control

- Most large-scale systems consist of networks of interconnected/interacting subsystems
  - Chemical plants, electrical power grids, water distribution networks, . . .
- Traditional approach: Decentralized control
  - Wealth of literature from the early 1970's on improved decentralized control [a]
  - Well known that poor performance may result if the interconnections are not negligible

[a] (Sandell Jr. et al., 1978; Šiljak, 1991; Lunze, 1992)

MPC at the large scale

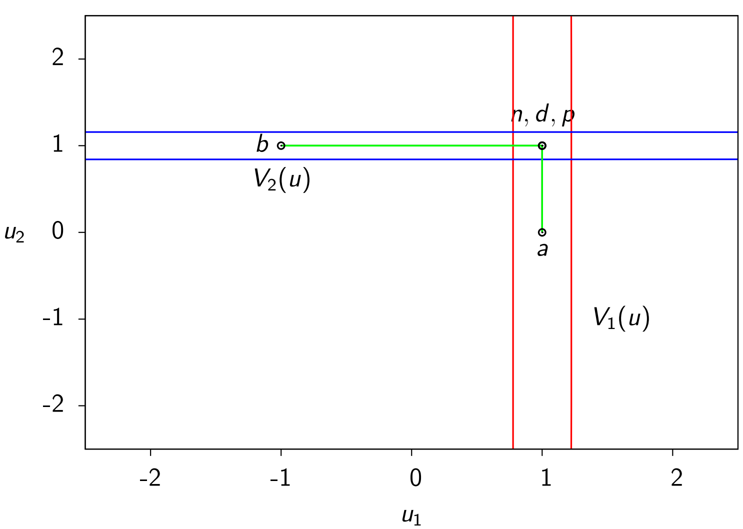
Centralized Control

- Steady increase in available computing power has provided the opportunity for centralized control
- Most practitioners view centralized control of large, networked systems as impractical and unrealistic
- A divide and conquer strategy is essential for control of large, networked systems (Ho, 2005)
- Centralized control: A benchmark for comparing and assessing distributed controllers

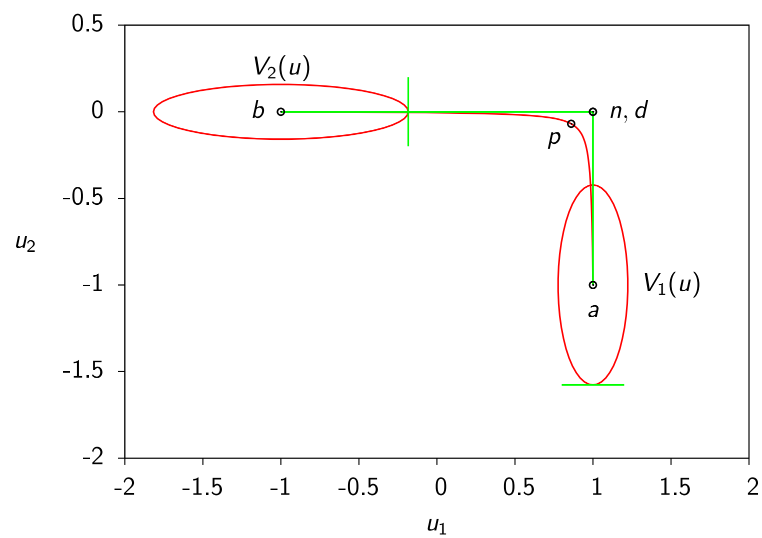
Nomenclature: consider two interacting units

Objective functions	$V_1(u_1, u_2), \; V_2(u_1, u_2)$
and	$V(u_1, u_2) = w_1 V_1(u_1, u_2) + w_2 V_2(u_1, u_2)$
decision variables for units	$u_1 \in \Omega_1, \quad u_2 \in \Omega_2$
Decentralized Control	$\min_{u_1 \in \Omega_1} \widetilde{V}_1(u_1) \quad \min_{u_2 \in \Omega_2} \widetilde{V}_2(u_2)$
Noncooperative Control	$\min_{u_1 \in \Omega_1} V_1(u_1, u_2) \quad \min_{u_2 \in \Omega_2} V_2(u_1, u_2)$
(Nash equilibrium)	
Cooperative Control	$\min_{u_1 \in \Omega_1} V(u_1, u_2) \quad \min_{u_2 \in \Omega_2} V(u_1, u_2)$
(Pareto optimal)	
Centralized Control	$\min_{u_1, u_2 \in \Omega_1 \times \Omega_2} V(u_1, u_2)$
(Pareto optimal)	

