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Expected Problems When Pairing on Negative RGA-elements

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Expected problems when pairing on negative RGA-elements

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Thank you all!

-Martin Jonathan Bland

Abstract

This project has been prepared as part of the MSc degree in Chemical Engineering from NTNU.

The project looks at process control of multiple input multiple output (MIMO) processes with negative relative gain array (RGA) elements. Control structures paired on these negative elements, using proportional-integral (PI) regulators, are tested and simulated using Simulink and MATLAB. The project looks at three case studies: a static gain process with delay, two coupled tanks and a binary distillation column.

Control that tracks set-points can be achieved with pairings on negative RGA elements, but should be avoided if possible. Compared to pairings on positive RGA elements negative elements gives poor failure tolerance and tends to give initial inverse responses. Negative pairings may be preferable where the process dynamics are considerably faster than for the positive elements.

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1 Introduction

The relative gain array, RGA, was introduced by Bristol in 1966 [1]. It provides a tool for assessing the steady state gains from input to output for multi-variable coupled systems. The RGA is a tool for assessing and picking the control structures that look most promising to achieve the control objectives.

One of the assertions emerging from Bristol's paper was that it was very undesirable to couple on negative RGA elements. Shinskey also extends the RGA analysis looking at the consequences of pairing feedback controllers on negative RGA elements [2]. The main counterpoints against negative pairings that emerge from these works are:

1. That the transfer function between input/output for a negative element will be non-minimal phase or unstable
2. Pairing a two-by-two process on a negative element with feedback controllers forces one of the controllers to have positive feedback. Thus if one of the controllers then goes off-line, for example due to saturation or being put in manual by operators, the control structure is left in a highly undesirable state where the remaining controllers gain has the wrong sign.
3. Pairing on negative elements results in inverse responses

Grosdidier, Morari and Holt expands the RGA analysis in their paper by methodically testing and proving, the qualitative arguments made by Bristol and Shinskey [3].

This project looks at the performance of controllers coupled on negative RGA-elements.

2 Theory

Often in many process control applications for multiple-input-multiple-output (MIMO) systems only steady state gains or simple models such as first order plus time delay (FOPTD) models are available for evaluation [3]. Such a system may be represented by a transfer function matrix $\mathbf{G}(s)$ that relates input \mathbf{U} to output \mathbf{Y} .

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s) \quad (2.1)$$

Where \mathbf{Y} is a vector of m output states, in this context controlled variables (CV's). \mathbf{U} is a vector of n input variables, or manipulated variables (MV's). The matrix of transfer functions $\mathbf{G}(s)$ has a dimension $n \times m$, where element $g_{ij}(s)$ of $\mathbf{G}(s)$ relates input u_i to output y_j . All transfer functions are given as Laplace transforms.

The state state gain matrix, \mathbf{K} , can be obtained directly from process data or as the limit of transfer functions in \mathbf{G} :

$$\mathbf{K} = \lim_{t \rightarrow \infty} \mathbf{G}(t) = \lim_{s \rightarrow 0} \mathbf{G}(s) = \mathbf{G}(0) \quad (2.2)$$

For the scope of this paper only systems with two inputs and outputs (2×2 systems) have been considered. This kind of system is illustrated in Figure 1 which illustrates a simple binary distillation column. The aim with control of the distillation column is to control the composition of light component in the bottom and top product from the column (mole fraction X_b and Y_d). This is achieved by adjusting the two MV's: the vapour flow (V) from the reboiler and liquid reflux flow (L) from the condenser. This example is studied in Section 3.3.

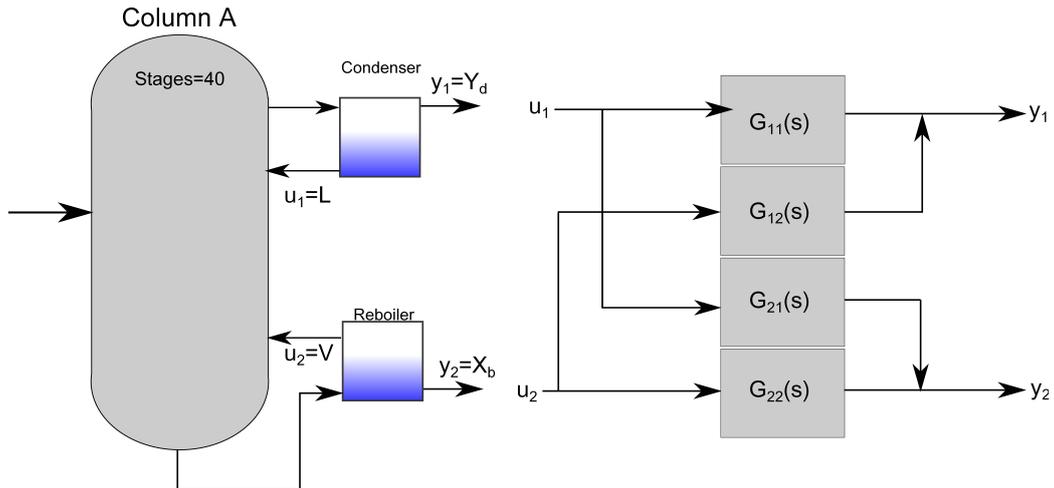


Figure 1: A schematic representation of a distillation column and its representation in a block diagram as a 2×2 -system

2.1 SISO Control of MIMO systems

Process control of systems with a single input that controls a single output, SISO control, has been extensively studied for optimal and stable operation. Procedures for designing and tuning SISO controllers are readily available from literature such as the Skogestad/Simple internal model control (SIMC). This method tunes the controller based on a first (FOPTD) or second order (SOPTD) transfer function with time delay. All controllers in this paper are tuned using the SIMC method. This is outlined in section 2.7 of the book *Multivariable Feedback Control, analysis and design* [4].

However most real world problems involve multiple MV's and target CV's. Some degree of interactions between inputs and outputs is expected for most control problems, such as V effecting both compositions X_b and Y_d the distillation column in Figure 1. This means that the controllers in the plant will disturb each other as they try to reach the set-point for their CV, which can severely impair controller performance [5].

The RGA provides a measurement of the degree of interactions (discussed in Section 2.3). Other multivariable techniques such as decoupling and cascade control aim to reduce the impact of the interactions. These are not considered in this paper, which looks at SISO controller performance in a MIMO (2×2) system with controller pairings on negative RGA elements.

For square systems with as many MV's as CV's ($n = m$), there are $n!$ possible control structures [2]. Most of these designs will be poorly suited for control. The RGA provides a simple and powerful tool to evaluate and choose candidate control structures for further analysis. Figure 2 shows the two possible control structures for a 2×2 system. The controller is a proportional-integral controller or pure integral controller (Section 3.1). The controller on cascade Laplace form is given as

$$C_i(s) = K_c \frac{\tau_i s + 1}{\tau_i s} \quad (2.3)$$

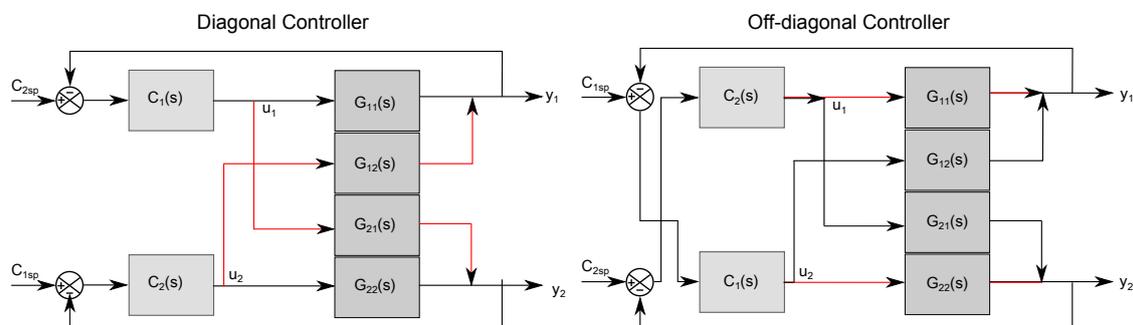


Figure 2: SISO control options for a 2×2 -system. The red lines indicate the interaction from one controller to the other output.

