EXAM IN COURSE 43917
MULTIVARIABLE CONTROL USING FREQUENCY DOMAIN METHODS
Friday 07 June 1996
Time period: 09.00 - 14.00

Closed book exam

Problem 1. Controllability analysis.
Compute the poles and zeros, and for the ones in the RHP the associated ouput direction, for the following three plants.

(a)

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{s+3}{(s+1)(s-2)} \\ \frac{10}{s-2} & \frac{5}{s+3} \end{bmatrix}$$

(b)

$$G(s) = \left[\begin{array}{cc|cc} 1 & 2 & 2 & 2 \\ 1 & 0 & 3 & 4 \\ \hline 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 0 \end{array}\right]$$

(c)

$$G(s) = \frac{10}{(s+1)(s+4)} \begin{bmatrix} s+1 & 3 \\ 3 & s+1 \end{bmatrix}$$

Do you expect any particular control problems for the three plants?

(d) For the plant in (c) there are three disturbances,

$$g_{d1} = \frac{1}{10s+1} \begin{bmatrix} 10 \\ 10 \end{bmatrix}, \; g_{d2} = \frac{1}{s+1} \begin{bmatrix} 10 \\ 10 \end{bmatrix}, \; g_{d3} = \frac{1}{s+1} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

Is acceptable disturbance rejection possible for each of the three disturbances? (Hint: you may want to make use of Equation (1) given below).

Problem 2. General control formulation.
Consider a stable uncertain plant

$$G_p = G(I + W_1\Delta_1) + W_2\Delta_2, \quad \|\Delta_i\|_\infty \leq 1$$

The robust performance (RP) objective is to achieve

$$\|W_3 S_p\|_\infty \leq 1, \; \forall G_p$$

where $S_p = (I + G_p K)^{-1}$.

(a) Make a (conventional) block diagram of the closed-loop system with all weights included.

(b) Derive the generalized plant P and the associated interconnection matrices N and M.

(c) State the RS- and RP-conditions in terms of N and M.

(d) Derive analytical expressions for RS and RP for the case of a SISO plant.

Problem 3. Various.

(a) Let $G = (I - H\Delta)^{-1}$. Find J such that $G = F_u(J, \Delta)$.

(b) Given the "true" plant

$$G'(s) = \frac{3e^{-0.1s}}{(2s+1)(0.1s+1)^2}$$

Sketch the multiplicative uncertainty weight when the nominal model is $G(s) = 3/(2s+1)$.

(c) Consider a conventional feedback system with the uncertain plant

$$G_p(s) = \begin{bmatrix} (1+\Delta_{11})g_{11} & (1+\Delta_{12})g_{12} \\ (1+\Delta_{21})g_{21} & (1+\Delta_{22})g_{22} \end{bmatrix}$$

Collect the uncertain Δ_{ij}'s into a diagonal matrix Δ and derive the interconnection matrix M for studying RS.

(d) Prove or disprove

i. $\mu(A) = 0 \Rightarrow A = 0$

ii. $\mu(A+B) \leq \mu(A) + \mu(B)$

iii. $\rho(AB) \leq \rho(A)\bar{\sigma}(B)$

(e) Explain and derive the condition

$$|y_z^H g_d(z)| < 1 \tag{1}$$

(f) Define and briefly explain the difference between the H_2-, H_∞- and Hankel norms of a transfer function $G(s)$.

(g) Consider a system where the disturbance enters at the plant input, $G_d = G$. What is a reasonable form for the controller if we want acceptable performance with minimum input usage.

(h) Discuss advantages and disadvantages of inverse-based controllers (decouplers).