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Optimal operation of energy storage in buildings

Use of hot water systems

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Abstract

The purpose of this study is to maximize the potential which lies in domestic hot water heaters. Today the temperature function in the hot water tank is constant throughout the day and the hot water demand is met by an electrical heater. Implementing a temperature which has the freedom to change in the tank, the energy demand can be met by both the energy stored in the hot water and the electrical heater. This means that if the electricity is expensive and the load on the mains are high, the hot water heater can shift load the electrical consumption from this time period to another by benefiting from the energy storage. This should result in a cost reduction since the consumption of electricity is moved to a time period with cheaper electricity. In addition, the consumption from the consumers can be altered if the electrical storage is properly used.

The main challenges with hot water tanks are the unpredictability of future demands and time varying electricity prices. The consumer is free to benefit from desired amounts of hot water at any time of day. In addition, the future electricity prices are unknown. Despite these uncertainties the hot water tank should always be able to deliver sufficient and hot enough water.

To optimize the hot water heater in a proposed optimal strategy was implemented for minimizing the electrical cost of heating water. The optimal operation was compared to other simple policies to see if they were better, worse or equally good strategies. From the result in the report the proposed strategy reduced the cost compared to all suggested policies. The highest savings were compared to having a constant temperature in the hot water tank. The electrical consumption was also proven to change if the policies were provided with the opportunity of benefiting from the energy stored in the water compared to only having electrical heating. This implies that one solution to shave the peak demands in the electricity is by utilising the energy stored in the hot water.

Sammen drag

Ideen bak denne studien har vært å maksimere potensialet i varmtvannstanker. I dag kontrolleres temperaturen i varmtvannstanker av en konstant temperatur gjennom hele dagen. Dette tvinger energibehovet til å bli møtt av elektrisitet gjennom hele døgnet. Ved å implementere en temperaturstyring med friheten til å forandre seg som en funksjon av tid kan energibehovet bli møtt av to strategier. Den ene vil være den lagrede energien som finnes i det varme vannet, mens den andre vil være elektrisitet. Varmtvannsberederen kan da velge å bruke energilageret i det varme vannet når strømprisene er dyre og forbruket høyt, for siden å gjenopprette lageret når det er færre som bruker strømmettet og prisene er lavere. Hvis strømprisene også reflekterer strømforbruket vil topplasten i nettet kunne bli redusert gjennom å flytte strømforbruket til andre tider hvor nettet er mindre belastet.

De største utfordringene med en varmtvannsbereder er uforutsigbarheten ved forbruket, siden forbrukeren står fritt til å benytte seg av varmtvann til alle døgnet tider og hvilken mengde de ønsker. I tillegg er fremtidens elektrisitetsprisene uvisse.

For å optimalisere varmtvannstanken i dette studiet er det blitt foreslått en optimal strategi for hvordan temperaturen i varmtvannstanken skal styres. Denne er sammenlignet med andre enkle strategier for å kvantifisere hvorvidt de er like bra, bedre eller dårligere en den foreslåtte optimale strategien. Fra resultatene vises det at den foreslåtte optimale styringsstrategien reduserer kostnaden sammenlignet med alle de foreslåtte strategiene. Den største besparelsen, sammenlignet med å ha konstant temperatur i varmtvannstanken igjennom hele døgnet, ble funnet. Strømlasten på nettet viste seg også å variere hvis varmtvannsberederen kunne benytte seg av lagret energi fra varmtvannet.

Preface

This thesis is written as a final work of the master program *Industrial Chemistry and Biotechnology* at the Norwegian University of Science and Technology, 2013.

I would like to thank my supervisor Sigurd Skogestad for being open minded and giving me the opportunity to combine elite sports with the thesis work.

A special thanks goes out to my co-supervisor Vinicius de Oliveira who has been invaluable throughout the entire project. He has always been helpful, positive and shown interest in my work.

Declaration of Compliance

I hereby declare that this is an independent work according to the exam regulations of the Norwegian University of Science and Technology (NTNU).

A handwritten signature in black ink, reading "Emma Johanson". The signature is written in a cursive style with a large initial 'E'.

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List of Symbols and Abbreviations

Symbol	Explanation	Unit
AMS	Avansert måle- og styringssystem	[-]
CV	Controlled variable	[various]
DMS	Demand side management	[-]
DOF	Degrees of freedom	[-]
DR	Demand reduction	[-]
EWB	Electrical water heater	[-]
IMC	Internal model control method	[-]
MV	Manipulated variable	[various]
PID	Proportional-integral-derivative	[-]
SIMC	Skogstads IMC rules	[-]
TOU	Time of use	[-]
τ_1	Integral time	[sec]
τ_c	Response time	[sec]
τ_d	Derivative time	[sec]
θ	Time delay	[sec]
A	Heat exchange area	[m^2]
c	Controller unit	[-]
c_p	Heat capacity	[kJ/kg,K]
d	Disturbances	[various]
$e(t)$	Error signal to controller	[°C]
E_s	Energy storage	[kJ]
ΔE_p	Deficiency penalty	[NOK]
g	Equality constraints vector	[various]
g	System	[-]
g_d	Disturbance	[various]
$G(s)$	Transferfunction	[various]
h	Inequality constraints vector	[various]
H	Enthalpy	[kJ/mol]
J	Cost function	[NOK]

k	Slope	[various]
K_c	Controller gain	[various]
p	Steady state value	[various]
$p(t)$	Electricity price	[NOK/kWh]
p^*	Penalty factor	[-]
$P_{threshold}$	Price threshold	[NOK]
$P(T)$	Penalty	[NOK/kWh]
q_{cw}	Cold water flow	[l/s]
q_{in}	Inlet flow tank	[l/s]
q_{out}	Outlet flow tank	[l/s]
q_{hw}	Flow to consumer	[l/s]
Q	Heat input	[W]
Q_{demand}	Energy demand	[kW]
Q_{loss}	Heat loss to surroundings	[kW/h]
ρ	Density	[kg/l]
t	Time	[sec]
T	Water temperature tank	[°C]
T_{buffer}	Temperature set-point	[°C]
T_{cw}	Cold water temperature	[°C]
T_{hw}	Water temperature to consumer	[°C]
$T_{initial}$	Initial temperature	[°C]
T_{out}	Outlet temperature tank	[°C]
T_{set}	Temperature set-point	[°C]
T_{surr}	Temperature of the surroundings	[°C]
$Time_{int}$	Time interval	[h]
u	Inputs	[various]
u	Degrees of Freedom (DOF)	[-]
u_t	Manipulated variables	[various]
U	Overall heat transfer coefficient	[kW/°Cm ²]
V	Volume	[l]
y	Output	[various]
y_s	Output set-point	[various]
x	State variables	[various]

1 | Introduction

Energy is one of the most essential needs in modern daily life. In order to be able to provide future generations with the same energy sources as today a sustainable energy consumption is important. To be able to conduct such a consumption, energy sources should be fully utilized and unnecessary losses eliminated. Possibilities for new energy sources should also be investigated as the need for power is increasing [10]. The previous measures all involve the production of power whereas the consumption is the throughput manipulator for the power generation. If the consumption increases the production has to increase if it is fully utilized. Therefore investigation how to create measures for trying to decrease the growth of consumption are of interest.

In 1991 the Norwegian electricity sector was deregulated and liberalized. Norway was integrated with the Swedish, Finnish and Danish markets to become the first common, inter-country electric market in the world, the Nordic electricity market [10]. After the merge the electricity prices decreased as a result of competition, overcapacity and wet years [1]. This led to limited investments in power generation, resulting in available capacity being more or less the same for 20 years [10]. *However*, the consumption has not been constant over the last 20 years. Consumption of electricity at peak hours in Norway has been increasing and is expected to continue to increase in the coming years [2]. In Figure 1.1 the annually peak loads compared to the available capacity from 1981 to 2005 is depicted. The peaks differ significantly from year to year due to climatic variation, in particular the variation of outside temperature as electricity is primarily used for space heating in Norwegian households [11]. Despite these fluctuations, Figure 1.1 shows how the peak loads has a trend of growing, and forecasts expects it to continue increasing further [1].

As the peak load approaches the total capacity of the system the probability for a capacity deficit occurring increases. Either the demand needs to be reduced or the hydro power distribution expanded, otherwise the situation might become critical in a very dry year [1]. To solve this problem peak shifting strategies can be applied for levelling the peak loads.

In many European Countries consumers benefit from tariffs based on TOU (time of use) and DR (demand reduction) if the electricity is used during off-peak hours [3]. This can reduce the load and benefit the utilities by avoiding the costs of installing new power units. In addition in some industrial countries direct control

of electrical appliances is common practice to decrease the peak load [12, 13].

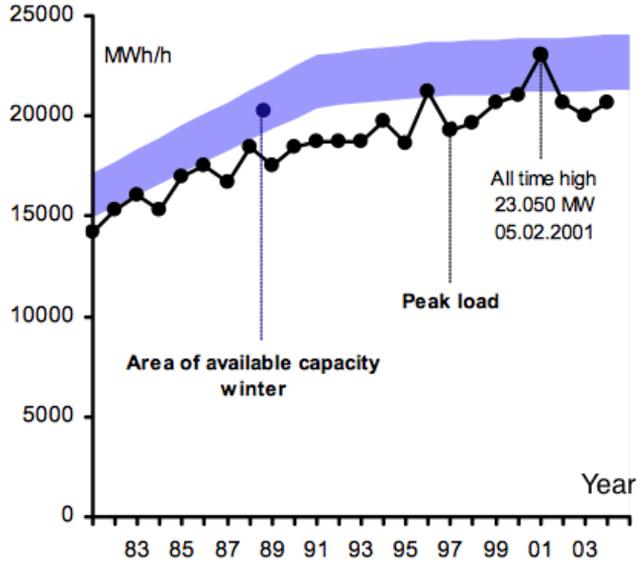


Figure 1.1: Available capacity and annual peak load in Norway [1].

One example is in Australia where 355,000 hot water heaters are involved in a direct load control program. The control strategy reduces the peak electricity consumption by 389 MW. The incentive for the consumer to participate in the program is a lower rate for hot water heating [12].

In Norway load management is being introduced through advanced measuring and controlling system units, AMS (avansert måle- og styringssystem). These will be implemented in households within the end of 2018 [14]. They will provide the consumers with more accurate information and awareness on their energy consumption [14]. The consumer will be able to see the consumption continuously during the day in addition to the current electricity price. With expansion packages, which can be bought, the consumer holds the possibility of utilise energy units when the price is low and alter their consumption patterns and save money. The power companies hope that this incentive will lead consumers to benefit of the AMS system, resulting in a less fluctuating electricity consumption, resulting in a decrease in the need for new capacity investments due to the increase in peak-demand.

For this system to work the cost savings should be made more available for the consumer. It is important for the utilities to provide an energy distributions per unit in a household. Every unit requires a specific amount of electricity to meet the requirements of the residents. Sintef Energiforskning AS in Trondheim has distributed a pie chart of the distribution of electricity consumption in Norwegian households [11]. Approximately 80 % of the total energy consumption in residential homes is provided by hot water heaters and domestic heating [11]. Of this percentage the hot water heater constitutes for approximately 15 % [11]. Load control of hot water heaters may therefore have a large demand response potential which should be quantified for. If fully utilised, a hot water heater should be able contribute to a decrease in the peak demands and slow down the need for new capacity investments. And as an incentive for the consumer a decrease in demand response should be met by a economical advantage.

1.1 Previous Work

Having introduced the need for utilising electrical units in Norway it should be mentioned that this is not a new topic. Several studies have been conducted and books published.

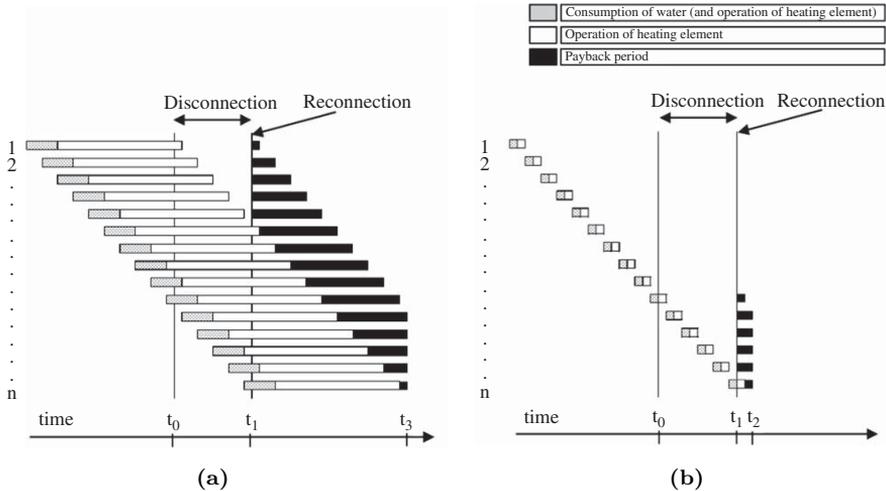


Figure 1.2: Energy recovering of water heaters with and without disconnections for households with a high level of hot water consumption 1,...,n (a), and low level of hot water consumption (b) [2].

An example is a large scale Norwegian project where load control was applied on domestic hot water heaters [2]. Electrical consumption of 475 households were investigated over a six month period from November 2003 to May 2004. The results show significant peak shavings in consumption during disconnection of hot water heaters. However the results also indicates an increase in consumption when the water heaters were reconnected, shown in Figure 1.2, essentially meaning that the energy not supplied to the heaters during disconnection will be required after reconnection.

Another study applied on water heaters performed by U. Atikol explores the possibility of applying a time controlled DSM (demand side management) program for reducing the peak demand generated by electrical water heaters, see Figure 1.3 [3]. The result shows that it is possible to operate the water heater once or twice a day and still meet the daily demand if the DSM is carefully designed for each household¹.

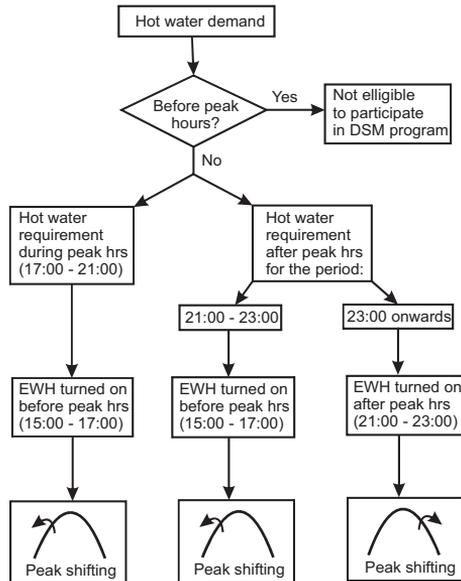


Figure 1.3: Simplified timer-setting strategies for different times of hot water requirements on typical days to be applied in a DSM program [3].

¹This study was conducted with the assumption of a infrastructure and technical expertise which were not favourable for more advanced options.

1.2 Project Scope

The studies mentioned in Section 1.1 primarily involve focus on levelling the consumption demand by switching the heater on and off. In the present study the objective is on the cost of heating water and not on the demand. The strategies applied are however assumed to have a demand shifting effect.

The idea is to minimize the cost of heating water by utilizing the heat capacity of water to store energy. To utilize the storage the temperature in the tank is adjusted as a function of time. The temperature will change during time of usage providing the unit to benefit from both the stored energy in the water as well as the electrical heater. Meaning that if the electricity is expensive, the goal is to shift load the energy consumptions to a time-interval where the price is cheaper. This should result in a reduction in cost and hopefully in peak demand.

If the result shows that both the cost and demand have positive outcome, the proposed optimal operational strategy will give an incentive for the consumer to benefit from such a system, which would make it easier for the distributor to implement it.

The structure of the report is presented as following. First the reader is provided with an explanation of the operational system followed by the assumption, equations and constraints, Chapter 2.2. Followed by the consumption profile for residential homes and electricity price profiles. Having introduced the essentials of the system the procedure of optimization is described followed by the control structure, Chapter 5 and 6 respectively. Finally the proposed optimal operation applied on the hot water heater is introduced in details in Chapter 7. In the same chapter there are also several case studies presented which proposes other simple policies for controlling a hot water heater. The optimal operation is compared to these cases and the results are presented in the same chapter. The discussion and conclusion are found in the end of the report, in Chapter 8 and 9, respectively.

2 | Operational System

The operational system of the present hot water heater consists of an electrical hot water heater used for domestic usage. A schematic process flow sheet of the hot water heater is presented in Figure 2.1 where all the important notations are present.

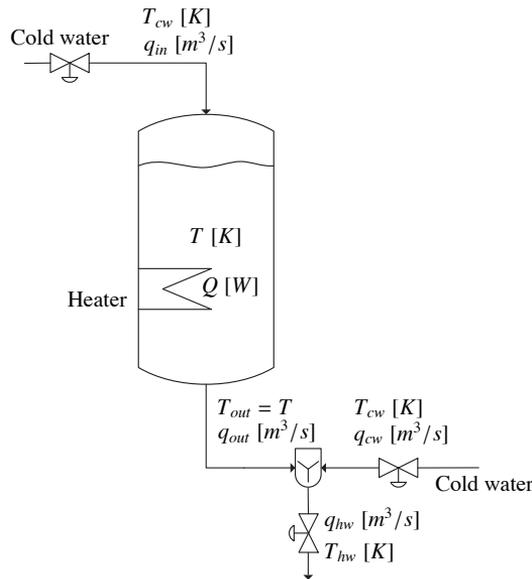


Figure 2.1: Operating system.

The system contains a cold water source, a thermally isolated tank, an electrical coil with adjustable power and control valves on the streams entering and leaving the tank. After leaving the tank the outlet stream is mixed with a cold water stream before reaching the consumer. The idea behind the mixing is that the storage tank can contain water with a higher temperature than the desired temperature of the consumer. That is

$$T_{out} > T_{hw} \quad (2.1)$$

where T_{out} is the tank temperature and T_{hw} is the temperature of the water reaching the consumer. The cold water stream, q_{cw} , is controlled by a valve and drawn from a cold water reservoir. The system also holds other specifications which differ from a standard hot water heater, presented in Table 2.1.

Table 2.1: System Specifications

Parameter	Standard [15, 2]	Project
q_{in}	Bottom	Top
q_{out}	Top	Bottom
V	200 l	200 l
Q_{max}	2,0 kW	5,5 kW
Q_{loss}	0,1 kW/h	0 kW/h

From Table 2.1 the system contains a hot water tank which is provided with cold water from the top of the tank while hot water exit from the bottom. This is contrary to what is normal in standard hot water heaters, where cold water enters at the bottom of the tank and hot waters is withdrawn from the top [2]. However this change in design will not effect the thermodynamics of the system due to perfect mixing. It is assumed that when entering the tank the cold water is perfectly mixed with the hot water resulting in a negative temperature change if,

$$T_{cw} \leq T \tag{2.2}$$

where T_{cw} is the temperature of the cold water and T the water temperature in the tank. From Table 2.1 other parameters also deviate from standard values. The maximum heat input, Q_{max} is more than two times higher than the standard value. This value is increased to *boost* the response on the temperature as the heat input is turned on. The heat loss is also neglected. The tank containing water is thermally isolated and is assumed to have no heat loss. In reality the heat loss from a tank is approximately 0.1 kWh/h from Table 2.1, which implies that it will take approximately 2.3 hours for a full tank to drop 1 °C in stand-by mode (i.e. when no water is consumed from the tank).

2.1 Assumptions

The water heater process has initially four DOF (degrees of freedom), Q , q_{cw} , q_{hw} and q_{in} . Two of the DOF are removed to satisfy the demand requirements for the assumption that the hot water flow rate q_{hw} and the hot water temperature set-point $T_{hw,s}$ are directly controlled by the consumer. This implies that they can not be controlled by the system. From a control point of view these inputs are defined as disturbances. In addition the temperature into the tank, T_{in} , and the price, p , are also classified as disturbances.

Defining q_{hw} and $T_{hw,s}$ as disturbances, there remains only two DOF left for optimization. The model equation can now be compactly written as Equation (2.3) with the state, input and disturbance vectors defined in Equations (2.4).

$$\frac{dx}{dt} = f(x, u, d) \quad (2.3)$$

$$x = \begin{bmatrix} V \\ T \end{bmatrix}, u = \begin{bmatrix} Q \\ q_{in} \end{bmatrix}, d = \begin{bmatrix} q_{hw} \\ T_{hw,s} \\ T_{in} \\ p \end{bmatrix} \quad (2.4)$$

Having assuming that $T_{hw,s}$ and q_{hw} are controlled by the consumer, these variables are the measurement of success, implying that the system assumes perfect control whenever feasible to satisfy the requirements of the consumer. Perfect control is defined in Equation (2.5),

$$T_{hw} = T_{hw,s} \quad \text{and} \quad q_{hw} = q_{hw,s} \quad (2.5)$$

where T_{hw} is the temperature while $T_{hw,s}$ is the set-point for the temperature of the hot water. The hot water flow rate q_{hw} should also be at the consumers desired flow rate, $q_{hw,s}$, when perfect control is reached. In cases where perfect control is not feasible, the aim will be to deliver desired flow rate but with a lower temperature, shown in Equation (2.6).

$$T < T_{hw,s} \quad \text{and} \quad q_{hw} = q_{hw,s} \quad (2.6)$$

2.1.1 Constraints

Operating the system entails keeping within the limits of the systems constraints. A system can have different limitations, for example regarding safety or physical properties.