2.2 Calculation of the RGA

The relative gain array, Λ , is given as the open loop gain over the closed loop gain for each element in \mathbf{Y} and \mathbf{U} . The open loop gain is the gain in an output y_i for an input u_j with all other inputs, u 's, constant, whilst the closed loop gain is the gain in y_i for input u_j with all other outputs, y 's constant. This can be written as

$$\lambda_{ij} = \frac{\text{Open-loop gain}}{\text{Closed-loop gain}} = \frac{(\partial y_i / \partial u_j)_{u_k \neq j}}{(\partial y_i / \partial u_j)_{y_i \neq i}} \quad (2.4)$$

The total RGA for a process with m output elements in \mathbf{Y} and n in inputs \mathbf{U} is then

$$\Lambda = \begin{matrix} & \begin{matrix} u_1 & u_2 & \cdots & u_n \end{matrix} \\ \begin{matrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{matrix} & \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{m1} & \lambda_{m2} & \cdots & \lambda_{mn} \end{bmatrix} \end{matrix} \quad (2.5)$$

2.2.1 RGA for 2×2 -system

Given a steady-state model of a 2×2 -system, for example obtained from Figure 1 by setting $s=0$ [5]

$$y_1 = K_{11}u_1 + K_{12}u_2 \quad (2.6)$$

$$y_2 = K_{21}u_1 + K_{22}u_2 \quad (2.7)$$

The open loop gain is simply

$$\left(\frac{\partial y_i}{\partial u_j} \right)_{u_k \neq j} = K_{ij} \quad (2.8)$$

The method for obtaining the closed loop gain for $(\partial y_1 / \partial u_1)_{y_2}$ is shown below:

$$y_2 = 0 \Rightarrow \quad (2.9)$$

$$u_2 = -\frac{K_{21}}{K_{22}}u_1 \Rightarrow \quad (2.10)$$

$$y_1 = \left(K_{11} - \frac{K_{12}K_{21}}{K_{22}} \right) u_1 \Rightarrow \quad (2.11)$$

$$\left(\frac{\partial y_1}{\partial u_1} \right)_{y_2} = K_{11} \left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}} \right) \quad (2.12)$$

This gives the following open and closed loop gains

$$\begin{aligned}
 \left(\frac{\partial y_1}{\partial u_1}\right)_{u_2} &= K_{11} & \left(\frac{\partial y_1}{\partial u_1}\right)_{y_2} &= K_{11}\left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right) \\
 \left(\frac{\partial y_1}{\partial u_2}\right)_{u_1} &= K_{12} & \left(\frac{\partial y_1}{\partial u_2}\right)_{y_2} &= K_{12}\left(1 - \frac{K_{11}K_{22}}{K_{12}K_{21}}\right) \\
 \left(\frac{\partial y_2}{\partial u_1}\right)_{u_2} &= K_{21} & \left(\frac{\partial y_2}{\partial u_1}\right)_{y_1} &= K_{21}\left(1 - \frac{K_{11}K_{22}}{K_{12}K_{21}}\right) \\
 \left(\frac{\partial y_2}{\partial u_2}\right)_{u_1} &= K_{22} & \left(\frac{\partial y_2}{\partial u_2}\right)_{y_1} &= K_{22}\left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right)
 \end{aligned} \tag{2.13}$$

Which gives the RGA

$$\Lambda = \begin{bmatrix} \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}} & \frac{1}{1 - \frac{K_{11}K_{22}}{K_{12}K_{21}}} \\ \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}} & \frac{1}{1 - \frac{K_{11}K_{22}}{K_{12}K_{21}}} \end{bmatrix} = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix} \tag{2.14}$$

Where $\lambda = \lambda_{11}$

The RGA of larger systems can be determined by the following expression [4]

$$\Lambda = \mathbf{K} \otimes (\mathbf{K}^{-1})^T \tag{2.15}$$

Where \otimes is the Haddamard/Schur product, which gives the product of each corresponding element in two matrices of the same dimensions. For non-square systems, with an unequal number of elements ($n>m$ or $n<m$), the calculation above can be performed with the pseudo inverse of \mathbf{K} [6].

2.3 Properties of the RGA

The RGA has several important properties.

1. It is normalized as all elements in each column are equal to one [6]. For square systems ($n=m$) the sum of each row is also equal to one. This greatly increases the ease of calculation as not all elements have to be determined to obtain the full array, as can be seen from equation 2.14 (only λ_{11} is necessary to compute).
2. From the definition of the RGA it clear that the elements, λ_{ij} are dimensionless and independent of units and scaling of the variables.
3. Presents an evaluation of the sensitivity to uncertainty in the gain \mathbf{K} . If a single element K_{ij} is changed to $K_{ij}(1 - \lambda_{ij})$ the gain matrix \mathbf{K} becomes singular, this is illustrated in Section 3.1.1.

4. Provides a measurement of two-way interaction, how the inputs interact and effect outputs in relation to each other. Large RGA elements, greater than 5-10, indicate strong interactions between the manipulated variable (u's) and outputs (y's) [4].

The recommendation for pairing controllers is to

1. Pair CV's with MV's on RGA elements as close to one as possible (at frequencies of control interest) [5]. This is to minimise controller interaction.
2. Avoid negative negative RGA elements at steady state $\lambda_{ij} > 0$ this is to avoid right hand plane (RHP)-zeros, see Section 3.2 [4].

2.3.1 Zero Elements, One Way and Non-interacting Processes

The RGA for systems with a limited or no degree of interaction is simply the identity matrix. This is shown for a 2×2 -system with no degree of interaction, Equation 2.16, and one way interaction, Equation 2.17. Note that the one-way interactive plant may be upper or lower triangular and still give an identity RGA matrix [3].

$$\mathbf{K} = \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix} \Rightarrow \lambda = 1 \Rightarrow \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I} \quad (2.16)$$

$$\mathbf{K} = \begin{bmatrix} K_{11} & K_{12} \\ 0 & K_{22} \end{bmatrix} \Rightarrow \lambda = 1 \Rightarrow \Lambda = \mathbf{I} \quad (2.17)$$

Surprisingly feedback control on the "wrong" elements where there is no or only one way interaction between the CV and MV can be made to work, because the controller introduces the necessary interaction [4]. An example of this is shown in Section 3.2 which looks at zero couplings for a coupled tank system.

2.3.2 Negative Elements

From the properties of the RGA it is clear that for any system with an element $\lambda_{ij} > 1$ there must be a corresponding negative element in the same column or row. For the 2×2 -system this will be when the product of the off-diagonal steady state gains are greater than their diagonal counterparts

$$K_{12}K_{21} > K_{11}K_{22} \quad (2.18)$$

Feedback control on negative RGA elements causes positive feedback through the indirect feedback loop (red lines in Figure 2). This can be mitigated by changing the sign of one of the controllers. However this structure is failure sensitive, if the other loop then is opened the remaining controller

has the wrong gain and gives positive feedback which causes the controller to fail [2]. This effect is shown for the Model Column A in Figure 8 for pairing on negative off-diagonal RGA-elements which fails for Y_d when the X_b controller is put in manual.

Another issue related to control pairings on negative RGA-elements are inverse responses. The negative values indicates that numerator and denominator of the open loop gain have opposite gains. For the 2×2 -case interaction from other control loops drives the output in the opposite of the desired direction. The indirect response has to travel through two process and a controller block before the output moves in the correct direction, giving an initial response away from the desired value, an inverse response [2]. Inverse responses are indicated by right hand plane (RHP)-zeroes [7]. A negative RGA element does not indicate the presence of a RHP-zero directly, it is a necessary but not sufficient criteria [3] [4]. An example of an inverse response is available in Figure 3

3 Case Studies

3.1 A Two way Interactive Plant

The results in this section is an expansion of Example 10.16 in Multivariable Feedback and Control [4]. Consider the two way interactive plant (delay with gain)

$$\mathbf{G}(s) = e^{-s} \begin{bmatrix} 1 & g_{12} \\ 5 & 1 \end{bmatrix} \quad (3.1)$$

3.1.1 Illustration of RGA Property 3

The inverse gain matrix and RGA are given by the following

$$\mathbf{G}(0)^{-1} = \frac{1}{1-5g_{12}} \begin{bmatrix} 1 & -g_{12} \\ -5 & 1 \end{bmatrix} \quad (3.2)$$

$$\Lambda = \frac{1}{1-5g_{12}} \begin{bmatrix} 1 & -5g_{12} \\ -5g_{12} & 1 \end{bmatrix} \quad (3.3)$$

The plant is singular for $g_{12} = 0.2$, this can be seen as the RGA elements greatly increase as $g_{12} \rightarrow 0.2$

$$\Lambda(g_{12} = 0.17) = \begin{bmatrix} 6.7 & -5.7 \\ -5.7 & 6.7 \end{bmatrix} \quad \Lambda(g_{12} = 0.19) = \begin{bmatrix} 20 & -19 \\ -19 & 20 \end{bmatrix} \quad (3.4)$$

3.1.2 Control with Pairing on Negative-element

The increasing elements of the RGA indicate an increasing degree of interaction between the two outputs, meaning that two SISO controllers trying to keep their set-point will be "fighting" each other for control of their CV. To counteract this effect the two I-controllers are decomposed in time with $\tau_{I_1} = 5$ and $\tau_{I_2} = 1$ for the diagonal pairing and $\tau_{I_1} = -1$ $\tau_{I_2} = 5$ for the off-diagonal (The controller in "possession" of the largest element, $g_{21} = 5$, is set to be the slowest). Figure 3 shows the step responses for the pairings on the positive and negative RGA elements. Both controllers work but the pairing on the negative element gives inverse response which is highly undesirable for control. Both controllers are also slow at reaching their set-points due to the very interactive nature of the plant.