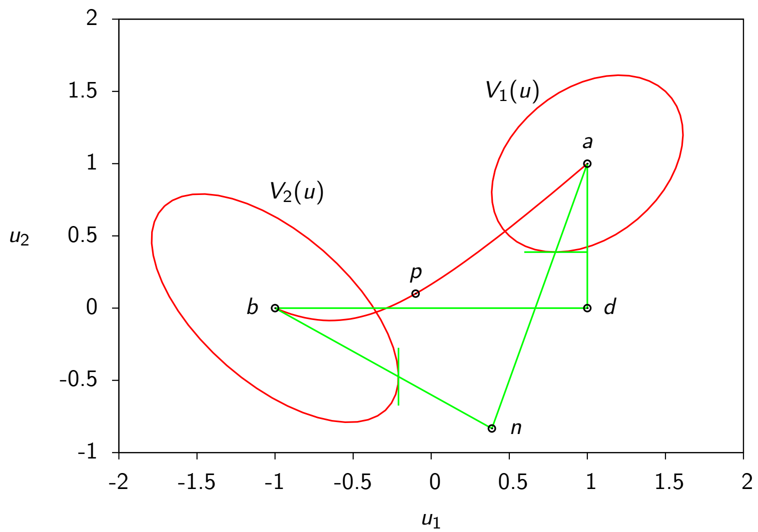
Noninteracting systems



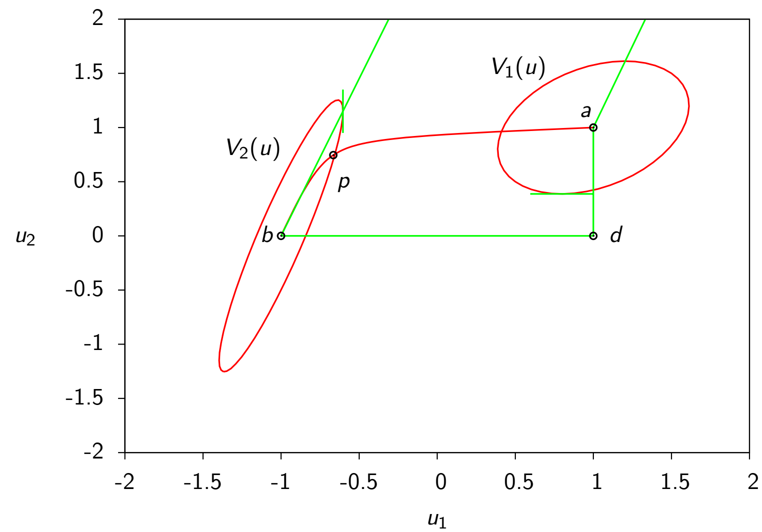
Weakly interacting systems



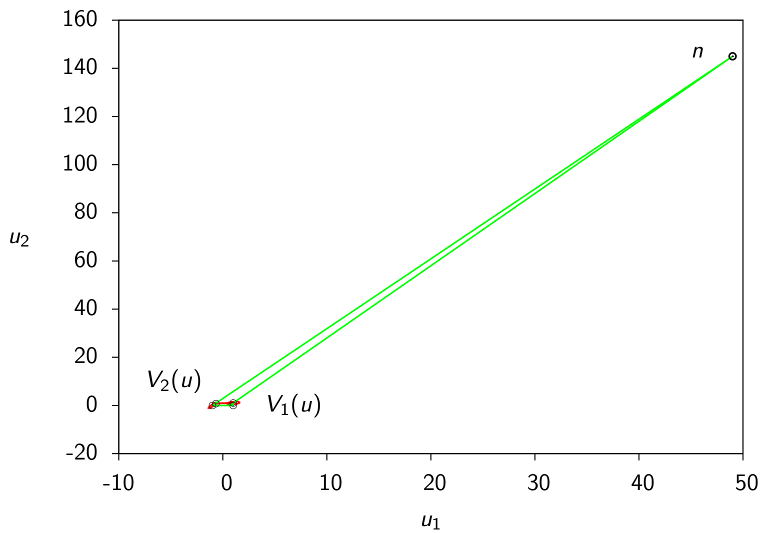
Moderately interacting systems



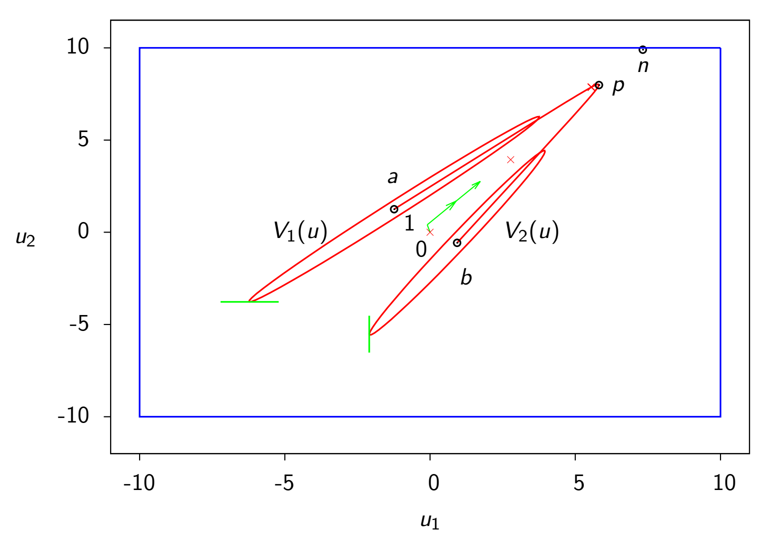