The inputs of this system, Q and q_{in} have the following limitations, presented in Equations (2.7)-(2.8).

$$0 \leq Q \leq Q_{max} \quad (2.7)$$

$$0 \leq q \leq q_{max} \quad (2.8)$$

The constraints for the outputs are defined in Equations (2.9)-(2.10). The temperature should be constrained on an upper bound for safety regulations. If the temperature exceeds 100 °C the water will start boiling and water vapour will accumulate in the tank in addition to other hazards. The lower bound of the temperature is to avoid temperatures freezing. The constrains for the volume are naturally controlled by the size of the tank.

$$T_{min} \leq T \leq T_{max} \quad (2.9)$$

$$V_{min} \leq V \leq V_{max} \quad (2.10)$$

The values of the constraints on the inputs and outputs in Equations 2.7-2.10 are shown in Table 2.2.

Table 2.2: System Constraints

Parameter	Description	Value	Unit
Q_{min}	Minimum power	0	kW
Q_{max}	Maximum power	5.5	kW
q_{min}	Minimum inlet flow	0	l/s
q_{max}	Maximum inlet flow	10	l/s
T_{min}	Temperature lower bound	0	°C
T_{max}	Temperature upper bound	90	°C
V_{min}	Volume lower bound	50	l
V_{max}	Volume upper bound	200	l

2.2 Mass- and Energy balances

The dynamic model for the system can be derived from mass and energy balances. The notations in the equations are the same as in Figure 2.1. New notations will be explained. Starting with the tank, the mass balance is presented in Equation (2.11),

$$\frac{d(\rho V)}{dt} = \rho_{in} q_{in} - \rho_{out} q_{out} \quad (2.11)$$

where the fluid density, ρ is assumed constant. The energy equation for the tank is simplified by neglecting the kinetic and potential energies, Equation (2.12),

$$\frac{dH}{dt} = H_{in} - H_{out} + Q - Q_{loss} \quad (2.12)$$

where Q_{loss} is the heat loss to the surroundings, H is the enthalpy of the system, whereas H_{in} and H_{out} are the enthalpies of the streams and Q is added heat. The heat loss is further defined in Equation (2.13),

$$Q_{loss} = UA(T - T_{surr}) \quad (2.13)$$

where U is the overall heat transfer constant, A is the heat exchange area and T_{surr} are the temperature of the surroundings. Assuming good isolation of the tank Q_{loss} is neglected ($Q_{loss} = 0$). The enthalpies for the system and the streams are shown in Equation (2.14), where the heat capacity c_p is assumed constant in addition to no phase change.

$$\begin{aligned} H &= \rho V c_p (T - T_{ref}) \\ H &= \rho q_{in} c_p (T_{in} - T_{ref}) \\ H &= \rho q_{out} c_p (T - T_{ref}) \end{aligned} \quad (2.14)$$

Here T_{ref} is a fixed reference temperature while q is the flow rate. Further the tank is assumed to be perfectly mixed, implying that $T_{out} = T$.

Combining the mass and energy balances for the volume and temperature the resulting state equations for the system becomes

$$\frac{dV}{dt} = q_{in} - q_{out} \quad (2.15)$$

$$\frac{dT}{dt} = \frac{1}{V}(q_{in}(T_{in} - T) + \frac{Q}{\rho c_p}) \quad (2.16)$$

where V denotes the volume of the tank and T the temperature.

The system also holds a mixer which combines the hot and cold streams before reaching the consumer. This unit is assumed to be a static process with constant heat capacity and no pressure drop. Equations (2.17)-(2.18) shows the equation for the mix unit.

$$q_{hw} = q_{out} + q_{cw} \quad (2.17)$$

$$T_{hw} = \frac{q_{out}T + q_{cw}T_{in}}{q_{hw}} \quad (2.18)$$

Energy Storage and Demand

In the introduction the objective function in this study was said to be minimized by utilizing energy storage to meet the consumers demand in addition to the electrical heater. In this subsection the energy storage is introduced and an alternative energy balance is provided for the whole system. The storage, E_s is assumed to be relative to the current cold water supply temperature (T_{in}), as shown in Equation 2.19.

$$E_s = \rho c_p V (T - T_{in}) \quad (2.19)$$

From Equation (2.19) the upper and lower bounds for stored energy can be found as shown in Equations (2.20) and (2.21).

$$E_{max} = \rho c_p V_{max} (T_{max} - T_{cw}) \quad (2.20)$$

$$E_{min} = \rho c_p V_{min} (T_{hw,s} - T_{cw}) \quad (2.21)$$

From evaluating these boundaries the maximum available energy storage which can be used as buffer in the hot water tank is presented in Equation (2.22).

$$\Delta E = E_{max} - E_{min} \quad (2.22)$$

From the previous equations the energy demand at any given time can be derived, Equation (2.23). This is the energy that needs to be provided if there were no energy stored, i.e. $T = T_{hw,s}$.

$$Q_{demand} = \rho c_p q_{hw} (T_{hw} - T_{cw}) \quad (2.23)$$

From the following definition the overall energy balance for the tank and mixer system are presented in Equations (2.24)-(2.26).

$$\frac{dH}{dt} = H_{in} - H_{cw} - H_{hw} + Q \quad (2.24)$$

Here H_{cw} and H_{hw} are the enthalpies of the cold and hot streams respectively. By implementing E_s and Q_{demand} the energy balance can be written as shown in Equation (2.25).

$$\frac{dE}{dt} = Q - Q_{demand} - \rho V c_p \frac{dT_{cw}}{dt} \quad (2.25)$$

This is the final alternative equation for the whole system (tank and mixer). Notice that if the cold water is assumed to have a constant temperature the energy equation will be simplified to

$$\frac{dE}{dt} = Q - Q_{demand} \quad (2.26)$$

3 | Consumption Profile

The main aspects of demand shifting strategies are to reduce the demand peaks. The peaks are caused by many people benefiting from electricity at the same times. The consumptions coincides due to similar behaviour pattern resulting in a high demand on certain times of day. However, close up the demand is somewhat random if the time scale is small enough. This means that the accident related to hot water usage does not happen on the same time and in the same amount every day. This results in a pattern that is difficult to estimate. In the following section the uncertainty of hot water demand is discussed.

In reality future hot water consumption profiles are truly unknown and can not be exactly predicted. However, recovery of data from past consumption can be used to generate a probability profile for the future demand. To do this there are several methods to chose among.

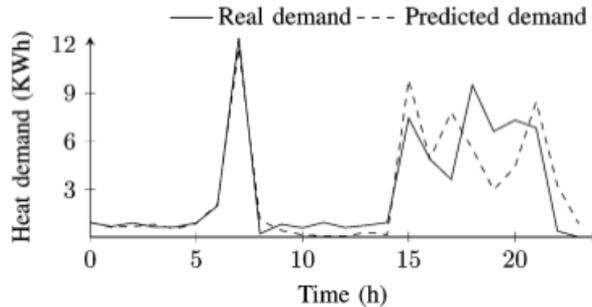


Figure 3.1: Heat demand prediction for a household on 22 November 2007 [4].

One example providing prediction models is neural networks techniques [4]. These models are based on biological neurons and are able to learn, to generalize or to cluster and organise data [16]. The networks need to be configured (trained) such that the application of the network combined with a set of given inputs generates the desired outputs. In the case presented in Chapter 2.2, the output of the prediction model can be either the flow or the heat demand which both provides the consumption demand directly or indirectly, respectively.

To be able to implement and use such a system a close study of a resident needs to be done. The behaviour of the residents, historical heat demand and weather forecasts are only a few input data that need to be collected [16]. An example of a generated consumption profile from [4] is presented in Figure 3.1.

Another way of predicting consumption profiles is by generating random profiles based on hot water consumption events [5]. e.g. calculating the amount of water used for a hand wash and counting occurring accidents per day and time of day. Collecting this data and combining it gives a good starting point for a probability function of hot water activities occurring.

The latter approach for finding a hot water demand profile is chosen as predictor in this study due to availability and time limitations. A predictor base on neural network is thought to be too time consuming and the data available is limited.

The load profiles from the predictor are based on a mean flow rate, Equation 3.1 [5],

$$\text{prob}(\dot{V}) = \frac{1}{\sqrt{2\pi}\alpha} \exp\left(-\frac{(\dot{V} - V_{mean})^2}{2\alpha^2}\right) \quad (3.1)$$

where \dot{V} is the mean flow rate and α is the duration of a load. The mean flow is combined with a probability function, Equation 3.2 [5].

$$\text{prob} = \text{prob}(\textit{accident}) \quad (3.2)$$

To visualize the profiles a 72 hour sequence of a generated profile from [5] with a mean load volume of 200 litres per day are shown in Figure 3.2. For comparison two generated consumption profiles from this report are shown in Figure 3.3 (a) and (b). The profiles were created using MATLAB scripts produced by Vinicius De Oliveira ,and are to be found in Appendix D.1.

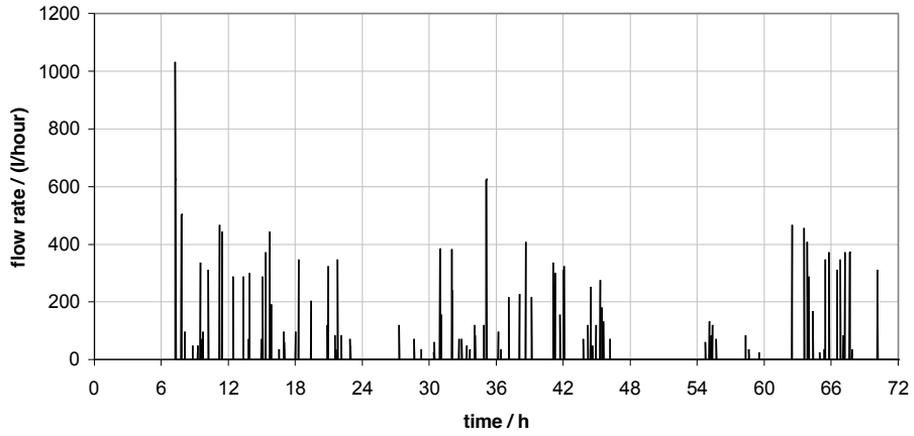


Figure 3.2: Hot water load profile generated for 72 hours [5].

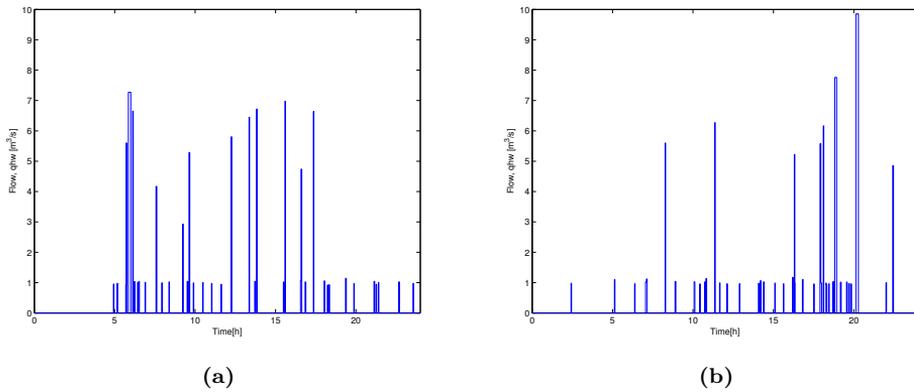


Figure 3.3: Two generated hot water load profiles generated from this report (a) and (b).

3.1 Power Consumption Profile

So far the hot water consumption has been revolving around the amount of water without evaluating the the heat input. Consumption of hot water leads to a decrease in volume, forcing cold water to enter to keep the volume at desired set-point. To make up for lost heat from the mixing of hot and cold water the heater is turned on. When the temperature reaches the set-point the heater is turned off. For a single household Q is consonantly turned on or off by the controller and the profile should look similar to the demand profiles in Figure 3.3. However, both the heater profile and demand profile are more continuous when investigating on a larger scale. Figure 3.4 shows the average daily profile for a domestic hot water heater from a study performed on hundreds of consumers in the province of Québec. This profile reflects the peak demands of electricity more clearly and reveals why DSM (demand side management) strategies need to be quantified.

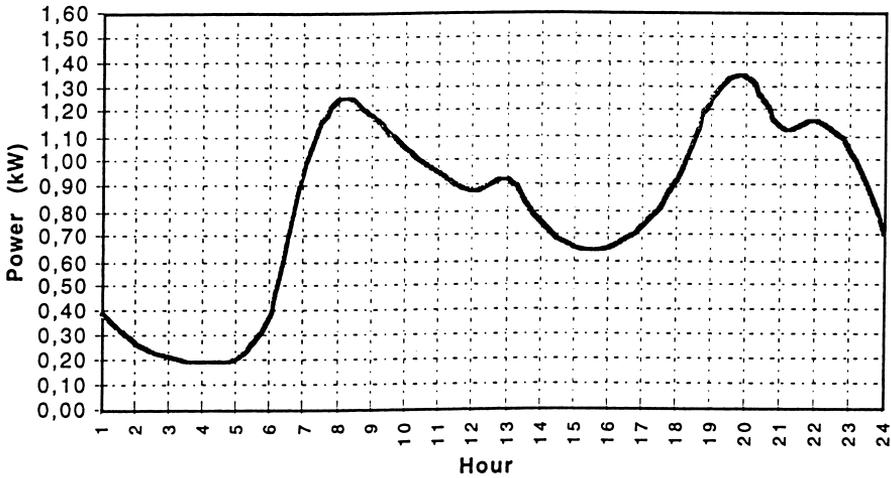


Figure 3.4: Average daily profile of the power consumption by electric hot water heaters in Canada[6].

4 | Price Profile

The objective of the study is the cost, which is directly effected by the electricity price provided. To be able to minimize the cost a closer look on the electricity prices is required.

The strategy behind the study is to shift load the energy consumptions to a time-interval where the prices are cheaper. This would result in a lower cost of heating. If the prices also reflects the consumption pattern of the society the peak demand should be shifted. To be able to benefit from such a strategy, the prices need to deviate throughout the day.

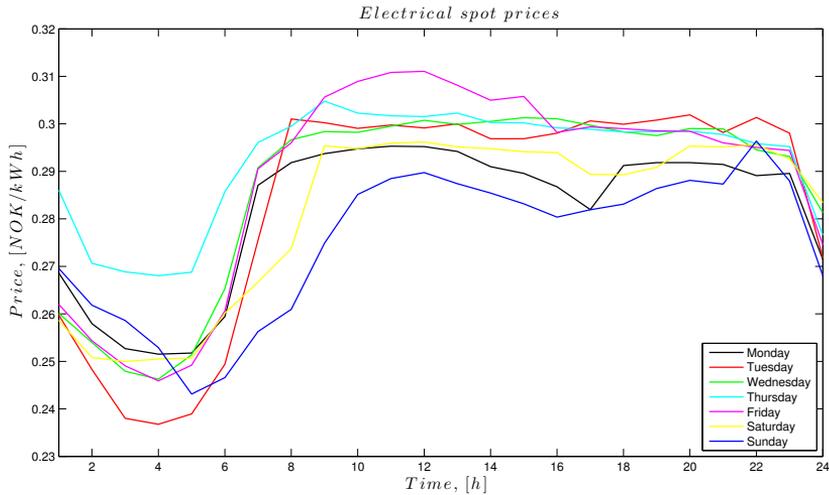


Figure 4.1: Electrical spot prices during week 32 (August 5. to 11.) in 2013.

If the electricity prices are constant all day, this strategy would not provide a lower cost. Thus, it would still be able to provide a change in electrical consumption and load shift demand.

Looking at the electricity prices from Figure 4.1 the prices change during the day. Many electrical companies have rate schedules of electricity prices that change with season of the year and time of the day. This causes a fluctuation in the

cost of energy and lead to the inequality such that $a kW \neq a kW$ [17]. Further explanation of this statement is that the energy units are the same, but the time-of-use will change the value of the energy. Generally low prices of electricity occur during the night time when few people are awake and the consumption of electricity is low (see Figure 3.4), these time periods are referred to as off-peak. In the day time the energy consumption is higher and the generating capacity and distribution of energy might be limited and can lead to higher prices. This time interval is referred to as on-peak. These notations will be used throughout the report.

To make the simulations throughout the report as realistic as possible the energy prices are actual prices collected from Nordpoolspot [18]. The month of August in 2013 was selected for Trondheim city, and a small presentation of some of the days in this month are depicted in Figure 4.1. The rest of the electricity prices used in the simulations are to be found in Appendix A.

5 | Optimization

In the optimization the focus is on the economical aspect of the process. From Skogestad [19] the goal of an optimization problem is to minimize an objective function J to its given constraints g and h , Equations (5.1) - (5.3). When evaluating the subject to optimization the equality and inequality constraints need to be satisfied in order for the optimization to be valid within the limits of the system. The constraints for this system are given in Chapter 2.2.

$$\text{minimum } J(x, u_t, d) \tag{5.1}$$

with subject to

$$\text{equality constraints: } g(x, u_t, d) = 0 \tag{5.2}$$

$$\text{inequality constraints: } h(x, u_t, d) \geq 0 \tag{5.3}$$

Here x are the state variables, u_t the manipulated variables, d the disturbances and J the cost. From a control perspective the optimization is to decide what to control with the available degrees of freedom.

The equality constraints g include the model equations, presented in Section 2.2, whereas the inequality constraints, presented in Section 2.1.1, mark the boundaries of the system.

5.1 Objective Function

The objective function of the system is the cost function in Equation (5.4).

$$J = \int_{t_0}^{\infty} p(t)Q dt + \int_{t_0}^{\infty} P(T)Q_{demand} dt \tag{5.4}$$

Here J is the cost, t_0 and ∞ are the time intervals, $p(t)$ is the electricity price, Q the heat input, $P(T)$ the penalty and Q_{demand} the energy demand. The

time limit in the objective function is from time t_0 to infinity, i.e. the unknown future. The penalty function, $P(T)$, for delivering lower temperature than the specification temperature set by the consumer consists of a linear and a quadratic term. The linear term is to prevent the system from having the optimal case where $V \leq V_{max}$ to keep $T = T_{hw}$ slightly below $T_{hw,s}$ in order to save energy since $Q_{demand} = \rho c_p q_{hw}(T_{hw} - T_{cw})$ drops linearly in T when $T_{hw} = T$ is reduced [20].

$$P(T) = \begin{cases} 0 & \text{if } T \geq T_{hw,s} \\ p *_1 (T_{hw,s} - T)^2 + p *_2 (T_{hw,s} - T) & \text{if } T < T_{hw,s} \end{cases} \quad (5.5)$$

Here $p*_1$ and $p*_2$ are penalty factors which controls the size of the penalty. These should always be high enough for the objective function to benefit from keeping the temperature, T , above the desired hot water temperature $T_{hw,s}$.

5.2 Problem Formulation

The problem of optimization is defined in Figure 5.1. Where Q and q_{in} are the DOF (degrees of freedom), T_{in} , $T_{hw,s}$, $p(t)$ and q_{hw} are disturbances and T and V are the outputs (and states) together with the objective function J .

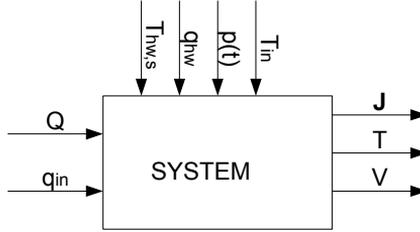


Figure 5.1: Initial optimization problem.

The objective of the problem of optimization is the cost, J , which is a non linear problem (Section 5.1). The most important parameter for the cost is the electrical heat input Q , which should have a main policy of being used when the electrical price, $p(t)$, is cheap. The water refilling, q_{cw} , affects the water temperature, T , whereas other effects are limited. From Equation 2.25 this stream does not affect the energy stored in the tank, E . The optimal solution is the minimal value of J with respect to the decision variables of the system, Equation (5.6).

$$\min_{Q, q_{in}} J \quad (5.6)$$

The two DOF in Figure 5.1 make out the MV's (manipulated variables) available for controlling the outputs, T and V . To control the outputs two controllers need to be implemented and set-points for the states defined. First a controller for the volume is introduced into the system, $c1$, shown in Figure 5.2. Since the heat loss, $Q_{loss} = 0$ is neglected, there is no disadvantage of having a high tank temperature. According to the equation for the the stored energy in the tank, Equation 2.25, there are an infinite number of combinations of T and V that will provide a certain energy storage. If the heat loss were included, which would be the case in real life, this would imply that it would always be optimal to keep the temperature in the tank as low as possible to minimize the losses. To achieve this the tank filling should be maximized resulting in selecting a volume set-point of V_{max} . The set-point for the volume is therefore selected to be V_{max} controlled by the inlet stream q_{in} . This specification results in one DOF left for optimization. The optimal control problem with the remaining DOF is shown in Equation (5.7),

$$\min_Q J \quad (5.7)$$

leaving Q as decision variable for the optimal control problem. Figure 5.2 depicts how the optimization problem is now redefined.

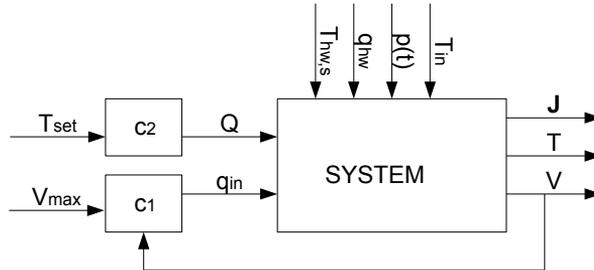


Figure 5.2: Optimization problem with only one DOF.