The Simulink model and MATLAB code for the simulation are available in Appendix C.1

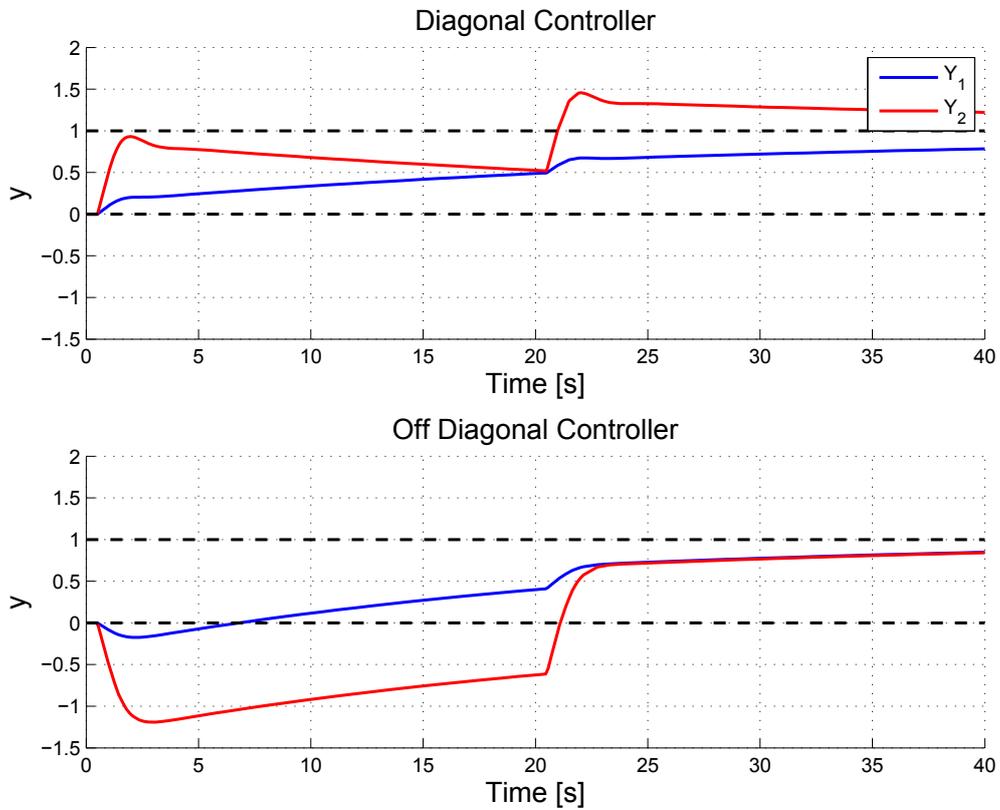


Figure 3: Step responses for diagonal positive RGA pairing and off diagonal negative pairing with $g_{12} = 0.17$. Step in Y_1 after 0 s and Y_2 after 20 s

3.2 Two Coupled Tanks: Pairing on Zero Elements

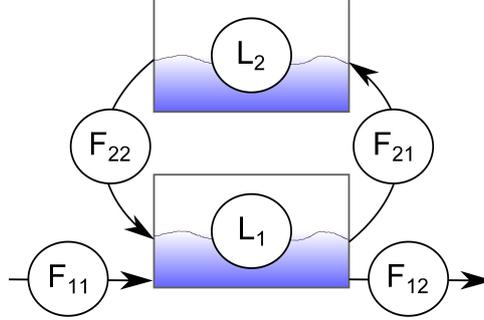


Figure 4: Schematic of two tanks with levels L_1 and L_2 connected together with instream F_{11} outstream F_{12} and recycle streams F_{21} and F_{22}

An example of a scenario where couplings on zero RGA elements are possible is the two tank configuration in Figure 4. We want to control the levels of both tanks for disturbances to magnitude of the feed and recycle stream. There are four control structures that can be conceived for this example where the input to the system, the throughput manipulator (TPM/feed disturbance), is kept fixed in F_{11} . The control structures are available in Figure 5.

Structure 1 and 3 are diagonal couplings on RGA elements where $\lambda = 1$, see equations 3.6 and 3.8. For these independent loop design is possible for the controllers. Structures 2 and 4 are couplings on zero diagonal RGA elements. This is shown in Equation 3.7 and 3.9. The controller for level 1 has to be closed and in operation for the level 2 controller to work.

3.2.1 Mathematical Model for Coupled Tanks

The transfer functions from the input streams F_{ij} to the tank level L_i is expressed by the following integrating plus time delay process

$$\mathbf{L} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \frac{e^{-\theta s}}{s} \begin{bmatrix} F_{11} - F_{12} - F_{21} + F_{21} \\ F_{21} - F_{22} \end{bmatrix} \quad (3.5)$$

Neglecting the TPM's (disturbances), the controller transfer functions, $\mathbf{G}(s)_i = \mathbf{L}(s)/\mathbf{u}(s)_i$, and RGA's are:

For structure 1, $\mathbf{u}_1 = [F_{12} \ F_{21}]^T$

$$\mathbf{G}_1(s) = \frac{e^{-\theta s}}{s} \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.6)$$

For structure 2, $\mathbf{u}_2 = [F_{21} \ F_{12}]^T$

$$\mathbf{G}_2(s) = \frac{e^{-\theta s}}{s} \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \quad \Lambda_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (3.7)$$

For structure 3, $\mathbf{u}_3 = [F_{12} \ F_{22}]^T$

$$\mathbf{G}_3(s) = \frac{e^{-\theta s}}{s} \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \quad \Lambda_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.8)$$

For structure 4, $\mathbf{u}_4 = [F_{22} \ F_{12}]^T$

$$\mathbf{G}_4(s) = \frac{e^{-\theta s}}{s} \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \quad \Lambda_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (3.9)$$

3.2.2 Simulation Results

Figure 6 shows the tank level response for step changes in the TPM's. All four control structures work, but the direct acting structures, 1 and 3, are far better at controlling the tank levels. The zero elements

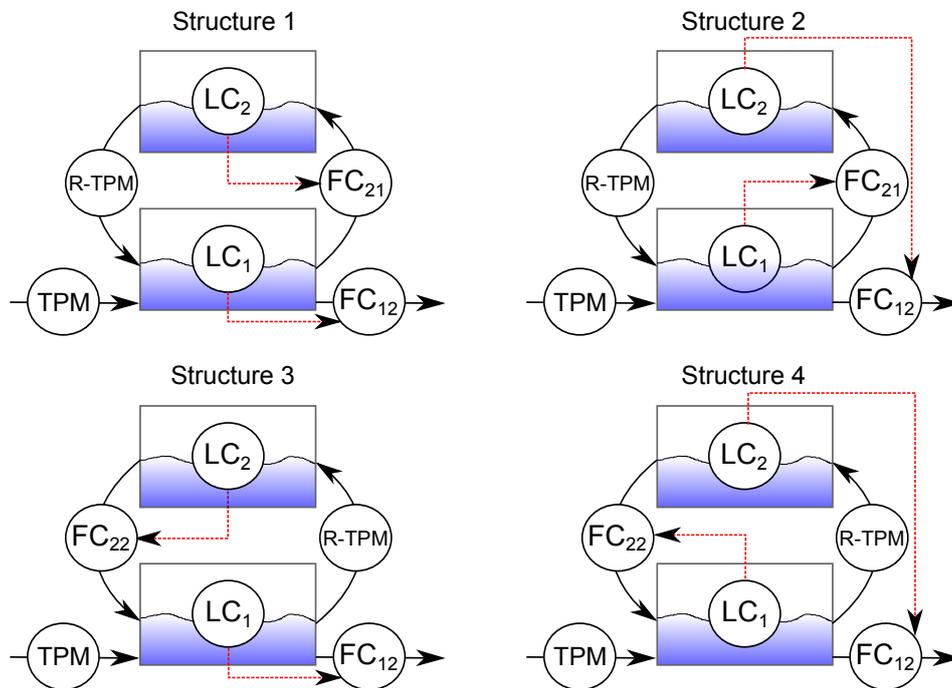


Figure 5: Control schemes for feedback control on the two tank levels. The TPM (throughput manipulator) block symbolises the disturbance feed to the tank, the Recycle-TPM, R-TPM, symbolises the recycle disturbance. Note that all the controllers are diagonal for these structures.

coupling, structures 2 and 4, for level L_2 controlled by stream F_{12} only works with the L_1 controller in operation. Should this controller somehow fail for example by saturation or being put in manual the L_2 controller will fail. The poor performance of structures 2 and 4 becomes even more distinct with larger delays as the control system has to wait for the delays in both tanks before it can take corrective action for the level of tank 2. This effect is shown in for control structure 1 and 2 with delays of 0 s and 2 s in Figure A.1 in Appendix A.

The Simulink model and MATLAB code for generating these results are available in Appendix C.2. The level controllers tuning and the derivation of the closed loop transfer functions for structure 2 and 4 are available in Appendix A.

