1 Abstract

This report presents optimization and control structure suggestions for a model of petroleum production wells. The model presented in the report is based the semi-realistic model of the Troll west oil rim form the work of Gunnerud and Foss(2009).

With active capacity constraints *maximization of oil production* defined as the optimization objective, an optimization routine that works well and in compliance with model specifications and optimization objective has been developed.

In the report, disturbance of well-composition was found to affect the set-points for optimal operation. The null space method has been applied in search for a self-optimizing structure for the model. By loss evaluation we have obtained suggestions for a self optimizing structure that would give a loss less than one third of what obtained by keeping the control vales at the at the original position under impact of disturbances. This implies that a further evaluation of self-optimizing control structures for the model could be interesting.

2 Acknowledgement

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3 Introduction

Increasing the oil recovery from existing reservoirs is one of the greatest challenges in oil production today. As the reservoir pressure drops production difficulties arises and according to current estimations presented by the Norwegian petroleum directorate 50% of the oil originally in place in the reservoirs will be left under ground when production is ceased.

To maintain the pressure and oil dispersion in the reservoirs, injection of water and/or gas are widely used methods. As a natural consequence to this, however, the production of water and gas is also increased, which represents a challenge both for the production facilities an for the environment.

The economical profit of increasing the recovery rate depends on the size of the field as well as production costs and future oil prices. On important oilfields where the recovery factor has been increased by 1% the gross value would be as much as 16-10 billion NOK at an oil price of 570 NOK/barrel.

The paper can be divided in three parts. In the first part of this paper we will present a model of petroleum production wells based the work of Gunnerud and Foss(2009). In this part data form Troll has also been processed and presented such that it can be applied in computation programs as MATLAB.

In the second part we define the optimization objective: *maximization of oil production*, and present the result obtained with active capacity constraints.

The third part we investigate the effect of disturbances and propose a self-optimizing control structure for the model, found by applying the null space method and loss evaluation.

4 Model

The production process is dependent on the conditions in the reservoir, wells, production pipeline as well as the separator. We therefore need a model that describes all these relations. The model that will be presented in this report describes:

1. The flow-pressure relations in four oil wells.
2. The pressure in the manifold.
3. The pressure drop in the production pipeline as a function of the flow and flow composition.
4. Capacity constraints in the separator.

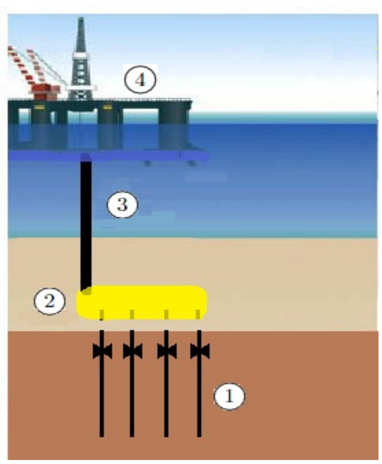

Figure 1: The figure shows the parts of the rig that will be described our model .

Figure 1 shows which parts of the rim that will be relevant to – and described in our model. The semi-realistic model of the Troll west rim is modelled in the numerical computing program MATLAB. As we look at a RTPO problem and expect little change in the process conditions, the process is modelled at steady state.

The model is based model equations from the work by Gunnerud and Foss (2009) as well as two datasets that consist of flow- and pressure measurements from the Troll west rim. The first dataset is used for modelling the flow-pressure relations in the four wells and the second dataset is used for modelling the pressure drop in the production pipeline.

In the following two sections we will elaborate on how the data was processed and applied in our model. The first section will describe relations in the wells and manifold, and the second part will describe relations in the production pipeline and separator. Figure 2 shows the pressure- and flow variables that are relevant to our problem. They are either given, or will be calculated in the model. The variables are also listed in table 1 together with an explanation.

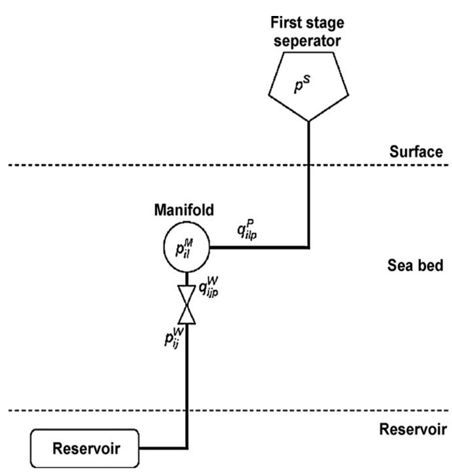

Figure 2: The figure shows the relevant variables that are given or will be calculated in the model.

Table 1: Model Variables

Variables	Definition
p_i^W	Choke pressure in well i
q_{ij}^W	Flow of component j in well i
p^M	Manifold pressure
q_j^P	Flow of component j in production pipe
p^S	Separator pressure

4.1 Wells and Manifold

The streams in the four wells are compositions of oil, gas and water. The flow in each well can be controlled individually by a choke valve as illustrated in figure 1. In our model the valve positions will be described in terms of well pressures P_i^W, which are the only variables in our system that can be manipulated, i.e we have 4 degrees of freedom. The well flow- and composition is decided by the pressure in these valves. The four wells flow through the manifold before entering the common production pipe.

Well Performance Curves From flow- and pressure measurements(Appendix I) well performance curves(WPC's) were generated for the four wells. The available flow data is for pressures between 20 and approximately 100 bar, so the WPC's are therefore only valid in this pressure interval. To obtain a continuous model, linear interpolation was applied between the measured points. The flow-pressure relations in the respective wells shown in Figure 3 is further used to optimize the four well pressures in order to maximize the production of oil.