Next step is to implement a controller for the temperature of the tank, $c2$. Figure 5.3 shows that the heat input is chosen as CV for the temperature. The provided temperature set-point for the controller is T_{set} which indirectly controls the heat input Q through the error signal from the controller. The heat input can be said

to be the main throughput manipulator for the cost, J . For the optimization the heat input is however not chosen as decision variable, the temperature set-point is, T_{set} . Since Q is indirectly controlled by T_{set} the set-point is proposed to implement a control structure for the heat input so that the hot water demand can be met by either the energy storage or the electrical heat. The new decision variable for the optimization is then changed to T_{set} , given in Equation (5.8), and will be the final formulation for the optimal operation.

$$\min_{T_{set}} J \quad (5.8)$$

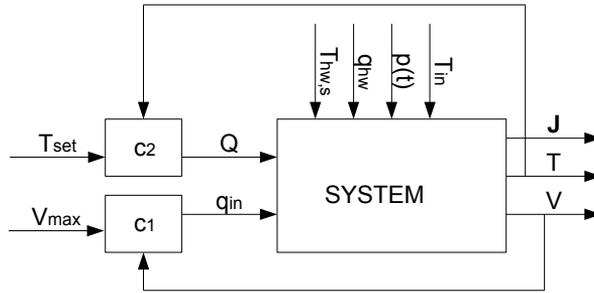


Figure 5.3: Final optimization formulation.

6 | Feedback Control

From the previous chapter two controllers were implemented into the system to be able to control the states. Both controllers are feedback controllers which measures the output and afterwards adjusts the input. In the following section a more thorough explanation of the controllers are presented.

6.1 Feedback Controller

The two dominating types of feedback control are proportional-integral-derivative control (PID) and on-off control [21]. On-off control is controlled by switching the manipulated variable on or off, while PID controllers are more complex and will be explained in more detail further down. A schematic presentation of a feedback controller is found in Figure 6.1, where c is the controller, g is the system and g_d is the disturbance.

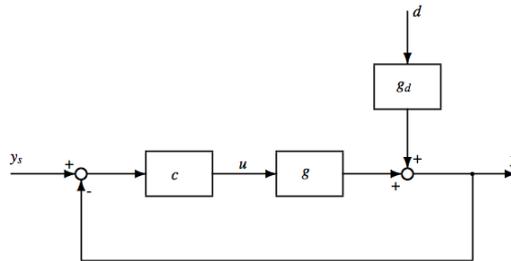


Figure 6.1: A schematic feedback controller loop [7].

The objective in feedback control is to reduce the error signal to zero. The error signal is defined in Equation (6.1)

$$e(t) = y_s(t) - y(t) \quad (6.1)$$

where $e(t)$ is the error signal between the set-point of the output $y_s(t)$ and the measured output $y(t)$.

PID controllers consist of three basic control modes, namely proportional, integral and derivative. The simplest is the proportional control where the output of the controller is proportional to the error signal, Equation (6.2) [21]

$$p(t) = p + K_c e(t) \quad (6.2)$$

where $p(t)$ is the controller output, p the steady-state value and K_c the controller gain. In proportional control the gain of the controller can be manipulated in order to make the output of the controller as sensitive as required to alterations between the set-point and controlled variable (CV). The sign of the gain, K_c , can also be selected to make the controller output increase (or decrease) as the error signal increases. A drawback of only using proportional control is that a steady-state error (off-set) occurs after a set-point change or a permanent disturbance.

In integral control the output depends on the integral of the error signal over time, Equation (6.3)

$$p(t) = p + \frac{1}{\tau_I} \int_0^t e(t) dt \quad (6.3)$$

where τ_I is integral time whereas the other parameters are the same as in proportional control. The usage of integral control is frequently used as it can eliminate the steady-state error. Thus only a small amount of control action takes place until the error signal is sustained over time. Integral control is therefore often combined with proportional control which has a immediate corrective action when an error is encountered for.

In derivative control the function is to anticipate the future behaviour of the error signal by measuring the rate of change, Equation (6.4)

$$p(t) = p + \tau_D \frac{de(t)}{dt} \quad (6.4)$$

where τ_D is the derivative time while the other parameters are the same as in proportional and integral control. From Equation (6.4) the derivative controller output is equal to the output steady-state value, p , as long as the error is constant. Due to this, derivative control is often combined with proportional or proportional-integral control [21]. By anticipating the future behaviour of the error signal the derivative control tends to stabilize the system. In addition it can improve the dynamic response of the controlled variable by shortening the process settling time, the time it takes to reach steady-state.

In most cases where PID-controllers are used the time domain is replaced by the Laplace transform in order to simplify differential equations to algebraic equations [21]. This can simplify the mathematical problem as well as the mathematical manipulations required to find a solution or to perform an analysis.

The controller output in the time domain for a PID-controller is the combination of Equations (6.2)-(6.4) and is shown in Equation (6.5).

$$p(t) = p + K_c(e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt + \tau_D \frac{de(t)}{dt}) \quad (6.5)$$

The corresponding PID equation with Laplace transformation is presented in Equation (6.6).

$$G_c = K_c(1 + \frac{1}{\tau_I s} + \tau_D s) \quad (6.6)$$

Selection of Controller

The PID controller consists of three terms which all contribute to optimal control of an output. It is however not always suitable to implement all terms to achieve good control. In this study the controllers are chosen to be PI controllers. The expression for a PI-controller is the same as for a PID-controller except that the τ_D is omitted, Equation(6.7) [21].¹

$$G_c = K_c(1 + \frac{1}{\tau_I s}) \quad (6.7)$$

6.1.1 Tuning parameters

To be able to control the states with the implemented PI controllers the parameters in Equation (6.7) are derived. These parameters are not easy to find without systematic approach, thus the SIMC rules, Skogestad's IMC rules will be applied to solve the problem. The first step is finding an approximate first (or second) order time delay model as shown in Equation (6.8) [7].

¹Due to the assumption of perfect mixing in the tank an on-off controller could also have been used as the heater only consists of one element. The heat consumption would then have a different profile in the results.

$$G(s) = \frac{ke^{-\theta s}}{\tau_1 s + 1} \quad (6.8)$$

Where k is the plant gain, τ_1 is the dominant lag time constant, θ is the effective time delay of the system and s is the Laplace parameter which has replaced the time t . To obtain the parameters in Equation (6.7) there are three possibilities, namely:

- Open-loop step response
- Closed loop set-point response with P-controller
- Approximation of effective delay using half rule

In this study the open-loop step response will be applied for finding tuning parameters.

First-order process

The open-loop step response experiment for a first-order process is shown in Figure 6.2.

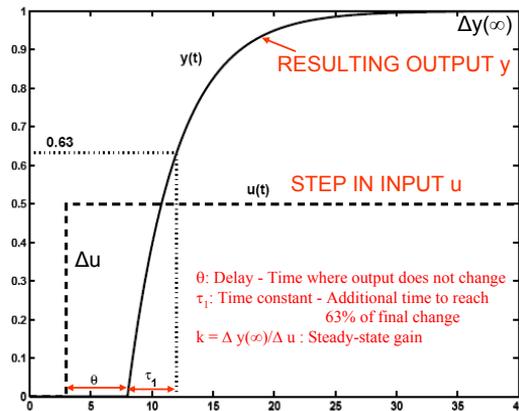


Figure 6.2: Open-loop step response test for a first-order process model [7].

From the depicted results of the open-step experiment in Figure 6.2, the model parameters can be obtained. Figure 6.2 gives a good explanation of several of the tuning parameters. To start with the steady state gain,

$$k = \frac{\Delta y(\infty)}{\Delta u} \quad (6.9)$$

is the final value of the change in the output, $\Delta y(\infty)$ as a fraction of the size of the step in the input, Δu . The effective time delay, θ , is the time it takes for the output to respond to a change in the input. The time constant, τ_1 is the time it takes to reach 63% of the final output value after θ .

Having introduced the approach for finding the tuning parameters, the SIMC rules for a first-order-plus-dead-time model are presented in Equations (6.10)-(6.11). These rules can be applied for a PI controller which is present in the control structure in this study.

$$K_c = \frac{1}{k'} \frac{1}{(\theta + \tau_c)} \quad , \quad k' = \frac{k}{\tau_I} \quad , \quad \tau = \tau_I \quad (6.10)$$

$$\tau_I = \min(\tau_1, 4(\tau_c + \theta)) \quad (6.11)$$

The response time is τ_c and must be in the range of $-\theta < \tau_c < \infty$ to get a positive and non-zero controller gain [7].

Integrating Process

The tuning parameters for a first order process are derived, however, in this study the first order process is approximated as an integrating process as shown in Equation (6.12).

$$G(s) = \frac{K e^{-\theta s}}{\tau s + 1} \approx \frac{k' e^{-\theta s}}{s} \quad (6.12)$$

The parameter for the slope, k' , can then be found as shown in Figure 6.3. The notations for the input and the output are the same as in first-order model.

In an integrating process the controller gain, K_c , can be calculated the same way as in an first-order process, Equation (6.10).

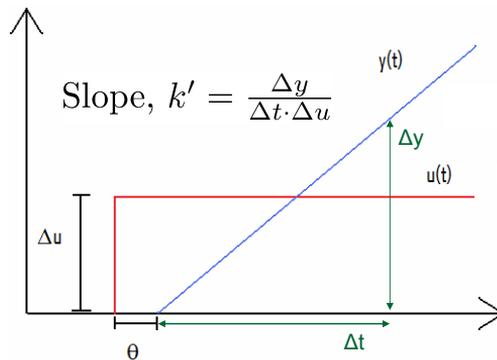


Figure 6.3: Open loop step experiment to obtain tuning parameters k' and θ for an integrating process [7].

The equation for the time constant, τ_1 , is different as it will go towards infinity and will always be larger than $4(\tau_c + \theta)$. The integral time is therefore given by

$$\tau_1 = 4(\tau_c) + \theta \quad (6.13)$$

The calculations and values of the tuning parameters in from Equations (6.10) - (6.13) are to be found in Table B.2 in Appendix B.2.

7 | Case Studies

In the following chapter the proposed control structure for the hot water tank is optimized and compared to other proposed policies. All the cases are provided with explanations of control structure, simulation data and results. Complete results for each case are presented in each section, in addition to selected figures. In Sections 7.2-7.5 the results are also compared to the optimal strategy to quantify which is most beneficial. All cases were solved using MATLAB_R2012b and Simulink, and all scripts are to be found in the Appendix D.

7.1 Optimal operation

The optimization objective of the study is to minimize the energy cost of heating water with the requirement that the uncertain demand should be attended at any time. The objective function is the cost function presented as in Section 5.1, Equation (5.4). The decision variable is the set-point of the temperature, T_{set} , which indirectly controls the heat input of the tank through the controller. The system for the optimal operation is presented in Figure 7.1. It is proposed that the hot water tank is controlled by a time dependent temperature set point, $T_{set}(t)$. This implies that the set-point can change with the inputs and disturbances of the system resulting in different strategies for how to meet the demand. Either the demand can be met by the energy storage, E_s , or the electrical heater. In addition, if the electricity prices applied reflect the demand load of the society, the demand peaks can be shaved, resulting in a reduced peak load demand.

7.1.1 Discretization

First the objective function is simplified by adjusting the time interval of the integral. The initial objective function is the future cost of indefinite time. However, due to the uncertainty of future electricity prices and consumption demand this is a truly unknown cost. To simplify the objective function the time interval is changed to one day (24 hours), assuming that the electricity price is known one day ahead. This simplifies the objective function as presented in Equation (7.1),

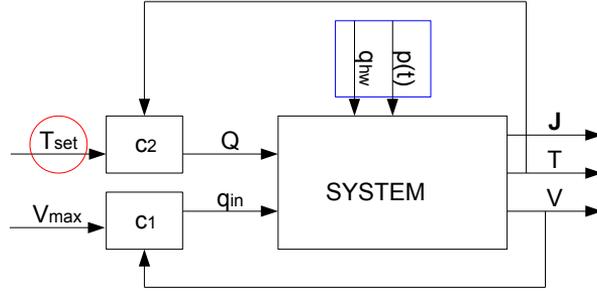


Figure 7.1: Optimal operation with marked decision variable, T_{set} , (red), disturbance vector, q_{hw} and $p(t)$, (blue) and objective function, J , (bold black).

$$J = \int_{t_0}^{t_1} p(t)Q dt + \int_{t_0}^{t_1} P(T)Q_{demand} dt \quad (7.1)$$

where the final value of a day is expressed by t_1 . The system is now more definite, and the optimal solution can be found for this renewed objective function. Further, the optimal operation assumes that the best way of controlling the system is by discretizing it into time intervals. This implies that the cost function will now provide the optimal solution for this particular optimization and will not be the global optimal solution as this is in continuous and infinite time. Equation (7.2) shows the formulation of the final optimal operation,

$$\min_u J \quad (7.2)$$

where T_{set} is the decision variable denoted by u in the equation and J is the manipulated cost function from Equation (7.1). Due to the discretization the decision variable is divided into time intervals, Equation (7.3), where each u is represented with a set of predetermined time intervals.

$$u = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad u_6 \quad u_7 \quad u_8] \quad (7.3)$$

The time intervals are chosen based on Norwegian consumption profiles from a typical Norwegian household, and are found in Equation (7.4). The values are in hours after midnight, where 0 is midnight.

$$Time_{int} = [0 \quad 5 \quad 9 \quad 14 \quad 17 \quad 20 \quad 21 \quad 24] \quad (7.4)$$

This time interval was further investigated to see if changing the intervals in $Time_{int}$ would have a significant impact on the optimization. The results show that for the three different time intervals applied, the cost deviated poorly from the cost found with $Time_{int}$ in Equation (C.1). This study can be found in the Appendix C.1 together with a more detailed explanation of the time intervals.

7.1.2 Disturbances

The system of operation is explained in Chapter 2.2, as well as the assumptions and constraints of the system. From the model equation (Equation (2.3)) the system is compactly written with states, inputs and disturbances in addition to the associated vectors for the parameters. The disturbance vector is reintroduced as shown in Equation (7.5).

$$d = \begin{bmatrix} q_{hw} \\ T_{hw,s} \\ T_{in} \\ p \end{bmatrix} \quad (7.5)$$

The system contains four disturbances; the consumption, q_{hw} , the hot water set-point, $T_{hw,s}$, the inlet temperature, T_{in} , and the electricity price, p . The temperature set-point is controlled by the consumer and is therefore considered a disturbance. In the simulations this parameter is assumed constant, implying that the consumer has the same predetermined desired hot water temperature over time. The inlet temperature is also considered to be constant. This last simplification is due to the time of simulation as T_{in} is expected to change with outside temperature. All the simulations are performed within the same month and the deviation will therefore be small enough to be neglected. Having a constant inlet temperature also implies having a constant cold water temperature as these are considered to come from the same source. The cold water temperature, as previously mentioned in Section 2.2, is present in the equation for the energy storage, and by assuming it is constant the the change in stored energy will be the difference between the current heating, Q and the current use, Q_{demand} , Equation (2.26). Table 7.1 presents the value of the two constant disturbances.

Table 7.1: System Specifications Optimal Case

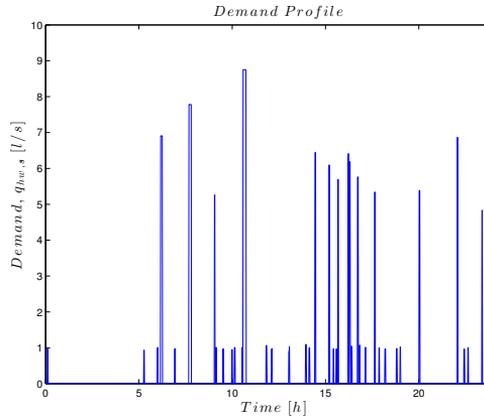
Variable	Value	unit
T_{in}	5	[°C]
$T_{hw,s}$	50	[°C]

Having assumed that two of the disturbances are constant, two are still remaining which are presented in the following sections.

Consumption profile

q_{hw} from the disturbance vector is the hot water demand from the consumer, and is one of the main challenges of controlling a hot water tank. q_{hw} denotes the uncertain demand of hot water usage at any time of day. The consumer is free to benefit from hot water usage at any time of the day and in any size of load. This makes the consumption unknown and a substantial disturbance. To be able to optimize such a system a predicted consumption profile is generated which should reflect the actual consumption profiles found in residential homes. In Chapter 3 two ways on constructing such a profile is presented and the actual approach used in this report is explained.

To be able to compare the optimal operation with other policies, the generated consumption is kept constant in every case of simulation. The profile provided is depicted in Figure 7.2.

**Figure 7.2:** Hot water consumption profile for a domestic household.

Electricity Prices

The remaining disturbance is the electricity price that is said to be known the following day. To make the simulation more accurate and compatible to reality the prices used in simulations are actual electricity prices from Trondheim in August 2013. All the prices applied in the simulations are shown in Appendix A. To get a preview of the prices two days are depicted in Figure 7.3.

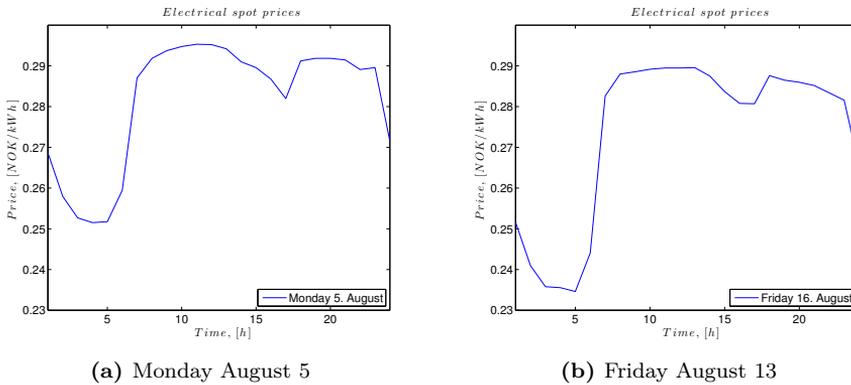


Figure 7.3: Electricity prices in Trondheim 2013.

7.1.3 Simulation

For simulation and optimization the built-in function `fminsearch` in MATLAB was used in combination with Simulink. To start with the system was designed in Simulink with the present inputs, disturbances, states and controllers. For the system (and the optimization) to be valid the constraints were implemented. The values for the constraints were presented in Chapter 2.1.1 in Table 2.2. A presentation of the MATLAB scripts and the Simulink model are to be found in Appendix D.3 and E, respectively. Further, a `sfunction` was introduced into the system block in Simulink and the state equations were implemented.

`fminsearch`

As a solver for the optimization problem the MATLAB function `fminsearch` was applied to the system. This solver is a gradient free solver which finds the minimum values of a non linear system without constraints [22]. The function tracks

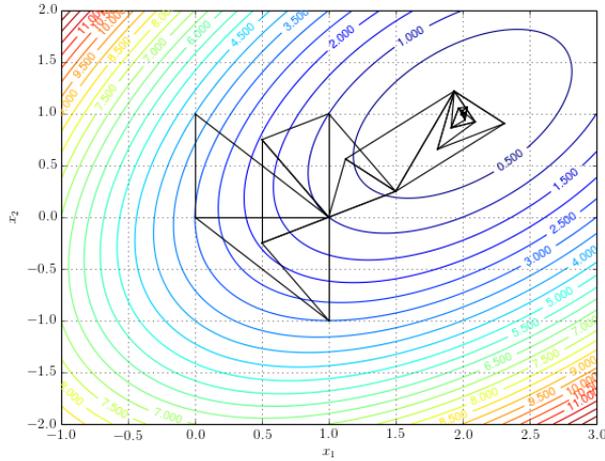


Figure 7.4: The path of a simplex algorithm with a global minimum at $x=(2,1)$, [8].

the optimal operating conditions by constructing a simplex (a n -dimensional version of a triangle) around the initial guess. For each step in the simplex a solution is calculated. In the reflection point (a specific point reflected from the simplex) a solution $f(r)$ is calculated. This solution is compared to other solutions obtained in the simplex, and according to the value of $f(r)$, one of the sides in the simplex be reduced, expanded or reflected [22]. This process will continue until a stopping criterion is met [22]. To get a better view of how the algorithm works Figure 7.4 shows the path of a simplex algorithm with a global minimum at point $x=(2,1)$.

The procedure `fminsearch` uses is the Nelder-Mead method which is a heuristic search method, meaning experience-based, and is used to speed up the process of finding a satisfactory solution. The drawback with the method is that it does not guarantee a global minimum[23]. The final result depends on the initial guess. Therefore changing the initial guess is a good way of verifying if a global or local minimum is located [23].

T_{set} Constraints

The decision variable was decided in Section 5.2. Initially T_{set} has no constraints, the constraint lie on the temperature of the tank T . If T_{set} is left free of constraints the response should be faster since ΔT between T_{set} and T can be higher. Whereas due to the limitations of heating ($Q_{max} = 5,5kWh$) the heater can not heat more even if ΔT is increased. In addition having T_{set} point within the bound-

Table 7.2: Initial Values

Parameter	Initial value	Unit
T	90	°C
J	0	NOK
Q	0	kW

aries of liquid water (0-100 °C) makes the decision variable more understandable for a possible consumer. However, the T_{set} constraints should not exceed the constraints for the water temperature, T , so the upper boundary should be limited by T_{max} . Based on the previous arguments constraints for T_{set} are introduced and presented in Equation (7.6).

$$0\text{ °C} \leq T_{set} \leq T_{max} \quad (7.6)$$

To solve this in the simulations a price penalty was implemented on T_{set} if it exceeds the boundaries in Equation (7.6). An addition benefit of having boundaries on T_{set} is the formation of Legionella. If the temperature in the hot water tank is too low over a long time the possibilities for Legionella pneumophila increases [24] and can lead to human diseases [25].