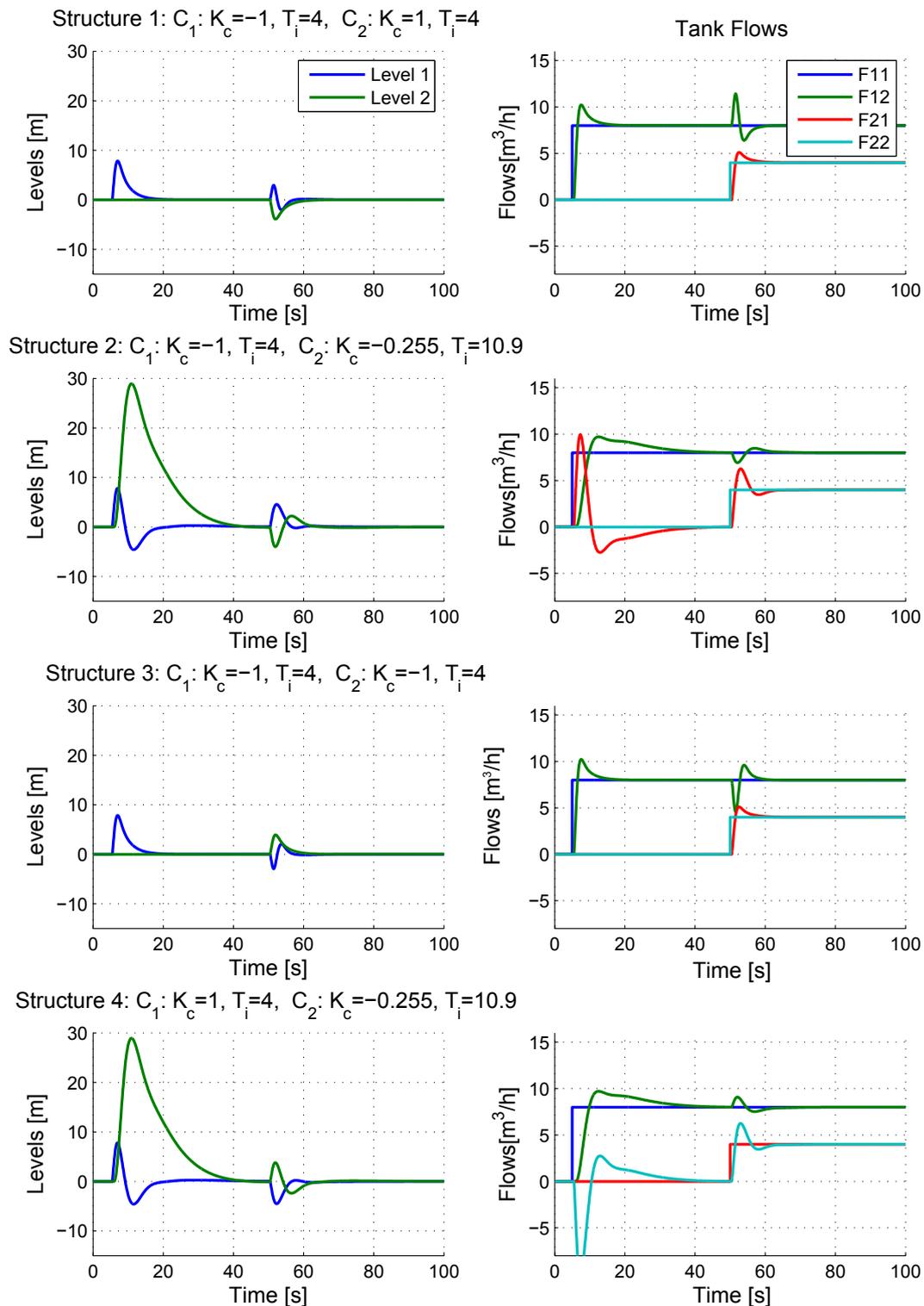


Figure 6: Simulation results for the various control structures of the two tank example with $\theta = 0.5s$. Stepchanges of magnitude 1 m are made for the TPM after 5 seconds and the R-TPM after 50 seconds.

3.3 Distillation Column A: Pairing on Negative Elements

A practical example of where one might be forced to pair on negative RGA-elements is a binary distillation column such as in Figure 1. The scenario is that in previous operation the column has been prone to saturate the boil up rate V and the composition control of the valuable bottom product (X_b) has been coupled with reflux flow L . This is the "wrong" coupling in the sense that it is slow and paired far away from the CV that is supposed to control. For a reduced feed rate the reboiler is less prone to max out and composition control of the top product, Y_d is also desirable and possible.

We wish control the composition of X_b and Y_d , the molar fraction (dimensionless) of light component in the bottom and top product, by adjusting the reflux flow L and the vapour boil up rate V (both kmol/min). Equation 3.10 shows the transfer functions with the corresponding RGA-matrix for Model Column A. [8]. The delay of 3 minutes is added to element g_{21} to model the time it takes from adjusting reflux L till it affects the bottom composition x_B .

$$\begin{bmatrix} Y_d \\ X_b \end{bmatrix} = \frac{1}{50s+1} \begin{bmatrix} 0.878 & -0.864 \\ 1.082e^{-3s} & -1.096 \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix} \quad \Lambda = \begin{bmatrix} 35.1 & -34.1 \\ -34.1 & 35.1 \end{bmatrix} \quad (3.10)$$

The compositions controllers are tuned using the SIMC method. For this scenario the X_B controller is already in place and it is tuned to be as fast as possible with $\tau_c = \theta = 3$ min. Then the transfer function for Y_d/V is estimated as a FOPTD transfer function by a step change in V . The Y_d controller is tuned on the basis of this estimation. The transfer function estimation and controller tunings are available in Appendix B.

The controller configuration is simulated for a step of magnitude 0.01 in X_b after 5 minutes and Y_d after 500 minutes, Figure 7 shows these step responses. For comparison the controller performance paired on the diagonally positive RGA-elements is also simulated with the same step changes. Figure 8 shows the controller performance with X_b in manual for both the off-diagonal and diagonal pairings.

The large RGA elements indicates a large degree of interaction between the manipulated flows L and V on both the compositions, this is also clear from the simulation where it takes a long time to reach the set-point especially for Y_d . For the off diagonal pairings a set-point change in one of the CV's has an adverse effect on the set-point tracking of the other variable. This effect is almost invisible with the diagonal pairing for the tracking of X_b with a set-point change in Y_d . The pairing on the negative elements give an inverse response for Y_d and this controller fails when X_b is put in manual (Figure 8). Whilst the controller for Y_d coupled on the diagonal positive elements pairing still tracks the set-point.

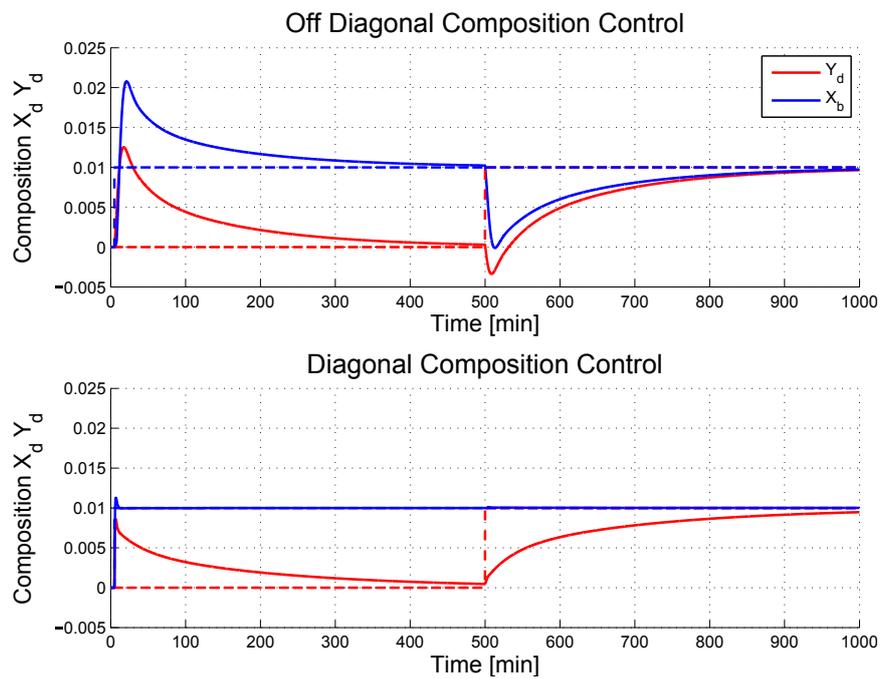


Figure 7: Step responses for off-diagonal and diagonal control of model Column A. Step in X_b after 5 min and Y_d after 500 min. Compositions in deviation variables