Strongly interacting (conflicting) systems



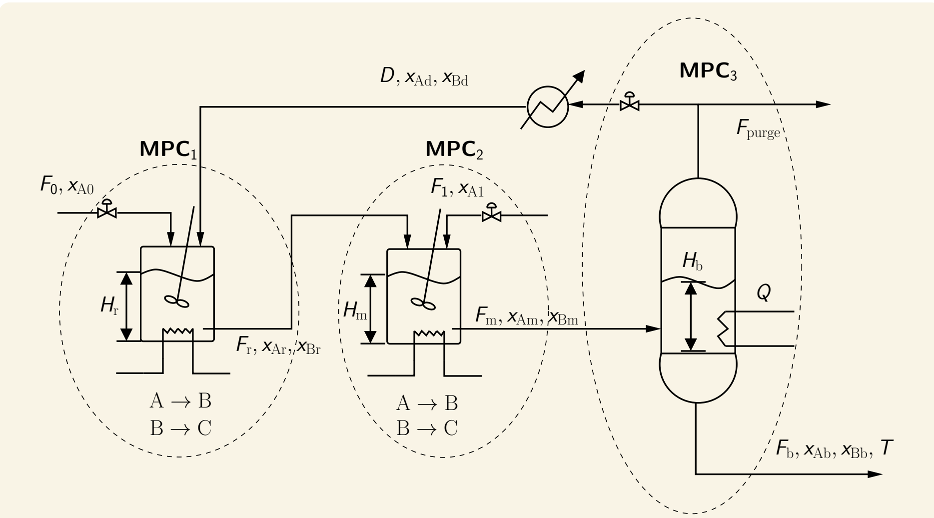
Strongly interacting (conflicting) systems



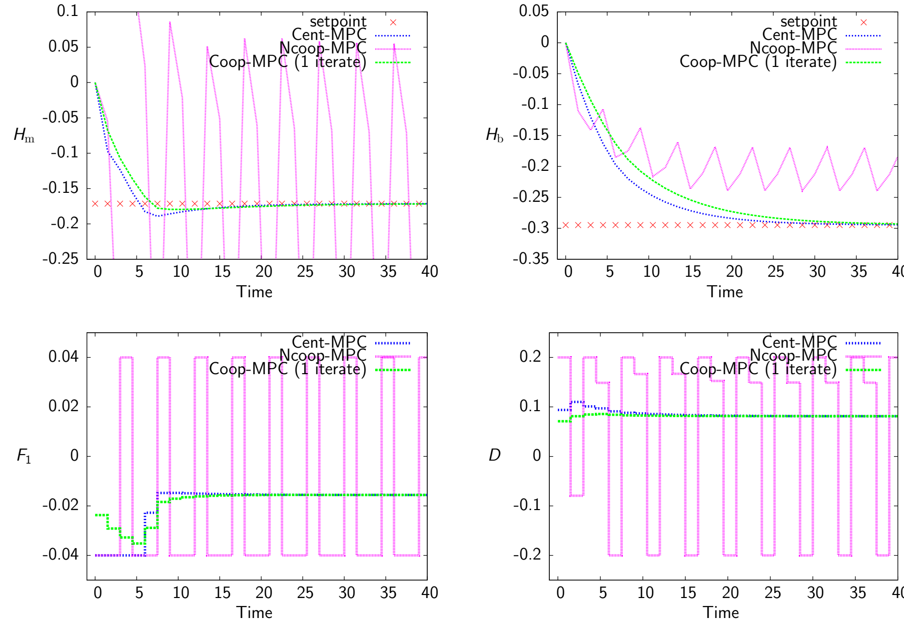
Geometry of cooperative vs. noncooperative MPC