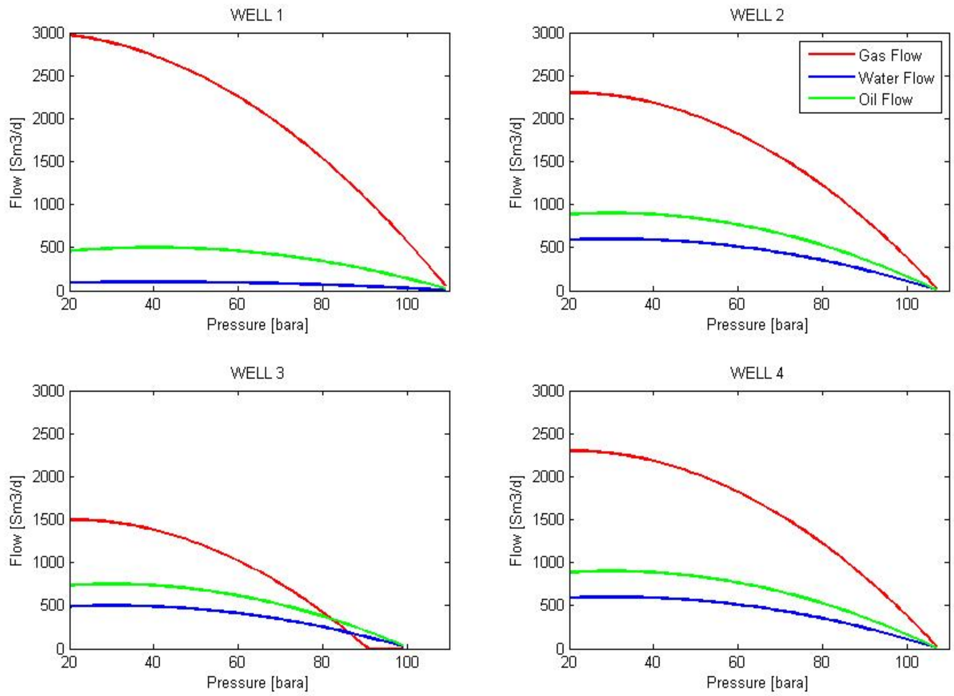

Figure 3: The figure shows the well performance curves for the respective wells.

Manifold Constraints In the manifold we have our first model constraint. To ensure that the optimization will provide well pressures that are physical feasible, we need to implement a pressure constraint in the manifold. None of the well pressures(pressures upstream the choke) can be lower than the pressure in the manifold, thus we have to set the constraint:

$$p^M \leq p^W \tag{1}$$

The manifold pressure is decided by the separator pressure and the pressure drop in the production pipeline. The pressure in the separator is given and is sat to be 20 bar. The pressure drop in the pipeline is calculated from a dataset of measured flow- and pressure drop relations. This method will be explained in the section 3.2.

$$p^M = p^S + \triangle p^P \tag{2}$$

4.2 Production Pipeline and Separator

In the production pipeline the streams from the four wells are mixed. This means that both the total flow and the component flow will be a summation of the flows in well 1 – well 4.

$$q_j^P = \sum_i q_{ij}^W \tag{3}$$

There is also a pressure drop across the pipeline. The pressure drop will vary with the flow and flow composition in the pipeline. 27000 combinations of this non-linear relation have been measured and is provided as a second dataset. The pressure drop in the pipeline will be a limiting factor in the optimization problem because it will decide the manifold pressure and, hence, also our first constraint.

In the separator we find two more model constraints. These are the water and gas capacities which will limit the the maximum total flowrate, thus also the maximum oil flowrate.

Pressure Drop in Production Pipeline As mentioned above, the pressure drop in the pipeline is dependent on the oil, gas and water flowrate. To be able to use the provided data, it is structured in a 4-dimensional matrix with the flowrate values of oil,water and gas at the x-,y- and z-axes respectively.

As the dataset is discrete it is made continuous by linear integration between the measured points. The resulting model will for any given flow and flow composition, which lies within the range of the dataset, provide a pressure drop close to the realistic value.

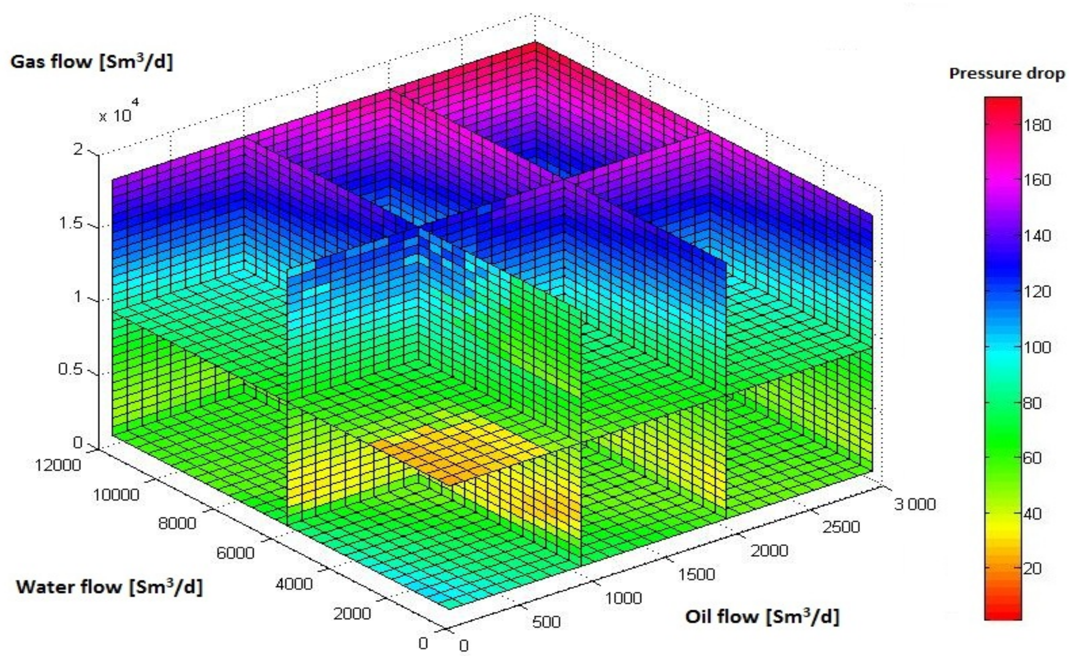

Figure 4: The figure illustrates how oil,water and gas flowrate is related to the pipeline pressure drop.

The colorbar in Figure 4 translates the colour-code into pressure drop values. The pressure drop in our model ranges from approximately 20 bar to 180. From Figure 4 it is obvious that high flows corresponds to high pressure drops. In the model presented in this report we will operate in the low- to mid region, corresponding to a pressure drop between 20 and 60 bar.

Capacity Constraints in the Separator The separator has a maximum capacity of processing of water and gas.

$$q^{P}_{water} \leq C^{Max}_{water} \tag{4}$$

$$q^{P}_{gas} \leq C^{Max}_{gas} \tag{5}$$

As the amount of water and gas in the wells are relatively high, the capacity constraints will also be limiting for the oil production.

5 Optimization

The general procedure for solving an optimization problem is approached by 3 steps:

1. Define the objective
2. Formulate objective function
3. Find variable values which give an optimum of this function

Define Objective When optimizing a system we first need to define the objective. By defining the objective you define what you want to achieve by the optimization, which for a production process is often to maximise profit. When the optimization problem is solved the variable values will be adjusted in order to satisfy this objective.

Define Objective Function When the object is decided the second step is to make the objective function. The objective function is a system of equations that describes relationships and constraints in the optimization problem. These equations contain the variables that is manipulated to optimise the objective function. In simple cases the objective function should be maximized or minimized, which will be the case for our objective function.