Initial Values

To start the simulation initial values needs to be chosen. The initial values for the states of tank and the cost function are to be found in Table 7.2.

The starting temperature of the water holds 90 °C. This implies that the tank is fully loaded at the beginning of the simulation. Meaning that the water is limited by the constraint to have a higher temperature, and the energy storage in the water is fully utilised. The initial values for the cost and the heat input are naturally zero.

7.1.4 Results

The simulations were performed on the first 20 days of August and complete results are presented in Table 7.4. To give a more visualized result, 1. of August is also presented in Figures 7.5-7.6 . Here the cost, J , the temperature in the tank, T , the T_{set} -vector and the demand, Q are depicted.

First presented is the cost as this is the objective function and our main interest lies here. The cost is a direct function of the electricity price and the heat input as long as the temperature in the tank is higher than $T_{hw,s}$. If the temperature is lower than $T_{hw,s}$ the penalty is > 0 and the cost increases drastically. To reach the optimal solution the penalty should therefore be avoided. The cost in Figure 7.5 (a) shows how the penalty is avoided as no drastic costs are present. The profile of the cost also reveals the strategy of the system. The cost increases in the morning from approximately 5 am to 11 am, whereas after this the slope of the cost function evens out and the total cost at the end of the day is 3,64 NOK. From the cost profile it might seem as if the consumption is almost zero in the afternoon, however the presented consumption profile shows that the highest consumption is in the evening from approximately 15 pm until 19 pm. This implies that the system first uses the electrical heater to meet the hot water demand, whereas in the afternoon the heat is met by the energy storage. To verify this statement the temperature in the tank and heat input requires a closer look.

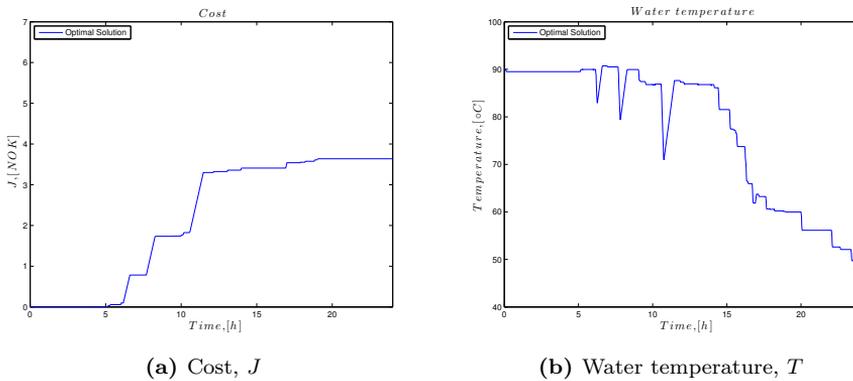


Figure 7.5: Result from the optimization of the optimal operation on August 1.

From Figure 7.5 (b) the temperature of the water in the tank decreases throughout the day. At the end of the day (time 24h) the temperature is equal to $T_{hw,s}$, meaning that the energy storage is fully utilized at the end of simulation. If a consumption would occur after this the electrical heater would have to meet the heating demand. An additional consumption would also lead to a significant increase in cost since $T \leq T_{hw,s}$. The temperature profile also confirms the alleged strategy from the cost. The temperature is kept at the initial value in the morning and in the evening the temperature drops as the heating coils is turned

on and cold water enters to keep $V = V_{max}$.

From the control structure the temperature is controlled by the temperature set-point as shown in Figure 7.1. T_{set} is the decision variable for the optimization thus the strategy is determined and controlled from this parameter. To validate this statement, the temperature set-point is presented Figure 7.6 (a). The profile follows the same pattern as the the temperature of the hot water in the tank.

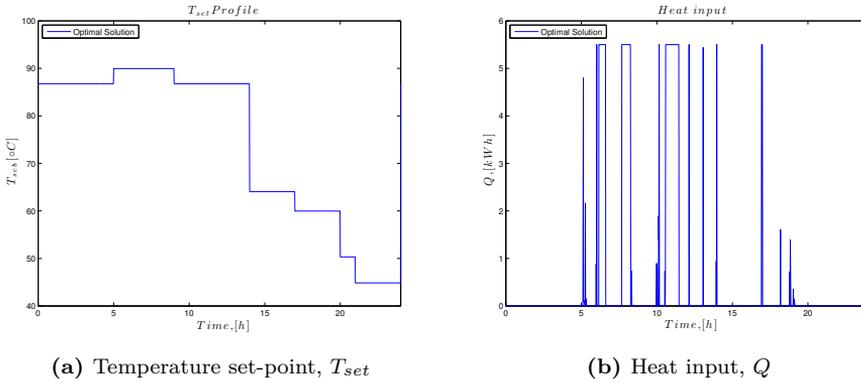


Figure 7.6: Result from the optimization of the optimal operation on August 1.

As a final validation the heat input is mentioned. The heat input is the main attribute to the cost in addition to the electricity prices. From Figure 7.6 the heat input is frequently used in the morning, whereas in the evening, when the demand increases, the heat input is more or less turned of except for a few incidents. Having confirmed that the heat profile matches the strategy visualized in the cost profile the operational strategy is summarized in Table 7.3.

Table 7.3: Operational Strategy of Hot Water Heat demand, Optimal Operation

	Morning	Evening
J	Electrical heat	Stored heat

This strategy is to be found in the remaining simulations of the optimal operation. However, it is not depicted in the report because of the amount of space it would take up. It can be seen in Table 7.4 where the T_{set} -vector is shown in addition to which day the simulation is performed on and the final cost. The T_{buffer} vector

values for most of the simulations have only a small deviation from each other. This is because of the same consumption profiles and the similarity in electricity prices during the 20 days of simulation which causes the optimal solutions to be of the same character.

Table 7.4: Results Optimal Operation

Day	J_{opt} , [NOK]	T_{set} - vector
1	3,64	[81 90 85 64 59 43 47 82]
2	3,62	[87 88 86 63 60 48 43 87]
3	3,52	[41 84 89 69 64 59 25 66]
4	3,32	[69 86 88 65 56 56 38 89]
5	3,62	[87 88 86 63 60 48 43 89]
6	3,65	[87 90 87 64 60 47 45 87]
7	3,67	[87 90 87 64 60 47 45 87]
8	3,71	[87 90 86 64 60 48 45 87]
9	3,75	[87 90 87 64 60 47 45 87]
10	3,56	[86 90 86 62 60 47 45 88]
11	3,43	[87 90 87 64 60 48 46 87]
12	3,61	[87 90 86 64 59 48 45 87]
13	3,47	[87 90 87 64 60 47 45 87]
14	3,45	[87 90 87 64 60 47 45 87]
15	3,54	[87 90 87 64 60 47 45 87]
16	3,55	[87 90 87 64 60 47 45 87]
17	3,35	[87 90 87 64 60 47 45 87]
18	2,85	[87 90 87 64 60 48 45 87]
19	3,63	[87 90 87 64 60 47 45 87]
20	3,55	[83 85 81 64 60 55 47 90]

7.2 Case I: Switching Between Two Set-points

In this section a new policy for operation of a hot water heater is introduced. The idea is to see whether it is a better, equally good or worse alternative than the proposed optimal strategy.

In this case a **switch** is implemented into the system. The policies is having a switch that can change between two set-points for the temperature where one set-point is the the maximum temperature of the hot water, T_{max} while the other one is a unknown constant temperature, T_{buffer} . The system of operation is presented in Figure 7.7.

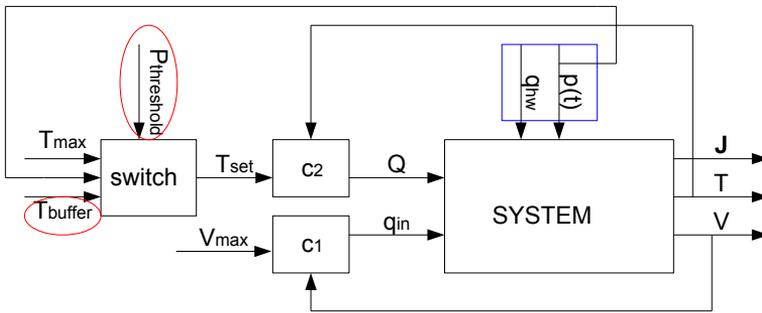


Figure 7.7: Optimization control problem with two set-points, Case I.

The disturbances and the initial values are the same as in the optimal operation study, whereas the time varying temperature set-point T_{set} is changed. In this case the T_{set} has the ability to continuously changed between to constant set-points through a **switch**. The **switch** is time independent, meaning that the switch can change between the two set-points at any time of simulation.

Due to these changes the problem of optimization is now redefined compared to the optimal operation. The objective function is discretized into only one interval resulting in constant decision variables throughout the day, see Equation (7.7). The new decision variables are the T_{buffer} and $P_{threshold}$ from Figure 7.7 and presented in Equation (7.8).

$$\min_u J \tag{7.7}$$

where

$$u = \begin{bmatrix} T_{buffer} \\ P_{threshold} \end{bmatrix} \quad (7.8)$$

For this strategy to work there has to be introduced a function, a threshold or a set of rules for the operation of the **switch**. In the following section the operation conditions for this unit is explained.

7.2.1 Switch

The **switch** is assumed to be controlled by the electrical price as shown in Figure 7.7. Looking at the prices from week 34 in August (19th -25th), Figure 4.1, the prices varies continuously between values of approximately 0.24 kWh to 0.30 kWh without any clear limit of when the prices are assumed to be high or low. In order to be able to optimize the system by controlling the set-point of the temperature with a **switch**, definitions of boundaries for the price needs to be introduced. The **switch** will then be able to select the set-points based on the current electricity price. Introducing this freedom, electricity is thought to be used at more beneficial periods and should result in both cost saving and a decrease in peak consumption compared to having a constant temperature throughout the day.

First the set-points for the temperature are defined based on a price threshold, $P_{threshold}$, which is the boundary between assumed high and low electricity price, Equation (7.9) .

$$T_{set} = \begin{cases} \text{if } p < P_{threshold} \text{ then } T_{set} = T_{max} \\ \text{if } p \geq P_{threshold} \text{ then } T_{set} = T_{buffer} \end{cases} \quad (7.9)$$

If the price, p , is lower than $P_{threshold}$ the price is cheap and the implemented **switch** changes the set-point to T_{max} . Contrary, if the price is higher than $P_{threshold}$ the price is expensive and set-point is changed to T_{buffer} . The value of T_{max} is 90 °C from Table 2.2, whereas the T_{buffer} is left for optimization.

Having introduced the purpose of $P_{threshold}$ the value of it stands to be evaluated. The price threshold has three possibilities. The first is $P_{threshold}$ always lower than the electricity price, selecting $T_{set} = T_{buffer}$, Equation (7.10). The second one is having a threshold which is always higher than the current electricity price, resulting in $T_{set} = T_{max}$. The last one is in between the maximum and

minimum of the electricity price, Equation (7.12), introducing a possible **switch** of set-points during the time of simulation.

$$P_{threshold} \leq p(t) \quad (7.10)$$

$$P_{threshold} \geq p(t) \quad (7.11)$$

$$p_{max} \geq P_{threshold} \geq p_{min} \quad (7.12)$$

For the two first cases (Equation 7.10 and 7.11) the boundary is easy to find by investigating past electricity prices ¹ and are shown in Table 7.5.

Table 7.5: Values For The Price Threshold, $P_{threshold}$

$P_{threshold} \leq p(t)$	Higher than highest measured value of electricity price
$P_{threshold} \geq p(t)$	Lower than lowest measured value of electricity price

For the last case the value for $P_{threshold}$ needs to be further investigated in a greater study in order to find a suitable global value (or function) for all days of the year. There are many factors to account for in order to be able to keep the threshold within the boundary of the prices at all times. Some factors change drastically from day to day, season variations and weather forecasts.

To shown the difficulty of finding a valid threshold an example is presented with a $P_{threshold}$ based on an average electricity price from the previous week as threshold for the days in the next week is shown in Figure 7.8. The electricity spot prices from Monday to Sunday in week 34 and the average value from week 33 in Trondheim in 2013 are shown.

¹Alternatively the implemented **switch** can be removed and a constant T_{set} can be introduced.

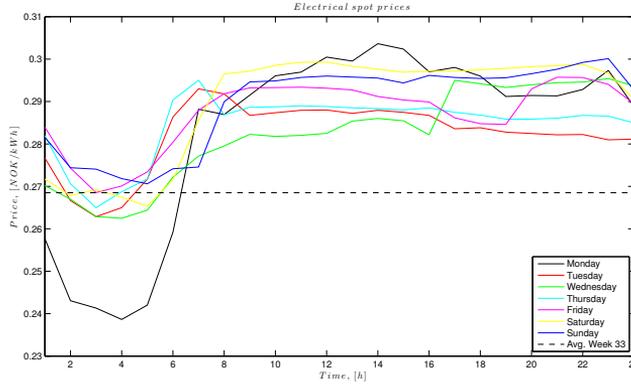


Figure 7.8: Electricity prices during week 34 compared to average electricity price of week 33.

The average value of week 33 is a suitable threshold for five out of seven days in week 34. However on Friday and Sunday the electricity prices is constantly higher than $P_{threshold}$ resulting in $P_{threshold}$ breaking the constraints in Equation (7.12). This small attempt to create a boundary reveals how sensitive the boundary is for changes in the electricity price. Small deviations from week to week are big enough to affect the threshold and break the constraints. This small study also concludes that the average electricity price from week 33 can not be the boundary for all days in week 34.

7.2.2 Simulation

To find the optimal solution in Case I the Brute Force method was used. This method is the most general and exact method for finding the optimal sets of CVs, it is also the most time consuming [26]. The method finds the optimal value of a set of $P_{threshold}$ and T_{buffer} by trying out all combinations and calculating the cost in every case.

The simulations were performed on the first 20 days of August the same as in the case the proposed optimal operation. The cost was calculated for 40 different $P_{threshold}$ s combined with 40 different T_{buffer} s between the boundaries of the decision variables shown in Table 7.6 on all days of simulation. The CV for optimal solution was then found by localizing the minimum of the objective function by using the built in function `min` in MATLAB. The MATLAB scripts for the simulations are presented in Appendix D.4.

Table 7.6: Brute Force Parameters

Parameter	Min. value	Max. value
$P_{threshold}$	0.20	0.40
T_{buffer}	25 °C	90 °C

7.2.3 Results

The results from the simulations are summarized and compared to the optimal operation in Table 7.7. In addition in Table 7.9, the corresponding CV, T_{buffer} and $P_{threshold}$, giving this particular minimum are presented. It is also denoted which of the possibilities in Section 7.2.1 $P_{threshold}$ is within, denoted by I, II or III according to the order of Equations (7.10)-7.12).

Table 7.7: Results Case I

Day	J, [NOK]	J_{opt} , [NOK]	ΔJ [%]
1	4,84	3,64	33,0%
2	4,72	3,62	30,5 %
3	3,64	3,52	3,5 %
4	3,72	3,32	12,2 %
5	4,72	3,62	30,5 %
6	4,84	3,65	32,7 %
7	4,86	3,67	32,3 %
8	4,91	3,71	32,1 %
9	4,91	3,75	31,1 %
10	3,96	3,56	11,1 %
11	4,68	3,43	36,5 %
12	4,73	3,61	31,2 %
13	4,59	3,47	32,3 %
14	4,59	3,45	32,9 %
15	4,70	3,54	32,9 %
16	4,64	3,55	30,8 %
17	4,56	3,35	35,9 %
18	3,26	2,85	14,5%
19	4,83	3,63	32,9 %
20	4,65	3,55	30,9 %

The result show that the policies applied in Case I are not better nor equally good compared to the optimal operation. The cost increases in all cases within a range of 3,5%-36,5% relative to the cost in the optimal case. These results should be expected as the stored energy from the initial temperature in the tank can not be fully utilised. Meaning that the available energy storage generated by the temperature must be of higher character at all time when the T_{set} is constant. If the temperature in the tank falls to $T_{hw,s}$ and a hot water incident occurs the water will immediately be to cold leading causing the penalty, $P(T)$, > 0 , which increases the cost. Therefore the T_{set} should be high enough to manage an unexpected consumption from the consumer. To give the reader a more presentable view of this argument Figure 7.9 shows the cost and temperature from Case I compared to the optimal operation.

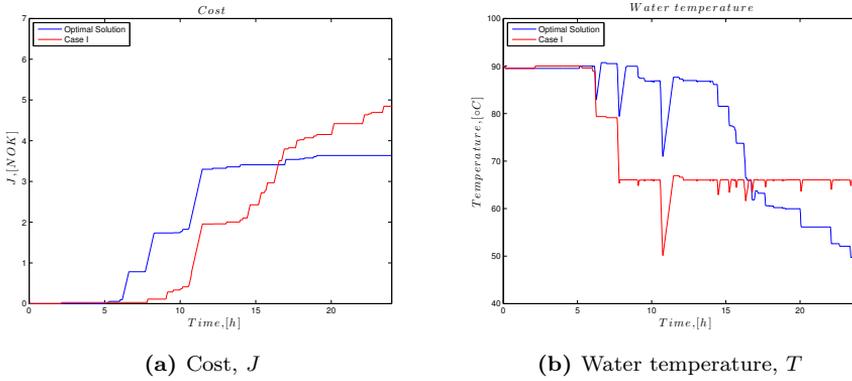


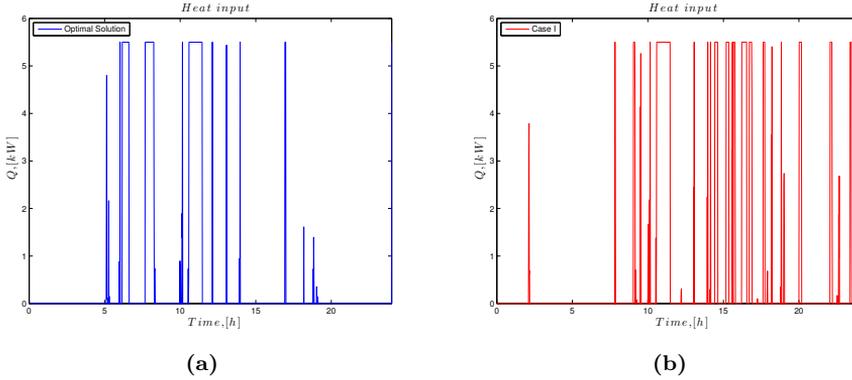
Figure 7.9: Comparison of cost and temperature from the optimal operation and Case I on August 1.

The cost profile in this case increases more continuously compared to the optimal case. The cost is lower in the morning and the break-even point where the costs are equal is not until late afternoon. These differences in profiles are caused by the energy storage strategy. In the present case the energy storage is utilized in the morning while the demand is met by the electrical heater in the evening. This is contrary to the optimal case. Table 7.8 shows the strategy in case I.

Table 7.8: Operational Strategy of Hot Water Heat Demand, Case I

	Morning	Evening
J	Stored heat	Electrical heat

The temperature in the tank shown in Figure 7.9 (b) confirms the strategy. In both cases the temperature is almost equal to the initial temperature until 5 am as the consumption until this time is almost zero. After this point the temperature in Case I drops down to the optimal T_{buffer} found in the optimization and the restoring heat demand is met by the electrical heater. Whereas in the optimal case the electrical heater is used until the storage is big enough to meet the final demand of the day.

**Figure 7.10:** Comparison of Q for (a) optimal operation and (b) for Case I on August 1.

It has also been mentioned that the optimal operation hopefully would effect the time heat input, Q . In this case the heat inputs are different from the optimal operation as depicted in Figure 7.10. The heat profiles reflects the operational strategy of the system. Case I has a low electricity consumption in the morning whereas the optimal strategy has a high consumption in the morning and contrary in the afternoon.

Looking at the temperature profiles once more. The difference in cost is caused by the difference in available energy storage. The optimal strategy can utilize more of the energy stored from the start due to varying T_{set} . In Case I the temperature

set-point can only change between two set-points which is the highest possible temperature and a T_{buffer} found in the optimization. This restriction causes the system to prevent it self from breaking the temperature constraint for the penalty, so that it avoids a higher cost, by always having an energy buffer if a consumption should occur.

Table 7.9: Optimization CVs, Case I

Day	T_{buffer} [°C]	$P_{threshold}$ [NOK/Kwh]	Possibility (I, II,III)
1	66	0,25	III
2	66	0,25	I
3	55	0,26	III
4	55	0,26	III
5	66	0,25	III
6	66	0,24	III
7	66	0,26	III
8	66	0,20-0,25	I
9	66	0,25	III
10	55	0,27	III
11	66	0,20-0,24	I
12	66	0,24	III
13	66	0,24	III
14	66	0,23	III
15	66	0,2-0,24	I
16	66	0,24	III
17	66	0,23	I
18	52	0,29	II
19	66	0,27	III
20	66	0,26	I

7.3 Case II:

T_{set} set to T_{max}

Until now the temperature set-point has had the ability to change throughout the day. In Case II the temperature is controlled by only one T_{set} , the maximum allowed temperature in the tank, T_{max} . Keeping T_{set} at a constant maximum temperature is assumed to be the easiest control structure since it does not imply optimization. The temperature reaches for the highest possible temperature at any given time of simulation. The set-point temperature is also identical to the initial temperature, implying that the results should be valid for every case where T_{set} is equal to the initial value throughout the entire simulations. Having T_{set} equal to the initial temperature also means that the hot water needs to be heated with the heater at any time of consumption in order to keep T at T_{set} . The control structure used on water heaters in Norway today are based on $T_{set} = T_{initial}$, causing the heating demand to be met by the electrical element [27]. Given the present arguments the cost is predicted to be higher when keeping the temperature equal to the initial temperature. The system in Case II is shown in Figure 7.11.