Two reactors with separation and recycle



Two reactors with separation and recycle

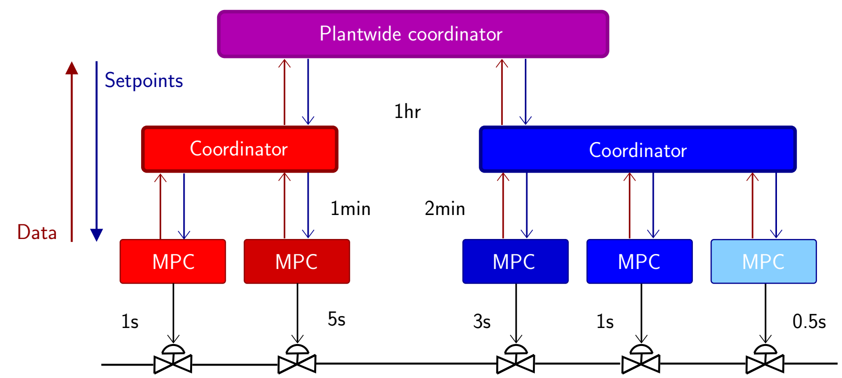


Two reactors with separation and recycle

Performance comparison

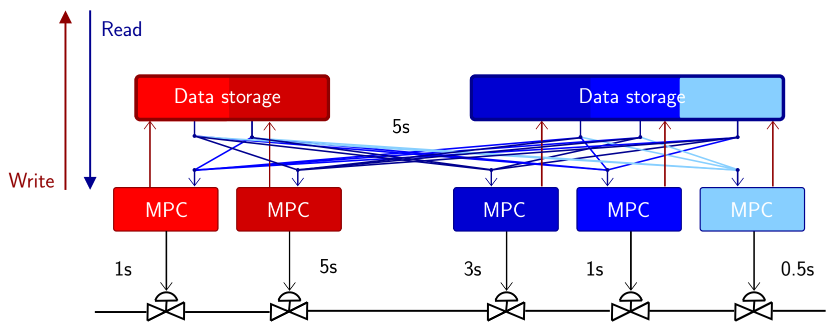
	Cost ($\times 10^{-2}$)	Performance loss
Centralized MPC	1.75	0
Decentralized MPC	∞	∞
Noncooperative MPC	∞	∞
Cooperative MPC (1 iterate)	2.2	25.7%
Cooperative MPC (10 iterates)	1.84	5%

Traditional hierarchical MPC



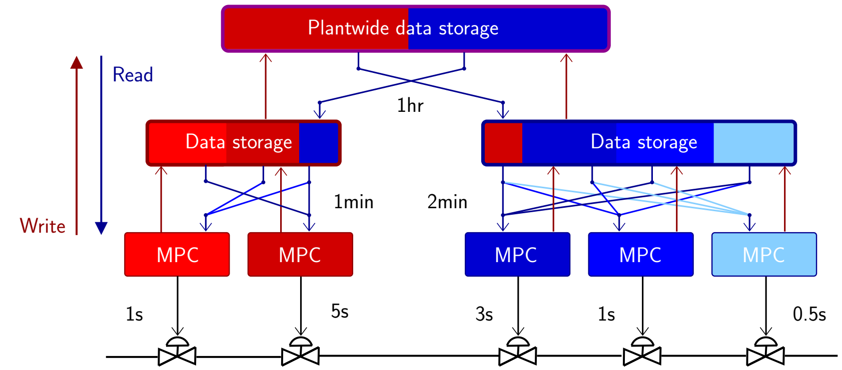
- Multiple dynamical time scales in plant
- Data and setpoints are exchanged on slower time scale
- Optimization performed at each layer