Variable Values The third step is to solve the set of equations in order to maximize or minimize the objective function. The variables can only take on values within a certain region termed the feasible region. This region is defined by one or more constraint functions. The methods for solving the function will depend on the model and on the complexity of the system. Our optimization problem will be solved by nonlinear programming, a process that will be described in the following section.

5.1 Nonlinear Programming(NLP)

Nonlinear programming(NLP) is the process of solving constrained nonlinear optimization problems. That an optimization problem is nonlinear means it has nonlinear objective and/or constraint functions, this requires a nonlinear solver.

Given in equation 6 is the general form of a nonlinear programming problem:

$$Min_x f(x) \tag{6}$$

Subject to:

$$h(x) = 0 \qquad (Model\ equations)$$

$$g(x) \leq 0 \qquad (Operational\ Constraints)$$

In equation 6 f(x) is the objective function to be minimized by optimizing the vector of continuous variables x. The inequality constraint functions are given by the vector g(x), and the equality constraints by h(x). In our problem these will represent the operational constraints and model equations respectively. Nonlinear programming is in general the procedure of finding a a local minimum point x^* for a feasible region defined by the constraint functions.

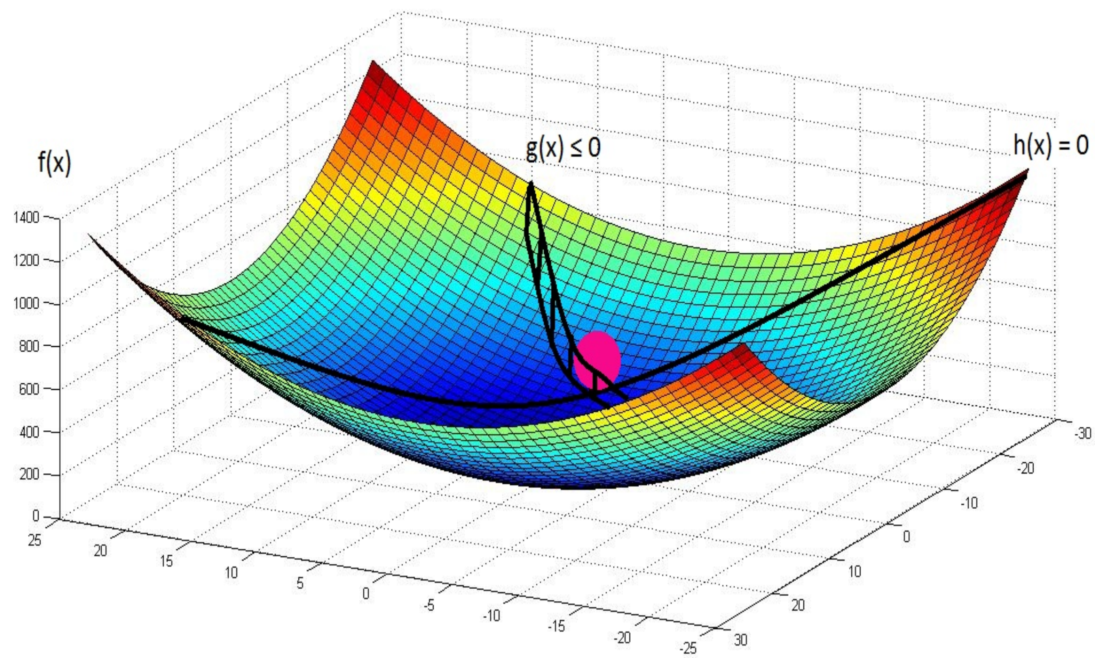

Figure 5: The figure illustrates the procedure of NLP.

Figure 5 illustrates how the procedure of NLP works. The solution $f(x^*)$ has to be a value along the "line" defined by the equality constraints h(x) and at the "right side" of the region defined by the inequality constraint g(x). Satisfying these conditions the the solution should at the same time be in the lowest possible point on the surface defined by f(x).

Active constraints From figure 5 we see that the solution $f(x^*)$ is not in the lowest point of the surface f(x) – it is constrained by $g(x^*)$, thus we say that $g(x^*)$ is an *active* constraint.

Optimality conditions In the reminder of this section we will generalize the concept of constrained minimization and describe the optimality conditions referred to as Kuhn Tucker (KT) conditions. To simplify notation we define the objection function as a Legrange function:

$$L(x, \mu, \lambda) = f(x) + g(x)^T \mu + h(x)^T \lambda = 0 \tag{7}$$

The vectors μ and λ are weights referred to as Kuhn Tucker multipliers.

The solution of the NLP satisfies optimality conditions referred to as the Kuhn Tucker(KT) condition(Biegler et.al(1997)). Satisfying these conditions are necessary for optimality.

1. Linear dependence of gradients

$$\nabla L(x^*, \mu^*, \lambda^*) = \nabla f(x^*) + \nabla g(x^*)^T \mu^* + \nabla h(x^*)^T \lambda^* = 0 \tag{8}$$

   Satisfying the first condition ensures that the objective function is minimized within the linear and nonlinear constraints. In terms of the illustration this means that the ball is at the lowest possible point – balanced by gravitational and normal forces.

2. Feasibility of NLP solution

$$g(x^*) \leq 0 \ , \ \ h(x^*) = 0 \tag{9}$$

The second condition ensures that the linear and nonlinear constraints are satisfied.

3. Complementarity condition

$$\mu^{*T} g(x^*) = 0 \tag{10}$$

The complimentary condition states that either the inequality constraint is inactive $(g(x^*) < 0)$ and the corresponding multiplier(μ) is zero, or, the constraint is active($g(x^*) = 0$) and the multiplier can be positive.

4. Nonnegativity of inequality constraint multipliers

$$\mu^* \geq 0 \tag{11}$$

The fourth condition ensures that the problem solution is on the right side of the inequality constraint.

5.2 Our Optimization Problem

We have a constrained nonlinear function which we want to solve such that the amount of produced oil is maximized. Mathematically the problem can be described as in equation 5.2.

$$Min_x(-q_o^P(x)) \tag{12}$$

Subject to:

Model equations:

$$Q_i^W = f(P_i^W, WPC_i) \qquad i \in \{1,2,3,4\}$$

$$Q^P = \sum_i Q_i^W \qquad i \in \{1,2,3,4\}$$

$$dP = f(Q_j^P) \qquad j \in \{water, gas, oil\}$$

$$P^M = P^S + dP \qquad j \in \{water, gas, oil\}$$

Operational Constraints

$$P^M \leq P_i^W \qquad i \in \{1,2,3,4\}$$

$$P_j^P \leq C_j^{Max} \qquad j \in \{water, gas\}$$

For solving our optimization problem we have used the MATLAB-solver*fmincon.* fmincon attempts to find a constrained minimum of the objective function by using a method referred to as nonlinear programming.