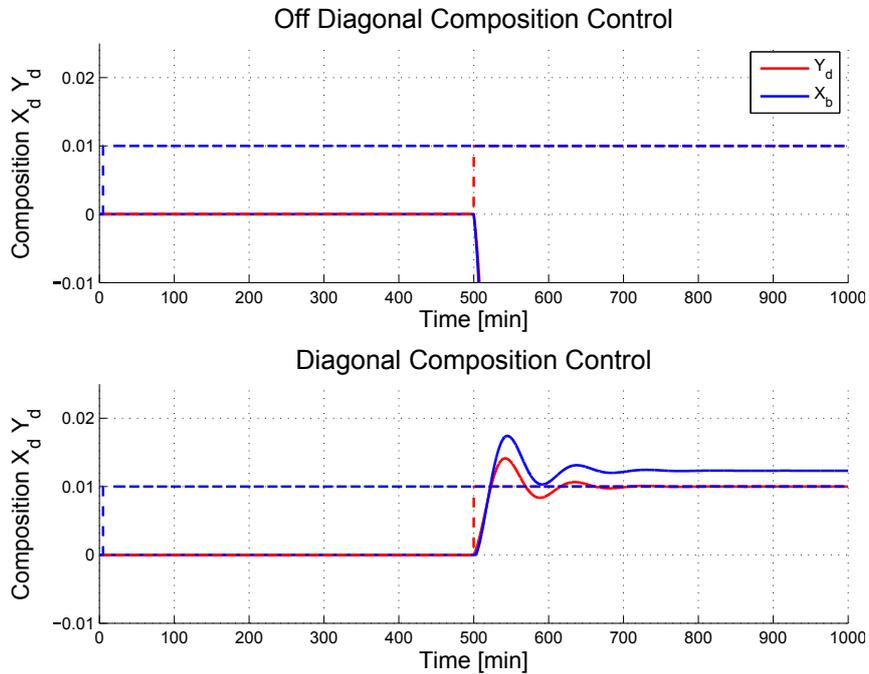


Figure 8: Step responses for off-diagonal and diagonal control of model Column A with the X_b controller in manual for both pairings. Same compositions steps as for Figure 7 and compositions in deviation variables

4 Conclusion

Designing stable control structures paired on negative RGA elements is possible, as all the negatively paired simulations show. Negative pairings may be beneficial over positive ones, where the dynamics are beneficial (faster) than for the positive elements. This is a weakness of the RGA as it was defined by Bristol in Equation 2.4 that it does not consider the dynamics of the process.

Negative pairings introduce inverse responses for processes with considerable time delays. Control paired on negative elements structures are also failure sensitive to other controllers being deactivated. These are serious limitations to pairing on negative RGA-elements.

5 Further Work

Further work would be to look at cases with faster dynamics for negative RGA pairings, such as Gagnepain and Seborgs paper [9] and a deeper look at RHP-zeroes and their impact on controller performance with regards to inverse response and controller stability.

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A Coupled Tanks Calculations

A.1 Tuning Controllers

The level controller for L_1 is tuned independently with the SIMC method on element $g_{11}(s) = \pm e^{-\theta s}/s$ for all four control structures. Which gives the following settings for $\tau_{c,L_1} = \theta$:

$$K_{c,L_1} = \frac{1}{k'} \frac{1}{2\theta} \quad (\text{A.1})$$

$$\tau_{c,L_1} = \min(\tau = \infty, 4(2\theta)) = 8\theta \quad (\text{A.2})$$

For control structure 1 and 3 the controller settings for L_2 follows the same procedure as for L_1 .

A.1.1 Control Structure 2

After closing the loop for level control of L_1 we can obtain the transfer function for L_2 , with its MV F_{12} , \hat{g}_{22}

$$\hat{g}_{22} = g_{22} - \frac{g_{21} c_1 g_{12}}{1 + c_1 g_{11}} = -g_{21} \frac{c_1 g_{11}}{c_1 g_{11} + 1} \quad (\text{A.3})$$

With $g_{22} = 0$, $g_{11} = g_{12}$, $c_1 = K_{c_1}(\tau_{c_1} s + 1)/s$, $K_{c_1} = 1/2\theta$, $\tau_{c_1} = 8\theta$ and linearising the expression for the delay in the denominator $e^{-\theta s} = 1 - \theta s$

$$\frac{c_1 g_{11}}{c_1 g_{11} + 1} = \frac{(8\theta s + 1)e^{-\theta s}}{(16\theta^2 s^2 + 8\theta s)e^{-\theta s}} = \frac{(8\theta s + 1)e^{-\theta s}}{8\theta^2 s^2 + 7\theta s + 1} = \frac{(8\theta s + 1)e^{-\theta s}}{(1.44\theta s + 1)(5.56s\theta s + 1)} \approx \frac{8}{5.56} \frac{e^{-\theta s}}{(1.44\theta s + 1)} \quad (\text{A.4})$$

$$\hat{g}_{22} = 1.44 \frac{e^{-2\theta s}}{s(1.44\theta s + 1)} \approx 1.44 \frac{e^{-2.72\theta s}}{s} \quad (\text{A.5})$$

With the SIMC rules for $\tau_{c_2} = 2.72\theta$ this gives the following controller settings

$$K_{c_2} = \frac{1}{1.44} \frac{1}{2 \times 2.72\theta} \quad \tau_{i_2} = \min(\tau = \infty, 4(\tau_{c_2} + 2.72\theta)) = 8 \times 2.72\theta \quad (\text{A.6})$$

A.1.2 Control Structure 4

For control structure $g_{11} = -g_{12}$ and we get a gain reversal for \hat{g}_{22} and K_{c_2} . compared to control structure 2

A.2 Effect of Delay

The effect of increasing delay on control structure 1 and 2 are shown in A.1. The response for structure 2 with a 2 second delay is out of scale compared to the other results. This shows the serious impairment of performance for increasing delay for zero element pairings

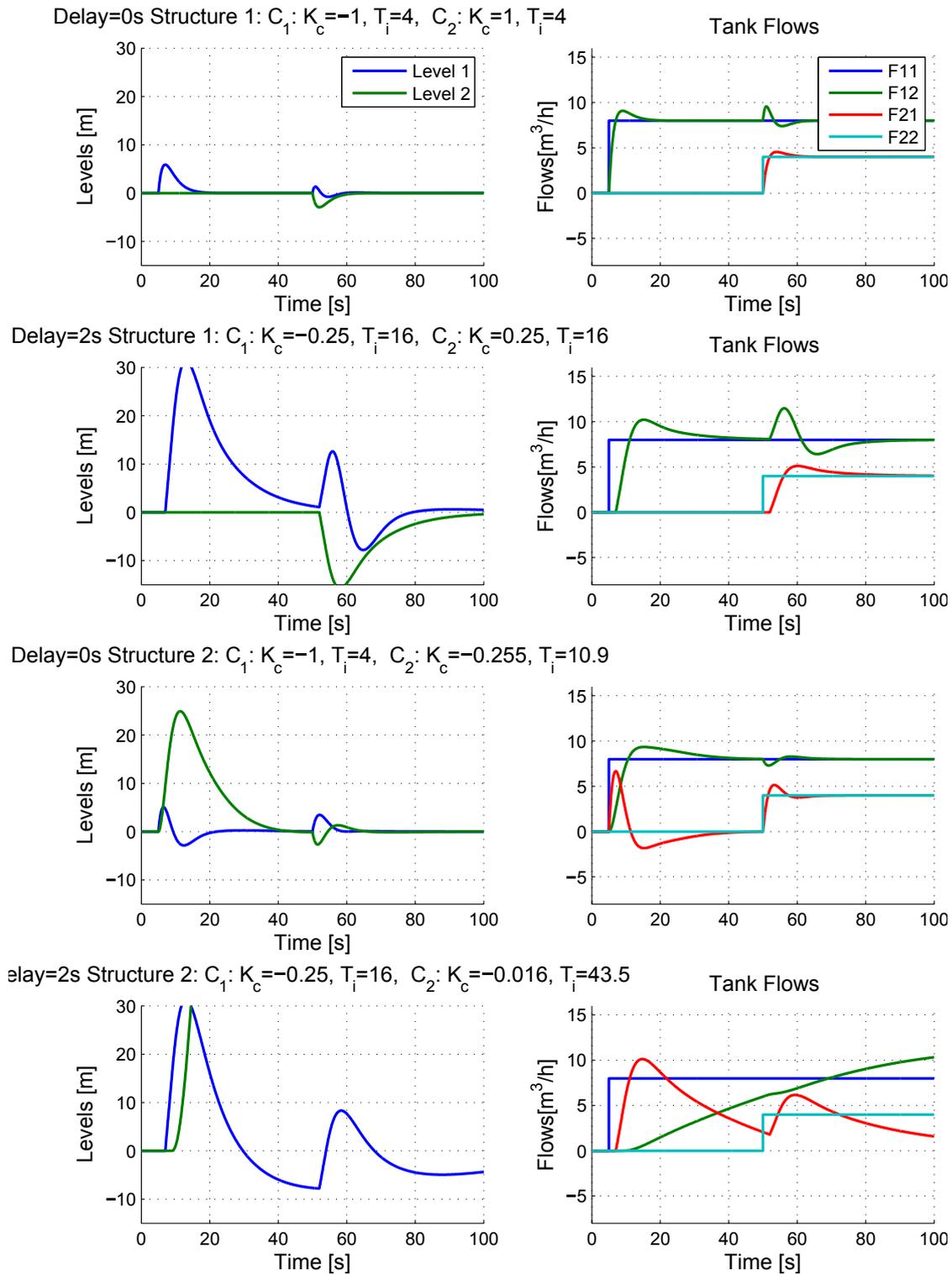


Figure A.1: Effect of delay on level control for structure 1 and 2 for stepchanges in TPM and R-TPM

B Distillation Column A Calculations

B.1 Controller Settings

B.1.1 Off-diagonal controllers

The controller settings for the off-diagonal couplings are for the fast X_b controller coupled with L ($\tau_{cX_b} = \theta_{21} = 3$)

$$K_{cX_b} = \frac{1}{K_{21}} \frac{\tau_{21}}{\tau_{cX_b} + \theta_{21}} = \frac{1}{1.082} \frac{50}{2 * 3} = 7.7 \quad (\text{B.1})$$

$$\tau_{IX_b} = \min(\tau_{21}, 4(\tau_{cX_b} + \theta_{21})) = 8\theta_{21} = 24 \quad (\text{B.2})$$