Cooperative MPC data exchange



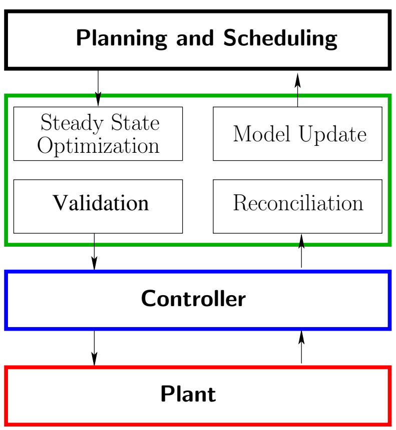
- All data exchanged plantwide
- Slowest MPC defines rate of data exchange

Cooperative hierarchical MPC



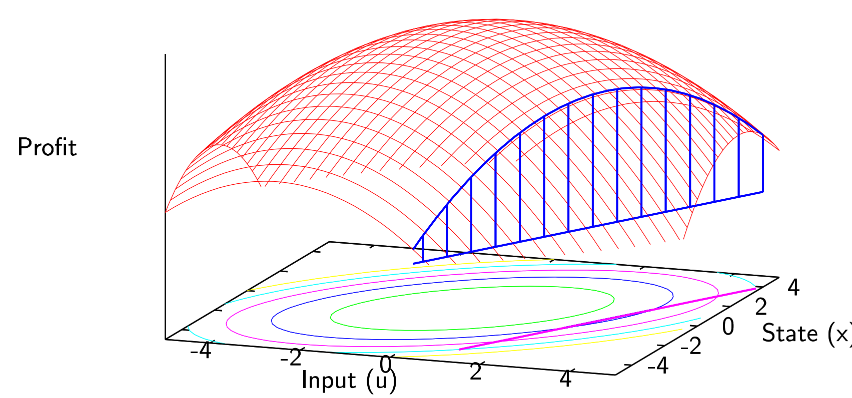
- Optimization at MPC layer only
- Only subset of data exchanged plantwide
- Data exchanged at slower time scale

Optimizing economics: Current industrial practice



- Two layer structure
- Drawbacks
  - Inconsistent models
  - Re-identify linear model as setpoint changes
  - Time scale separation may not hold
  - Economics unavailable in dynamic layer

Motivating the idea



Economics controller

$$\min_{u(t)} \int_0^T L(x,u)dt \qquad \text{subject to:} \quad \begin{aligned} \dot{x} &= f(x,u) \\ y &= g(x,u) \end{aligned}$$

- Target tracking (standard)

$$L(x,u) = |y_{sp} - g(x,u)|_Q^2 + |u_{sp} - u|_R^2$$

- Economic optimization (new)
  L is the negative of economic profit function

$$L(x,u) = -P(x,u)$$

Strong duality and asymptotic stability

Strong Duality

If there exists a λ such that the the following problems have the same solution

$$\begin{aligned} &\min_{x,u} L(x,u) \\ &f(x,u) = 0 \\ &h(x,u) \leq 0 \end{aligned} \qquad \begin{aligned} &\min_{x,u} L(x,u) - \lambda(f(x,u)) \\ &h(x,u) \leq 0 \end{aligned}$$

- Asymptotic stability of the closed-loop economics controller with a strictly convex cost and linear dynamics (Rawlings et al., 2008)
- Asymptotic stability of the closed-loop economics controller with strong duality in the steady-state problem (Diehl et al., 2011)

Example

$$x_{k+1} = \begin{bmatrix} 0.857 & 0.884 \\ -0.0147 & -0.0151 \end{bmatrix} x_k + \begin{bmatrix} 8.565 \\ 0.88418 \end{bmatrix} u_k$$