5.3 Optimization Results

It is economical to set the capacity limit as close to the real production as possible. The capacity constrain for water in the separator is set to 1800 [Sm^3/day] which results in the production flow presented in table 2. The gas capacity is adjusted close to the production flow of gas so that both constrains can be considered as active.

Table 2: Capacity constraints and optimized flow values

	Capacity Constraints	Production Flow
Gas	3800	3792
Water	1800	1800
Oil	–	1189

With constraints as given in table 2 the optimized oil production is 1189 [Sm^3/day]. This oil production can under the given conditions be attained by setting the well pressures as listed in table 11.

Table 3: Optimized well pressures

	Well 1	Well 2	Well 3	Well 4
Well pressure	109.00	76.72	58.47	76.53

5.4 Optimization Discussion

By comparing the optimal well pressures to the well performance curves in figure 3 we see that a pressure of 109 bar in well 1 corresponds to a fully closed valve. Figure 3 also shows that well 1 produces the largest gas flow and at the same time has a relatively low oil composition. From these observations we can conclude that a closed valve in well 1 will agree with the objective of our optimization problem.

Further, we see that the optimal well pressure in well 3 is 58.47 which corresponds to a valve position that is fully open. This result agrees with the well performance curve in Figure 3 that shows that well 3 has lowest flow of gas, and thus the highest composition of oil. Well 2 and well 4 have approximately the same compositions and behaviour, and will therefore operate at more or less the same optimal well pressure.

We can from these results conclude that the optimization routine works well and in compliance with model specifications and optimization objective. However, as mentioned in the previous part, the optimal well pressures correspond to two valve positions that have been driven into full saturation. This gives us two active constraints, and leave us with only two remaining degrees of freedom.

From a control perspective it is desirable to have as many degrees of freedom as possible, so for the following sections it should be kept in mind to try to stay in a region where maximum one of the valve positions have to stay fully opened/closed.

6 System Disturbance

In a realistic case the well performance will decrease during production. The wells will after some time of production produce a higher water and gas composition than what is presented in the well performance curves which is the pressure-flow relations at production start. A change in the process conditions implies that the values for optimal well pressures also will change as illustrated in figure 6.

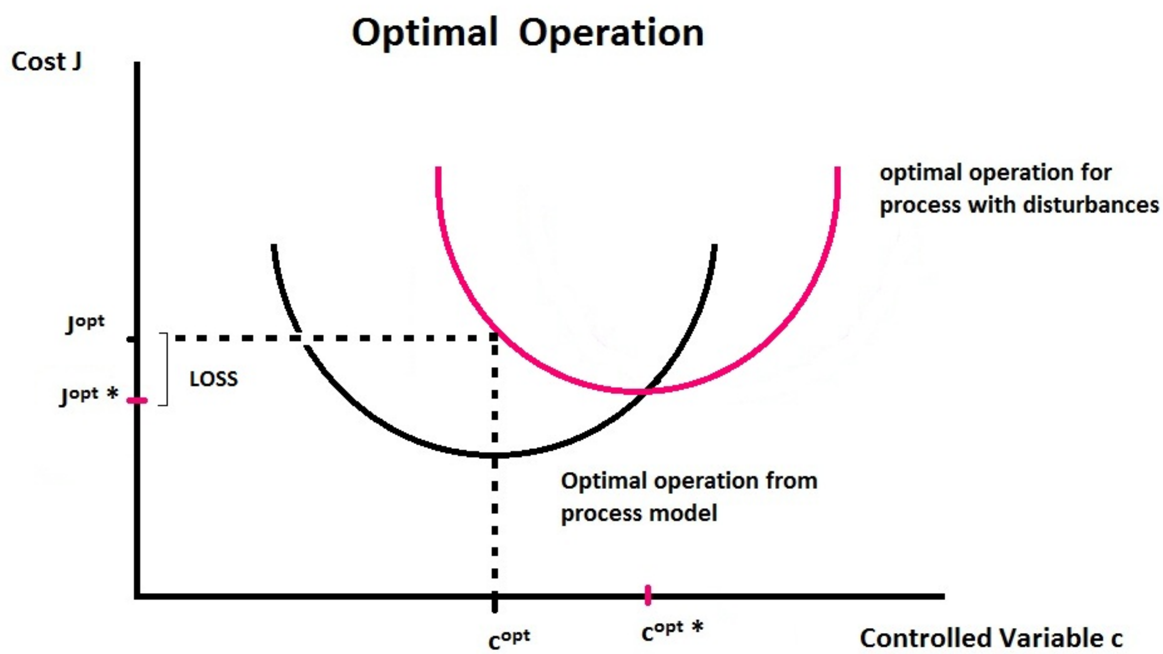

Figure 6: The figure illustrates the how process disturbence effects optimal operating conditions.

Figure 6 illustrates how the optimal operational conditions shift under impact of disturbances. Keeping the controlled variables at the their "old" optimal values will make the process run suboptimal and therefore lead to a loss. To keep the process at optimal operation, a frequent re-optimizing of the system is needed. This is costly, time consuming, and requires a very good model for process and disturbances.

To avoid re-optimizing it would also be beneficial to control variables where the optimal point changes minimally due to the effect of disturbances. Variables with this property are termed *self optimizing variables.* The concept of self-optimizing variables will be elaborated on it the next section.

6.1 Self-optimizing Control

The idea of self-optimizing control is, as mentioned in the previous section, to control certain variables that makes it possible to circumvent the whole process of continuous re-optimization. We want to control variables which, when held at a constant set-point, can keep the process close to the optimum and this way keep the loss at an acceptable level.

The procedure of finding self-optimizing variables requires a process model and a cost function to be minimized(Skogestad, 1999). A self-optimizing variable should satisfy the following requirements:

1. Its optimal value is insensitive to disturbances i.e $\frac{\triangle c_{opt}}{\triangle d} = small$
2. It should be easy to measure and control
3. Its value should be sensitive to input changes i.e $\frac{\triangle c}{\triangle u} = large$

There is a handful methods for identifying self optimizing structure. Examples are: *"Brute force"*, *"Maximum gain rule"* *"Null space method"* and *"Exact local method"*.

"Brute force" and *"Maximum gain rule"* are easy and intuitive, however tedious methods. Here, one CV-candidate is held constant at the time, while the loss and gain is calculated under the impact of the different disturbances. The methods also requires that all CV-candidates are provided. As the self-optimizing variable could be a ratio or a function as well as a single measurement, this means that the result will depend strongly on the process/control insight of the person applying the method.