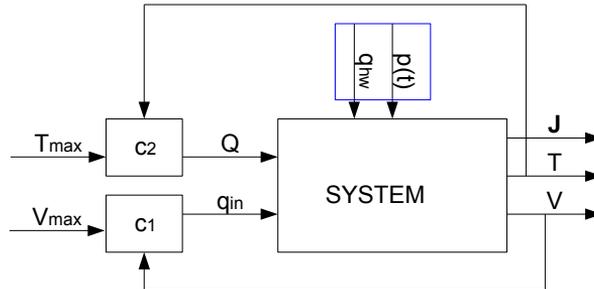


Figure 7.11: Operational system with $T_{set} = T_{max}$, Case II.

7.3.1 Simulation

The simulation in this case requires no optimization since the set-points for both states are prefixed leaving no DOF left for optimization. The simulation was performed using MATLAB and Simulink and the scripts for the MATLAB files are to be found in Appendix D.3. The initial conditions and assumptions are the same as in the optimal case as well as the time of simulation.

7.3.2 Results

The results from the simulations are presented and compared to the proposed optimal operation in Table 7.10. The cost reveals that there is a significant increase in cost compared to the optimal case. The increase is in the range of 65,3-86,8%.

Table 7.10: Results Case II

Day	J, [NOK]	J_{opt} , [NOK]	ΔJ [%]
1	6,42	3,64	76,4%
2	6,29	3,62	74,1%
3	5,81	3,52	65,3%
4	5,98	3,32	80,2%
5	6,29	3,62	74,1%
6	6,43	3,65	76,2%
7	6,46	3,67	75,8%
8	6,49	3,71	74,8%
9	6,53	3,75	74,3%
10	6,30	3,56	76,6%
11	6,07	3,43	77,2%
12	6,31	3,61	75,0%
13	6,10	3,47	75,8%
14	6,09	3,45	76,3%
15	6,20	3,54	75,2%
16	6,19	3,55	74,5%
17	5,96	3,35	77,6%
18	5,31	2,85	86,6%
19	6,40	3,63	76,3%
20	6,22	3,55	75,3%

The results from 1. of August are also depicted in Figures 7.12-7.13 to give a more explanatory result. To start with, the cost and temperature profiles are to be found in Figure 7.12 (a) and (b), respectively. The profiles reveals the strategies applied in both cases and are almost identical at the beginning of simulation. However, at approximately 2 pm the cost keeps on increasing in Case II whereas for the optimal case the slope of the cost evens out. This difference is caused by the change in strategy. Case II keeps on meeting the demand with the electrical heater, while the optimal case uses the stored energy. This is also confirmed when

looking at the profiles for the electrical heater, Q , in Figure 7.13 (b).

The heater is turned on in the morning when the consumption exceeds zero and is frequently used throughout the day as it can not benefit from the stored energy in the water at any time since $T_{set} = T_{initial}$.

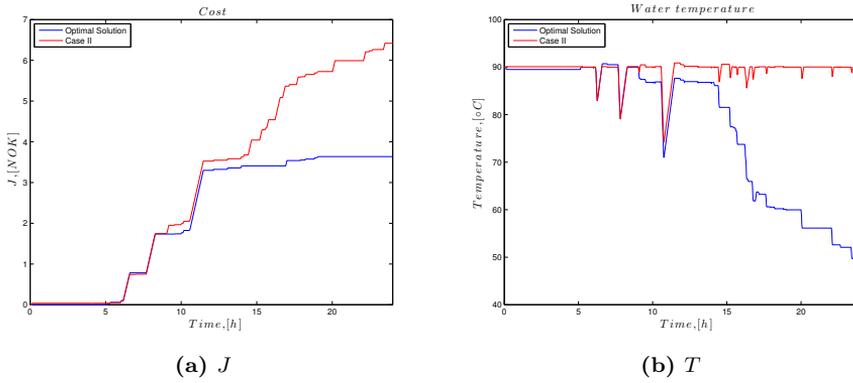


Figure 7.12: Comparison of cost and temperature from the optimal operation and Case I on August 1.

This electrical heat profile is valid for all similar cases, meaning that if the stored energy in the hot water can not be utilized Q will have this particular profile if the consumption is identical to the one applied in this study. Q will also have a similar profile if the consumption pattern is similar to this one, however, no identical. This would cause all electrical heaters to benefit from electrical power at approximately the same time of the day, and result in demand peaks on the power grids.

Table 7.11: Operational Strategy of Hot Water Heat Demand, Case II

	Morning	Evening
J	Electrical heat	Electrical heat

For comparison, the profile for the electrical heater from the optimal case is presented in Figure 7.13. This profile resembles the profile from Case II in the morning hours, thus in the evening the energy demand is met by the stored energy for the optimal case. This differences in profiles reveals the possibility

that lies in hot water heaters. The consumption patten for the electricity can be manipulated if the stored energy in the water is made available and shift load the energy consumption resulting in shaving the peak demands. The strategy for this simulation is presented in Table 7.11.

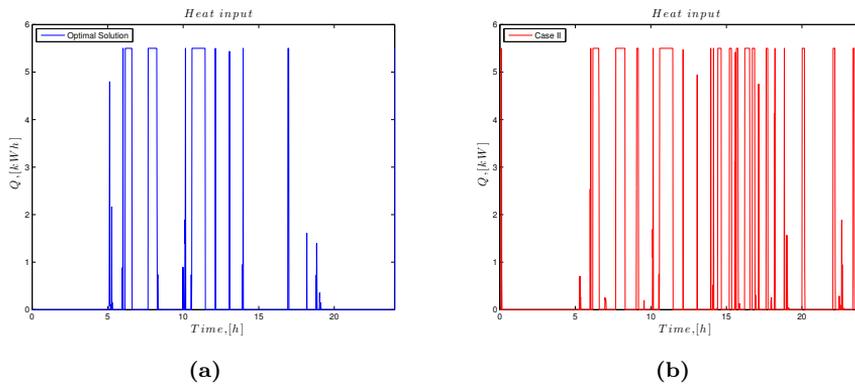


Figure 7.13: Comparison of Q for (a) optimal operation and (b) for Case I on August 1.

7.4 Case III:

Decision Variables, $P_{threshold}$ and T_{buffer}

In this section the decision variable for the optimization of the system is changed to $P_{threshold}$ and T_{buffer} to quantify whether these variables provide an equally good solution compared to the optimal. The system is identical to Case I with the implemented `switch` and the two set-points T_{max} and T_{buffer} . Otherwise the operational conditions are the same as in the optimal case with the same initial values and disturbances. Figure 7.14 gives a presentation of the system.

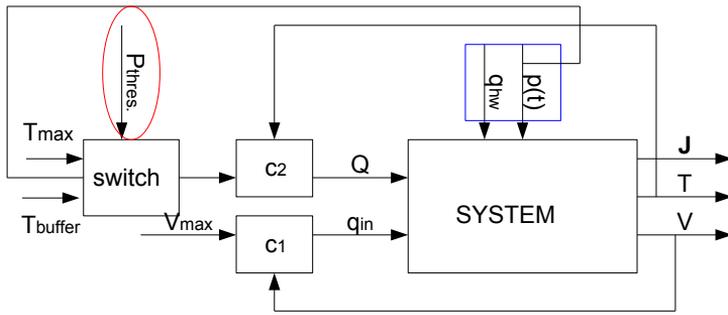


Figure 7.14: Optimization control problem with $P_{threshold}$ and T_{buffer} as decision variables, Case III.

The `switch` is operated the same way as in Case I, Section 7.2.1, with one small change. The value of $P_{threshold}$ is not the same throughout the simulation, it is discretized into time intervals similar to T_{set} in the optimal case, shown in Equation (7.15). T_{buffer} is also denoted as a decision variable for the optimization, it thought to have a constant value throughout the simulation contrary to the optimal operation, were the value is time dependent. The equation for the optimization is presented in Equation (7.13).

$$\min_u J \quad (7.13)$$

where

$$u = \begin{bmatrix} T_{buffer} \\ P_{vector} \end{bmatrix} \quad (7.14)$$

$$P_{vector} = [P_1 \ P_2 \ P_3 \ P_4 \ P_5 \ P_6 \ P_7 \ P_8] \quad (7.15)$$

The policy of changing $P_{threshold}$ implies moving the boundary between assumed cheap and expensive electricity price as a function of time. This will change the definition of the price from cheap to expensive throughout the optimization, meaning that one price might be thought to be expensive in the morning whereas in the afternoon this same price is assumed cheap by the threshold. $P_{threshold}$ also provides the control structure for the switch as describes in Case I. If the boundary defines the current electricity price as cheap the set-point for the temperature will be T_{buffer} , otherwise the set-point will be T_{max} , shown in Figure 7.14

7.4.1 Simulation

The simulation in this case was performed with the same initial conditions as the the proposed optimal operation. The decision variables were changed to $P_{threshold}$ and T_{buffer} and implemented into the Simulink structure. The MATLAB scripts for the simulations are to be found in Appendix D.5.

7.4.2 Results

The results from the simulations are compared to the results from the optimal case and are presented in Table 7.12. The results show that the cost increased in the range of 0,2-33,1% in all cases except two, where the cost was identical or decreased with 0,5%.

Additional Figures from the optimization for the 1. of August are presented in Figures 7.15-7.17 and compared to the optimal operation.

First the decision variables, $P_{threshold}$ and T_{buffer} , are shown in Figure 7.15 (a) and (b), respectively. The price threshold, $P_{threshold}$, has a more or less equal value throughout the simulation. It has a constantly lower value then the electricity price except for a small time interval in the morning. This results in a change in T_{set} in the early morning, whereas after this small time interval the set-point is T_{buffer} for the rest of the simulation, shown in Figure 7.15 (b).

From Table 7.12 the cost increased in almost all days of simulation. For the 1. of August the cost increased with 32,4% compared to the optimal case, this is shown in Figure 7.16 (a). The cost for Case III is lower in the beginning of the simulation until the break-even point at approximately 5 pm, and after this point the optimal case has a lower total cost. This is caused by the applied strategies

from the optimization. The system in Case III benefits the stored energy in the morning until the temperature drops to T_{buffer} . From this point the heating demand is met by the electrical heater. The strategy for the optimal case is presented in Table 7.3.

Table 7.12: Results Case III

Day	J , [NOK]	J_{opt} , [NOK]	ΔJ [%]
1	4,82	3,64	32,4 %
2	4,69	3,62	29,8 %
3	3,52	3,52	0,2 %
4	3,62	3,32	9,1 %
5	4,69	3,62	29,8 %
6	4,81	3,65	31,9 %
7	4,84	3,67	31,6 %
8	4,84	3,71	30,3 %
9	4,89	3,75	30,5 %
10	4,74	3,56	33,1 %
11	3,43	3,43	0,0 %
12	4,71	3,61	30,6 %
13	4,56	3,47	31,5 %
14	4,56	3,45	32,1 %
15	4,64	3,54	30,9 %
16	4,61	3,55	30,0 %
17	3,60	3,35	7,2 %
18	2,83	2,85	-0,5 %
19	4,81	3,63	32,4 %
20	4,63	3,55	30,4 %

Case III was conducted to see whether this optimization with these decision variables would provide an equally good solution as the proposed optimal operation. However, the system in Case III is not able to benefit from all the stored energy in the tank at the start of the simulation which was the main idea. This is due to the fact that $P_{threshold}$ can only change at the end of each time interval in the discretization. If T_{buffer} were 50 °C, which is $T_{hw,s}$ the hot water heater would be able to utilise the entire energy storage. However, the temperature would immediately need to switch to T_{max} when a consumption would occur, otherwise the penalty would make the cost increase drastically. In this case however the switch between the set-points can only occur at the end of the present time interval which causes the need for a hot water storage big enough to meet the

unexpected demand at any time. The optimization has found that for this case the storage is big enough if the T_{buffer} is $66\text{ }^{\circ}\text{C}$.

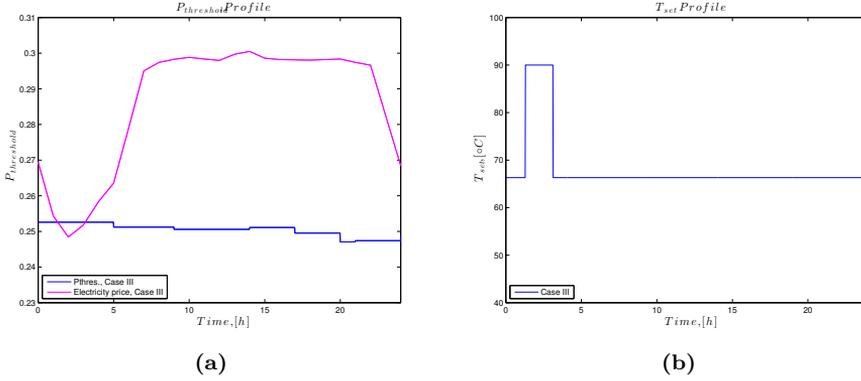


Figure 7.15: Presentation of electricity price and $P_{threshold}$ -vector in (a) and T_{set} profile in (b) on August 1, Case III.

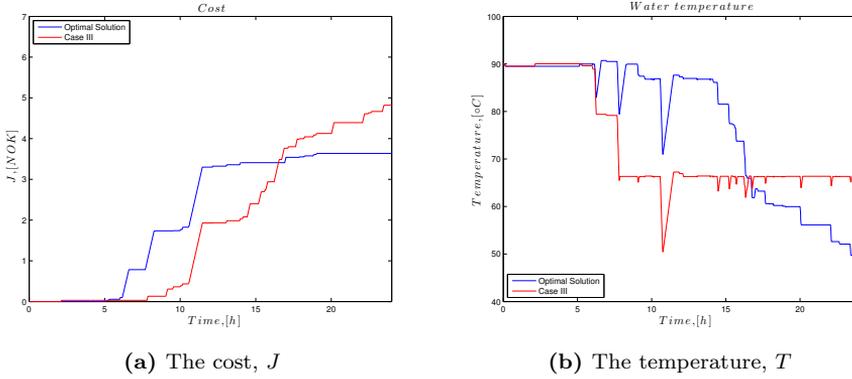


Figure 7.16: Comparison of J and T between the optimal operation and Case III on August 1.

The strategy of the system in Case III is the same as in Case I, Table 7.8. This would imply that if the T_{buffer} values are the same the final cost would be equal. To validate this statement the cost and temperature profiles from Case I are depicted together with the proposed optimal case and Case III in Figure 7.17.

Both the cost and the temperature profiles are almost identical for Cases I and III. This is probably caused by the size and amount of time intervals which the problem of optimization is discretized into. If the the amount of intervals increased the strategy would be able to have a higher utilization of the energy storage and T_{buffer} would be able to have a lower value. This might result in a lower cost and should be further investigated.

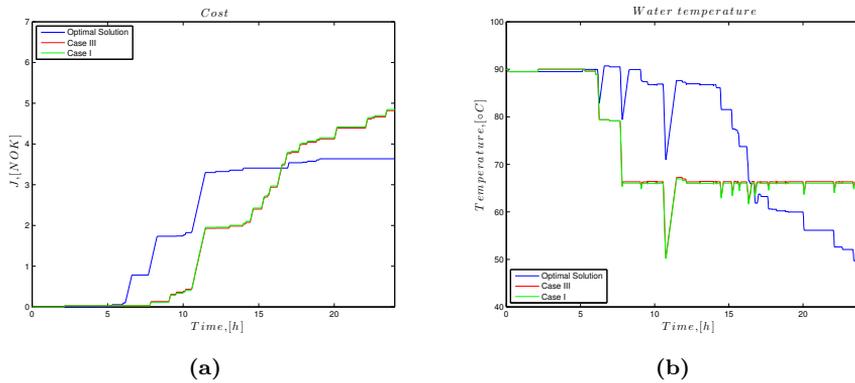


Figure 7.17: Comparison of J and T between the optimal operation, Case I and Case III on August 1.

7.5 Case IV: Implementing Final Constraint, ΔE_s

In all cases except Case II, the final value of the temperature is lower than the initial. The available stored energy from the start is distributed throughout the day to meet the hot water demand and is not restored at the end of the day. Meaning that there is no final constraint on the temperature in the simulations, causing a lower temperature at the end compared to the start. This is shown in Figures 7.5 and 7.9 in Section 7.1 and 7.2, respectively. This deficiency causes the hot water tank to start at a lower initial temperature the next day if the simulations were performed in sequence². As a consequence of this the cost will be higher the following day and result in a lower savings or even no saving at all. To make up for this deficiency and quantify the actual cost, a penalty is implemented. The penalty is added to the final cost in Section 7.1 and is shown in Equation (7.16).

$$\Delta E_p = c_p \rho V \Delta T p(t) \quad (7.16)$$

Here c_p is the specific heat capacity of water, ρ the density of water, V the volume of the tank, $p(t)$ the electricity price and ΔT is the temperature difference shown in Equation (7.17).

$$\Delta T = T_{initial} - T_{end} \quad (7.17)$$

The penalty calculates the additional cost for heating the water from the final to the initial temperature and will increase the costs found in the optimal operation.

Time of restoring

One aspect of reheating the water is that it is time consuming. For the water to reach $T_{initial}$ (90 °C) it will take approximately 2 hours if the final temperature at the end of the day is 50 °C and the consumption is zero. If the consumption exceeds zero the time of reheating will be prolonged exponentially with the increase of consumption. The restoring of energy should therefore take place in the night time when few hot water incidents occur. In this study the consumption profile presented in Section 7.1.2 has almost no consumption from the start of simulation to 5 am, which gives time to heat the hot water and restore the

²which they are in real life

energy storage. It is therefore suggested that the reheating should occur in this time interval if it was implemented into the model, however, the penalty is in this case only added to the final value instead of reconstructing the problem of optimization.

For the consumer to benefit from the proposed optimal operation with additional penalty there should be an incentive for the consumer. It is suggested that the incentive is reheating the water at night at a low price. The price is chosen to be the lowest price from the month of August in Trondheim 2013, which is 0.127 NOK/kWh. This will benefit the consumer since the actual electricity price at heating is higher than this value.

7.5.1 Simulation

The simulation was performed with the exact same conditions as the optimal. The penalty was added as a final value to the result and does not affect the electrical heater nor the temperature in the simulations. This choice was based on the objective function, J , as this is the main subject of the study. However, to get a more correct outcome regarding the temperature and heat demand, the final constraints should be implemented to give the actual profiles. The optimization was performed with the same solver as in the optimal operation, `fminsearch`, and MATLAB scripts can be found in Appendix D.3.

The simulations were performed on the first 20 days in Trondheim in August 2013 and January 2011. The prices from January are significantly higher than the prices from August 2013, causing the incentive for the consumer to be of more significant value. These prices are also collected from Nordpoolspot [18], and the highest and lowest prices of each day can be found Table A.4 in the Appendix A.

7.5.2 Results

The purpose was to add the additional cost of heating the water to the initial cost found in Section 7.1. The results are presented in Tables 7.13-7.14 for August and January, respectively. For comparison the results found in the optimal case are also presented in these tables. In the case of January, an additional simulation was made to be able to compare these results to the optimal case. The simulation is presented in Appendix D.3. To provide the reader with a more visual result the costs for the 1. of August and January are depicted in Figures 7.18 (a) and (b), respectively. The prices starts at ΔJ to show the cost required at night time provided by electrical heating.

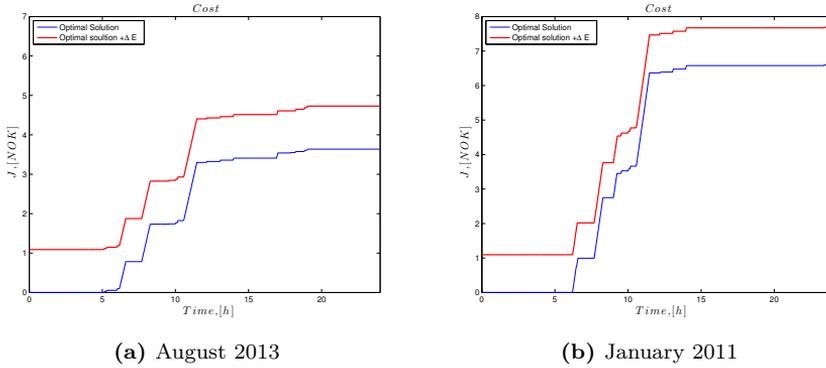


Figure 7.18: Comparison of J at optimal operation with and without ΔE for August 2013 and January 2011 presented by (a) and (b) respectively.

From Figure 7.18 the additional costs for August and January increase the same amount. This is because the additional heat applied is distributed with the same cost. However, the electrical price in January 2011 is almost twice as expensive as in August 2013, causing ΔE to be a smaller fraction of the total cost. The incentive for the consumer will therefore have a more significant affect. This is more accurately presented in Tables 7.13-7.14 where the increase ΔJ is calculated in percentage for August and January.

