NORGES TEKNISK-
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EKSAMEN I FAG 43917 / DIE3917 – MULTIVARIABEL FREKVENSANALYSE

Tirsdag 25. mai 1999

Tid: kl. 0900 - 1400

Tillatte hjelpemidler : Godkjent lommekalkulator, Ingen trykte eller håndskrevne hjelpemidler.
Merk: Du kan gjerne svare på norsk.

Problem 1. Pole uncertainty.
The set Π of possible plants is given by

$$g_p(s) = \frac{1}{s+a}; \qquad -2 \leq a \leq 1$$

(a) Explain why you cannot model this uncertainty as additive or multiplicative (relative) uncertainty.
(b) Model the set of plants in terms of inverse multiplicative uncertainty. Is this a tight uncertainty description in this case? Find the uncertainty weight $w_i(s)$ and sketch it as a function of frequency.
(c) Derive the robust stability condition. Is it possible to find a controller $k(s)$ which satisfies it ?

Problem 2. Gain uncertainty.
Consider a system with a nominal loop transfer function $L_0(s)$, and a real (perturbed) transfer function

$$L_p(s) = kL_0(s)$$

where k is an uncertain gain. For a stable plant the usual ("upper") gain margin GM is defined such that the closed-loop system corresponding to $L_p(s)$ is stable for $k < GM$ and unstable for $k > GM$. For an unstable plant, one has in addition a "lower" gain margin (gain reduction margin) GM' defined such we have instability for $k < GM'$. In summary, we have stability for

$$GM' < k < GM$$

(a) Illustrate these gain margins by making a sketch of the Nyquist plot of L_0 and computing the upper and lower gain margins for a system with

$$L_0(s) = \frac{2}{s-1}e^{-0.34s} \tag{1}$$

(recall that $|L_0(j\omega)| = 2/\sqrt{\omega^2+1}$; $\angle L_0 = -\pi + \arctan(\omega) - 0.34\omega$).
(b) As discussed above, studying reductions in the gain is important when studying robust stability of unstable plants. What about a stable plant; is there any reason to study the effect of reductions in the gain?

(c) Consider a system $L_p = kL_0$ with gain variations

$$k_{min} < k < k_{max}$$

It has been suggested to represent these variations using an uncertainty description of the form

$$k = \bar{k}(1 + r_k\Delta); \quad \|\Delta\|_\infty \leq 1 \tag{2}$$

where

$$\bar{k} = \frac{k_{max} + k_{min}}{2}; \quad r_k = \frac{k_{max} - k_{min}}{k_{max} + k_{min}}$$

Is this a tight uncertainty description? Formulate the robust stability test with this uncertainty description when Δ is complex. Discuss the effect of using real or complex Δ.
(d) Consider a with "symmetric" gain variations

$$1/G < k < G$$

where $G \geq 1$. Derive an uncertainty description on the form in (2). Choose $G = 2$ and formulate the robust stability test for complex perturbations when L_0 is given in (1). Is the system predicted to be robustly stable (consider at least the condition at steady-state)? Is the test conservative or optimistic? Comment on your answers.

Problem 3. Controllability analysis.
(a) Consider the plant (time in seconds)

$$G(s) = \frac{-s+1}{(s+1)(0.01s+1)}$$

for which we want tight control at high frequencies, $\omega > 10$ rad/s. Is this possible (explain)? Suggest a controller (e.g., $K(s) = K_c$, $K(s) = K_c/s$ or $K(s) = K_c s$; choose a reasonable gain K_c), and for this controller compute the closed-loop poles and sketch $|L|$ and $|S|$ as a function of frequency.

(b) Let

$$G(s) = \begin{bmatrix} \beta & 1 \\ \frac{1}{s+1} & \beta \end{bmatrix}$$

(i) Discuss the controllability of the the plant. Which value of β is the worst if we want tight control at steady-state? (ii) Suppose $\beta = 1$. Which output is the most difficult to control?

(c) Consider a SISO plant with model $G(s)$ and G_d(s). $G(s)$ may have a RHP-pole p and/or it may have a RHP-zero z. Does the presence of an instability, or a RHP-zero or a large disturbance by themself (one at a time) present a problem?
Then discuss and possibly give conditions for: (i) Output performance for a plant with a RHP-zero in the presence of disturbances. (ii) Input performance for a plant with a RHP-zero in the presence of disturbances. (iii) Output performance for an unstable plant in the presence of disturbances. (iv) Input performance for an unstable plant in the presence of disturbances. (v) Output performance for an unstable plant with a RHP-zero.

Comment: You may consider SG_d or S for output performance, and KSG_d or KS for input performance (ideally, all of these should be small for good performance), and you should state if there are any lower bounds on these transfer functions.

(d) Define the output directions of poles, zeros and disturbances. Give a simple example. What does it mean if the directions are the same; what if the directions are very different (i.e. orthogonal)?

Problem 4. General control formulation for inverse problem.
Assume that you want to identify (estimate) the inputs u (idependent variables) given some noisy measurements of the outputs. Specifically, the model is

$$y = Gu; \quad y_m = y + Nn$$

and your estimate $\hat{u}$ of the inputs is

$$\hat{u} = Ky_m$$

and you want to design the estimator K. Make a conventional block diagram (with G, N and K). Then set the poblem up in standard form (find the generalized plant P). Discuss the use of the standard PK-form to design K (do you need to make additional assumptions about signals, weights, etc.?).

Problem 5. Multivariable stability conditions.
Consider stability of the standard $M\Delta$-structure, where Δ is any stable transfer matrix that satisfies $\bar{\sigma}(\Delta) \leq 1, \forall\omega$. A robust stability condition in terms of the spectral radius is

$$\rho(M\Delta) < 1 \qquad \forall\omega \qquad \forall\Delta \tag{3}$$

a) Define the spectral radius. Is the spectral radius a norm? Is this condition necessary or sufficient for robust stability? Do you need any additional conditions?
b) Prove (3) mathematically *or* explain in words why (3) is reasonable.
c) (3) may be used to derive more conservative conditions such as $||M\Delta||_i < 1$, where i denotes an induced norm. Is this a spacial or temporal norm? Define induced norms and give three examples of induced norms. Prove that $\rho(A)$ is a lower bound on any induced norm $||A||_i$.
d) Consider the matrix

$$A = \begin{bmatrix} 0 & -0.5 \\ 2 & 0 \end{bmatrix}$$

Compute i) $\rho(A)$, ii) $\bar{\sigma}(A)$, and iii) $\mu(A)$ with respect to a diagonal, and iv) full matrix.