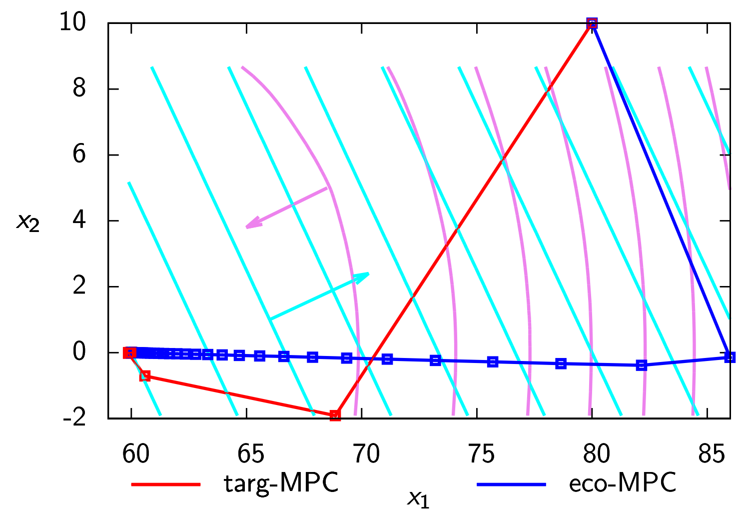
Input constraint: $-1 \leq u \leq 1$

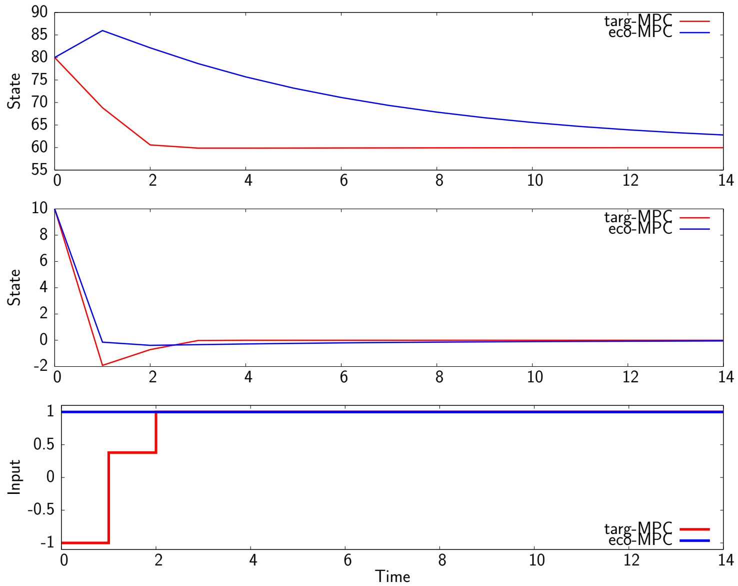
Economics

- $L_{eco} = \alpha' x + \beta' u$
- $\alpha = \begin{bmatrix} -3 & -2 \end{bmatrix}' \qquad \beta = -2$

Tracking

- $L_{targ} = |x - x^*|_Q^2 + |u - u^*|_R^2$
- $Q = 2I_2 \qquad R = 2$
- $x^* = \begin{bmatrix} 60 & 0 \end{bmatrix}' \qquad u^* = 1$





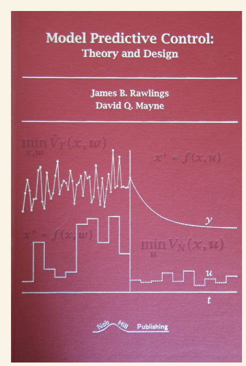
Conclusions

- Optimal dynamic operation of chemical processes has undergone a total transformation in the last 20 years. Both in theory and in practice.
- The currently available theory splits the problem into state estimation and regulation. Both are posed and solved as online optimization problems. Basic properties have been established. Lyapunov functions are the dominant theoretical tool for analysis and design.
- Industrial implementations and vendor software are basically keeping pace with the best available theory and algorithms. That is a surprising and noteworthy outcome!

Future directions — Current research in MPC

- Distributed versions of MPC
  - Controlling large-scale systems composed of many small-scale MPCs
  - How to structure the small-scale MPCs so they cooperate on plantwide objectives
- Optimizing economics with MPC
  - The optimal economic point is not necessarily a steady state
  - Allows removal of the steady-state economic optimization layer
  - Dynamic economic optimization subject to settling at the optimal steady state

New MPC graduate textbook



- 576 page text
- 214 exercises
- 335 page solution manual
- 3 appendices on web (133 pages)
- `www.nobhillpublishing.com`

Further reading I

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Further reading II

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