Having corrected for the deficiency of not having a final constraint on the temperature, the question rises whether the proposed optimal operation is actually better compared to the previously cases. In cases I and III new simulations needs to be run and ΔE_p added as the final temperatures are not the same as the initial one. Whereas in Case II $T_{end} = T_{initial}$ and the result can be directly compared to the results found in this case. The results in cases I and III are however not without value. Having a quick look at the results the costs are almost the same compared to Case IV, implying that the proposed solutions are equally good to the optimal one. However, due to the fact that they also benefit from the storage without recovering it, ΔE_p , will cause the costs to increase. Thus, ΔE_p would have a lower value compared to Case IV since the constant temperatures found for T_{buffer} are higher than the final temperature found in the simulations for the optimal case. Adding this additional cost will increase the cost and imply that the proposed optimal strategy is still be a better operational strategy when evaluating the cost.

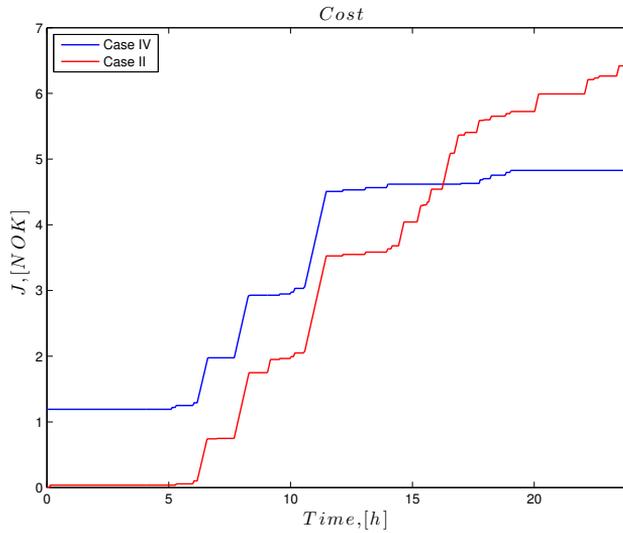


Figure 7.19: Comparison of cost between Case IV and II.

In Case II the final and initial values of the temperatures were equal, and can be compared directly to Case IV. The results for the 1. of August are depicted in Figure 7.19.

Comparing the costs show that the proposed operation is still beneficial even with the final constraint. The break-even point for the costs are somewhat moved to the right compared to the initial result with out the constraint, but the saving is still significant. This simulation was not only performed on one day. To get a valid conclusion the rest of the days of August were simulated and the are presented in Table 7.15.

Table 7.13: Results Case IV, August 2013

Day	J_{opt} , [NOK]	$J_{\Delta E}$, [NOK]	ΔJ [%]
1	3,64	4,83	32,7%
2	3,62	4,79	32,5%
3	3,52	4,43	26,0%
4	3,32	4,50	35,6%
5	3,62	4,79	32,5%
6	3,65	4,84	32,6%
7	3,67	4,86	32,3%
8	3,71	4,90	31,9%
9	3,75	4,94	31,8%
10	3,56	4,76	33,5%
11	3,43	4,62	34,8%
12	3,61	4,80	33,1%
13	3,47	4,66	34,3%
14	3,45	4,64	34,4%
15	3,54	4,73	33,6%
16	3,55	4,73	33,4%
17	3,35	4,55	35,6%
18	2,85	4,04	41,9%
19	3,63	4,82	32,8%
20	3,55	4,74	33,6%

Table 7.14: Results Case IV January 2011

Day	J_{opt} , [NOK]	$J_{\Delta E}$, [NOK]	ΔJ [%]
1	6,59	7,79	18,2%
2	8,35	9,54	14,3%
3	8,80	9,99	13,6%
4	8,65	9,85	13,8%
5	8,23	9,43	14,6%
6	7,72	8,92	15,6%
7	7,88	9,07	15,1%
8	7,63	8,83	15,7%
9	6,91	8,1	17,3%
10	7,18	8,38	16,8%
11	6,97	8,16	17,1%
12	6,90	8,09	17,3%
13	6,93	8,12	17,2%
14	6,87	8,06	17,3%
15	6,58	7,76	18,0%
16	6,18	7,38	19,4%
17	6,37	7,56	18,7%
18	6,39	7,58	18,7%
19	6,35	7,54	18,7%
20	6,31	7,51	19,0%

Table 7.15: Result Case IV (August 2013) compared to Case II.

Day	$J_{\Delta E}$, [NOK]	J , [NOK]	ΔJ [NOK]
1	4,73	6,42	35,8 %
2	4,68	6,29	34,4 %
3	4,33	5,81	34,2 %
4	4,40	5,98	35,8 %
5	4,68	6,29	34,4 %
6	4,74	6,43	35,6 %
7	4,76	6,46	35,6 %
8	4,80	6,49	35,1 %
9	4,84	6,53	35,0 %
10	4,66	6,30	35,2 %
11	4,52	6,07	34,4%
12	4,70	6,31	34,4 %
13	4,56	6,10	33,7 %
14	4,54	6,09	34,0%
15	4,63	6,20	33,9 %
16	4,63	6,19	33,6 %
17	4,45	5,96	34,1 %
18	3,94	5,31	34,9 %
19	4,72	6,40	35,6 %
20	4,64	6,22	33,9 %

8 | Discussion

The objective in the study has been to minimize the cost of heating water for a hot water tank in a typical domestic household. The proposed optimal operation resulted in significant savings compared to other proposed policies in this study. The result with an additional penalty for not restoring the initial temperature at the end of simulation also indicated in this result.

The biggest saving was found comparing J_{opt} to having T_{set} set to T_{max} . The tank in this case had the same temperature set-point as the initial temperature and can not benefit from the stored energy in the water, which causes the heat demand to be met by the electrical heater at any time of consumption. With this policy, which is applied in hot water heaters in Norway [27], the total cost can not be manipulated. The electrical coil can not be turned on when prices are cheap and turned off when prices are expensive due to the fixed temperature set-point. This control structure causes the energy storage to be unavailable and will result in similar consumption patterns between consumers presented by demand peaks.

The proposed optimal operation tries to avoid this by implementing a time dependent temperature set-point. This introduces the possibility to meet the hot water demand by both the electrical supply and stored energy by manipulating the set-point. The results showed that by implementing this control structure the heating demand was divided by the electrical heater in the morning and the energy storage in the evening. However, the same peak demand problem should occur if all consumer would apply the proposed optimal operation since the strategies would be the same for every household. The morning peak would still be present and afternoon peak would only re-occur in the night when the initial temperature is restored. To avoid this the optimal strategy should be divided into several different time intervals which might differentiate the consumption in a wider time scale, resulting in a flatter demand. In addition the strategy can be changed, similar costs should be found by utilizing the storage in the morning and benefiting the electrical coil in the evening, as long as the entire storage is utilized. Some consumers can then benefit of the storage in the morning whereas some in the evening resulting in shaving the peak demand. The restoring of the initial temperature can also be distributed between households throughout the night. It was estimated that it would take 2 hours to restore the water to $T_{initial}$ which is enough time to restore at several times in the night without overlap.

The time intervals which the objective function was discretized into consisted

of seven intervals. This limitation was caused by simulation problems. When the number of intervals exceeded seven the optimization broke the constraints and the minimum cost was not found. Using an other function to solve the optimization might solve these problems and increase the amount of intervals, one solver is e.g. GlobalSearch. This function was not used in this report due to limitations of software. Increasing the intervals would give the discretized optimization problem a more continuous profile and the temperature set-point will be able to change more rapidly throughout the day. This result might lead to an additional decrease in cost, but the policies might be more complex due to the increase in time intervals. The size of the savings might not be of significant order in comparison of increasing the complexity. Discretizing the problem into less intervals was not desired as this would be more limiting in the optimization.

One of the policies investigated was changing the decision variable to $P_{threshold}$ and T_{buffer} in the optimization. This policy was thought to be an equally good solution to the optimal operation. However, the result showed that the cost was higher compared to the optimal case as the energy storage could not be fully utilized. The problem seem to be the discretization which kept the system from benefiting the entire energy storage. The switching between set-points were controlled not only by the price threshold but also by the time intervals set by the discretization which controlled the value of $P_{threshold}$. This policies might provide a better or equally good result as the proposed optimal if the objective function was discretized into more intervals or even in continuous time. However, the applied policies were supposed to be simpler than the optimal operation which may not be the case if the amount of time intervals are increased.

As a final simulation the decrease in temperature from the start to the end in the simulation was accounted for. The optimal operation had a major deficiency when allowing the system to benefit from the initial energy storage without having to restore it at the end of simulation. This caused the results to look better than they actually were. To solve this a penalty was implemented which calculated the addition cost required for heating up the water from the final temperature to the initial temperature. Giving the next day the same initial conditions and result. As an incentive for the consumer the price for restoring the storage was set to be the lowest price found in the month of August 2013 for Trondheim. Even with this addition penalty for lacing final constraints the results showed that the assumed optimal operation still provided the best result when focusing on the objective function. However, the size of the penalty is directly controlled by the electricity of restoring the initial temperature. If this price was not as beneficial as it is in this study, the cost would be higher and this strategy might not prove to be the best alternative any more. The question is then what is a suitable price for this extra penalty? The price should be low enough for the consumer

to make a significant savings, whereas the profit for the distributor should be high enough for it to be valuable. To show the value of the electricity price applied, additional simulation were performed on January 2011. The electricity prices in this month were significantly higher compared to August 2013 and the incentive would therefore higher. The additional penalty decreased in percentage from 32,7% to 18,2% which is a decrease of almost 50 %. The incentive for the consumer is therefore of higher value when the electricity prices are high if the price is equal throughout the year, which might not be favourable.

As a final aspect is was mentioned in the introduction that the proposed optimal strategy of how to control a hot water heater might also alter the consumption pattern, resulting in decreasing in the peak demand. From the results in all cases the consumption demand can be manipulated if the stored energy is made available. The critical part is managing the controlled structures correct to avoid all households from benefiting the same strategies at the same time of day.

8.1 Further work

There are many interesting aspects of hot water heaters and much more to investigate. From this report the calculations in all cases including the optimal one should be conducted with a final constraint in order to really determined the profiles for the energy demand. The case with $P_{threshold}$ and T_{buffer} as decision variables for the optimization should also be redefined to get a more continuous profile for $P_{threshold}$. These results should then give a valid answer whether this solution can be equally good to the proposed optimal one.

The price for restoring the heat to the initial value should be investigate. What is a suitable value for this price to make the incentive high enough for the consumer to be willing to participate and the distributor to profit.

Also, the cost for heating the hot water tank was optimized by manipulating the temperature set-point, not the heat input, Q , which might be the more natural way of doing it. It should therefore be investigated to see if this optimization is equally good, worse of better compared to optimizing the cost by with Q as decision variable.

9 | Conclusion

The objective of the optimization was to minimize the energy cost of heating water. The proposed optimal operation of a hot water heater was proven successful. The decision variable for the optimization, the temperature set-point, was discretized into time intervals resulting in utilising the stored energy in the hot water tank. The initial results were extremely good and the biggest saving was found when comparing the optimal strategy with having a constant temperature set-point in the hot water tank equal to the initial value. The cost savings were in the range of 39,5 - 46.4 % in compared to the optimal case. However the final temperature had no constraint leading to a very low temperature at the end of the day. This would affect the next day if the simulations were performed in a sequence. To solve this a penalty was implemented, where consumer could restore their hot water storage by heating the hot water at night to a very beneficial price. After implementing this penalty the result still showed that the proposed operation was beneficial, the savings would be in the range of 33,6-35,8% compared to the worst case.

The applied optimal control structure also altered the consumption pattern for the electrical heater which can result in demand shaving if used correctly.

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A | Electricity Prices

The simulations performed in the report are based on real electricity prices collected from Nordpoolspot [18]. The prices for all the days of simulation in August are presented in Tables A.2-A.3. As a more quick overview the minimum and maximum values per day for August and January are summarized in Tables A.1 and A.4, respectively.

Table A.1: Minimum and Maximum Electricity Prices August 2013

Day	Min_{Price} [NOK,kWh]	Max_{price} [NOK,kWh]	Diff. Price [NOK,kWh]	Diff. Price [%]
1	0,248	0,300	0,052	21,0 %
2	0,252	0,295	0,043	17,1 %
3	0,224	0,292	0,068	30,4 %
4	0,237	0,297	0,060	25,3 %
5	0,252	0,295	0,043	17,1 %
6	0,237	0,302	0,065	27,4 %
7	0,246	0,301	0,055	22,4 %
8	0,268	0,305	0,037	13,8 %
9	0,246	0,311	0,065	26,4 %
10	0,250	0,296	0,046	18,4 %
11	0,243	0,296	0,053	21,8 %
12	0,240	0,294	0,054	22,5 %
13	0,242	0,285	0,043	17,8 %
14	0,225	0,286	0,061	27,1 %
15	0,247	0,289	0,042	17,0 %
16	0,235	0,290	0,055	23,4 %
17	0,231	0,287	0,056	24,2 %
18	0,127	0,288	0,161	126,8 %
19	0,239	0,304	0,065	27,2 %
20	0,263	0,293	0,030	11,4 %

Table A.2: Electricity Prices for Trondheim 2013, 1.-10. of August

Hour	Thu. 1.	Fri. 2.	Sat. 3.	Sun. 4.	Mon. 5.	Tue. 6.	Wed. 7.	Thu. 8.	Fri. 9.	Sat. 10.
0	0,27	0,27	0,26	0,27	0,27	0,26	0,26	0,29	0,26	0,26
1	0,25	0,26	0,25	0,25	0,26	0,25	0,25	0,27	0,25	0,25
2	0,25	0,25	0,24	0,25	0,25	0,24	0,25	0,27	0,25	0,25
3	0,25	0,25	0,23	0,24	0,25	0,24	0,25	0,27	0,25	0,25
4	0,26	0,25	0,22	0,24	0,25	0,24	0,25	0,27	0,25	0,25
5	0,26	0,26	0,22	0,24	0,26	0,25	0,27	0,29	0,26	0,26
6	0,28	0,29	0,23	0,24	0,29	0,28	0,29	0,30	0,29	0,27
7	0,30	0,29	0,24	0,25	0,29	0,30	0,30	0,30	0,30	0,27
8	0,30	0,29	0,26	0,26	0,29	0,30	0,30	0,30	0,31	0,30
9	0,30	0,29	0,27	0,28	0,29	0,30	0,30	0,30	0,31	0,29
10	0,30	0,30	0,28	0,28	0,30	0,30	0,30	0,30	0,31	0,30
11	0,30	0,30	0,28	0,29	0,30	0,30	0,30	0,30	0,31	0,30
12	0,30	0,29	0,27	0,29	0,29	0,30	0,30	0,30	0,31	0,30
13	0,30	0,29	0,27	0,28	0,29	0,30	0,30	0,30	0,30	0,29
14	0,30	0,29	0,27	0,28	0,29	0,30	0,30	0,30	0,31	0,29
15	0,30	0,29	0,27	0,28	0,29	0,30	0,30	0,30	0,30	0,29
16	0,30	0,28	0,27	0,28	0,28	0,30	0,30	0,30	0,30	0,29
17	0,30	0,29	0,27	0,28	0,29	0,30	0,30	0,30	0,30	0,29
18	0,30	0,29	0,29	0,28	0,29	0,30	0,30	0,30	0,30	0,29
19	0,30	0,29	0,29	0,30	0,29	0,30	0,30	0,30	0,30	0,30
20	0,30	0,29	0,29	0,30	0,29	0,30	0,30	0,30	0,30	0,30
21	0,30	0,29	0,29	0,30	0,29	0,30	0,29	0,30	0,30	0,30
22	0,30	0,29	0,29	0,30	0,29	0,30	0,29	0,30	0,29	0,29
23	0,28	0,27	0,28	0,30	0,27	0,27	0,28	0,28	0,27	0,28

Table A.3: Electricity Prices for Trondheim 2013, 11.-20. of August

Hour	Sun. 11.	Mon. 12.	Tue. 13.	Wed. 14.	Thu. 15.	Fri. 16.	Sat. 17.	Sun.18.	Mon. 19.	Tue. 20.
0	0,27	0,26	0,25	0,25	0,26	0,25	0,27	0,25	0,26	0,28
1	0,26	0,25	0,25	0,24	0,25	0,24	0,26	0,23	0,24	0,27
2	0,26	0,24	0,24	0,23	0,25	0,24	0,24	0,21	0,24	0,26
3	0,25	0,24	0,24	0,22	0,25	0,24	0,23	0,18	0,24	0,26
4	0,24	0,24	0,24	0,23	0,25	0,23	0,23	0,16	0,24	0,27
5	0,25	0,27	0,25	0,25	0,27	0,24	0,23	0,13	0,26	0,29
6	0,26	0,29	0,27	0,27	0,28	0,28	0,24	0,14	0,29	0,29
7	0,26	0,29	0,28	0,28	0,28	0,29	0,25	0,20	0,29	0,29
8	0,27	0,29	0,28	0,28	0,29	0,29	0,27	0,23	0,29	0,29
9	0,29	0,29	0,28	0,28	0,29	0,29	0,28	0,25	0,30	0,29
10	0,29	0,29	0,28	0,28	0,29	0,29	0,29	0,26	0,30	0,29
11	0,29	0,29	0,28	0,28	0,29	0,29	0,29	0,26	0,30	0,29
12	0,29	0,29	0,28	0,28	0,29	0,29	0,29	0,26	0,30	0,29
13	0,29	0,29	0,28	0,28	0,29	0,29	0,29	0,25	0,30	0,29
14	0,28	0,29	0,28	0,28	0,29	0,28	0,28	0,25	0,30	0,29
15	0,28	0,29	0,28	0,28	0,29	0,28	0,28	0,25	0,30	0,29
16	0,28	0,29	0,28	0,28	0,29	0,28	0,28	0,25	0,30	0,28
17	0,28	0,29	0,28	0,29	0,29	0,29	0,28	0,26	0,30	0,28
18	0,29	0,29	0,28	0,29	0,29	0,29	0,29	0,27	0,29	0,28
19	0,29	0,29	0,28	0,29	0,28	0,29	0,29	0,28	0,29	0,28
20	0,29	0,29	0,28	0,29	0,28	0,29	0,29	0,28	0,29	0,28
21	0,30	0,29	0,28	0,29	0,28	0,28	0,29	0,29	0,29	0,28
22	0,29	0,29	0,28	0,28	0,28	0,28	0,28	0,29	0,30	0,28
23	0,27	0,27	0,26	0,27	0,27	0,27	0,26	0,27	0,29	0,28

Table A.4: Minimum and Maximum Electricity Prices January 2011

Day	<i>Min_{Price}</i> [NOK,kWh]	<i>Max_{price}</i> [NOK,kWh]	Diff. Price [NOK,kWh]	Diff. Price [%]
1	0,52	0,66	0,14	26,9%
2	0,64	0,71	0,07	10,9%
3	0,66	0,73	0,07	10,6%
4	0,67	0,71	0,04	6,0%
5	0,62	0,68	0,06	9,7%
6	0,59	0,64	0,05	8,5%
7	0,60	0,65	0,05	8,3%
8	0,60	0,63	0,03	5,0%
9	0,51	0,60	0,09	17,6%
10	0,50	0,59	0,09	18,0%
11	0,51	0,58	0,07	13,7%
12	0,53	0,57	0,04	7,5%
13	0,53	0,59	0,06	11,3%
14	0,53	0,56	0,03	5,7%
15	0,52	0,56	0,04	7,7%
16	0,50	0,53	0,03	6,0%
17	0,49	0,53	0,04	8,2%
18	0,48	0,54	0,06	12,5%
19	0,48	0,54	0,06	12,5%
20	0,48	0,53	0,05	10,4%

B | Tuning

In the report the approach for finding tuning parameters for a first order process and an integrating process were presented. In the following sections the transfer functions for the states are derived followed by the equations for the controllers.

B.1 Derivation of transfer functions

The transfer function of a system is the ratio between the state of the system and the input of the system, Equation (B.1) [28].

$$G(s) = \frac{X(s)}{U(s)} \quad (\text{B.1})$$

where $G(s)$ is the transfer function, $X(s)$ is the state and $U(s)$ is the input.

The transfer functions for the two states in the hot water system are derived in the following section. The two states V and T are controlled by the two inputs q_{in} and Q respectively. The state equation for the volume is presented in Equation (B.2) and is divided by 60 to make the time unit correct. From this equation the transfer equation is derived, Equation (B.3).

$$\frac{dV}{dt} = (q_{in} - q_{out}) \frac{1}{60} \quad (\text{B.2})$$

$$\frac{dV}{dt} = \frac{q_{in}}{60} \quad (\text{B.3})$$

Equation (B.3) is Laplace transformed and the resulting transfer equation for the volume is shown in Equation (B.4).

$$G(s) = \frac{V(s)}{q_{in}(s)} = \frac{1}{60s} \quad (\text{B.4})$$

The transfer function for the temperature is derived from the state equation in (B.5), and is more complex as it is a function of the inlet flow, q_{in} .

$$\frac{dT}{dt} = \frac{1}{V}(q_{in}(T_{in} - T) + \frac{Q}{\rho c_p}) \quad (\text{B.5})$$

The q_{in} parameter is time dependent and does not have a set-point as it interacts with the disturbance q_{hw} . To simplify the transfer function the q_{in} is assumed to have a set-point of zero as it is zero when the demand is zero (which it is much of the time). Equation (B.5) is then simplified to Equation (B.6) and the resulting transfer function for the temperature is to be found in Equation (B.7).

$$\frac{dT}{dt} = \frac{1}{V}\left(-\frac{q_{in}T}{60} + \frac{Q}{\rho c_p}\right) \quad (\text{B.6})$$

$$G(s) = \frac{T(s)}{Q(s)} = \frac{1}{c_p \rho V s} \quad (\text{B.7})$$

B.2 Tuning Parameters

To calculate the tuning parameters for the PI controllers the SIMC rules are applied. First, to ensure the value of the calculated transfer functions for the temperature are correct, due to the assumption of $q_{in} = 0$, a step response experiment was performed. In practise the tuning parameters for a first-order model are often obtained by this method [7]. Figure B.2 shows an open-loop experiment for an integrating process with a step in the input u . Skogestad states that for an integrating process to be able to obtain the tuning parameters the experiment does not need to run for longer than 10 times the effective delay [7]. The first order plus dead time model can then be approximated into an integrating model Equation (B.8), also shown in Figure B.1.

$$G(s) = \frac{K e^{-\theta s}}{\tau s + 1} \approx \frac{k' e^{-\theta s}}{s} \quad (\text{B.8})$$

From Equation (B.8) $k' = \frac{k}{\tau I}$ is the slope and can be calculate from the experiment by Equation (B.9), [7].