The settings for the Y_d -controller are based on the model in Equation B.9. For stable control the time constant was set to $\tau_{cY_d} = 140$

$$K_{cY_d} = \frac{1}{K_{\hat{K}_{12}}} \frac{\hat{\tau}_{12}}{\tau_{cY_d} + \hat{\theta}_{12}} = 5.6 \quad (\text{B.3})$$

$$\tau_{IY_d} = \min(\hat{\tau}_{12}, 4(\tau_{cX_b} + \hat{\theta}_{12})) = \hat{\theta}_{12} = 40 \quad (\text{B.4})$$

B.1.2 Diagonal controllers

The controller settings for the diagonal couplings are:

For the fast X_b controller coupled with V ($\tau_{cX_b} = 0.5$)

$$K_{cX_b} = \frac{1}{K_{22}} \frac{\tau_{22}}{\tau_{cX_b} + \theta_{22}} = \frac{1}{-1.096} \frac{50}{2 * 0.5} = -91.2 \quad (\text{B.5})$$

$$\tau_{IX_b} = \min(\tau_{22}, 4(\tau_{cX_b} + \theta_{22})) = 8\theta_{22} = 2 \quad (\text{B.6})$$

Y_d controller coupled with L ($\tau_{cY_d} = 2.5$)

$$K_{cY_d} = \frac{1}{K_{11}} \frac{\tau_{11}}{\tau_{cX_b} + \theta_{11}} = \frac{1}{0.878} \frac{50}{2 * 2.5} = 1.1 \quad (\text{B.7})$$

$$\tau_{IY_d} = \min(\tau_{11}, 4(\tau_{cY_d} + \theta_{11})) = 4\tau_{cX_b} = 10 \quad (\text{B.8})$$

B.2 Step response for Off-Diagonal Tuning

Figure B.1 shows the response for Y_d with a step response in V , this was used to estimate the FOPTD transfer function for $\hat{G}_{12} = Y_d/V$. The code for this simulation is available in Appendix C.3.2.

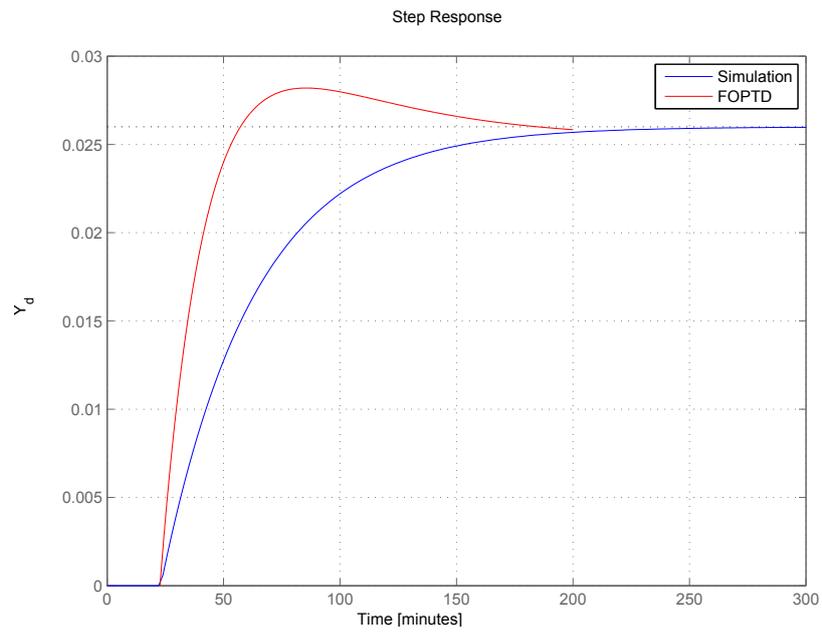


Figure B.1: Y_d response for step in V for Model Column A

$$\hat{G}_{12} = 0.026 \frac{e^{-23s}}{40s + 1} \quad (\text{B.9})$$

C Simulink Models and MATLAB code

C.1 Two Way Interactive Plant

C.1.1 Simulink Model

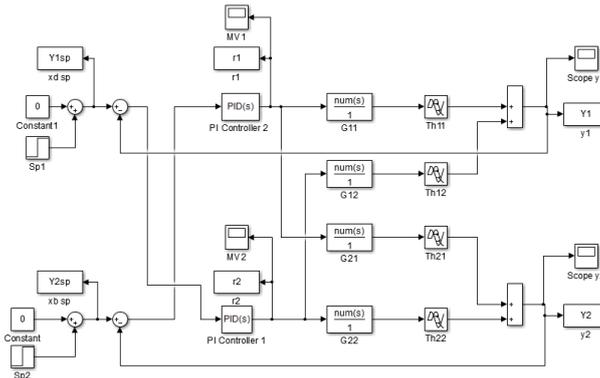


Figure C.1: Simulink model for the off diagonal controller of the coupled plant

C.1.2 MATLAB code

MATLAB code for simulating step responses for both diagonal and off diagonal controller couplings to generate Figure 3.

```

1 clear all
2 close all
3 clc
4 %Script for 2*2 Model execution Example 10.16 From Multivariable control
5 %Negative RGA diagonal pairing
6 %The process model:
7 %Generic transfer function
8 g12=0.17;      %%%Model parameter%%%
9 %The process model
10 K11=1;
11 K12=g12;
12 K21=5;
13 K22=1;
14
15 %Delays
16 Th11=0.5;
17 Th12=0.5;
18 Th21=0.5;

```

```
19 Th22=0.5;
20 %The RGA
21 K=[K11 K12;K21 K22];
22 RGA=K.*(pinv(K) .');
23 %Diagonal
24 %Simulation for positive Diagonal pairings
25 %Controller 1
26 Ti1=5;
27 %Controller 2
28 Ti2=1;
29 %Step 1
30 t_step1=0;
31 K_step1=1; %Step in y1
32 %Step 2
33 t_step2=20;
34 K_step2=1; %Step in y1
35 %Simulation
36 tn='Model2by2_DiagCtrl';
37 t_model=40;
38 sim(tn)
39 fig=figure;
40 subplot(211)
41 Y1ht=0:Y1.time((length(Y1.time)));
42 Y1h=ones(size(Y1ht));
43 Y2h=zeros(size(Y1ht));
44 %Plotting Results
45 hold on
46 plot(Y1.time,Y1.signals.values)
47 plot(Y2.time,Y2.signals.values,'r')
48 legend('Y_1','Y_2')
49 axis([0 t_model -1.5 2])
50 xlabel('Time [s]','fontsize',14)
51 ylabel('y','fontsize',14)
52 title('Diagonal Controller','fontsize',14)
53 plot(Y1ht,Y1h,'--k')
54 plot(Y1ht,Y2h,'--k')
55 grid
56 hold off
57 %Off Diagonal
58 %Simulation for Negative non-diagonal pairings
59 %Controller 1
60 Ti1=-1;
61 %Controller 2
62 Ti2=5;
63 %Simulation
64 tn='Model2by2_NonDiagCtrl';
65 sim(tn)
```

```

66 subplot(212)
67 Y1ht=0:Y1.time((length(Y1.time)));
68 Y1h=ones(size(Y1ht));
69 Y2h=zeros(size(Y1ht));
70 %Plotting Results
71 hold on
72 plot(Y1.time,Y1.signals.values)
73 plot(Y2.time,Y2.signals.values,'r')
74 axis([0 t_model -1.5 2])
75 xlabel('Time [s]','fontsize',14)
76 ylabel('y','fontsize',14)
77 title('Off Diagonal Controller','fontsize',14)
78 plot(Y1ht,Y1h,'--k')
79 plot(Y1ht,Y2h,'--k')
80 grid
81 hold off
82 thicklines
83 print(fig,'-dpdf','MFC1016step.eps')

```

C.2 Coupled Tanks

C.2.1 Simulink Model

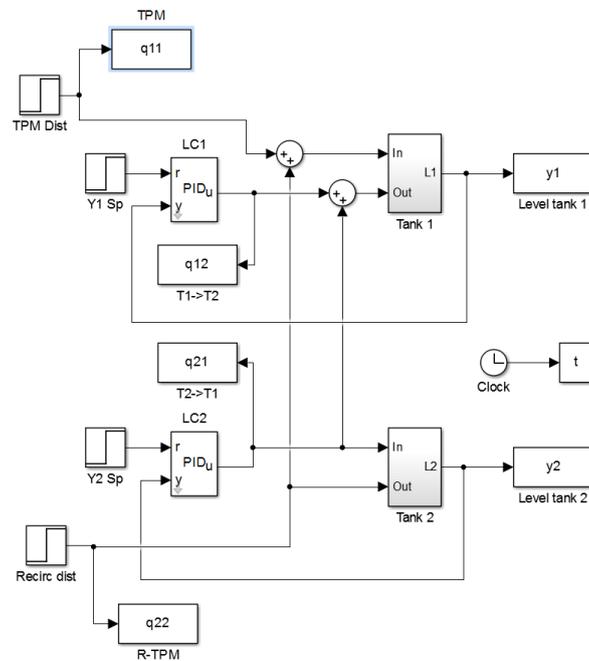


Figure C.2: Example Simulink model of control structure 1 for the coupled tanks simulation