With *"Null space method"* and *"Exact local method"* we find self-optimizing variables as an optimal combination of measurements. The difference between the methods is that Exact local method takes account for both disturbance and measurement noise where Null space method only accounts for disturbance. Also, the Null space method needs to satisfy: $n_y \geq n_u + n_d$. This is not the case for the exact local method.

As we can satisfy the criteria: $n_y \geq n_u + n_d$ and only look at disturbance in our problem,the Null space method is reasonable choice for evaluation self-optimizing variables.

6.2 Null Space Method

The null space method is a method for finding candidate control variables(c) for self-optimizing control. The variables may not only be single measurements, but can be a function or a ratio of measurements. The idea is to find a combination of measurements(y) that, at a constant set point, will keep the process at its optimum even under impact of disturbances.

$$\triangle c = h_1 \triangle y_1 + h_1 \triangle y_1 + ... = H \triangle y \tag{13}$$

Equation B shows that the control variable is a linear combination of the measurement vector y and H. The matrix H is a selection matrix where each row sums to 1.

As we want to look at the measurement's sensitivity to disturbance(d), we define the optimal sensitivity matrix:

$$F = \frac{\delta y^{opt}}{\delta d^T} \tag{14}$$

The H matrix that gives us an optimal combination of the measurements lies in the null space of F such that:

$$HF = 0 \tag{15}$$

By choosing this value for H, and fixing the value of c to its nominal optimal value, it will give a zero loss for disturbances.

For the null space method the following assumptions are made:

A1: Only steady state operation is considered.

A2: Only disturbances with effect on steady state operation are included.

A3: We assume to have the same active constraints for all values of disturbances.

A4: Implementation error is disregarded.

6.3 Disturbance evaluation

To see if implementation of a control structure could be relevant to our process, it is interesting to look at how disturbances affect our process. In this section follows an evaluation on how much the variation in well compositions can effect our optimal set points. To simulate these conditions a -10% disturbances is implemented in oil flow of all four wells respectively. The notation together with an explanation of the disturbance implementations are given in table 4.

Table 4: Disturbance nature and size

	Disturbance nature	Disturbance size
d1	oil flow in well 1	-10%
d2	oil flow in well 2	-10%
d3	oil flow in well 3	-10%
d4	oil flow in well 4	-10%

To see the individual effect of each disturbance, the oil flow will be decreased in one well at the time. The new optimal well pressures, manifold pressure and product flow can then be compared with their nominal values. The comparison is made in table 5.

Table 5: Optimal values for disturbed process

	Nominal value	d1	d2	d3	d4
p_1^{W*}	109.00	109.00	109.00	109.00	109.00
p_2^{W*}	76.72	76.31	85.84	62.00	62.00
p_3^{W*}	58.47	59.47	62.04	85.85	62.00
p_4^{W*}	73.53	73.31	62.00	62.01	85.86
p^{M*}	58.47	58.46	58.41	58.44	58.41
q_g^{P*}	3792	3789	3800	3800	3800
q_w^{P*}	1800	1800	1799	1799	1799
q_o^{P*}	1189	1188	1159	1169	1160
$ActiveConstraints$	q_w^P	q_w^P	q_g^P	q_g^P	q_g^P

From table 5 we see that a disturbance in well 1 has a negligible effect on the process and on the optimal values of the well pressures. This is expected as the optimal valve position in well 1 is fully closed. Disturbances in the other well compositions gives, on the other hand, significant changes in the optimal well pressures. This means that the process will operate suboptimal if the valve positions are not adjusted, and implies that it could be useful to look at a control strategy for the valves.

7 Loss Evaluations and Results

7.1 Applying Null Space Method to our Problem

Form table 5 we note that the active constraint region change for some of the implemented disturbances. To apply the null space method , and to be able to compare the results we need to remain in the same active constraint region. Before proceeding with the analysis for the self optimizing variables we therefore need to define conditions where we will keep within the same active constraint region, also as disturbances are implemented.

The two capacity constraints as equality constraints in stead of inequality constraints as for the previous optimization. In other words we will control the two constraints to be active:

$$q^P_{water} = C^{Max}_{water} \tag{16}$$

$$q^P_{gas} = C^{Max}_{gas} \tag{17}$$

By controlling the two constraints we ensure that these always are active. However, disturbance implementation can also cause other constraints to become active, which will lead to an invalid solution(Assumption 3). To keep away from the other active constraint regions we need to define a new set of capacity limits. The new capacity constraints are given in table 6

Table 6: New capacity constraints in the separator

	Separator Capacity
Gas flow	3300
Water flow	1600

With new capacity constraints and also a some smaller disturbance value, we repeat the disturbance evaluation from section 6.3. The size and nature of the disturbances implemented to the system is presented in table 7.

Table 7: Disturbance nature and size

	Disturbance nature	Disturbance size
d1	oil flow in well 1	-7%
d2	oil flow in well 2	-7%
d3	oil flow in well 3	-7%
d4	oil flow in well 4	-7%

The details in the null space method calculations are given in Appendix II. From these calculations we obtain four optimal CV's:

$$c_1 = [\ 0.0031 \quad -0.5773 \quad -0.5773 \quad -0.5773 \quad 0.0102 \quad 0 \quad -0.00003 \quad 0]$$

$$c_2 = [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$$

$$c_3 = [\ 0.0031 \quad 0.9999 \quad 0.0018 \quad 0.0018 \quad 0.0018 \quad 0 \quad -0.00003 \quad 0]$$

$$c_4 = [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$$

The combination vectors c1 and c3 suggest to different a combination of the measurements as self-optimizing variables, while c2 and c4 suggest to keep q_g^{P*} and q_o^{P*} constant respectively.

Please note that the vector of c1 does not sum to one. This will be discussed in the end of the report.

7.2 Evaluation by Loss Function

As we have two remaining degrees of freedom, we can use two of the measurement combinations found above to keep the process as close to optimum as possible. To evaluate which combination that gives the best result we look at the total loss for each combination. The loss is calculated for each disturbance implementation by equation 18.

$$L = J(c, d) - J(c^{opt}(d), d) \tag{18}$$

We first find the optimal values (given in $[Sm^3/day]$) of the cost function for the nominal point and for implemented disturbances.

Table 8: Optimal values for nominal and disturbed process

	nom	d1	d2	d3	d4
J_{opt}	-1055.5	-1055.2	-1037.3	-1032.3	-1037.3

The same cost calculation is done for each variable combination and presented in table 9. We see that the combination of c_2,c_4 gives infeasible solutions for disturbance in well 3 and well 4, and is therefore disregarded in the loss evaluation.