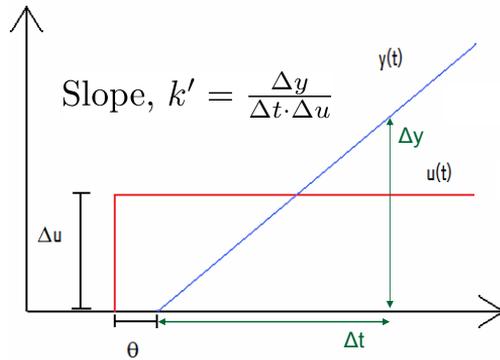


Figure B.1: Open loop step experiment to obtain tuning parameters k' and θ for an integrating process

$$k' = \frac{\Delta y}{\Delta t \Delta u} \quad (\text{B.9})$$

To be able to calculate the tuning parameters from the given equation an open loop experiment was performed on the system. A closed loop with a PI-controller was implemented on the volume with q_{in} as MV, whereas the temperature operates in an open loop. A step was implemented in the MV for the temperature (Q). The results are presented in Figure B.2.

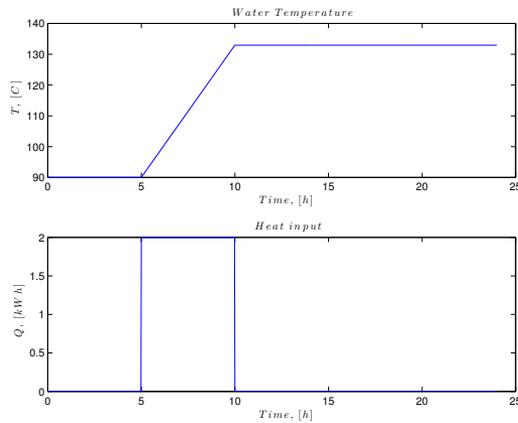


Figure B.2: Open step experiment with without disturbances

Table B.1: Open Loop Experiment Parameters

Parameter	Value
Δy	(130-90) °C
Δu	(2-0 k)W
Δt	3600s · 5
θ	0 s

The values found from the experiment are presented in Table B.1.

From Table B.1 the value of k' can be calculated for an integrating process, Equation (B.11).

$$k' = \frac{\Delta y}{\Delta t \Delta u} = \frac{130 - 90}{5 \cdot 3600 \cdot 2} = 0.001 \quad (\text{B.10})$$

To compare the experimental value of k' with the calculated value from the derived transfer function, k' is calculated from Equation (B.7). The values for c_p , ρ and V is to be found in Table C.1 and the result is

$$k' = \frac{1}{V \rho c_p} = \frac{1}{200 \cdot 4.19 \cdot 1} = 0.001 \quad (\text{B.11})$$

The values for the slope, k' , are the same in both the experimental and calculated case. This implies that the assumption made are reasonable.

Having checked that the value of the slope is equal the equations for the tuning parameters for the controllers can be derived from the Equations (B.12)-(B.13). The value of the response time, τ_c , is decided to be 20 for the volume and 175 for the temperature, whereas the integral time, τ_I , is ∞ .

$$G(s) = K_c \left(1 + \frac{1}{\tau_I s} \right) \quad (\text{B.12})$$

$$K_c = \frac{1}{k' (\tau_c + \theta)} \quad (\text{B.13})$$

$$\tau_I = \min\{\tau_1, 4(\tau_c + \theta)\}$$

The tuning parameters for the controller of the temperature is first derived,

$$K_c = \frac{1}{k'} \frac{1}{(\tau_c + \theta)} = \frac{1}{\frac{1}{c_p \rho V}} \frac{1}{(175 + 0)} = 0,21 \quad (\text{B.14})$$

$$\tau_I = \min\{\tau_1, 4(\tau_c + \theta)\} = 4(20 + 0) = 80$$

The resulting equation for the PI-controller for the temperature in the tank is to be found in Equation (B.15). The values for c_p and ρ can be found in Table C.1.

$$c(s)_T = K_c \left(1 + \frac{1}{\tau_I s}\right) = 0.21 \left(1 + \frac{1}{80s}\right) \quad (\text{B.15})$$

The same procedure is conducted on the tuning parameters for the volume and the final equation for the PI-controller is shown in Equation (B.17). The slope for the volume is $k' = \frac{1}{60}$ and is found in Equation (B.4).

$$K_c = \frac{1}{k'} \frac{1}{(\tau_c + \theta)} = \frac{1}{\frac{1}{60}} \frac{1}{(20 + 0)} = 3 \quad (\text{B.16})$$

$$\tau_I = \min\{\tau_1, 4(\tau_c + \theta)\} = 4(20 + 0) = 80$$

$$g(s)_V = 3 \left(1 + \frac{1}{80s}\right) \quad (\text{B.17})$$

As a final overview of the calculated tuning parameters all the values are summarized in Table B.2.

Table B.2: Tuning Parameters

Parameter	Description	Value
V	Volume	
θ	Time delay	0
τ_c	Response time	20
K_c	Controller gain	3
τ_I	Integral time	80
T	Temperature	
θ	Time delay	0
τ_c	Respon time (selected)	20
K_c	Controller gain	0.21
τ_I	Integral time	80

C | Case Studies

In Chapter 2.2 the mathematical equations for the system are derived. The parameters presented in these equations are shown in Table C.1. Other values for the remaining parameters applied are presented in the main report.

Table C.1: System Parameters

Parameter	Description	Value	Unit
c_p	Heat capacity water	[4,19	kJ/kg,K]
ρ	Density water	1	[kg]/l
V	Tank volume	200	[l]

C.1 Time Intervals

To be able to decide at what time T_{buffer} should change the general consumption of a Norwegian household is investigated. Figure C.1 depicts a typical weekday (green) and a typical weekend (red) in Norwegian households. Both the weekday and the weekend has a high demand in the morning and in the evening. In the week day mornings the peak comes earlier due to early working hours. In the evening the peak is similar for both cases. Deciding on the time intervals for T_{buffer} these peaks are considered. The resulting time intervals is to be found in Equation (C.1).

$$Time_{int} = [0 \quad 5 \quad 9 \quad 14 \quad 17 \quad 20 \quad 21 \quad 24] \quad (C.1)$$

Changing time intervals

The optimal solution is constructed with time intervals based on consumption patterns in Norway, previous section. These intervals are not optimized and might have a significant impact on the cost if they were changed. In order to investigate this uncertainty the assume optimal operation is run with different $Time_{int}$ in order to see the impact on the cost.

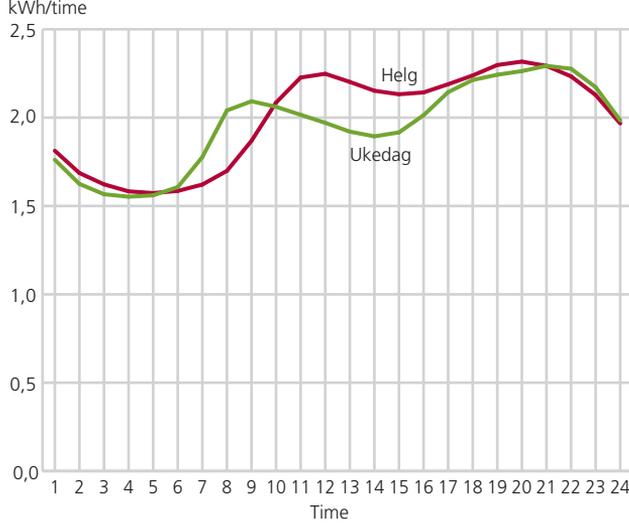


Figure C.1: General consumption profile for a typical Norwegian household [9]

The operational conditions are the same as in optimal operation.

The intervals are presented in Equations (C.2)-(C.5) where $Time_{int}$ is the time interval applied in the main report. The change in cost compared to assumed optimal operation for each case are presented in Table C.2.

$$Time_{int} = [0 \quad 5 \quad 9 \quad 14 \quad 17 \quad 20 \quad 21 \quad 24] \quad (C.2)$$

$$Time_{int-1} = [0 \quad 4 \quad 6 \quad 8 \quad 11 \quad 16 \quad 20 \quad 24] \quad (C.3)$$

$$Time_{int-2} = [0 \quad 2 \quad 5 \quad 10 \quad 14 \quad 19 \quad 22 \quad 24] \quad (C.4)$$

$$Time_{int-3} = [0 \quad 6 \quad 9 \quad 12 \quad 16 \quad 20 \quad 23 \quad 24] \quad (C.5)$$

Table C.2: ΔJ compared to J_{opt} with different time intervals.

Day	ΔJ		
	$Time_{int-1}$ %	$Time_{int-2}$ %	$Time_{int-3}$ %
1	0,32%	0,33 %	0,03 %
2	-0,50%	-1,14%	-0,01%
3	-7,09%	-8,51%	-7,51%
4	0,64 %	-0,89%	-0,04%
5	-0,50%	-1,14%	-0,01%
6	-0,05%	0,50 %	0,19 %
7	0,49 %	0,16 %	0,09 %
8	-0,07%	-0,23%	0,04 %
9	-0,33%	-1,16%	0,10 %
10	-0,10%	-0,34%	0,02 %
11	0,05 %	-0,61%	0,11 %
12	0,19 %	-0,04%	0,07 %
13	0,18 %	0,15 %	0,09 %
14	0,16 %	0,39 %	0,20 %
15	0,21 %	-0,28%	0,06 %
16	-0,31%	-0,65%	0,20 %
17	0,18 %	-1,03%	0,11 %
18	0,28 %	-0,07%	0,46 %
19	0,54 %	-0,18%	0,01 %
20	0,15 %	-0,33%	-0,12%

$$\Delta J = \frac{(J_{opt} - J) \cdot 100}{J_{opt}} \quad (C.6)$$

C.2 Case IV: Optimal simulation January 2011

To be able to compare the result in Case IV in the report with the proposed optimal operation simulations were made with prices from January 2011.

The formulation of the optimization problem, the assumptions and initial values are the same as in Section 7.1. The only difference is the electricity price which is here changed to January 2011. The minimum and maximum values are presented in Table A.4, whereas complete prices on hourly basis can be found in [18].

Results

The cost found are shown in Table C.3 together with the day the simulation was performed. Naturally these results are higher due to expensive electricity. The profiles for consumption and temperature are however similar to the ones found for August, shown in Figure C.2.

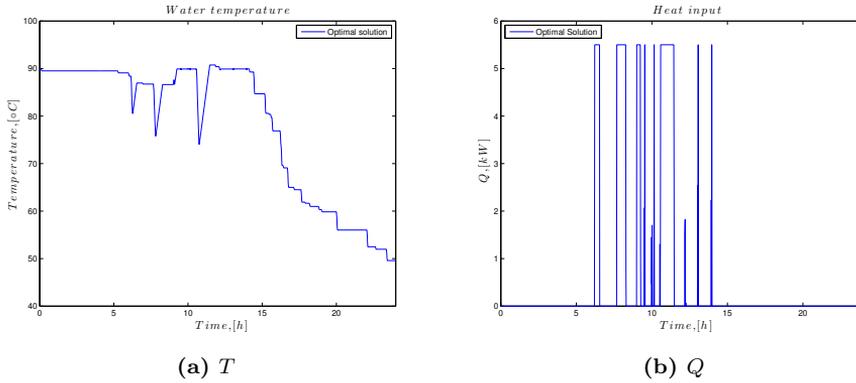


Figure C.2: Temperature and heat input profiles from the optimal operation in January 2011.

Table C.3: Results Optimal Operation January 2011

Day	J_{opt} , [NOK]
1	6,59
2	8,35
3	8,80
4	8,65
5	8,23
6	7,72
7	7,88
8	7,63
9	6,91
10	7,18
11	6,97
12	6,90
13	6,93
14	6,87
15	6,58
16	6,18
17	6,37
18	6,39
19	6,35
20	6,31

D | MATLAB scripts

D.1 Demand Profile

The script from the demand profiles are generated from the following MATLAB script.

```
1 %%Water consumption profile
2
3 % Master thesis 2013
4 % Written by Vinicius de Oliveira
5
6 function day=getProfile
7
8 dist=prob_dist_shower;
9 shower_dist = make_prob_table(dist);
10 prob_dist_small_medium
11 small_medium_dist = make_prob_table(dist);
12 prob_dist_bath
13 bath_dist = make_prob_table(dist);
14
15
16 day.time = 0:1/60:24;
17 day.flow = zeros(length(day.time),1);
18
19 % flow types, number of incidents/day
20 % short, 28
21 % medium, 12
22 % bath, .143
23 % shower, 2
24
25
26
27
28 % generate a normal day
29 average_inc_pr_day = 40;
30
31 inc_today = ceil(average_inc_pr_day + 4*rand);
32 inc_dist = [28,12,.143,2];
33 inc_dist = inc_dist./sum(inc_dist);
34 inc_prob = cumsum(inc_dist);
35 inc_dist_today = [0,0,0,0];
36
37 for i = 1:inc_today
```

```

38     index = find(inc_prob > rand,1);
39     inc_dist_today(index) = inc_dist_today(index) + 1;
40 end
41
42 disp(inc_dist_today)
43
44
45 for i = 1:4
46 number_of_inc_today = inc_dist_today(i);
47 switch i
48     case 1
49         mean_flow = 1;
50         std_flow = .05;
51         mean_duration = 1;
52         std_duration = .05;
53         time_dist = small_medium_dist;
54     case 2
55         mean_flow = 6;
56         std_flow = 1;
57         mean_duration = 1;
58         std_duration = .05;
59         time_dist = small_medium_dist;
60     case 3
61         mean_flow = 14;
62         std_flow = 2;
63         mean_duration = 10;
64         std_duration = 2;
65         time_dist = bath_dist;
66     case 4
67         mean_flow = 8;
68         std_flow = 1;
69         mean_duration = 5;
70         std_duration = 2;
71         time_dist = shower_dist;
72 end;
73
74 if number_of_inc_today > 0;
75     for n = 1:number_of_inc_today
76         %hour of the usage
77         time = time_of_usage(time_dist);
78         flow = mean_flow + std_flow*randn; %l/m
79         flow = max(flow,0);
80         duration = mean_duration + ceil(std_duration*randn); %min
81         duration=duration/60;%[now in hours]
82         day.flow(( day.time >= time & day.time <=time+duration))=...
            flow;
83     end
84 end
85
86 end
87
88 % stairs(day.time,day.flow)

```

```

89 % axis([0 24 0 10])
90 % xlabel('Time[h]')
91 % ylabel('Flow, qhw [m^3/s]')

```

```

1
2 function [duration_vec,flow_vec,timestart_vec]=getProfileDetails
3
4 dist=prob_dist_shower;
5 shower_dist = make_prob_table(dist);
6 prob_dist_small_medium
7 small_medium_dist = make_prob_table(dist);
8 prob_dist_bath
9 bath_dist = make_prob_table(dist);
10
11
12 day.time = 0:1/60:24;
13 day.flow = zeros(length(day.time),1);
14
15 % flow types, number of incidents/day
16 % short, 28
17 % medium, 12
18 % bath, .143
19 % shower, 2
20
21
22
23
24 % generate a normal day
25 average_inc_pr_day = 20;
26
27 inc_today = ceil(average_inc_pr_day + 4*rand);
28 inc_dist = [28,12,.143,2];
29 inc_dist = inc_dist./sum(inc_dist);
30 inc_prob = cumsum(inc_dist);
31 inc_dist_today = [0,0,0,0];
32
33 duration_vec=[];
34 flow_vec=[];
35 timestart_vec=[];
36
37 for i = 1:inc_today
38     index = find(inc_prob > rand,1);
39     inc_dist_today(index) = inc_dist_today(index) + 1;
40 end
41
42 disp(inc_dist_today)
43
44
45 for i = 1:4
46     number_of_inc_today = inc_dist_today(i);

```

```

47 switch i
48     case 1
49         mean_flow = 1;
50         std_flow = .05;
51         mean_duration = 1;
52         std_duration = .05;
53         time_dist = small_medium_dist;
54     case 2
55         mean_flow = 6;
56         std_flow = 1;
57         mean_duration = 1;
58         std_duration = .05;
59         time_dist = small_medium_dist;
60     case 3
61         mean_flow = 14;
62         std_flow = 2;
63         mean_duration = 10;
64         std_duration = 2;
65         time_dist = bath_dist;
66     case 4
67         mean_flow = 8;
68         std_flow = 1;
69         mean_duration = 5;
70         std_duration = 2;
71         time_dist = shower_dist;
72 end;
73
74 if number_of_inc_today > 0;
75     for n = 1:number_of_inc_today
76         %hour of the usage
77         time = time_of_usage(time_dist); %[h]
78         flow = mean_flow + std_flow*randn; %l/m
79         flow = max(flow,0);
80         duration = mean_duration + ceil(std_duration*randn); %min
81         duration=duration/60;%[now in hours]
82         duration_vec=[duration_vec duration*3600]; %[in seconds]
83         flow_vec=[flow_vec flow];
84         timestart_vec=[timestart_vec time*3600 ];
85
86     end
87 end
88
89 end
90 timestart_vec=sort(timestart_vec)
91 % stairs(day.time,day.flow)

```

```

1 function dist = make_prob_table(dist)
2
3 %ensuring probabillity sum to one
4 total_prob = trapz(dist.t,dist.p);

```

```

5 dist.p = dist.p/total_prob;
6
7 % interpolates the date one minute scale
8 inter_p.t = 0:1/60:24;
9 inter_p.p = zeros(length(inter_p.t),1);
10 for i = 1:length(inter_p.t)
11     inter_p.p(i) = interp1(dist.t,dist.p,inter_p.t(i));
12 end
13 % calculating the probability minute by minute
14 % discrete probability
15 inter_p.P = zeros(length(inter_p.t)-1,1);
16 for i = 1:length(inter_p.t)-1
17     inter_p.P(i) = trapz(inter_p.t(i:i+1),inter_p.p(i:i+1));
18 end
19
20 dist.prob_table.p = cumsum(inter_p.P);
21 dist.prob_table.t = inter_p.t;
22
23 end

```

```

1 dist.t = [
2     0
3     7
4     8
5    14
6    17
7    19
8    21
9    23
10   24
11   ];
12
13 dist.p = [
14     0
15     0
16    .01
17    .06
18    .05
19    .22
20    .03
21    .01
22     0
23   ];

```

```

1
2 function dist=prob_dist_shower
3 dist.t = [
4     0

```

```
5     5
6     6
7     7
8     8
9     9
10    18
11    19
12    19.5
13    21
14    23
15    24
16    ];
17
18    dist.p = [
19        0
20        0
21        .15
22        .25
23        .15
24        .02
25        .02
26        .05
27        .09
28        .02
29        .02
30        0
31    ];
```

```
1    dist.t = [
2        0
3        4.9
4        5
5        23
6        23.1
7        24
8    ];
9
10   dist.p = [
11       .01
12       .01
13       .05
14       .05
15       .01
16       .01
17    ];
```

```
1    function time = time_of_usage(dist)
2
```

```
3 time_index = find(dist.prob_table.p > rand, 1);  
4 time = dist.prob_table.t(time_index);  
5  
6 end
```

D.2 S-Function

```

1  function [sys,x0,str,ts] = sfunctionmodel(t,x,u,flag,par)
2
3
4  switch flag,
5
6      %%%%%%%%%%%%%%%%%%%%%%%%%%
7      % Initialization %
8      %%%%%%%%%%%%%%%%%%%%%%%%%%
9      case 0,
10         [sys,x0,str,ts]=mdlInitializeSizes;
11
12         %%%%%%%%%%%%%%%%%%%%%%%%%%
13         % Derivatives %
14         %%%%%%%%%%%%%%%%%%%%%%%%%%
15         case 1,
16             sys=mdlDerivatives(t,x,u,flag,par);
17
18         %%%%%%%%%%%%%%%%%%%%%%%%%%
19         % Update &terminate %
20         %%%%%%%%%%%%%%%%%%%%%%%%%%
21         case {2,4,9}
22             sys=[]; %Unused flags
23
24         %%%%%%%%%%%%%%%%%%%%%%%%%%
25         % Outputs %
26         %%%%%%%%%%%%%%%%%%%%%%%%%%
27         case 3,
28             sys=mdlOutputs(t,x,u,flag,par);
29
30
31         otherwise
32             DASTudio.error('Simulink:blocks:unhandledFlag', num2str(flag))...
33                 ;
34     end
35
36 % end sfunctionmodel
37
38
39 %=====
40 % mdlInitializeSizes
41 % Return the sizes, initial conditions, and sample times for the S...
42 % -----function.
43
44 function [sys,x0,str,ts]=mdlInitializeSizes
45