C.2.2 MATLAB code

MATLAB code for generating the delay effect plot in Figure A.1. The code for evaluating all structures used for Figure 6 follows the same structure.

```

1 % Script: Simulation of recirculation tank system
2 % Model file: pairing01MJB.mdl, pairing02MJB.mdl, pairing03MJB.mdl, pairing04MJB.mdl
3 % Calls: scenariodefMJB.m, plotsimMJB.m
4 %
5 % =====
6 % (c) Krister Forsman, Martin Bland
7 % =====
8 % Created 2014-09-11; Latest modifications. 2014-09-23
9 clear all
10 close all
11 % Define scenario
12 sd=scenariodefMJB;
13
14 %Main figure
15 %# centimeters units
16 X = 21;           %# A4 paper size
17 Y = 29.7;        %# A4 paper size
18 xMargin = 0.5;   %# left/right margins from page borders
19 yMargin = 0.5;   %# bottom/top margins from page borders
20 xSize = X - 2*xMargin; %# figure size on paper (width & height)
21 ySize = Y - 2*yMargin; %# figure size on paper (width & height)
22
23 fig = figure('MenuBar','none');
24 set(fig, 'PaperUnits','centimeters')
25 set(fig, 'PaperSize',[X Y])
26 set(fig, 'PaperPosition',[xMargin yMargin xSize ySize])
27
28 %Subplot 1
29 %Control Structures 1 \theta=0
30 tn='pairing01MJB';
31 % Control parameters [Kc, Ti, TD, a]
32 % [Kc Ti TD a]
33 %Delay
34 td=0.0;
35 c1=[-1 4 0 1];
36 c2=[ 1 4 0 1];
37 %Simulation
38 sim(tn)
39 %Levels
40 h1=subplot(421);
41 hold on

```

```

42 plot(t,[y1 y2])
43 axis([0 sd.Tdur -15 30])
44 grid
45 str=sprintf(['Delay=0s Structure 1: C_1: K_c=%0.3g, T_i=%0.3g, C_2: '...
46         'K_c=%0.3g, T_i=%0.3g'],c1(1:2),c2(1:2));
47 title(str,'interpreter','tex','fontsize',12)
48 legend({'Level 1','Level 2'},'location','northeast');
49 xlabel('Time [s]','fontsize',12);
50 ylabel('Levels [m]','fontsize',12);
51 hold off
52 %Flows
53 h2=subplot(422);
54 hold on
55 plot(t,[q11 q12 q21 q22])
56 axis([0 sd.Tdur -8 16])
57 grid
58 str=sprintf('Tank Flows');
59 title(str,'interpreter','tex','fontsize',12)
60 legend({'F11','F12','F21','F22'},'location','northeast');
61 xlabel('Time [s]','fontsize',12);
62 ylabel('Flows[m^3/h]','fontsize',12);
63 thicklines
64 %Subplot 2
65 %Control Structure 1 \theta=2
66 tn='pairing01MJB';
67 % Control parameters [Kc, Ti, TD, a]
68 % [Kc Ti TD a]
69 td=2;
70 c1=[-0.25 16 0 1];
71 c2=[ 0.25 16 0 1];
72 %Simulation
73 sim(tn)
74 %Levels
75 h1=subplot(423);
76 hold on
77 plot(t,[y1 y2])
78 axis([0 sd.Tdur -15 30])
79 grid
80 str=sprintf(['Delay=2s Structure 1: C_1: K_c=%0.3g, T_i=%0.3g, C_2: '...
81         'K_c=%0.3g, T_i=%0.3g'],c1(1:2),c2(1:2));
82 title(str,'interpreter','tex','fontsize',12)
83 xlabel('Time [s]','fontsize',12);
84 ylabel('Levels [m]','fontsize',12);
85 hold off
86 %Flows
87 h2=subplot(424);
88 hold on

```

```
89     plot(t,[q11 q12 q21 q22])
90     axis([0 sd.Tdur -8 16])
91     grid
92     str=sprintf('Tank Flows');
93     title(str,'interpreter','tex','fontsize',12)
94     xlabel('Time [s]','fontsize',12);
95     ylabel('Flows[m^3/h]','fontsize',12);
96     thicklines
97     hold off
98 %Subplot 3
99 %Control Structure 2 \theta=0
100 tn='pairing02MJB';
101     % Control parameters [Kc, Ti, TD, a]
102     %   [Kc Ti TD a]
103     td=0;
104     c1=[-1 4 0 1];
105     c2=[-0.255 10.9 0 1];
106 %Simulation
107     sim(tn)
108 %Levels
109     h1=subplot(425);
110     hold on
111     plot(t,[y1 y2])
112     axis([0 sd.Tdur -15 30])
113     grid
114     str=sprintf(['Delay=0s Structure 2: C_1: K_c=%0.3g, T_i=%0.3g, C_2: '...
115         'K_c=%0.3g, T_i=%0.3g'],c1(1:2),c2(1:2));
116     title(str,'interpreter','tex','fontsize',12)
117     xlabel('Time [s]','fontsize',12);
118     ylabel('Levels [m]','fontsize',12);
119     hold off
120 %Flows
121     h2=subplot(426);
122     hold on
123     plot(t,[q11 q12 q21 q22])
124     axis([0 sd.Tdur -8 16])
125     grid
126 %Subplot 4
127 %Control Structure 2 \theta=2
128 tn='pairing02MJB';
129     % Control parameters [Kc, Ti, TD, a]
130     %   [Kc Ti TD a]
131     td=2;
132     c1=[-0.25 16 0 1];
133     c2=[-0.016 43.52 0 1];
134 %Simulation
135     sim(tn)
```

```

136 %Levels
137 h1=subplot(427);
138 hold on
139 plot(t,[y1 y2])
140 axis([0 sd.Tdur -15 30])
141 grid
142 str=sprintf(['Delay=2s Structure 2: C_1: K_c=%0.3g, T_i=%0.3g, C_2: '...
143           'K_c=%0.3g, T_i=%0.3g'],c1(1:2),c2(1:2));
144 title(str,'interpreter','tex','fontsize',12)
145 xlabel('Time [s]','fontsize',12);
146 ylabel('Levels [m]','fontsize',12);
147 hold off
148 %Flows
149 h2=subplot(428);
150 hold on
151 plot(t,[q11 q12 q21 q22])
152 axis([0 sd.Tdur -8 16])
153 grid
154 str=sprintf('Tank Flows');
155 title(str,'interpreter','tex','fontsize',12)
156 xlabel('Time [s]','fontsize',12);
157 ylabel('Flows[m^3/h]','fontsize',12);
158 thicklines
159 print(fig,'-dpdf','All_structures.eps')

```

C.3 Distillation Column A

C.3.1 Simulink Models

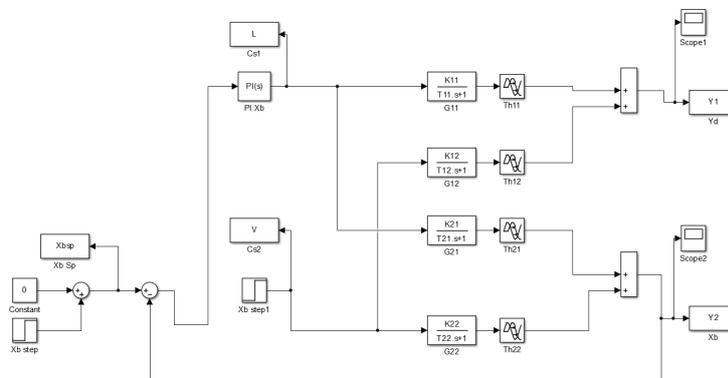


Figure C.3: Simulink model for generating FOPTD model for Y_d for the off diagonal Distillation control

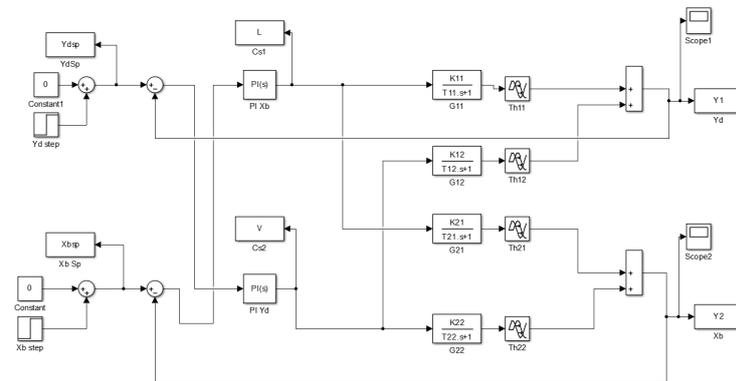


Figure C.4: Simulink model for simulating step responses for off diagonal pairings for Distillation Column A. The diagonal pairing was simulated by changing the feedback structure