Table 9: Optimal oil flow for all combinations of c

	J_{d1}	J_{d2}	J_{d3}	J_{d4}
$c1, c2$	-1055.1	-1037.2	-1032.3	-1033.5
$c1, c3$	-1053.6	-1024.5	-1030.2	-1033.5
$c1, c4$	-1055.1	-1037.2	-1032.1	-1033.5
$c2, c3$	-1055.1	-1036.9	-1030.7	-1025.0
$c2, c4$	-1055.2	-1037.3	–	–
$c3, c4$	-1055.1	-1036.9	-1030.7	-1025.0
$P_{W,const}{}^*$	-1055.2	-1032.4	-1028.2	-1032.4

The loss for each combination is given in table 10. The total loss indicates which combination of variables that keeps operating conditions closest to optimum for the given disturbances. To be able to evaluate if control of self-optimizing variables is beneficial, the total loss for process with constant well pressures was also calculated.

Table 10: Loss for all combinations of c

	L_{d1}	L_{d2}	L_{d3}	L_{d4}	Total Loss	% Total Loss
$c1, c2$	0.1	0.1	0	3.8	4	0.000961 %
$c1, c3$	1.6	12.8	2.1	3.8	20.3	0.00489 %
$c1, c4$	0.1	0.1	0	3.8	4	0.000961 %
$c2, c3$	0.1	0.4	1.6	12.3	14.4	0.00346 %
$c3, c4$	0.1	0.1	1.6	12.3	14.4	0.00346 %
${P_{W,const}}^*$	0	4.9	4.1	4.9	13.9	0.00334 %

7.3 Discussion of Results

The results imply a potential for implementation of a control structure in our model. This is also the result that should be given the most focus in this part, in stead of the individual values form the loss evaluation and the suggestions of CV-combinations. When considering the following results one should keep in mind that the c1-vector does not sum to zero, which can imply that something could be wrong in this evaluation. In further work a more detailed analysis of the problem should be done, which hopefully can provide a loss evaluation and CV-suggestions with lower insecurity .

From Table 10 we see that the combinations that give the lowest total loss is c_1,c_2 and c_1,c_4 implying that this would be the most beneficial structures to implement. The control variable c1 is a part of both combinations. The vector c1 keep a constant ratio between 6 of the 8 measurements, where c2 and c4 only keep one of the measurements constant constants.

From the results it can look like it is beneficial to control a ratio an a constant at the same time. The cobmination of two constant measurements c2,c4, gave an infeasible solution. This is reasonable as this combination suggests keeping a constant gas- and oil rate, which are two variables that highly depend on each other as the component ratio in each well is given for a given pressure.

8 Conclusion and suggestion to further work

In the final part of this report we will give some concluding remarks as well as some aspects and ideas on further work on this problem

8.1 Concluding remarks

In this paper we have made a model of the flow-pressure drop relation in a well-model based on data form the Troll west rig. As expected the general trend was increasing pressure drop with increasing total flow, but there will be slight differences depending on the flow composition.

Further an optimization problem has been defined, with objective to maximize the oil flow up from the reservoirs. The optimization routine developed in MATLAB works well and in compliance with model specifications and optimization objective. The optimal well-pressures that were found at capacity constraints $C_{water}^{Max} = 1800$ and $C_{gas}^{Max} = 3800$ are given in Table 11 :

Table 11: Optimized well pressures

	Well 1	Well 2	Well 3	Well 4
Well pressure	109.00	76.72	58.47	76.53

From a disturbance evaluation it was concluded that disturbance in the well compositions would effect the optimal operating set-points for our process. In trying to find a self-optimizing structure for the well-model, the null space method was applied to find suggestions for self-optimizing variables.

The loss was evaluated for combinations of the self-optimizing variables found from the null space method. The loss evaluation implied that the implementation of c1,c2 or c1,c4 would give the lowest loss, and more than three times lower loss than with valve positions set at the original optimum. This implies an economical potential in implementing a control structure in the model.

8.2 Further Work

The results and conclusion in this report suggests potential in looking at control structures for the model. It could be interesting to investigate further on self-optimizing structures for the model, in attempt to find a structure that could keep the operations close to optimal at a constant set-point. Further, also control structures in different active constraint regions, and shift between these, could be investigated.

However, as mentioned in the previous part, the c1-vector does not sum to zero, which can imply that something could be wrong in this evaluation. Before building further on these results, a more detailed analysis of the problem should be done, which hopefully can provide a loss evaluation and CV-suggestions with lower insecurity.

9 References

Alstad, V., Skogestad, S. (2007). Null space method for selecting optimal measurement combinations controlled variables.

Gunnerud,V.,Foss,B.(2009). Oil production optimization A piecewise linear model, solved with two decomposition strategies.

Norwegian Petroleum directorate.(2011).Petrolium resources on the Norwegian continental shelf.

Skogestad, S.(2000). Plantwide control: The search for the self-optimizing control structure.

Lorentz T. Biegler et.al(1997). Systematic methods of chemical process design.