```

```

46
47 sizes = simsizes;
48
49 sizes.NumContStates = 3;
50 sizes.NumDiscStates = 0;
51 sizes.NumOutputs    = 3;    %Number of outputs
52 sizes.NumInputs     = 6;    %Number of inputs
53 sizes.DirFeedthrough = 0;   %Put to one if input is used as output
54 sizes.NumSampleTimes = 1;   %at least one sample time is needed
55
56 sys = simsizes(sizes);
57 x0 = [200 273+90 0];
58 str = [];           %str is always an empty matrix
59 ts = [0 0];        %initializes the array of sample times
60
61
62
63 % end mdlInitializeSizes
64
65
66 %=====
67 % mdlDerivatives
68 % Return the derivatives for the continuous states.
69 %=====
70
71 function sys=mdlDerivatives(t,x,u,flag,par)
72
73 %Identify inputs
74 Q = u(1);
75 qin = u(2);
76 qhw = u(3);
77 Thws = u(4);
78 Tin = u(5);
79 p = u(6);
80
81 %Identify states
82 V = x(1);
83 T = x(2);
84 J = x(3);
85
86 %Equations
87
88 Q_demand = par.ro.water*par.cp.water*qhw*(Thws-Tin);
89
90 p1 = 0.005;
91 p2 = 0.05;
92
93 if T >= Thws
94 pp = 0;
95 else
96     pp = p1*(Thws-T)^2 + p2*(Thws-T);
97 end

```

```
98
99
100 %qout = qhw*((Thws-par.Tcw)/(T-par.Tin))
101 qins = qin/60;
102 qhws = qhw/60;
103
104 %Volume
105 dxdt(1) = qins-qhws*((Thws-Tin)/(T-Tin));
106 %Temperature
107 dxdt(2) = 1/V*((qins*(Tin-T))+Q/(par.ro.water*par.cp.water));
108 %Price
109 dxdt(3) = p*Q/3600 + pp*Q_demand/3600; %Divides with 3600 to get ...
      in hours
110
111 sys = [dxdt(1) dxdt(2) dxdt(3)];
112
113
114 %=====
115 % mdlOutputs
116 % Return the block outputs.
117 %=====
118 %
119 function sys=mdlOutputs(t,x,u,flag,par)
120
121 %States
122 %V = x(1);
123 %T = x(2);
124 %J = x(3)
125
126 sys = x; %[V; T];
127
128 %end mdlOutputs
```

D.3 Optimal operation, Case II and IV

The following MATLAB script are used for the proposed optimal operation, Case II and IV.

```

1  %File Tbuffer.m
2  clear all
3  close all
4
5  %Tbuffervector = [90 90]+273; %Tbuffer init., x1 int.
6  %Tbuffervector = [90 65 80]+273; %Tbuffer init., x2 int.
7  %Tbuffervector = [90 90 65 90]+273; %Tbuffer init., x3 int.
8  %Tbuffervector = [90 90 80 65 90] +273; %Tbuffer init., x4 int.
9  %Tbuffervector = [90 90 80 65 90 65]+273; %Tbuffer init., x5 int.
10 %Tbuffervector = [90 90 80 65 90 65 90]+273; %Tbuffer init., x6 ...
    int.
11 Tbuffervector = [87 90 87 64 60 47 45 87]+273; %Tbuffer init.,x7 ...
    int.
12 %Tbuffervector = [90 90 90 90 80 80 70 90 90 90 70 65]+273;%...
    Tbuffer init.,x13 int.
13
14
15 %% Case optimal
16
17 %Always expensive energy, Switch goes to T_buffer
18 Pricethresholdvector = linspace(0.1,0.1,20);
19 Pthreshold = Pricethresholdvector(1);
20
21 %% Case 2
22
23 % Always cheap energy, Switch goes to Tmax
24 % Pricethresholdvector = linspace(1,1,20);
25 % Pthreshold = Pricethresholdvector(1);
26
27 %% Input data
28
29 prompt = {'Enter start day','Enter days of simulation'};
30 dlg_title = 'Simulation data';
31 num_lines = 1;
32 def = {'1','1'};
33 answer = inputdlg(prompt,dlg_title,num_lines,def);
34
35 [start, days] = answer{:};
36
37 start = str2num(start);
38 days = str2num(days);
39
40 if start > 31
41     display('Start is to high')

```



```

6
7 %Parameters
8 par.cp.water = 4.19; %[kJ/Kg,K] SI Chemical Data
9 par.ro.water = 1; %[g/cm^3] SI Chemical Data
10
11 %Saturation limits
12 Qmin = 0.0;      %[kW]
13 Qmax = 5.5;     %[kW]
14 qin_min = 0;    %[m^3/s]
15 qin_max = 10;   %[m^3/s]
16
17 Qlb = Qmin;
18 Qub = Qmax;
19 qinlb = qin_min;
20 qinub = qin_max;
21
22
23 load day.mat %Demand data
24
25         qhw=[];
26         p=[];
27
28 for i = start:(start+days-1);
29
30     tsim = 3600*24;
31
32     %constant demand
33
34     qhw = day.flow;
35     Thws = zeros(size(day.time))'+273+50;
36     Tin = zeros(size(day.time))'+273+5;
37
38
39 %     %constant price
40 %     day(i) = getProfile;
41 %     qhw(:,i) = day(i).flow;
42 %     Thws(:,i) = zeros(size(day(i).time))'+273+50;
43 %     Tin(:,i) = zeros(size(day(i).time))'+273+5;
44
45
46
47 % varying price
48 % PRICE: converting excel values into the dim of the ...
49 %     distrubance matrix
50
51 price_day(:,i) = price(:,i);
52 price_days(:,i) = interp1(time_excel,price_day(:,i),day.time,'...
53     linear','extrap');
54
55 %constant price
56 %p(:,i) = price_days(:,i)';
57

```

```

56     %constant demand
57     p=[p price_days(:,i)];
58
59     %% Disturbance matrix
60     %
61     %     %constant price
62     %     dsim.time = [day(i).time]*3600;
63     %     dsim.signals.values = [qhw(:,i) Thws(:,i) Tin(:,i) p(:,i)]; ...
64     %     % disturbances
65     %     dsim.dimensions = [4 1];
66
67     %constant demand
68     dsim.time = [day.time]*3600;
69     dsim.signals.values = [qhw Thws Tin p]; % disturbances
70     dsim.dimensions = [4 1];
71
72     %Tbuffer values
73
74     Tbuffertime = [0 5 9 14 17 20 21 24]'; %Time in h,x7, initial
75
76     %%case with different Time_int
77     %Tbuffertime = [0 4 6 8 11 16 20 24]'; %Time in h,x71
78     %Tbuffertime = [0 2 5 10 14 19 22 24]'; %Time in h, x72
79     %Tbuffertime = [0 6 9 12 16 20 23 24]'; %Time in h, x73
80
81
82
83     Tbuffer_vect = (zeros(size(Tbuffertime))+Tbufferi)';
84     Pthreshold_vect = (zeros(size(Tbuffertime))+Pthreshold)';
85
86
87     Pthreshold.time = [Tbuffertime]*3600;
88     Pthreshold.signals.values = [Pthreshold_vect]';
89     Pthreshold.dimensions = [1 1];
90
91
92     Tbuffer.time = [Tbuffertime]*3600;
93     Tbuffer.signals.values = [Tbuffer_vect]';
94     Tbuffer.dimensions = [1 1];
95
96
97     options = simset('SrcWorkspace','current');
98     sim('simulinkmodel_Tbuffer',[],options)
99     J=simout.signals.values(end,end);
100
101     penalty = 100;
102
103     %Implemented penalty constraint
104
105     numberofint = length(Tbufferi);
106

```

```

107     for m=1:numberofint;
108
109     Jp = max(Tbufferi(m)-90-273,0)*penalty+max(25+273-Tbufferi(m)...
110           ,0)*penalty;
111
112     J = J + Jp;
113
114     end
115
116     T = simout.signals.values(end,2);
117
118     plotres.qhw_case=simout1.signals.values;
119     plotres.JJ = simout.signals.values(:,3);
120     plotres.T=simout.signals.values(:,2);
121     plotres.t = simout.time/3600;
122     plotres.Tset = simout2.signals.values;
123     plotres.p = simout3.signals.values;
124
125     %Case 4
126     %Delta E penalty implemented for lacing final constraint.
127     %
128     %     deltaT = (90+273-T);
129     %     deltaE = par.cp.water*par.ro.water*deltaT*200*2.778*10^(-4)...
130     %     *0.127;
131     %
132     %     J = J + deltaE;
133     %
134     %     plotresJan.dEJJ =simout.signals.values(:,3) + deltaE;
135
136     save plotres
137
138     J;
139
140 end
141 end

```

D.4 Case I

The following MATLAB scripts were used for the simulations in Case I.

```

1 %File Tbuffer.m
2 % Finding the cost for all Tbuffers and Pthresholds
3 clear all
4 close all
5
6 Tbuffervector = linspace(60,90,20)+273;
7

```



```

59 for k=start:(start+days-1)
60 for j=1:length(Pricethresholdvector)
61
62 for i=1:length(Tbuffervector)
63
64
65     Pthresholdj = Pricethresholdvector(j)
66     Tbufferi=Tbuffervector(i);
67
68     Jkji=objective_general1(Tbufferi,Pthresholdj,start,days,price,...
        time_excel);
69
70     Jmatrix(k,j,i)=Jkji;
71
72 end
73 end
74 end
75
76 %%
77
78 %% Results saved
79 % Change day for every simulation
80
81 result.Tbuffer = Tbuffervector;
82 result.Pthreshold = Pricethresholdvector;
83 result.Jmatrix = Jmatrix(k, :, :);
84
85 %save('res_demand_31.mat','result')
86
87
88 %% Plot surf
89
90 surf(Pricethresholdvector,Tbuffervector-273,squeeze(Jmatrix(1, :, :)...
    ))
91 title('$Total$ $Cost$ $[NOK]$', 'interpreter', 'latex', 'FontSize'...
    ,14)
92 xlabel('$Price$ $Threshold$, $[NOK/kWh]$', 'interpreter', 'latex', '...
    FontSize',14)
93 ylabel('$T_{buffer}$, $[(\circ)C]$', 'interpreter', 'latex', '...
    FontSize',14)
94 zlabel('$Total$ $cost$, $[NOK]$', 'interpreter', 'latex', 'FontSize'...
    ,14)
95 axis tight

```

```

1 function [ J ] = objective_general1( Tbufferi,Pthreshold,start,...
    days,price,time_excel)
2 %Finds the objective function
3 %Takes in the Tbufferi and Pthresholdj, runs the simulink model ...
    and returns the cost
4

```

```

5
6 %Parameters
7 par.cp.water = 4.19; %[kJ/Kg,K] SI Chemical Data
8 par.ro.water = 1; %[g/cm^3] SI Chemical Data
9
10 %Saturation limits
11 Qmin = 0.0;      %[kW]
12 Qmax = 5.5;     %[kW]
13 qin_min = 0;    %[m^3/s]
14 qin_max = 10;   %[m^3/s]
15
16 Qlb = Qmin;
17 Qub = Qmax;
18 qinlb = qin_min;
19 qinub = qin_max;
20
21
22
23
24 load day.mat
25
26         qhw=[];
27         p=[];
28
29 for i = start:(start+days-1);
30
31     tsim = 3600*24;
32
33     %constant demand
34
35     qhw = day.flow;
36     Thws = zeros(size(day.time))'+273+50;
37     Tin = zeros(size(day.time))'+273+5;
38
39
40 %     %constant price
41 %     day(i) = getProfile;
42 %     qhw(:,i) = day(i).flow;
43 %     Thws(:,i) = zeros(size(day(i).time))'+273+50;
44 %     Tin(:,i) = zeros(size(day(i).time))'+273+5;
45
46
47
48 % varying price
49 % PRICE: converting excel values into the dim of the ...
50 %     distrubance matrix
51
52 price_day(:,i) = price(:,i);
53 price_days(:,i) = interp1(time_excel,price_day(:,i),day.time,'...
54     linear','extrap');
55
56 %p(:,i) = price_days(:,i)';

```

```

55     p=[p price_days(:,i)];
56
57     %% Distubance matrix
58     %
59     % %   %constant price
60     %     dsim.time = [day(i).time]*3600;
61     %     dsim.signals.values = [qhw(:,i) Thws(:,i) Tin(:,i) p(:,i)]; ...
62     %     % disturbances
63     %     dsim.dimention = [4 1];
64
65     %constant demand
66     dsim.time = [day.time]*3600;
67     dsim.signals.values = [qhw Thws Tin p]; % disturbances
68     dsim.dimention = [4 1];
69
70
71     %Tbuffer values
72     %Tbuffertime = linspace(0,23,24)';
73     Tbuffertime = [0 6 8 11 16 23 24]'; %Time in h
74     Tbuffer_vect = (zeros(size(Tbuffertime))+Tbufferi)';
75
76     Tbuffer.time = [Tbuffertime]*3600;
77     Tbuffer.signals.values = [Tbuffer_vect]';
78     Tbuffer.dimention = [1 1];
79
80
81     options = simset('SrcWorkspace','current');
82     sim('simulinkmodel_Tbuffer_case2',[],options)
83     J=simout(end,end)
84
85
86
87     end
88     end

```

D.5 Case III

In following MATLAB scripts are used for Case III.

```

1   %File Tbuffer.m
2   clear all
3   close all
4
5
6   %% Case 5
7

```



```

58
59
60 %% Cost calculation
61     %Jvector=[];
62     opts=optimset('TolFun',1e-2,'TolX',1e-1);
63     options = optimset('Display','iter');
64
65     for k=start:(start+days-1)
66
67         [Tbufferopt,J,EXITFLAG]= fminsearch(@(Tbufferi)...
            objective_generall(Tbufferi,start,days,price,time_excel),...
            decisionvector,opts)
68         %[Tbufferopt,J,EXITFLAG]= fmincon(@(Tbufferi)objective_generall...
            (Tbufferi,Pthreshold,start,days,price,time_excel),...
            Tbuffervector,[],[],[],[],LB,UB,[],options)
69     end
70
71     %%
72
73     % plot(Tbuffervector - 273,J)
74     % %axis([1 24 0.23 0.31])
75     % title('$Total$ $cost$ $vs$ $T_{buffer}$','interpreter','latex','...
        % FontSize',14)
76     % xlabel('$T_{buffer}$ , ${{\circ}C}$','interpreter','latex','...
        % FontSize',14)
77     % ylabel('$Total$ $cost$, $[NOK]$','interpreter','latex','FontSize...
        % ',14)

```

```

1
2 function [ J ] = objective_generall( decisionvector,start,days,...
    price,time_excel)
3
4 %Finds the objective function
5 %Takes in the Tbufferi and Pthreshold, runs the simulink model and...
    returns the cost
6
7 Tbuffervector = decisionvector(1);
8 Pthreshold_vector = decisionvector(2:end);
9
10
11 %Parameters
12 par.cp.water = 4.19; %[kJ/Kg,K] SI Chemical Data
13 par.ro.water = 1; %[g/cm^3] SI Chemical Data
14
15 %Saturation limits
16 Qmin = 0.0;      %[kW]
17 Qmax = 5.5;     %[kW]
18 qin_min = 0;    %[m^3/s]
19 qin_max = 10;   %[m^3/s]
20

```

```

21 Qlb = Qmin;
22 Qub = Qmax;
23 qinlb = qin_min;
24 qinub = qin_max;
25
26 load day.mat
27
28         qhw=[];
29         p=[];
30
31 for i = start:(start+days-1);
32
33     tsim = 3600*24;
34
35     %constant demand
36
37     qhw = day.flow;
38     Thws = zeros(size(day.time))'+273+50;
39     Tin = zeros(size(day.time))'+273+5;
40
41
42 %     %constant price
43 %     day(i) = getProfile;
44 %     qhw(:,i) = day(i).flow;
45 %     Thws(:,i) = zeros(size(day(i).time))'+273+50;
46 %     Tin(:,i) = zeros(size(day(i).time))'+273+5;
47
48
49
50 % varying price
51 % PRICE: converting excel values into the dim of the ...
52 %     distrubance matrix
53
54 price_day(:,i) = price(:,i);
55 price_days(:,i) = interp1(time_excel,price_day(:,i),day.time,'...
56 %     linear','extrap');
57
58 %constant price
59 %p(:,i) = price_days(:,i)';
60
61 %constant demand
62 p=[p price_days(:,i)];
63
64 %% Distubance matrix
65 %
66 %     %constant price
67 %     dsim.time = [day(i).time]*3600;
68 %     dsim.signals.values = [qhw(:,i) Thws(:,i) Tin(:,i) p(:,i)]; ...
69 %     % disturbances
70 %     dsim.dimensions = [4 1];

```

```

70     %constant demand
71     dsim.time = [day.time]*3600;
72     dsim.signals.values = [qhw Thws Tin p]; % disturbances
73     dsim.dimensions = [4 1];
74
75
76     Tbuffertime = [0 5 9 14 17 20 21 24]'; %Time in h,x7, initial
77     %Tbuffertime = [0 4 6 8 11 16 20 24]'; %Time in h,x71
78     %Tbuffertime = [0 2 5 10 14 19 22 24]'; %Time in h, x72
79     %Tbuffertime = [0 6 9 12 16 20 23 24]'; %Time in h, x73
80
81
82     %case 5
83     Tbuffer_vect = (zeros(size(Tbuffertime))+Tbuffervector)';
84     Pthreshold_vect = (zeros(size(Tbuffertime))+Pthreshold_vector...
85         ' ');
86
87     Pthreshold.time = [Tbuffertime]*3600;
88     Pthreshold.signals.values = [Pthreshold_vect]';
89     Pthreshold.dimensions = [1 1];
90
91
92     Tbuffer.time = [Tbuffertime]*3600;
93     Tbuffer.signals.values = [Tbuffer_vect]';
94     Tbuffer.dimensions = [1 1];
95
96
97     options = simset('SrcWorkspace','current');
98     sim('simulinkmodel_Tbuffer',[],options)
99     J=simout.signals.values(end,end);
100
101     penalty = 100;
102
103     numberofint = length(Tbuffervector);
104
105     for m=1:numberofint;
106
107     Jp = max(Tbuffervector(m)-90-273,0)*penalty+max(25+273-...
108         Tbuffervector(m),0)*penalty;
109
110     J = J + Jp;
111
112     end
113
114     J;
115
116     plotresPthres.qhw_case=simout1.signals.values;
117     plotresPthres.JJ = simout.signals.values(:,3);
118     plotresPthres.T=simout.signals.values(:,2);
119     plotresPthres.t = simout.time/3600;

```

```
120     plotresPthres.Pthres = simout2.signals.values;  
121     plotresPthres.Tset = simout3.signals.values;  
122  
123     save plotresPthres  
124  
125     end  
126     end
```

E | Simulink Model

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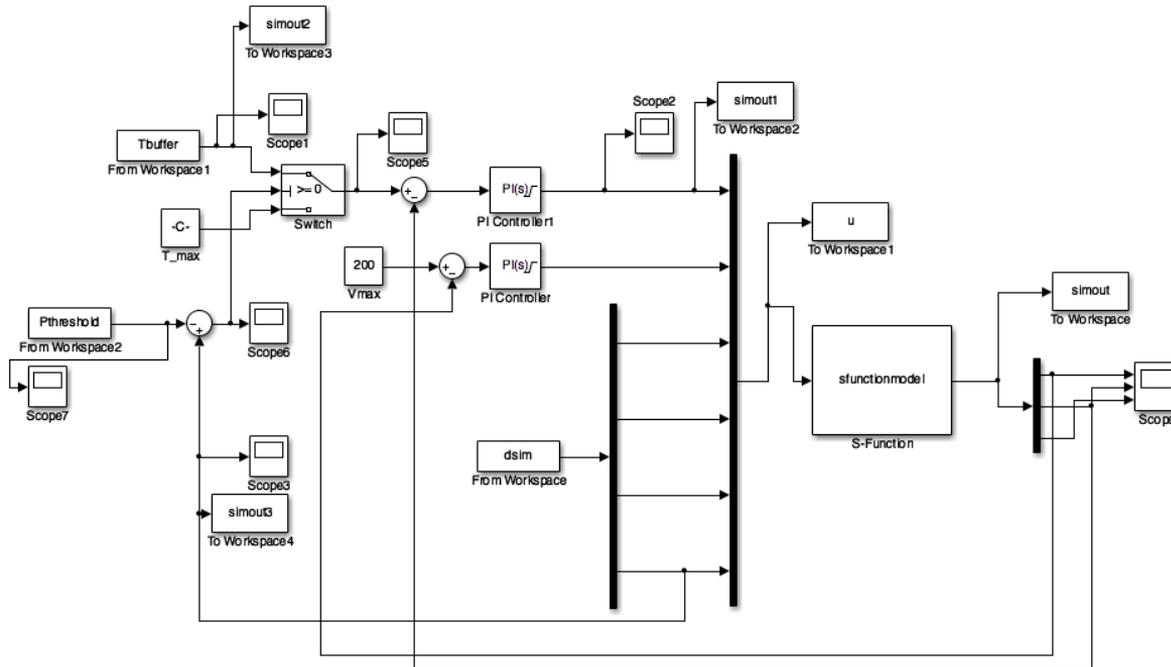


Figure E.1: Overview of the simulink model which solves the optimization problems.