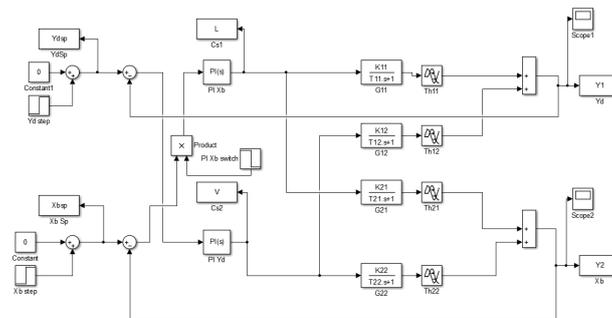


Figure C.5: Simulink model for simulating step responses for off diagonal pairings for Distillation Column A with X_b in manual. The diagonal pairing was simulated by changing the feedback structure

C.3.2 MATLAB Code

Script for obtaining step response for Y_d for step in V :

```

1 %Simulation settings
2 t_model=200;
3 t_stepV=0;
4 K_stepV=1;
5 t_stepXb=801;
6 K_stepXb=0.0;
7
8 %Model settings
9 %L->Xd Liquid downcomer with Xd
10 K11=0.878;
11 T11=50;

```

```

12 Th11=0;
13 %V->Xd vapour boilup with Xd
14 K12=-0.864;
15 T12=50;
16 Th12=0;
17 %L->Xb
18 K21=1.082;
19 T21=50;
20 Th21=3;
21 %V->Xb
22 K22=-1.096;
23 T22=50;
24 Th22=0;
25 %Controller X_B Coupling on G21 (SIMC-rules)
26 TcXb=Th21; %Tc=Th %%%Tuning knob%%%
27 KcXb=(1/K21)*(T21/(TcXb+Th21));
28 TiXb=min(T12,4*(TcXb+Th21));
29 tn='Model2by2_NonDiagCtrl_StepVXd';
30 sim(tn)
31 close all
32 hold on
33 plot(Y1.time,Y1.signals.values)
34 grid
35 s=tf('s');
36 Gh=0.026*exp(-23*s)/((40*s+1));
37 step(Gh)
38 %plot(V.time,V.signals.values)

```

Script for modeling step responses for the Distillation Column A:

```

1 %TKP 4550 Project
2 %Author: Martin Jonathan Bland, December 2014
3 %
4 %Disillation Model execution of both negative and positive RGA-pairings
5 %With the controller for Xb turned off
6 %Simulation settings
7 t_model=1000;
8 t_stepXb=5;
9 K_stepXb=0.01;
10 t_stepYd=500;
11 K_stepYd=0.01;
12 %L->Xd Liquid downcomer
13 K11=0.878;
14 T11=50;
15 Th11=0;
16 %V->Xd vapour boilup with Xd
17 K12=-0.864;

```

```

18 T12=50;
19 Th12=0;
20 %L->Xb
21 K21=1.082;
22 T21=50;
23 Th21=3;
24 %V->Xb
25 K22=-1.096;
26 T22=50;
27 Th22=0;
28 %----Off Diagonal Control----%
29 %Controller 1 L->XB Coupling on G21 (SIMC-rules)
30 Tc1=Th21; %Tc=Th %%%%Tuning knob%%%%
31 KcXb=(1/K21)*(T21/(Tc1+Th21));
32
33 TiXb=min(T12,4*(Tc1+Th21));
34 %Controller 2 V->XD Coupling on G12
35 % ^G(s)=Gh=0.026*exp(-23*s)/((40*s+1));;
36 K12m=0.026;
37 T12m=40;
38 Th12m=23;
39 Tc2=250; %%%% Tuning knob %%%%
40 KcYd=(1/K12m)*(T12m/(Tc2+Th12m));
41 TiYd=min(T12m,4*(Tc2+Th12m));
42 tn='Model2by2_NonDiagCtrl';
43 sim(tn)
44 fig=figure;
45 subplot(211)
46 hold on
47 plot(Y1.time,Y1.signals.values,'r')
48 plot(Y2.time,Y2.signals.values)
49 plot(Ydsp.time,Ydsp.signals.values,'--r')
50 plot(Xbsp.time,Xbsp.signals.values,'--b')
51 axis([0 t_model -0.005 0.025 ])
52 thicklines
53 legend('Y_d','X_b')
54 xlabel('Time [min]','fontsize',14)
55 ylabel('Composition X_d Y_d','fontsize',14)
56 title('Off Diagonal Composition Control','FontSize',16)
57 grid
58 %----Diagonal Control----%
59 %Controller 1 V->Xb Coupling on G22 (SIMC-rules)
60 Tc1=0.5; %Tc=Th %%%%Tuning knob%%%%
61 KcXb=(1/K22)*(T22/(Tc1+Th22));
62 TiXb=min(T22,4*(Tc1+Th22));
63 %Controller 2 V->Yd Coupling on G11 (SIMC-rules)
64 Tc2=2.5;

```

```

65 KcYd=(1/K11)*(T11/(T11+Th11));
66 TiYd=min(T11,4*(Tc2+Th11));
67 tn='Model2by2_DiagCtrl';
68 sim(tn)
69 subplot(212)
70 hold on
71 plot(Y1.time,Y1.signals.values,'r')
72 plot(Y2.time,Y2.signals.values)
73 plot(Ydsp.time,Ydsp.signals.values,'--r')
74 plot(Xbsp.time,Xbsp.signals.values,'--b')
75 axis([0 t_model -0.005 0.025 ])
76 thicklines
77 xlabel('Time [min]','fontsize',14)
78 ylabel('Composition X_d Y_d','fontsize',14)
79 title('Diagonal Composition Control','FontSize',16)
80 grid
81 print(fig,'-dpdf','SimpleDiststep.eps')

```

Code for simulation of column a with X_b controller in manual:

```

1 %TKP 4550 Project
2 %Author: Martin Jonathan Bland, December 2014
3 %
4 %Disillation Model execution of both negative and positive RGA-pairings
5 %With the controller for Xb turned off
6 t_switch=0;
7 t_model=1000;
8 t_stepXb=5;
9 K_stepXb=0.01;
10 t_stepYd=500;
11 K_stepYd=0.01;
12 %Process Model
13 %L->Xd
14 K11=0.878;
15 T11=50;
16 Th11=0;
17 %V->Xd
18 K12=-0.864;
19 T12=50;
20 Th12=0;
21 %L->Xb
22 K21=1.082;
23 T21=50;
24 Th21=3;
25 %V->Xb
26 K22=-1.096;
27 T22=50;

```

```

28 Th22=0;
29 %----Off Diagonal Control----%
30 %Controller 1 L->XB Coupling on G21 (SIMC-rules)
31 Tc1=Th21; %Tc=Th %%%Tuning knob%%%
32 KcXb=(1/K21)*(T21/(Tc1+Th21));
33 TiXb=min(T12,4*(Tc1+Th21));
34 %Controller 2 V->XD Coupling on G12
35 % ^G(s)=Gh=0.026*exp(-23*s)/((40*s+1));;
36 K12m=0.026;
37 T12m=40;
38 Th12m=23;
39 Tc2=125; %%% Tuning knob %%%
40 KcYd=(1/K12m)*(T12m/(Tc2+Th12m));
41 TiYd=min(T12m,4*(Tc2+Th12m));
42 tn='Model2by2_NonDiagCtrl_switch';
43 sim(tn)
44 fig=figure;
45 subplot(211)
46 hold on
47 plot(Y1.time,Y1.signals.values,'r')
48 plot(Y2.time,Y2.signals.values)
49 plot(Ydsp.time,Ydsp.signals.values,'--r')
50 plot(Xbsp.time,Xbsp.signals.values,'--b')
51 axis([0 t_model -0.01 0.025 ])
52 thicklines
53 legend('Y_d','X_b')
54 xlabel('Time [min]','fontsize',14)
55 ylabel('Composition X_d Y_d','fontsize',14)
56 title('Off Diagonal Composition Control','FontSize',16)
57 grid
58 %----Diagonal Control----%
59 %Controller 1 V->Xb Coupling on G22 (SIMC-rules)
60 Tc1=0.5; %Tc=Th %%%Tuning knob%%%
61 KcXb=(1/K22)*(T22/(Tc1+Th22));
62 TiXb=min(T22,4*(Tc1+Th22));
63 %Controller 2 V->Yd Coupling on G11 (SIMC-rules)
64 Tc2=2.5;
65 KcYd=(1/K11)*(T11/(T11+Th11));
66 TiYd=min(T11,4*(Tc2+Th11));
67 tn='Model2by2_DiagCtrl_Switch';
68 sim(tn)
69 subplot(212)
70 hold on
71 plot(Y1.time,Y1.signals.values,'r')
72 plot(Y2.time,Y2.signals.values)
73 plot(Ydsp.time,Ydsp.signals.values,'--r')
74 plot(Xbsp.time,Xbsp.signals.values,'--b')

```

```
75 axis([0 t_model -0.01 0.025 ])
76 thicklines
77 xlabel('Time [min]','fontsize',14)
78 ylabel('Composition X_d Y_d','fontsize',14)
79 title('Diagonal Composition Control','FontSize',16)
80 grid
81 print(fig,'-dpdf','SimpleDistSwitch.eps')
```