A APPENDIX I - WPC Data

	Well 1				Well 2				Well 3				Well 4		
Pressure	Gas Flow	Oil Flow	Water Flow	Pressure	Gas Flow	Oil Flow	Water Flow	Pressure	Gas Flow	Oil Flow	Water Flow	Pressure	Gas Flow	Oil Flow	Water Flow
20	2970	92	460,2	20	2300	590	885,3	20	1500	490	735,3	20	2300	590	885,3
21	2963,7	92,78	463,9	21	2299,7	591,9	887,85	21	1499,7	491,9	737,85	21	2299,7	591,9	887,85
22	2956,8	93,52	467,6	22	2298,8	593,6	890,4	22	1498,8	493,6	740,4	22	2298,8	593,6	890,4
23	2949,3	94,22	471,1	23	2297,3	595,1	892,65	23	1497,3	495,1	742,65	23	2297,3	595,1	892,65
24	2941,2	94,88	474,4	24	2295,2	596,4	894,6	24	1495,2	496,4	744,6	24	2295,2	596,4	894,6
25	2932,5	95,5	477,5	25	2292,5	597,5	896,25	25	1492,5	497,5	746,25	25	2292,5	597,5	896,25
26	2923,2	96,08	480,4	26	2289,2	598,4	897,6	26	1489,2	498,4	747,6	26	2289,2	598,4	897,6
27	2913,3	96,62	483,1	27	2285,3	599,1	898,65	27	1485,3	499,1	748,65	27	2285,3	599,1	898,65
28	2902,8	97,12	485,6	28	2280,8	599,6	899,4	28	1480,8	499,6	749,4	28	2280,8	599,6	899,4
29	2891,7	97,58	487,9	29	2275,7	599,9	899,85	29	1475,7	499,9	749,85	29	2275,7	599,9	899,85
30	2880	98	490	30	2270	600	900	30	1470	500	750	30	2270	600	900
31	2867,7	98,38	491,9	31	2263,7	599,9	899,85	31	1463,7	499,9	749,85	31	2263,7	599,9	899,85
32	2854,8	98,72	493,6	32	2256,8	599,6	899,4	32	1456,8	499,6	749,4	32	2256,8	599,6	899,4
33	2841,3	99,02	495,1	33	2249,3	599,1	898,65	33	1449,3	499,1	748,65	33	2249,3	599,1	898,65
34	2827,2	99,28	496,4	34	2241,2	598,4	897,6	34	1441,2	498,4	747,6	34	2241,2	598,4	897,6
35	2812,5	99,5	497,5	35	2232,5	597,5	896,25	35	1432,5	497,5	746,25	35	2232,5	597,5	896,25
36	2797,2	99,68	498,4	36	2223,2	596,4	894,6	36	1423,2	496,4	744,6	36	2223,2	596,4	894,6
37	2781,3	99,82	499,1	37	2213,3	595,1	892,65	37	1413,3	495,1	742,65	37	2213,3	595,1	892,65
38	2764,8	99,92	499,6	38	2202,8	593,6	890,4	38	1402,8	493,6	740,4	38	2202,8	593,6	890,4
39	2747,7	99,98	499,9	39	2191,7	591,9	887,85	39	1391,7	491,9	737,85	39	2191,7	591,9	887,85
40	2730	100	500	40	2180	590	885	40	1380	490	735	40	2180	590	885
41	2711,7	99,98	499,9	41	2167,7	587,9	881,85	41	1367,7	487,9	731,85	41	2167,7	587,9	881,85
42	2692,8	99,92	499,6	42	2154,8	585,6	878,4	42	1354,8	485,6	728,4	42	2154,8	585,6	878,4
43	2673,3	99,82	499,1	43	2141,3	583,1	874,65	43	1341,3	483,1	724,65	43	2141,3	583,1	874,65
44	2653,2	99,68	498,4	44	2127,2	580,4	870,6	44	1327,2	480,4	720,6	44	2127,2	580,4	870,6
45	2632,5	99,5	497,5	45	2112,5	577,5	866,25	45	1312,5	477,5	716,25	45	2112,5	577,5	866,25
46	2611,2	99,28	496,4	46	2097,2	574,4	861,6	46	1297,2	474,4	711,6	46	2097,2	574,4	861,6
47	2589,3	99,02	495,1	47	2081,3	571,1	856,65	47	1281,3	471,1	706,65	47	2081,3	571,1	856,65
48	2566,8	98,72	493,6	48	2064,8	567,6	851,4	48	1264,8	467,6	701,4	48	2064,8	567,6	851,4
49	2543,7	98,38	491,9	49	2047,7	563,9	845,85	49	1247,7	463,9	695,85	49	2047,7	563,9	845,85
50	2520	98	490	50	2030	560	840	50	1230	460	690	50	2030	560	840
51	2495,7	97,58	487,9	51	2011,7	555,9	833,85	51	1211,7	455,9	683,85	51	2011,7	555,9	833,85
52	2470,8	97,12	485,6	52	1992,8	551,6	827,4	52	1192,8	451,6	677,4	52	1992,8	551,6	827,4
53	2445,3	96,62	483,1	53	1973,3	547,1	820,65	53	1173,3	447,1	670,65	53	1973,3	547,1	820,65
54	2419,2	96,08	480,4	54	1953,2	542,4	813,6	54	1153,2	442,4	663,6	54	1953,2	542,4	813,6
55	2392,5	95,5	477,5	55	1932,5	537,5	806,25	55	1132,5	437,5	656,25	55	1932,5	537,5	806,25
56	2365,2	94,88	474,4	56	1911,2	532,4	798,6	56	1111,2	432,4	648,6	56	1911,2	532,4	798,6
57	2337,3	94,22	471,1	57	1889,3	527,1	790,65	57	1089,3	427,1	640,65	57	1889,3	527,1	790,65
58	2308,8	93,52	467,6	58	1866,8	521,6	782,4	58	1066,8	421,6	632,4	58	1866,8	521,6	782,4
59	2279,7	92,78	463,9	59	1843,7	515,9	773,85	59	1043,7	415,9	623,85	59	1843,7	515,9	773,85

