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EKSAMEN I FAG 43917 – MULTIVARIABEL FREKVENSANALYSE

Lørdag 7. juni 1997
Tid: kl. 0900 - 1400

Tillatte hjelpemidler : Godkjent lommekalkulator, Ingen trykte eller håndskrevne hjelpemidler.
Merk: Du kan gjerne svare på norsk.

Problem 1. Controllability analysis.
Compute the poles and zeros, and for the ones in the RHP the associated direction, for the following three plants:
(a)

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s-1} \\ \frac{s-2}{(s+1)(s-1)} & \frac{1}{s+1} \end{bmatrix}$$

(b)

$$G(s) = \begin{bmatrix} \frac{s+2}{s+1} & \frac{s+3}{s+2} \\ \frac{2s+1}{s+2} & \frac{10}{s+2} \end{bmatrix}$$

(c)

$$G(s) = \left[\begin{array}{cc|cc} -1 & -2 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ \hline 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right]$$

Do you expect any particular control problems for the three above plants?
(d) For the plant in (c) there are three disturbances,

$$g_{d1} = \frac{1}{10s+1}\begin{bmatrix} 10 \\ 10 \end{bmatrix}, \; g_{d2} = \frac{1}{10s+1}\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \; g_{d3} = \frac{1}{10s+1}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

In general, what are the limitations on disturbance rejection? Is acceptable disturbance rejection possible for each of the three disturbances?

Problem 2. Limitations of RHP zeros and RHP poles.

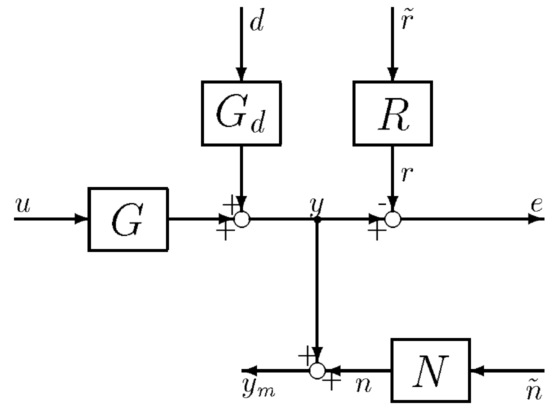

Figure 1: Model in terms of scaled variables

In the above figure we have scaled the variables such that

$$\|d\|_\infty \leq 1; \; \|\tilde{r}\|_\infty \leq 1; \; \|\tilde{n}\|_\infty \leq 1;$$

and our control objective is to manipulate u with $\|u\|_\infty \leq 1$ such that $\|e\|_\infty \leq 1$ (at least at most frequencies).
Thus R is the magnitude of the reference changes relative to the allowed control error, and N is the magnitude of the measurement noise relative to the allowed control error.
We use 1-DOF feedback control, $u = -K(y_m - r)$, to control the plant.
(a) Derive the closed-loop transfer functions from d, $\tilde{r}$ and $\tilde{n}$ to e and u.
(b) Explain why for acceptable control we must at least require that the H_∞-norm of the above six transfer functions are less than 1.
(c) The above requirements may be inconsistent with the restrictions imposed by RHP-zeros and RHP-poles. Consider first a SISO plant with a single RHP-zero z. Prove that

$$\|wS\|_\infty \geq |w(z)|$$

and use this to derive bounds on the allowed magnitudes of $R(z)$ and $G_d(z)$.
(d) Next, consider a SISO plant with a single RHP-pole p. Note that $KS = G^{-1}T$. Prove that $T(p) = 1$ and use this to show that

$$\|wKS\|_\infty \geq |w(p)| \cdot |(G^{-1})_m|_{s=p} \tag{1}$$

where $(G^{-1})_m$ is the minimum-phase version of G^{-1} (with the RHP-zeros mirrored into the LHP). For example, for $G(s) = 1/(s-10)$ we have $(G^{-1})_m = s + 10$ and $|(G^{-1})_m|_{s=p} = p + 10$.
(e) Use (1) to derive bounds on the allowed magnitude of $R(p)$, $G_d(p)$ and $N(p)$ (assuming $R(s), G_d(s)$ and $N(s)$ are stable).
(f) Consider a plant $G(s) = 1/(s-10)$ which is scaled as outlined above. Feedback is needed to stabilize the plant. Assume N is constant (independent of frequency). What is the maximum allowed value of N which is consistent with requiring $\|u\|_\infty \leq 1$?

Problem 3. Various.

(a) Consider a scalar uncertain plant

$$G_p = \frac{3(1+0.2\Delta_1)}{s+2(1+0.1\Delta_2)} \tag{2}$$

where $-1 \leq \Delta_i \leq 1$. (i) Formulate a test for robust stability in terms of $\mu(N) \leq 1$ (i.e., find N). (ii) Can $\mu(N)$ be computed using the upper bound $\min_D \bar{\sigma}(DND^{-1})$? (iii) Formulate a test for robust performance if the nominal performance condition is $\|w_P S\|_\infty \leq 1$.

(b) Consider again the plant in (2). Approximate the uncertainty with complex relative uncertainty, $G_p = G(1+w\Delta)$ (determine G and find w). Then redo the above problem.

(c) Define the RGA and explain how the RGA may be useful for analyzing the effect of uncertainty.

(d) Assume $G(s)$ is stable. What is the Youla-parameterization of all stabilizing controllers? Show that it may be written as an LFT.

(e) Why and when do we need feedback control (rather than only feedforward control)?

(f) Explain the main steps in DK-iteration.