60	2250	92	460	60	1820	510	765	60	1020	410	615	60	1820	510	765
61	2219,7	91,18	455,9	61	1795,7	503,9	755,85	61	995,7	403,9	605,85	61	1795,7	503,9	755,85
62	2188,8	90,32	451,6	62	1770,8	497,6	746,4	62	970,8	397,6	596,4	62	1770,8	497,6	746,4
63	2157,3	89,42	447,1	63	1745,3	491,1	736,65	63	945,3	391,1	586,65	63	1745,3	491,1	736,65
64	2125,2	88,48	442,4	64	1719,2	484,4	726,6	64	919,2	384,4	576,6	64	1719,2	484,4	726,6
65	2092,5	87,5	437,5	65	1692,5	477,5	716,25	65	892,5	377,5	566,25	65	1692,5	477,5	716,25
66	2059,2	86,48	432,4	66	1665,2	470,4	705,6	66	865,2	370,4	555,6	66	1665,2	470,4	705,6
67	2025,3	85,42	427,1	67	1637,3	463,1	694,65	67	837,3	363,1	544,65	67	1637,3	463,1	694,65
68	1990,8	84,32	421,6	68	1608,8	455,6	683,4	68	808,8	355,6	533,4	68	1608,8	455,6	683,4
69	1955,7	83,18	415,9	69	1579,7	447,9	671,85	69	779,7	347,9	521,85	69	1579,7	447,9	671,85
70	1920	82	410	70	1550	440	660	70	750	340	510	70	1550	440	660
71	1883,7	80,78	403,9	71	1519,7	431,9	647,85	71	719,7	331,9	497,85	71	1519,7	431,9	647,85
72	1846,8	79,52	397,6	72	1488,8	423,6	635,4	72	688,8	323,6	485,4	72	1488,8	423,6	635,4
73	1809,3	78,22	391,1	73	1457,3	415,1	622,65	73	657,3	315,1	472,65	73	1457,3	415,1	622,65
74	1771,2	76,88	384,4	74	1425,2	406,4	609,6	74	625,2	306,4	459,6	74	1425,2	406,4	609,6
75	1732,5	75,5	377,5	75	1392,5	397,5	596,25	75	592,5	297,5	446,25	75	1392,5	397,5	596,25
76	1693,2	74,08	370,4	76	1359,2	388,4	582,6	76	559,2	288,4	432,6	76	1359,2	388,4	582,6
77	1653,3	72,62	363,1	77	1325,3	379,1	568,65	77	525,3	279,1	418,65	77	1325,3	379,1	568,65
78	1612,8	71,12	355,6	78	1290,8	369,6	554,4	78	490,8	269,6	404,4	78	1290,8	369,6	554,4
79	1571,7	69,58	347,9	79	1255,7	359,9	539,85	79	455,7	259,9	389,85	79	1255,7	359,9	539,85
80	1530	68	340	80	1220	350	525	80	420	250	375	80	1220	350	525
81	1487,7	66,38	331,9	81	1183,7	339,9	509,85	81	383,7	239,9	359,85	81	1183,7	339,9	509,85
82	1444,8	64,72	323,6	82	1146,8	329,6	494,4	82	346,8	229,6	344,4	82	1146,8	329,6	494,4
83	1401,3	63,02	315,1	83	1109,3	319,1	478,65	83	309,3	219,1	328,65	83	1109,3	319,1	478,65
84	1357,2	61,28	306,4	84	1071,2	308,4	462,6	84	271,2	208,4	312,6	84	1071,2	308,4	462,6
85	1312,5	59,5	297,5	85	1032,5	297,5	446,25	85	232,5	197,5	296,25	85	1032,5	297,5	446,25
86	1267,2	57,68	288,4	86	993,2	286,4	429,6	86	193,2	186,4	279,6	86	993,2	286,4	429,6
87	1221,3	55,82	279,1	87	953,3	275,1	412,65	87	153,3	175,1	262,65	87	953,3	275,1	412,65
88	1174,8	53,92	269,6	88	912,8	263,6	395,4	88	112,8	163,6	245,4	88	912,8	263,6	395,4
89	1127,7	51,98	259,9	89	871,7	251,9	377,85	89	71,7	151,9	227,85	89	871,7	251,9	377,85
90	1080	50	250	90	830	240	360	90	30	140	210	90	830	240	360
91	1031,7	47,98	239,9	91	787,7	227,9	341,85	91	0	127,9	191,85	91	787,7	227,9	341,85
92	982,8	45,92	229,6	92	744,8	215,6	323,4	92	0	115,6	173,4	92	744,8	215,6	323,4
93	933,3	43,82	219,1	93	701,3	203,1	304,65	93	0	103,1	154,65	93	701,3	203,1	304,65
94	883,2	41,68	208,4	94	657,2	190,4	285,6	94	0	90,4	135,6	94	657,2	190,4	285,6
95	832,5	39,5	197,5	95	612,5	177,5	266,25	95	0	77,5	116,25	95	612,5	177,5	266,25
96	781,2	37,28	186,4	96	567,2	164,4	246,6	96	0	64,4	96,6	96	567,2	164,4	246,6
97	729,3	35,02	175,1	97	521,3	151,1	226,65	97	0	51,1	76,65	97	521,3	151,1	226,65
98	676,8	32,72	163,6	98	474,8	137,6	206,4	98	0	37,6	56,4	98	474,8	137,6	206,4
99	623,7	30,38	151,9	99	427,7	123,9	185,85	99	0	23,9	35,85	99	427,7	123,9	185,85
100	570	28	140	100	380	110	165					100	380	110	165
101	515,7	25,58	127,9	101	331,7	95,9	143,85					101	331,7	95,9	143,85
102	460,8	23,12	115,6	102	282,8	81,6	122,4					102	282,8	81,6	122,4
103	405,3	20,62	103,1	103	233,3	67,1	100,65					103	233,3	67,1	100,65
104	349,2	18,08	90,4	104	183,2	52,4	78,6					104	183,2	52,4	78,6
105	292,5	15,5	77,5	105	132,5	37,5	56,25					105	132,5	37,5	56,25
106	235,2	12,88	64,4	106	81,2	22,4	33,6					106	81,2	22,4	33,6
107	177,3	10,22	51,1	107	29,3	7,1	10,65					107	29,3	7,1	10,65
108	118,8	7,52	37,6												
109	59,7	4,78	23,9												

B APPENDIX II - Null Space Calculation

Our candidates for self-optimizing variables are:

$$y = \left| \begin{array}{cccccccc} p_1^{W*} & p_2^{W*} & p_3^{W*} & p_4^{W*} & p^{M*} & q_g^{P*} & q_w^{P*} & q_o^{P*} \end{array} \right|.$$

From simulations we find the following nominal values for our y-matrix(matrix of measurements/candidates for self-optimizing variables):

$$y_{nom} = \left| \begin{array}{cccccccc} 109.0000 & 85.5045 & 77.5209 & 63.9579 & 54.2464 & 3300 & 1055.5 & 1600 \end{array} \right|.$$

Implementing the disturbance, we obtain new optimal values for the measurements:

$$y(d)_{opt} = \left| \begin{array}{cccccccc} 109.0000 & 85.5045 & 77.5208 & 63.9580 & 54.2455 & 3300 & 1055.2 & 1600 \\ 109.0000 & 88.2364 & 69.7468 & 69.0002 & 54.1975 & 3300 & 1037.3 & 1600 \\ 109.0000 & 69.7469 & 88.2364 & 69.0000 & 54.2163 & 3300 & 1044.3 & 1600 \\ 109.0000 & 69.0000 & 69.7469 & 88.2364 & 54.1975 & 3300 & 1037.3 & 1600 \end{array} \right|.$$

First we calculate the sensitivity matrix

$$F = \frac{\delta y^{opt}}{\delta d^T} \tag{19}$$

$$F = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 390.27 & -225.10857 & -235.77857 \\ -0.00142857 & -111.05857 & 153.07857 & -111.05714 \\ 0.00142857 & 72.032857 & 72.03000 & 346.8357 \\ -0.012857 & -0.69857 & -0.4300 & -0.69857 \\ 0 & 0 & 0 & 0 \\ -4.2857 & -260.0000 & -160.0000 & -260.0000 \\ 0 & 0 & 0 & 0 \end{vmatrix}.$$

Further, we find the combination matrix H which lies in the null space of F:

$$HF = 0 \tag{20}$$

$$H = \begin{vmatrix} 0.0031 & -0.5773 & -0.5773 & -0.5773 & 0.0102 & 0 & -0.00003 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.999995 & 0.001789 & 0.001789 & 0.001789 & -0.00003 & 0 & 0.0000001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}.$$

This gives us the following values for c:

$$c_1 = [\ 0.0031 \quad -0.5773 \quad -0.5773 \quad -0.5773 \quad 0.0102 \quad 0 \quad -0.00003 \quad 0]$$

$$c_2 = [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$$

$$c_3 = [\ 0.0031 \quad 0.9999 \quad 0.0018 \quad 0.0018 \quad 0.0018 \quad 0 \quad -0.00003 \quad 0]$$

$$c_4 = [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$$